Local and Non-local Form Factors in $B \to K^*\ell\ell$

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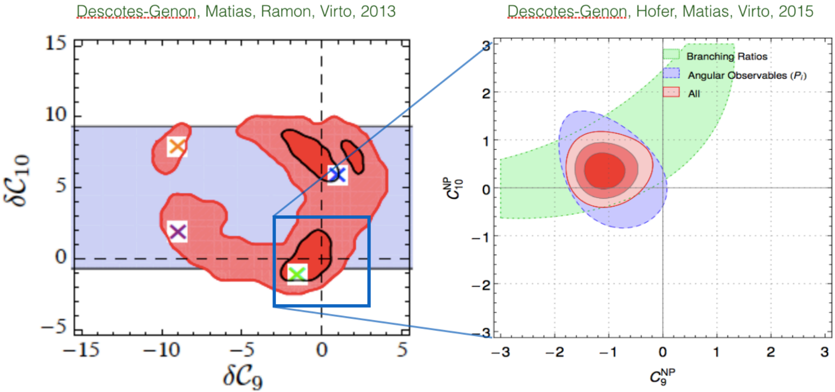
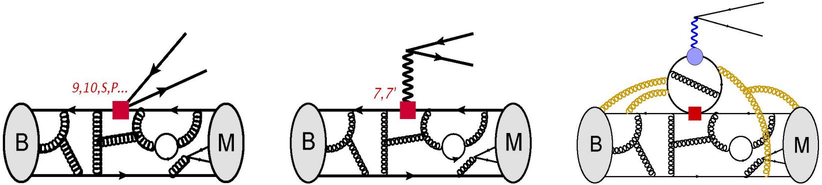
Hadronic Contributions to New Physics Searches 2019 – Tenerife – September 24, 2019

New Physics in $b \to s\ell\ell$



- Fit dominated by $B \to K^* \mu^+ \mu^-$
- Hadronic Contributions 2 New Physics Searches ✓
- Need theory predictions with $\sim 10\%$ (reliable) precision

Anatomy of $B \to M_\lambda \ell^+ \ell^-$ EFT Amplitudes



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

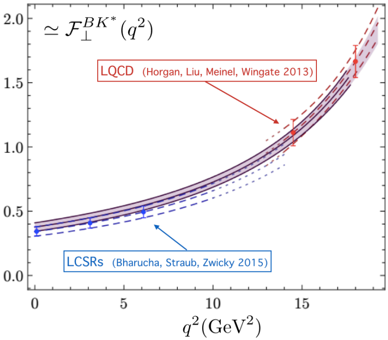
- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s}\, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | \, T\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

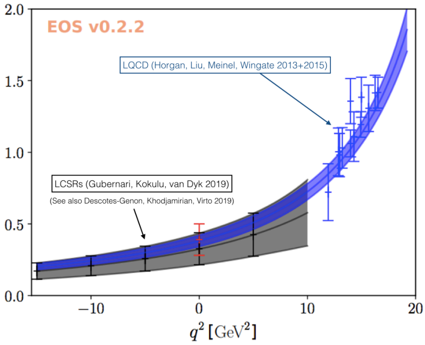
Outline

1. Local Form Factors (10’)

2. Non-Local Form Factors (10’)

Local Form Factors

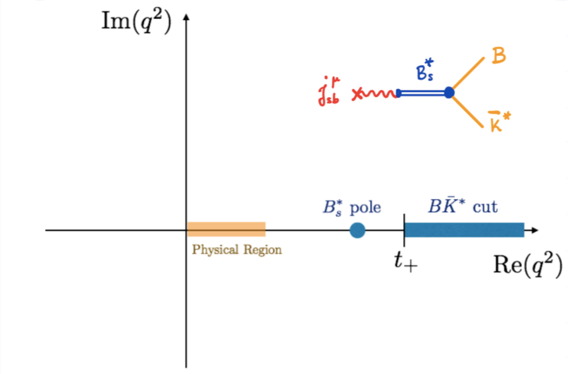




- Two main approaches: (1) Lattice QCD (large q^2) (2) LCSRs (low q^2)
- Two approaches to LCSRs, in terms of (1) K^* LCDAs (2) B LCDAs
- q^2 dependence can be parametrized model-independently

