The Role of IR-Improvement in Precision LHC/FCC Physics and in Quantum Gravity

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partly in collaboration with

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OUTLINE

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- Review of Exact Amplitude-Based Resummation Theory
- Applications in Precision LHC Physics
- Applications in Precision FCC Physics
- Applications in Quantum Gravity
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Introduction

WHAT IS RESUMMATION (IR, UV, CL)?

- FAMILIAR SUMMATION: \[ \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \]
- RESUMMATION:

\[
\sum_{n=0}^{\infty} C_n \alpha_s^n \begin{cases} 
= F_{\text{RES}}(\alpha_s) \sum_{n=0}^{\infty} B_n \alpha_s^n, & \text{EXACT} \\
\simeq G_{\text{RES}}(\alpha_s) \sum_{n=0}^{N} B'_n \alpha_s^n, & \text{APPROX.}
\end{cases}
\]
Introduction

  F. Berends and I considered, ’How Accurate Can Exponentiation (RESUMMATION) Really Be?’

- Would It Limit or Enhance Precision for a Given Level of Exactness: LO, NLO, NNLO, .... ?
'Two’ Realizations in Literature:
Jackson-Scharre (JS) (APPROX) vs YFS (EXACT)

JS → ’limit to precision’
YFS → ’no limit to precision’

See 1989 CERN Yellow Book article: Frits was almost convinced, but not completely!

Today, the analogous discussion continues to new paradigms: precision LHC/FCC physics and quantum gravity
Introduction

50 YEARS of $SU_{2L} \times U_1$, S. Weinberg, PRL 19 (1967)


(SM@50, B. Lynn et al., Case Western, June, 2018) ⇒

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Must Keep Historical Perspective

Must keep historical perspective.

Future Circular Collider (FCC)

<table>
<thead>
<tr>
<th></th>
<th>$s$</th>
<th>L/IP (cm$^{-2}$ s$^{-1}$)</th>
<th>Int. L/IP(ab$^{-1}$)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>e$^+$e$^-$</td>
<td>~90 GeV</td>
<td>230 x 10$^{34}$</td>
<td>5 ab$^{-1}$</td>
<td>2 experiments, Total ~ 15 years of operation</td>
</tr>
<tr>
<td></td>
<td>WW</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>8.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>t$\bar{c}$</td>
<td>1.5</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>pp</td>
<td>100 TeV</td>
<td>5 x 10$^{34}$</td>
<td>2.5 ab$^{-1}$</td>
<td>2+2 experiments, Total ~ 25 years of operation</td>
</tr>
<tr>
<td>PbPb</td>
<td>$\sqrt{s_{NN}} = 39$ TeV</td>
<td>3 x 10$^{29}$</td>
<td>100 nb$^{-1}$/run</td>
<td>1 run = 1 month operation</td>
</tr>
<tr>
<td>ep</td>
<td>3.5 TeV</td>
<td>1.5 x 10$^{34}$</td>
<td>2 ab$^{-1}$</td>
<td>60 GeV e$^-$ from ERL Concurrent operation with pp for ~ 20 years</td>
</tr>
<tr>
<td>e-Pb</td>
<td>$\sqrt{s_{eh}} = 2.2$ TeV</td>
<td>0.5 x 10$^{34}$</td>
<td>1 fb$^{-1}$</td>
<td>60 GeV e$^-$ from ERL Concurrent operation with PbPb</td>
</tr>
</tbody>
</table>

Sequential implementation, FCC-ee followed by FCC-hh, would enable:
- variety of collisions (ee, pp, PbPb, eh) → impressive breadth of programme, 6++ experiments
- exploiting synergies by combining complementary physics reach and information of different colliders → maximise indirect and direct discovery potential for new physics
- starting with technologically ready machine (FCC-ee); developing in parallel best technology (e.g. HTS magnets) for highest pp energy (100++ TeV)
- building stepwise at each stage on existing accelerator complex and technical infrastructure

Purely technical schedule, assuming green light to preparation work in 2020.
A 70 years programme

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020-2028</td>
<td>8 years preparation</td>
</tr>
<tr>
<td>2023-2028</td>
<td>10 years tunnel and FCC-ee construction</td>
</tr>
<tr>
<td>2028-2053</td>
<td>15 years FCC-ee operation</td>
</tr>
<tr>
<td>2053-2064</td>
<td>11 years FCC-hh preparation and installation</td>
</tr>
<tr>
<td>2064-2090</td>
<td>25 years FCC-hh operation pp/PbPb/eh</td>
</tr>
</tbody>
</table>

Also studied: HE-LHC: $\sqrt{s}$=27 TeV using FCC-hh 16 T magnets in LHC tunnel; L~1.6x10$^{35}$ → 15 ab$^{-1}$ for 20 years operation
Review of Exact Amplitude-Based Resummation Theory

\[
d\bar{\sigma}_{\text{res}} = e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy \cdot (p_1 + q_1 - p_2 - q_2 - \sum k_{j_1} - \sum k'_{j_2}) + D_{\text{QCED}}} \beta_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}, \tag{1}
\]

where *new* (YFS-style) *non-Abelian* residuals \( \beta_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \) have *n* hard gluons and *m* hard photons.
Here,

\[ \text{SUM}_{\text{IR}}(\text{QCED}) = 2\alpha_s \Re B_{\text{QCED}}^{\text{nls}} + 2\alpha_s \tilde{B}_{\text{QCED}}^{\text{nls}} \]

\[ D_{\text{QCED}} = \int \frac{d^3k}{k^0} \left( e^{-iky} - \theta(K_{\text{max}} - k^0) \right) \tilde{S}_{\text{QCED}}^{\text{nls}} \]

where \( K_{\text{max}} \) is “dummy” and

\[ B_{\text{QCED}}^{\text{nls}} \equiv B_{\text{QCD}}^{\text{nls}} + \frac{\alpha}{\alpha_s} B_{\text{QED}}^{\text{nls}}, \]

\[ \tilde{B}_{\text{QCED}}^{\text{nls}} \equiv \tilde{B}_{\text{QCD}}^{\text{nls}} + \frac{\alpha}{\alpha_s} \tilde{B}_{\text{QED}}^{\text{nls}}, \]

\[ \tilde{S}_{\text{QCED}}^{\text{nls}} \equiv \tilde{S}_{\text{QCD}}^{\text{nls}} + \tilde{S}_{\text{QED}}^{\text{nls}}. \]

“nls”\( \equiv \) DGLAP-CS synthesization.

Shower/ME Matching: \( \hat{\beta}_{n,m} \rightarrow \hat{\beta}_{n,m} \)
Applications in Precision LHC Physics

- IR-Improved DGLAP-CS Theory: Herwiri1.031
  Interfaced to MC@NLO and MG5_aMC@NLO:
  
  Z and W+jets Production, ...

- KKMC-hh: Exact $O(\alpha^2L)$ CEEX EW Corrections Interfaced to Herwig6.5 and Herwiri1.031

- In Z and W+ jets Production, IR-Improvement gives a comparable or better data fit without ad hoc parameters

- In KKMC-hh, IR-improvement allows to quantify role of ISR in precision predictions for Z production observables, as we now illustrate.
### Standard Model Input Parameters

DIZET uses a modified $G_{\mu}$ Scheme with an over-complete set of inputs to take advantage of precision measurements to the extent possible. The following input parameters are used, taken from the 2014 EW Benchmark study, S. Alioli *et al.*, CERN-TH-2016-137 / arXiv:1606.02330

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\alpha(0)$</td>
<td>137.03599991</td>
</tr>
<tr>
<td>$G_F$</td>
<td>$1.1663787 \times 10^{-5}$ GeV$^{-2}$</td>
</tr>
<tr>
<td>$\sin^2(\theta_W)$</td>
<td>0.2232290158</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>2.4952 GeV</td>
</tr>
<tr>
<td>$\Gamma_W$</td>
<td>2.085 GeV</td>
</tr>
<tr>
<td>$m_d$</td>
<td>4.7 MeV</td>
</tr>
<tr>
<td>$m_s$</td>
<td>0.15 GeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4.6 GeV</td>
</tr>
<tr>
<td>$m_e$</td>
<td>510.999 keV</td>
</tr>
<tr>
<td>$m_\tau$</td>
<td>1.777 GeV</td>
</tr>
<tr>
<td>$1/\alpha(M_Z)$</td>
<td>128.952</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>0.12018</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>91.1876 GeV</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.385 GeV</td>
</tr>
<tr>
<td>$M_H$</td>
<td>125 GeV</td>
</tr>
<tr>
<td>$m_u$</td>
<td>2.2 MeV</td>
</tr>
<tr>
<td>$m_c$</td>
<td>1.2 GeV</td>
</tr>
<tr>
<td>$m_t$</td>
<td>173.5 GeV</td>
</tr>
<tr>
<td>$m_\mu$</td>
<td>105.6583 MeV</td>
</tr>
</tbody>
</table>
Angular Variables for $pp \rightarrow Z/\gamma^* \rightarrow \ell\ell$

We will consider distributions of the angle $\theta_{CS}$ of the negative $\ell$ defined in the Collins-Soper frame: the CM frame of $\ell^\pm$, relative to a $\hat{z}$ axis oriented as shown relative to the proton beams.

If $P = p_\ell + p_{\bar{\ell}}$ and $p^\pm = p^0 \pm p^z$ in the lab,

$$\cos(\theta_{CS}) = \text{sgn}(P^z) \frac{p^+_\ell p^-_{\bar{\ell}} - p^-_\ell p^+_{\bar{\ell}}}{\sqrt{P^2}}$$

$A_{FB}, A_4$: See talk by S.Yost
Interplay of (IR-Improved) DGLAP-CS QCD Theory and Exact $\mathcal{O}(\alpha^2L)$ CEEX EW Corrections

- Consider recent ATLAS measurement of $M_W$, arXiv:1701.07240:
  $80370 \pm 7\text{(stat.)} \pm 11\text{(exp.syst.)} \pm 14\text{(mod.syst.)}\text{MeV} = 80370 \pm 19$
- $Z/\gamma^*$ data used to help get mod. syst.
Muon Transverse Momentum

Muon $P_T$ Distribution

- Red: CEEX2
- Violet: CEEX2 no IFI
- Blue: EEX1
- Green: EEX1 no ISR

$\sqrt{s} = 7000$ GeV

100M events

Ratios of $P_T^\mu$ Distributions to CEEX2

- Violet: CEEX2 no IFI
- Blue: EEX1
- Green: EEX1 no ISR

$\sqrt{s} = 7000$ GeV

100M events
Applicatons in Precision LHC Physics

Lepton Transverse Momentum: ATLAS, 1701.07240
Applications in Precision FCC Physics

- FCC <-> FCC-ee + FCC-hh
- IR-Improvement of even the FCC-hh discovery spectra is needed--see arXiv:1801.03303
- For FCC-ee, a key issue is the theoretical precision of the Luminosity.
- Today, for illustration, we address the latter concern.
- We review what is the current state-of-the-art.
- We show the path forward to 0.01%
### General context:

**QED uncertainties in EW observables**

To be discussed in the following

<table>
<thead>
<tr>
<th>Observable</th>
<th>From</th>
<th>Present {QED}</th>
<th>FCC stat.</th>
<th>FCC syst.</th>
<th>Now {QED}</th>
<th>FCC(exp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_Z ) [MeV]</td>
<td>Z linesh.</td>
<td>91187.5 ( \pm ) 2.1( (0.3) )</td>
<td>0.005</td>
<td>0.1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_Z ) [MeV]</td>
<td>Z linesh.</td>
<td>2495.2 ( \pm ) 2.1( (0.2) )</td>
<td>0.008</td>
<td>0.1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_h/\Gamma_l )</td>
<td>( \sigma(M_Z) )</td>
<td>20.767 ( \pm ) 0.025( (0.012) )</td>
<td>( 1 \cdot 10^{-4} )</td>
<td>( \sim 10^{-3} )</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>( N_{\nu} )</td>
<td>( \sigma(M_Z) )</td>
<td>2.984 ( \pm ) 0.008( (0.006) )</td>
<td>( 0.8 \cdot 10^{-4} )</td>
<td>( 4 \cdot 10^{-4} )</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>( N_{\nu} )</td>
<td>Z+( \gamma )</td>
<td>2.69 ( \pm ) 0.15( (0.06) )</td>
<td>( 1 \cdot 10^{-3} )</td>
<td>( &lt; 10^{-3} )</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>( \sin^2 \theta_W^{eff} )</td>
<td>( A_{FB}^{lept.} )</td>
<td>( 0.23099 \pm 0.00053( (06) )</td>
<td>( 0.6 \cdot 10^{-5} )</td>
<td>( &lt; 10^{-5} )</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>( \sin^2 \theta_W^{eff} )</td>
<td>( A_{FB}^{pol.} )</td>
<td>( 0.23159 \pm 0.00041( (12) )</td>
<td>( 0.6 \cdot 10^{-5} )</td>
<td>( &lt; 10^{-5} )</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>( M_W ) [MeV]</td>
<td>ADLO</td>
<td>80376 ( \pm ) 33( (7) )</td>
<td>0.3</td>
<td>0.5</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>( A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}} )</td>
<td>( \frac{d\sigma}{d\cos \theta} )</td>
<td>( \pm 0.020( (0.001) )</td>
<td>( 1.0 \cdot 10^{-5} )</td>
<td>( 0.3 \cdot 10^{-5} )</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>( \alpha^{-1}_{QED}(M_Z) )</td>
<td>( \leq 10\text{GeV} )</td>
<td>128.952 ( \pm ) 0.014</td>
<td>0.004</td>
<td>0.001</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1:** Experimental precision of electroweak observables, which are most sensitive to QED effects. In the braces \{\ldots\} in 3-rd column are estimates of the systematic error due to QED calculation uncertainty. The necessary improvement factors of QED calculations for FCCee experiments are shown in the last column. FCCee systematic is without QED component. Uncertain numbers are marked with the question mark.

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QED challenges at FCCee are of 2-fold type:

A. More higher (fixed) orders, better resummation, more sophisticated Monte Carlo programs

B. Possibly completely new methodology of the QED “deconvolution” and related new definition of the EW pseudo-observables (EWPO’s)

--See talk by S. Jadach

An illustrative example:
Low angle Bhabha for luminosity measurement which enters into many observables, notably neutrino counting.
Motivation: better measurement of invisible Z width from Z peak x-section

LEP legacy:

\[ R^0_{\text{inv}} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell}} = \sqrt{\frac{12\pi R^0_{\ell}}{\sigma^0_{\text{had}} m_Z^2}} - R^0_{\ell} - (3 + \delta_r) \]

assuming lepton universality

\[(R^0_{\text{inv}})_{\text{exp}} = N_\nu \left( \frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\ell}} \right)_{\text{SM}}\]

from LEP Z-peak measurements

\[
N_\nu = 2.9840 \pm 0.0082 \\
\delta N_\nu \approx 10.5 \frac{\delta n_{\text{had}}}{n_{\text{had}}} \oplus 3.0 \frac{\delta n_{\text{lept}}}{n_{\text{lept}}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}
\]

\[
\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \implies \delta N_\nu = 0.0046
\]

7.5x0.061%=0.0046. Shall we do better at FCCee??

In 1999 lumi TH error 0.061% was dominated by VP. No motivation to improve QED error components. At FCCee VP error will be reduced by 4-6! New reality!

Low angle Bhabha luminometer already defined, Mogens Dam, FCC Week 2018, this wkshp
• LEP legacy, lumi TH error budget

<table>
<thead>
<tr>
<th>Type of correction/error</th>
<th>LEP1 1996</th>
<th>LEP1 1999</th>
<th>LEP2 1996</th>
<th>LEP2 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Missing photonic $O(\alpha^2)$ [4,5]</td>
<td>0.10%</td>
<td>0.027%</td>
<td>0.20%</td>
<td>0.04%</td>
</tr>
<tr>
<td>(b) Missing photonic $O(\alpha^2L^3)$ [6]</td>
<td>0.015%</td>
<td>0.015%</td>
<td>0.03%</td>
<td>0.03%</td>
</tr>
<tr>
<td>(c) Vacuum polarization [7,8]</td>
<td>0.04%</td>
<td>0.04%</td>
<td>0.10%</td>
<td>0.10%</td>
</tr>
<tr>
<td>(d) Light pairs [9,10]</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td>(e) Z-exchange [11,12]</td>
<td>0.015%</td>
<td>0.015%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Total</td>
<td>0.11% [12]</td>
<td>0.061% [13]</td>
<td>0.25% [12]</td>
<td>0.12% [13]</td>
</tr>
</tbody>
</table>

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within $1^\circ$-$3^\circ$ (18-52 mrad), and for LEP2 energies up to 176 GeV and an angular range within $3^\circ$-$6^\circ$. Total uncertainty is taken in quadrature. Technical precision included in (a).