Form Factors : q^2-dependence from analyticity

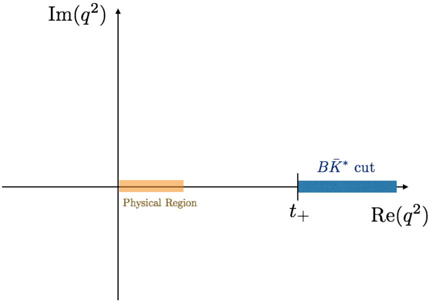
$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k)|\, \bar{s}\,\Gamma_\lambda^{(T)}\, b\,|\bar{B}(k+q)\rangle$: Analytic structure in q^2 :

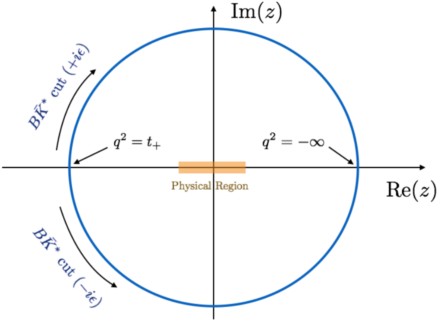


$\widehat{\mathcal{F}}_\lambda^{(T)}(q^2) \equiv (q^2 - m_{B_s^*}^2)\, \mathcal{F}_\lambda^{(T)}(q^2)$ has no pole, only cut.

Form Factors : q^2-dependence from analyticity

- Conformal mapping : $$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$





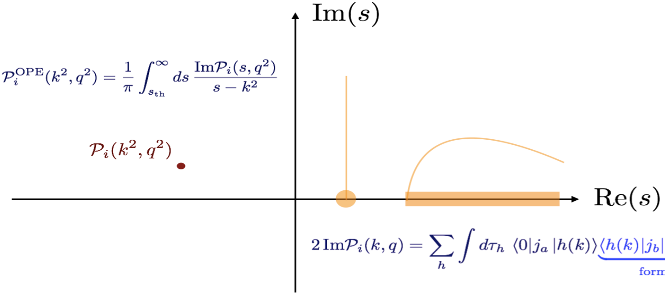
- "z-parametrization" : $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$ is **analytic in** $|z| < 1$ $\quad (|z_{\text{phys}}| < 0.15)$

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m_{B_s^*}^2)} \sum_k \alpha_k \, z(q^2)^k$$

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

Khodjamirian, Mannel, Offen 2006

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$

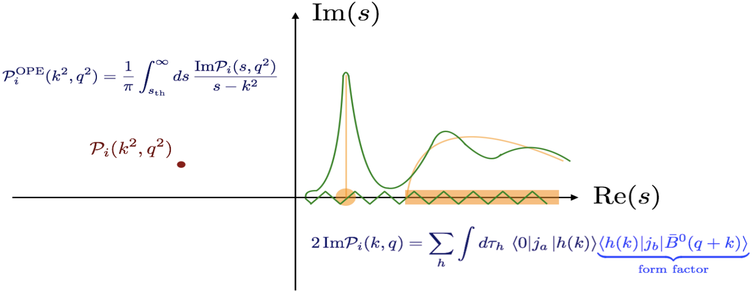


- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^* F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$

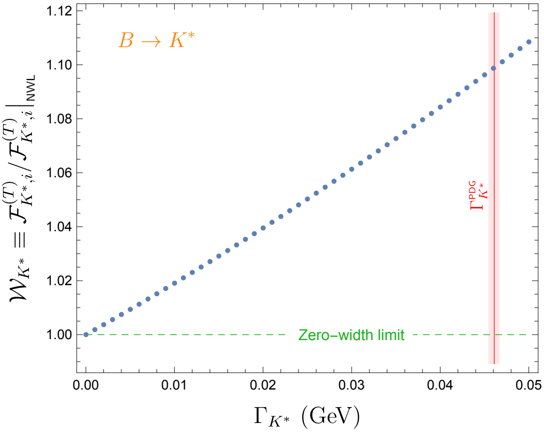


- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\, F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$
- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017 : $h(k) = K\pi + \cdots$

LCSRs with B-meson DAs, natural for this generalization.