• By the time of FCCee VP contribution will be merely 0.006% (F. Jegerlehner)

• QED corrections and Z contrib. come back to front!

• $Z$ contr. easy to master, even if rises at FCCee, because (28-58)→(64-86) mrad.

• Our FCCee forecast is 0.001%, provided QED is improved.

Bibliography in last slides
The Path to 0.01% Theoretical Luminosity Precision for the FCC-ee

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\textsuperscript{d}Max Planck Institute für Physik, München, Germany
\textsuperscript{e}The Citadel, Charleston, SC, USA

Abstract
The current status of the theoretical precision for the Bhabha luminometry is critically reviewed and pathways are outlined to the requirement targeted by the FCC-ee precision studies. Various components of the pertinent error budget are discussed in detail – starting from the context of the LEP experiments, through their current updates, up to prospects of their improvements for the sake of the FCC-ee. It is argued that with an appropriate upgrade of the Monte Carlo event generator BHLUMI and/or other similar MC programs calculating QED effects in the low angle Bhabha process, the total theoretical error of 0.01\% for the FCC-ee luminometry can be reached. A new study of the $Z$ and $s$-channel $\gamma$ exchanges within the angular range of the FCC-ee luminometer using the BHMIDE Monte Carlo was instrumental in obtaining the above result. Possible ways of BHLUMI upgrade are also discussed.
All of LEP/SLD luminosity QED error estimates represent corrections missing in BHLUMI v.4.04 Monte Carlo, used by all LEP and SLD collaborations.

BHLUMI features $O(\alpha^1)$ and $O(L_e^2 \alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between $e^+$ and $e^-$ lines, where $L_e = \ln(|t|/m_e^2)$.

Vacuum polarisation and pairs not dominant any more — QED photonic corrections and Z-exchange come back to front line!
1. Photonic corrections are large, but higher orders contrib. known, hence soft/collinear re-summation is mandatory!

2. M.E. in BHLUMI includes $O(\alpha^1)$ and $O(L_\gamma^2\alpha^2)$ corrections within YFS soft photon re-summation, neglecting photonics interferences between $e^+$ and $e^-$ lines (suppressed by $|t|/s$ factor).

3. Photonics 2nd order NLO $O(L_\gamma^2\alpha^2)$ and 3rd order LO $O(\alpha^3L_\gamma^3)$ corrections were calculated long ago [4], [6]. Presently they are not in BHLUMI v4.02 and accounted for in the error budget. Once included, error estimate is done for $O(L_\gamma^0\alpha^2)$, $O(\alpha^4L_\gamma^4)$ and $O(\alpha^3L_\gamma^2)$ corrections.

4. Corrections $O(L_\gamma^0\alpha^2) \sim 10^{-5}$ are not quoted in FCC error budget because are known.

5. Using scaling rules of thumb we estimate $O(\alpha^4L_\gamma^4)$ as $0.015\% \times \gamma = 0.6 \times 10^{-5}$ and $O(\alpha^3L_\gamma^2) \sim \gamma^2\alpha/\pi \sim 10^{-5}$.

6. N.B. BHLUMI with $O(L_\gamma^0\alpha^2)$ has been already realised but not published because VP was dominant in 1998.

\[ \gamma = \frac{\alpha}{\pi} \ln \frac{|t|}{m^2} = 0.042 \]

\[ |t|^{1/2} = \langle |t| \rangle^{1/2} \approx 3.25 \text{ GeV} \]


Z and s-channel gamma exchange for FCCee angular range 64-86mrad

1. With respect to dominant t-channel gamma exchange \( |\gamma|^2 = \gamma_t \otimes \gamma_t \), all other contributions are suppressed (near Z) by factor \( \langle |t|/s \rangle = 1.3 \times 10^{-3} \) (instead 0.4 \( \times 10^{-3} \) for LEP!)

2. However, resonant Zs exchange gets enhanced by \( \frac{M_{Z}}{\Gamma_{Z}} \) and \( \gamma_t \otimes Z_s \) term will be up to 1%. It is included in BHLUMI at the complete 1-st order level (with QED running couplings). Using results of ref. [11] its uncertainty due to QED corrections is presently estimate above as 0.090%

3. Non-resonant \( \gamma_t \otimes \gamma_s \sim 0.1 \% \) is included in BHLUMI, gets small QED cor. with uncertainty 0.01%

4. Other contributions not in BHLUMI are: \( Z_s \) ~0.1%, \( Z_t \sim 3 \times 10^{-5} \), \( |\gamma|^2 \sim 10^{-6} \) and \( |Z|^2 \sim 10^{-6} \)

5. It will be straightforward to reduce the above uncertainties to \( \sim 10^{-4} \) level by means of upgrade of the BHLUMI matrix element to the level of BHWIDE (EEX type).

6. With the implementation of the mat.el. of the CEEX type, as in KKMC, one could get for this group of contributions precision level of \( \sim 10^{-5} \).

Study of Z and s-channel gamma exchanges using BHWIDE

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>Update 2018</th>
<th>FCCee forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic ( O(L^4 \alpha^4) )</td>
<td>0.027%</td>
<td>6.0 \times 10^{-5}</td>
</tr>
<tr>
<td>(b) Photonic ( O(L^2 \alpha^3) )</td>
<td>0.015%</td>
<td>0.1 \times 10^{-4}</td>
</tr>
<tr>
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1. The error due to imprecise knowledge of the QED coupling constant for the t-channel exchange is

\[ \frac{\delta_{VP}\sigma}{\sigma} = 2 \frac{\delta\alpha_{eff}(t)}{\alpha_{eff}(t)} \]

2. With \( \Delta\alpha^{(5)}(-s_0) = (64.09 \pm 0.63) \times 10^{-4} \), of ref. [26], at \( s_0=2\text{GeV} \) we get \( (\delta_{VP}\sigma)/\sigma = 1.3 \times 10^{-4} \).

3. Anticipating improvement of hadronic \( e^+e^- \) cross section we expect by the FCCee time factor 2 improvement down to \( \delta_{VP}\sigma/\sigma = 0.65 \times 10^{-4} \).

4. N.B. The above is part of strategy of obtaining \( \alpha_{eff}(M_Z^2) \) in two steps:
   (a) obtaining \( \Delta\alpha^{(5)}(-s_0) \) from \( \sigma_{had}(s), s \leq 2.5 \text{GeV} \) using dispersion relations,
   (b) calculating \( \Delta\alpha^{(5)}(M_Z^2) - \Delta\alpha^{(5)}(-s_0) \) using perturbative QCD.
   Getting \( \Delta\alpha^{(5)}(-s_0) \) for Bhabha luminometry from \( \alpha_{eff}(M_Z^2) \) could be an interesting crosscheck:

---


1. Additional light fermion pair production in Bhabha process \( e^-e^+ \rightarrow e^-e^+ f\bar{f}, f = e, \mu, \tau, u, d, s \) together with the corresponding virtual correction (fermion loop on photon line) is a valid 2nd order correction.

2. Numerically most sizeable is electron pair production subprocess \( e^-e^+ \rightarrow e^-e^+ \gamma^*, \gamma^* \rightarrow e^-e^+ \) which very well known [9,10,18,19,53-60] and its precision is usually quoted to be \( \sim 0.5 \cdot 10^{-4} \).

3. Second pair production \( e^-e^+ \rightarrow e^-e^+ 2(e^-e^+) \) and addition photon production \( e^-e^+ \rightarrow e^-e^+e^-e^+\gamma \) are calculable [10,18,54] and quoted to be negligible.

4. Contributions from heavier leptons and light quarks \( f = \mu, \tau, u, d, s \) are typically \( \sim 0.8 \cdot 10^{-4} \) and in LEP context were entirely accounted as part of an error. They can be however calculated with the precision \( \ll 0.5 \cdot 10^{-4} \).

5. These corrections can be incorporated only partly in BHLUMI (electron pair exponentiation in [10]), most likely auxiliary MC programs will be needed to calculate them.

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1. From ref. [27] this photonics (1st order correction) is known to be \( \delta \sigma / \sigma \approx 0.07 |t|/s \) and for the luminometry it was negligible.

2. For FCCee it will come in a natural way in the upgrade M.E. of BHLUMI, to be done either as in BHWIDE or in KKMC.

3. We use conservatively factor \( 2 \gamma \approx 0.1 \) in its precision estimate.
1. Technical precision is the hardest problem!

2. In LEP workshop ref. [29] (1998) it was based on two pillars: comparison with semi-analytical calculation in ref. [45] and on comparison of BHLUMI with two hybrid MCs, LUMLOG+OLBBIS and SABSPV.

3. It was established to be 0.27%, together with missing photonics corrections.

4. Later on another BabaYaga MC was developed [20-24] based on the parton shower algorithm, and in principle could be used to evaluate technical precision independently.

5. However, once BHLUMI will be upgraded to include complete $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$ the problem will come back, because it will be much harder to upgrade BabaYaga to the same NNLO level due to known peculiarities of the parton shower methodology.

6. Alternative solution could/should be worked out.

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All of LEP/SLD luminosity QED error estimates represent corrections missing in BHLUMI v.4.04 Monte Carlo, used by all LEP and SLD collaborations.

BHLUMI features $O(\alpha^1)$ and $O(L_e^2\alpha^2)$ corrections with YFS resumation, neglecting photonics interferences between $e^+$ and $e^-$ lines, where $L_e = \ln(|t|/m_e^2)$.

One has to add to BHLUMI QED matrix element corrections of $O(L_e\alpha^2)$ and $O(\alpha^3L_e^3)$

They were calculated by Cracow-Knoxville collaboration long time ago (1996-99), but there was no strong motivation to publish them in the MC form, because of large VP uncertainty.

Interferences between $e^+$ and $e^-$ lines should be added at 1-st order, with resummation.

This class of corrections are implemented in the KKMC and BHWIDE since 1999.

Corrections due to Z exchange and s-chanel gamma are big but easy to master (ME upgrade).

There is (almost) enough auxiliary programs and calculations to control light pair corrections.

Summarising no hard obstacles are on the way to 0.01% QED precision on the theory side.

The sticky issue is that of “technical precision”.
If BabaYaga Monte Carlo team makes sufficient progress then this problem is solved.
But alternative solutions are available: comparing CEEX and EEX upgrades of BHLUMI,... .

We do need sufficient theory resources.
References


http://ez.r2.ifi.edu.pl/media/user/jadach/


Applications in Quantum Gravity

- Preliminary Remarks
- Overview of Resummed Quantum Gravity
- Planck Scale Cosmology
- An Estimate of $\Lambda$
- An Open Question?
- Einstein-Heisenberg Consistency Condition
- Constraints on SUSY GUTs
IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?

STRING THEORY: NO. You need superstrings, supersymmetric one-dimensional objects of Planck length size, $1.62 \times 10^{-33}$ cm.