Finite-width effects

Descotes-Genon, Khodjamirian, Virto, 1908.02267



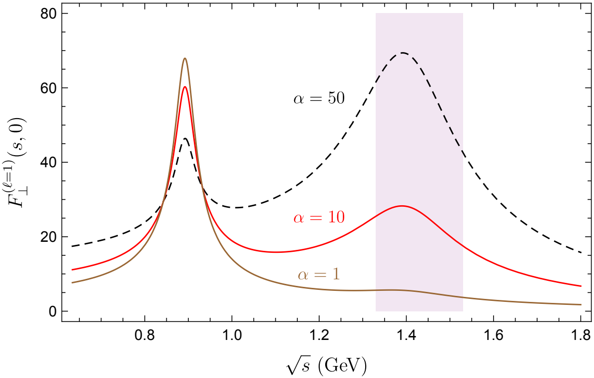
$$\mathcal{W}_{K^*} \simeq 1 + 1.9\,\frac{\Gamma_{K^*}}{m_{K^*}}$$

$$\mathcal{W}_{K^*} = 1.09 \pm 0.01$$

- $\mathcal{W}_{K^*}$ is independent of the form factor type
- $\mathcal{W}_{K^*}$ is indep. of q^2

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K^*}|^2 \simeq 1.2$. Ratios unaffected.

Set $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$ with α a floating parameter



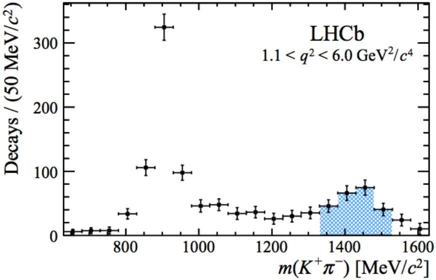
$\alpha = 1:\ \mathcal{F}_{K^*,\perp}(0) = 0.28\,;\quad \alpha = 10:\ \mathcal{F}_{K^*,\perp}(0) = 0.22\,;\quad \alpha = 50:\ \mathcal{F}_{K^*,\perp}(0) = 0.11\,.$

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell \, d\cos\theta_K \, d\phi$]

$$\frac{d\Gamma}{dq^2\,dk^2\,d\Omega} = \frac{1}{4\pi}\sum_{i=1}^{41} f_i(\Omega)\,\tilde{\Gamma}_i(q^2,k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2,k^2)$ have been measured by LHCb (arXiv:1609.04736) in the bins

$$\sqrt{k^2} \in [1.33, 1.53]\text{GeV}\ , \quad q^2 \in [1.1, 6]\text{GeV}^2$$



High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Descotes-Genon, Khodjamirian, Virto, 1908.02267

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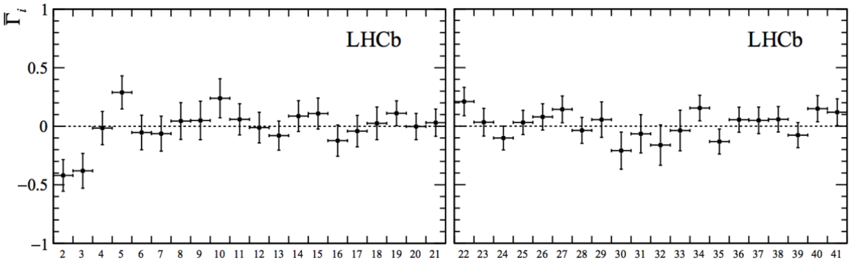