---

**Preliminary Remarks**

**IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?**

**STRING THEORY: NO. You need superstrings, supersymmetric one-dimensional objects of Planck length size, $1.62 \times 10^{-33}$ cm.**

---

### String Theory

- **Advantages:**
  - UV FINITENESS,
  - THEORY OF EVERYTHING: $SU_2 \otimes U_1 \otimes QCD \otimes QG$,
  - APPLIED PHYSICS IN PURE MATHEMATICS, ...

- **Disadvantages:**
  - So Many Solutions $\sim 10^{500}$ and counting $\Rightarrow$ ANTHROPICITY, ...

- **quark:**
  - QFT -
  - SST -

  $0$ size $\ell_P = 1.62 \times 10^{-33}$ cm
Preliminary Remarks

LOOP QUANTUM GRAVITY: NO. You need Planck length size loops that are the fundamental constructs for quantum gravity.

Loop Quantum Gravity

- Advantages:
  - Suspected UV FINITENESS,
  - LINEARIZED QG Constraints, ...
- Disadvantages:
  - UNKNOWN SEMI-CLASSICAL LIMIT,
  - QUESTIONS OF PRINCIPLE, ...

- graviton: QFT - LQG - ....

0 size $\ell_{Pl} = 1.62 \times 10^{-33} cm$
HORAVA-LIFSHITZ THEORY: NO. You need anisotropic scaling at Planck length scales:

- Time and space differ by a factor of $z$ in scale dimension at Planck length distances with $z = 3$ in the original proposal—this violates local Lorentz invariance.

### HORAVA-LIFSHITZ Quantum Gravity

- **Advantages:**
  - POWER COUNTING UV RENORMALIZABLE,
  - CONNECTIONS TO CONDENSED MATTER, ...
- **Disadvantages:**
  - VIOLATES LOCAL LORENTZ INVARIANCE,
  - SCALAR GRAVITON (PATHOLOGY?), ...
- **graviton:**
  - **RQFT** - $\frac{1}{\omega^2 - \vec{k}^2}$
  - **HLQG** - $\frac{1}{\omega^2 - \vec{k}^2 - G\vec{k}^6}$

Size propagator, $\ell_{Pl} = 1.62 \times 10^{-33} \text{ cm} = \sqrt{G}$
Preliminary Remarks

- Our Response: Exact Amplitude-Based Resummation of Feynman’s Formulation of Einstein’s Theory – Resummed Quantum Gravity (RQG)

- RESULT (1): UV Finiteness!
- RESULT (2): Constraints on SUSY GUT’s
- RESULT (3): Prediction for the Cosmological Constant $\Lambda$ with Relatively Small Theoretical Uncertainty.
- RESULT (4): Consistent with Weinberg’s Asymptotic Safety Ansatz, as realized by Exact Field Space Renormalization Group Program of Reuter et al.
RESULT (5): Consistent with Kreimer’s Leg Renormalizability Results...

Today we give highlights on the status and outlook for this new RQG approach.
Overview of Resummed Quantum Gravity

SM ⇔ Many Massive Point Particles.
Feynman: spin is an inessential complication – checked. We replace \( L_{SM}^G(x) \) with that a free physical Higgs field, \( \varphi(x) \), with a rest mass 125 GeV (ATLAS, CMS) \( \Rightarrow \) the representative model


\[
\mathcal{L}(x) = \frac{1}{2\kappa^2} R \sqrt{-g} + \frac{1}{2} \left( g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_0^2 \varphi^2 \right) \sqrt{-g}
\]

\[
= \frac{1}{2} \left\{ h_{\mu\nu,\lambda} h_{\mu\nu,\lambda} - 2 \eta^{\mu\mu'} \eta^{\lambda\lambda'} h_{\mu,\lambda'} h_{\mu',\sigma} \right\} + \frac{1}{2} \left\{ \varphi,\mu \varphi,\mu - m_0^2 \varphi^2 \right\} - \kappa h^{\mu\nu} \left[ \frac{\varphi,\mu \varphi,\nu}{\varphi,\mu \varphi,\nu} + \frac{1}{2} m_0^2 \varphi^2 \eta_{\mu\nu} \right]
\]

\[
- \kappa^2 \left[ \frac{1}{2} h_{\lambda\rho} h^{\rho\lambda} \left( \varphi,\mu \varphi,\mu - m_0^2 \varphi^2 \right) - 2 \eta_{\rho\rho'} h^{\mu\rho} h^{\rho\nu} \varphi,\mu \varphi,\nu \right] + \cdots
\]

(1)
where $\varphi,_{\mu} \equiv \partial_{\mu}\varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$

- $\bar{y}_{\mu\nu} \equiv \frac{1}{2} (y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu}y_{\rho\rho})$ for any tensor $y_{\mu\nu}$

- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^{\mu}\bar{h}_{\nu\mu} = 0$

$\iff$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.
For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in the Figure.

\[ \begin{align*}
q & \quad k \\
(k + q) & \quad (a) \\
& \\
& + \\
& \\
k & \quad (b)
\end{align*} \]

**Figure:** The scalar one-loop contribution to the graviton propagator.

\( q \) is the 4-momentum of the graviton.

**QG’s BAD UV behavior – unrenormalizable.**
YFS resum the propagators in the NON-ABELIAN gauge theory of QG:
⇒ from the YFS formula

\[ iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_{-1}^{-1}(p) - \Sigma'_F(p)}, \]  

we find for Quantum Gravity, proceeding as above, the analogue of

\[ \alpha B''_\gamma = \int \frac{d^4 \ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \chi^2 + i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)} \bigg|_{k = k'} \]

as \(-B''_g(k)\) with

\[ B''_g(k) = -2i\kappa^2 k^4 \frac{1}{16\pi^4} \frac{d^4 \ell}{\ell^2 - \chi^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \]

for \(\Delta = k^2 - m^2\) ⇒ for a scalar field

\[ i\Delta'_F(k)\big|_{\text{YFS-resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)}. \]
Expand theory with the 'improved Born' propagators

\[ iP_{\alpha_1 \ldots, \alpha'_1 \ldots} \Delta'_F(k) |_{YFS\text{-resummed}, \Sigma'=0} = \frac{iP_{\alpha_1 \ldots, \alpha'_1 \ldots} e^{B''_g(k)}}{(k^2 - m^2 + i\epsilon)} \]  \hspace{1cm} (5)

where in the DEEP UV we get

\[ B''_g(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right) , \]  \hspace{1cm} (6)

⇒ ALL PROPAGATORS FALL FASTER THAN ANY POWER OF \(|k^2|\) ⇒ QG IS FINITE (SEE MPLA 17 (2002) 2371; hep-ph/0607198)!
CONTACT WITH ASYMPTOTIC SAFETY APPROACH

OUR RESULTS IMPLY

\[ G(k) = G_N / (1 + \frac{k^2}{a^2}) \]

⇒ FIXED POINT BEHAVIOR FOR
\[ k^2 \to \infty, \]

OUR RESULTS ⇒ AN ELEMENTARY PARTICLE HAS NO HORIZON. THIS AGREES WITH BONANNO & REUTER THAT A BLACK HOLE WITH A MASS LESS THAN \[ M_{cr} \sim M_{Pl} \]
HAS NO HORIZON.
BASIC PHYSICS:
\[ G(k) \text{ VANISHES FOR } k^2 \to \infty. \]
Planck Scale Cosmology

- Bonanno and Reuter see arXiv.org:0803.2546, and refs. therein – phenomenological approach to Planck scale cosmology:

\[ \mathcal{L}(x) = \frac{1}{2\kappa^2} \sqrt{-g} (R - 2\Lambda) \quad (7) \]

PHENOMENOLOGICAL EXACT RENORMALIZATION GROUP FOR THE WILSONIAN COARSE GRAINED EFFECTIVE AVERAGE ACTION IN FIELD SPACE \( \Rightarrow \) RUNNING NEWTON CONSTANT \( G_N(k) \) AND COSMOLOGICAL CONSTANT \( \Lambda(k) \) APPROACH UV FIXED POINTS AS \( k \) GOES TO \( \infty \) IN THE DEEP EUCLIDEAN REGIME – \( k^2 G_N(k) \rightarrow g_*, \Lambda(k) \rightarrow \lambda_* k^2 \).

- Due to the thinning of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Foot et al. (PLB664(2008)199) are obviated. – See also MPLA 25(2010)607; SHAPIRO&SOLA, PLB682(2009)105
CONTACT WITH COSMOLOGY PROCEEDS AS FOLLOWS:

PHENOMENOLOGICAL CONNECTION BETWEEN THE MOMENTUM SCALE $k$ CHARACTERIZING THE COARSENESS OF THE WILSONIAN GRAININESS OF THE AVERAGE EFFECTIVE ACTION AND THE COSMOLOGICAL TIME $t$, B-R SHOW STANDARD COSMOLOGICAL EQUATIONS ADMIT (see also Bonanno et al., 1006.0192) THE FOLLOWING EXTENSION:

\[
\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{1}{3} \Lambda + \frac{8\pi}{3} G_N \rho
\]

\[
\dot{\rho} + 3(1 + \omega)\frac{\dot{a}}{a} \rho = 0
\]

\[
\dot{\Lambda} + 8\pi \rho G_N = 0
\]

\[
G_N(t) = G_N(k(t))
\]

\[
\Lambda(t) = \Lambda(k(t))
\]

FOR DENSITY $\rho$ AND SCALE FACTOR $a(t)$
Planck Scale Cosmology

WITH ROBERTSON-WALKER METRIC REPRESENTATION

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]  \hspace{1cm} (9)

\( K = 0, 1, -1 \Leftrightarrow \) RESPECTIVELY FLAT, SPHERICAL AND PSEUDO-SPHERICAL 3-SPACES FOR CONSTANT TIME \( t \)

FOR A LINEAR RELATION BETWEEN THE PRESSURE \( p \) and \( \rho \) (EQN. OF STATE)

\[ p(t) = \omega \rho(t). \]  \hspace{1cm} (10)
Planck Scale Cosmology

FUNCTIONAL RELATIONSHIP BETWEEN MOMENTUM SCALE $k$ AND COSMOLOGICAL TIME $t$ DETERMINED PHENOMENOLOGICALLY VIA

$$k(t) = \frac{\xi}{t} \quad (11)$$

WITH POSITIVE CONSTANT $\xi$.

Using the UV fixed points for $k^2 G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ B-R SHOW THAT (8) ADMITS, FOR $K = 0$, A SOLUTION IN THE PLANCK REGIME ($0 \leq t \leq t_{\text{class}}$, with $t_{\text{class}}$ a few times the Planck time $t_{Pl}$), WHICH JOINS SMOOTHLY ONTO A SOLUTION IN THE CLASSICAL REGIME ($t > t_{\text{class}}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.
PHENOMENOLOGICAL NATURE OF THE ANALYSIS: THE fixed-point results $g_\ast, \lambda_\ast$ depend on the cut-offs used in the Wilsonian coarse-graining procedure.