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The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ depend on S, P, D-wave amplitudes:

i	$f_i(\Omega)$	$\Gamma_i^{L,\mathrm{tr}}(q^2)/\mathbf{k}q^2$	$\eta_i^{L\to R}$
1	$P_0^0 Y_0^0$	$\left[\lvert H_0^L\rvert^2 + \lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2 + \lvert S^L\rvert^2 + \lvert D_0^L\rvert^2 + \lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2 \right]$	$+1$
2	$P_1^0 Y_0^0$	$2 \left[\frac{2}{\sqrt{5}} Re(H_0^L D_0^{L*}) + Re(S^L H_0^{L*}) + \sqrt{\frac{3}{5}}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) \right]$	$+1$
3	$P_2^0 Y_0^0$	$\frac{\sqrt{5}}{7} (\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2)$ - $\frac{1}{\sqrt{5}} (\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) + \frac{2}{\sqrt{5}} \lvert H_0^L\rvert^2 + \frac{10}{7\sqrt{5}} \lvert D_0^L\rvert^2 + 2\ Re(S^L D_0^{L*})$	$+1$
4	$P_3^0 Y_0^0$	$\frac{6}{\sqrt{35}} \left[- Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + \sqrt{3} Re(H_0^L D_0^{L*}) \right]$	$+1$
5	$P_4^0 Y_0^0$	$\frac{2}{7} \left[-2(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + 3\lvert D_0^L\rvert^2 \right]$	$+1$
6	$P_0^0 Y_2^0$	$\frac{1}{2\sqrt{5}} \left[(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + (\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - 2\lvert S^L\rvert^2 - 2\lvert D_0^L\rvert^2 - 2\lvert H_0^L\rvert^2 \right]$	$+1$
7	$P_1^0 Y_2^0$	$\left[\frac{\sqrt{3}}{5}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) - \frac{2}{\sqrt{5}}\, Re(S^L H_0^{L*}) - \frac{4}{5}\, Re(H_0^L D_0^{L*}) \right]$	$+1$
8	$P_2^0 Y_2^0$	$\left[\frac{1}{14}(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) - \frac{2}{7}\lvert D_0^L\rvert^2 - \frac{1}{10}(\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - \frac{2}{5}\lvert H_0^L\rvert^2 - \frac{2}{\sqrt{5}} Re(S^L D_0^{L*}) \right]$	$+1$
9	$P_3^0 Y_2^0$	$-\frac{3}{5\sqrt{7}} \left[Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + 2\sqrt{3}\, Re(H_0^L D_0^{L*}) \right]$	$+1$
10	$P_4^0 Y_2^0$	$-\frac{2}{7\sqrt{5}} \left[\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2 + 3\lvert D_0^L\rvert^2 \right]$	$+1$
11	$P_1^1 \sqrt{2}\, Re(Y_2^1)$	$-\frac{3}{\sqrt{10}} \left[\sqrt{\frac{2}{3}}\, Re(H_\parallel^L S^{L*}) - \sqrt{\frac{2}{15}}\, Re(H_\parallel^L D_0^{L*}) + \sqrt{\frac{2}{5}}\, Re(D_\parallel^L H_0^{L*}) \right]$	$+1$
12	$P_2^1 \sqrt{2}\, Re(Y_2^1)$	$-\frac{3}{[illegible]} \left[Re(H_\parallel^L H_0^{L*}) + \sqrt{\frac{5}{[illegible]}}\, Re(D_\parallel^L S^{L*}) + \frac{5}{[illegible]}\, Re(D_\parallel^L D_0^{L*}) \right]$	$+1$

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Descotes-Genon, Khodjamirian, Virto, 1908.02267

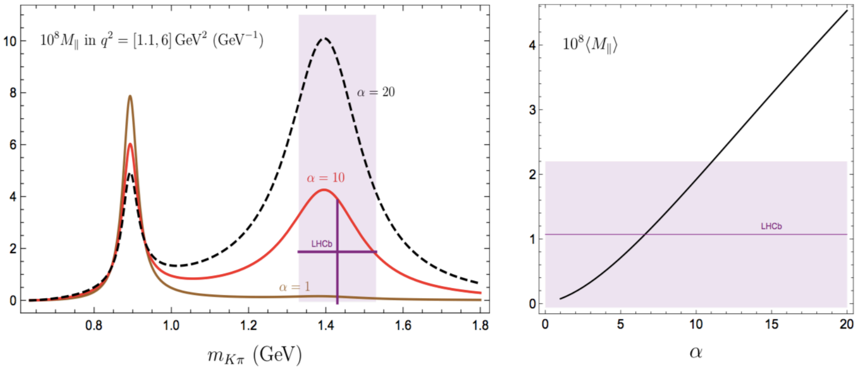
Combinations of moments depending only on P-wave:

$$
\begin{aligned}
|\widehat{A}_{\|}^{L}|^2 + |\widehat{A}_{\|}^{R}|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 - 5\sqrt{15}\tilde{\Gamma}_{19} + 35\sqrt{3}\tilde{\Gamma}_{21}) \\
|\widehat{A}_{\perp}^{L}|^2 + |\widehat{A}_{\perp}^{R}|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 + 5\sqrt{15}\tilde{\Gamma}_{19} - 35\sqrt{3}\tilde{\Gamma}_{21}) \\
\mathrm{Im}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} + \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) &= \frac{5}{36}(\sqrt{15}\tilde{\Gamma}_{24} - 7\sqrt{3}\tilde{\Gamma}_{26}) \\
\mathrm{Re}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} - \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) &= \frac{1}{36}(-5\sqrt{3}\tilde{\Gamma}_{29} + 7\sqrt{15}\tilde{\Gamma}_{31})
\end{aligned}
$$

Binned LHCb results (arXiv: 1609.04736) imply:

$$
\begin{aligned}
\tau_B \langle |\widehat{A}_{\|}^{L}|^2 + |\widehat{A}_{\|}^{R}|^2 \rangle &\equiv \langle M_{\|} \rangle = (1.07 \pm 1.13) \times 10^{-8} \\
\tau_B \langle |\widehat{A}_{\perp}^{L}|^2 + |\widehat{A}_{\perp}^{R}|^2 \rangle &\equiv \langle M_{\perp} \rangle = (0.94 \pm 1.06) \times 10^{-8} \\
\tau_B \langle \mathrm{Im}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} + \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) \rangle &\equiv \langle M_{\mathrm{im}} \rangle = (-0.75 \pm 0.79) \times 10^{-8} \\
\tau_B \langle \mathrm{Re}(\widehat{A}_{\perp}^{L}\widehat{A}_{\|}^{L*} - \widehat{A}_{\perp}^{R}\widehat{A}_{\|}^{R*}) \rangle &\equiv \langle M_{\mathrm{re}} \rangle = (0.27 \pm 0.50) \times 10^{-8}
\end{aligned}
$$

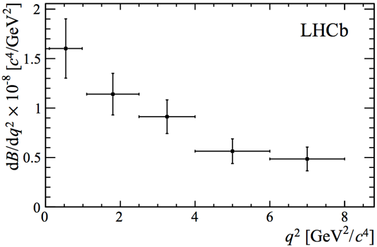
Example: $\langle M_{\|} \rangle$:



Bounds: From $\langle M_{\|} \rangle$: $\alpha \lesssim 11$; From $\langle M_{\perp} \rangle$: $\alpha \lesssim 17$; From $\langle M_{\rm re} \rangle$: $\alpha \lesssim 18$.

Upper bounds on P-wave from differential BR:

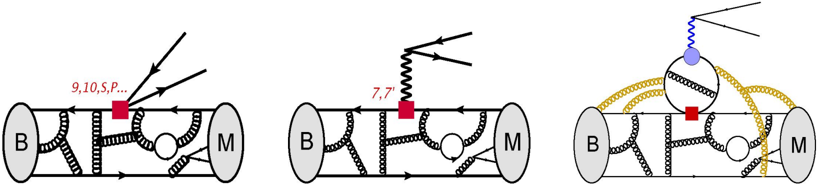
$$\frac{d\Gamma}{dq^2\, dk^2} = \tilde{\Gamma}_1 = |\widehat{A}_{\parallel}^{L}|^2 + |\widehat{A}_{\parallel}^{R}|^2 + |\widehat{A}_{\perp}^{L}|^2 + |\widehat{A}_{\perp}^{R}|^2 + |\widehat{A}_{0}^{L}|^2 + |\widehat{A}_{0}^{R}|^2 + \dots$$



$$\begin{aligned}
10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} &= 1.41 \pm 0.27 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} &= 1.60 \pm 0.29 \to \alpha \lesssim 6 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} &= 1.37 \pm 0.26 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} &= 1.12 \pm 0.26 \to \alpha \lesssim 4 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} &= 0.98 \pm 0.23 \to \alpha \lesssim 3
\end{aligned}$$

Bounds are easily improved with some info on S-wave form factors.