KEY PROPERTIES OF $g_\ast, \lambda_\ast$ USED FOR THE B-R ANALYSES: they are both positive and the product $g_\ast\lambda_\ast$ is cut-off/threshold function independent.
In Phys. Dark Univ. 2 (2013) 97, using (5) and (6) we get rigorous cut-off independent values for the fixed points $g_*, \lambda_*$ and the following estimate of $\Lambda$:

\[
\rho_\Lambda(t_0) \approx \frac{-M_{Pl}^4(1 + c_{2,\text{eff}}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64} \sum_j \left(-1\right)^{F_j n_j} \rho_j^2 \\
\times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3
\]

\[
\approx \frac{-M_{Pl}^2(1.0362)^2(-9.194 \times 10^{-3}) (25)^2}{64} t_0^2
\]

\[
\approx (2.4 \times 10^{-3}\text{eV})^4,
\]  

where the age of the universe is $t_0 \approx 13.7 \times 10^9$ yrs.

Compare: $\rho_\Lambda(t_0)|_{\text{expt}} \approx ((2.37 \pm 0.05) \times 10^{-3}\text{eV})^4$. 

B.F.L. Ward RADCOR2019
A MAIN UNCERTAINTY: $t_{tr}$
B-R: NUMERICAL STUDIES $\Rightarrow t_{tr} \approx 25/M_{Pl}$
IN GENERAL, A FACTOR of $\mathcal{O}(100)$ IS ALLOWED
CAN WE DO BETTER?
Recently, arXiv:1507.00661, we use the de Sitter space solutions of Duerr et al. to get the Einstein-Heisenberg consistency condition

\[ k \geq \frac{\sqrt{5}}{2w_0} = \frac{\sqrt{5}}{2} \frac{1}{\sqrt{3/\Lambda(k)}} \]  

(13)

from the Heisenberg uncertainty relation \( \Delta p \Delta q \geq \frac{1}{2} \), with \( \Delta p = k \) and \( (w_0 = \sqrt{3/\Lambda}) \)

\[ (\Delta q)^2 \cong \int_0^{w_0} dww^2w^2 < \cos^2 \theta > = \frac{1}{5} w_0^2. \]  

(14)

Violation of (13) ends Planck scale inflation: solving for \( k_{tr} \)

\( k_{tr} \cong M_{Pl}/25.3 \), in agreement with what Bonnano and Reuter suggested from numerical studies.

\( \Rightarrow \) uncertainty on our estimate of \( \rho_\Lambda \) is \( \mathcal{O}(10) \).
Note

\[ <0|\mathcal{H}|0> \sim \int^{M_{Pl}} d^3k \frac{1}{2} \omega(k) = \int^{M_{Pl}} d^3k \frac{1}{2} \sqrt{k^2 + m_t^2} \]


Intermediate Stage:

\[ SU_{2L} \times SU_{2R} \times U_1 \times SU(3)_c \]

SM Stage at \(~2\text{TeV} = M_R;\)

\[ SU_{2L} \times U_1 \times SU(3)_c \]

SUSY Breaking at EW scale \(~M_S;\)

\[ U_1 \times SU(3)_c \]
• Possible spectrum(?)

\[
\begin{align*}
m_{\tilde{g}} &\approx 1.5(10)\text{TeV} \\
m_{\tilde{G}} &\approx 1.5\text{TeV} \\
m_{\tilde{q}} &\approx 1.0\text{TeV} \\
m_{\tilde{\ell}} &\approx 0.5\text{TeV} \\
m_{\chi_i^0} &\approx \begin{cases} 0.4\text{TeV}, & i = 1 \\ 0.5\text{TeV}, & i = 2, 3, 4 \end{cases} \\
m_{\chi_i^{\pm}} &\approx 0.5\text{TeV}, \ i = 1, 2 \\
m_s &= 0.5\text{TeV}, \ S = A^0, H^\pm, H_2 \\
\end{align*}
\]

\[\Delta_{\text{GUT}} = \sum_{j \in \{MSSM \ low \ energy \ susy \ partners\}} \frac{(-1)^F n_j}{\rho_j^2} \approx 1.13(1.12) \times 10^{-2}\]
• Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to 
\[ \sim 0.05 \, M_{\text{GUT}} \sim 2 \times 10^{15} \, \text{GeV}. \]
• For approach (A), new quarks and leptons at 
\[ M_{\text{High}} \sim 3.4(3.3) \times 10^3 \, \text{TeV}, \]
scalar partners at \[ \sim 0.5 \text{TeV} = M_{\text{Low}} \]
What About EW, QCD, GUT Symmetry Breaking Scales?

Consider GUT symmetry breaking:
It gives a $M_{\text{GUT}}^4 / (0.01 M_{\text{Pl}}^4/64) < 10^{-6}$
correction, which we drop here.
The other breaking scales are even smaller and hence their corrections
are even less significant in our result for $\rho_\Lambda$. 
Covariance Issues for $\dot{\Lambda}, \dot{G}_N \neq 0$:

Bianchi’s Identity,

$$D^\nu (\Lambda g_{\nu\mu} + 8\pi G_N T_{\nu\mu}) = 0,$$

allows

$$\dot{\rho} + 3\frac{\dot{a}}{a}(1+\omega)\rho = -\frac{\dot{\Lambda} + 8\pi \rho \dot{G}_N}{8\pi G_N},$$

a more general form of the new Friedmann eqns: qualitatively the same but details differ -- see arXiv:0907.4555,1103.4632,1202.5097....

Our estimate uses the more general form.
Summary

- Precision Quantum Field Theory: EW, QCD, QG \equiv \text{Control all limits:}

  \begin{align*}
  \text{IR (} z \to 1 \text{)} \\
  \text{and} \\
  \text{Collinear (} p_T \to 0 \text{)} \\
  \text{UV limit}
  \end{align*}

- We now have control over all aspects of the QG corrections.

- Toward quantitative understanding of $\rho_\Lambda$ along with other precision observables.