Non-Local Form Factors



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

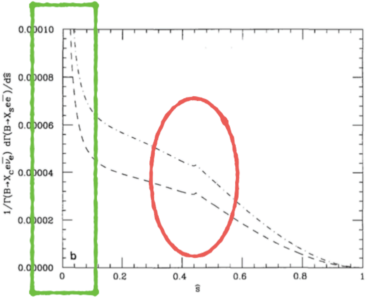
- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s}\, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | \, T\{ \mathcal{J}_{\rm em}^\mu(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

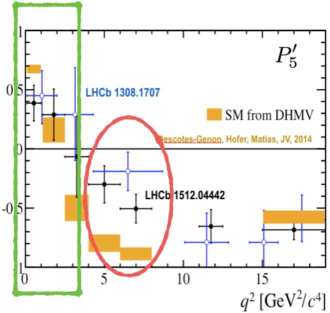
Non-local form factors

- QCD Factorization   tiny Beneke, Feldmann, Seidel 2001

$$\mathcal{H}_\lambda(q^2) \sim \Delta C_9^\lambda(q^2)\,\mathcal{F}_\lambda(q^2) + \frac{1}{q^2}\Delta C_7^\lambda(q^2)\,\mathcal{F}_\lambda^T(q^2) + HSS + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

- It is assumed that the charm loop is dominated by short distances





- Kink at $q^2 = 4m_c^2$ symptom of breaking of perturbativity

Non-local form factors: Strategy

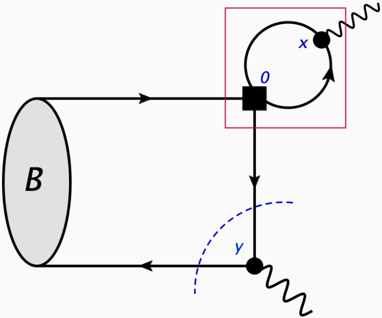
Similar to local form factors:

- Calculation at low-q^2 (LCSRs) and high-q^2 (LQCD)
- Interpolation with analytic expansion (z-expansion)

We can apply the same approach to Non-Local form factors:

- Calculate non-local ME at very low q^2
- Access to $q^2 > 0$ via analytic continuation + data

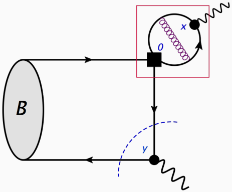
- LCSRs with B-meson DAs   Khodjamirian, Mannel, Pivovarov, Wang

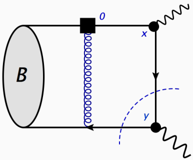


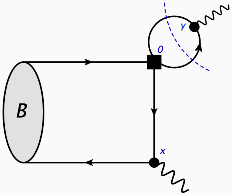
LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$

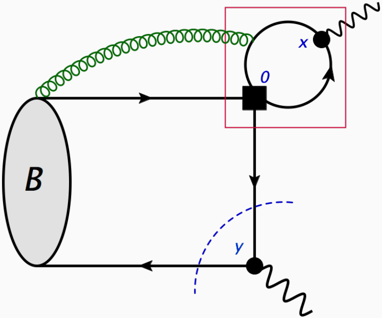






Charm-loop at very low q^2

- LCSRs with B-meson DAs — Khodjamirian, Mannel, Pivovarov, Wang



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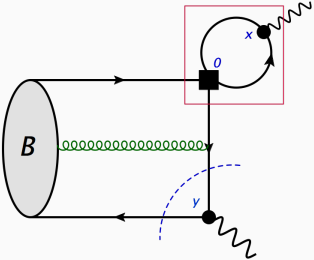
$$+ \,(\text{coeff}) \times [\,\bar{s}_L \gamma^\alpha (i n_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,] \;+ \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



Charm-loop at very low q^2

Recalculation of charm-loop effect Gubernari, van Dyk, Virto, in preparation

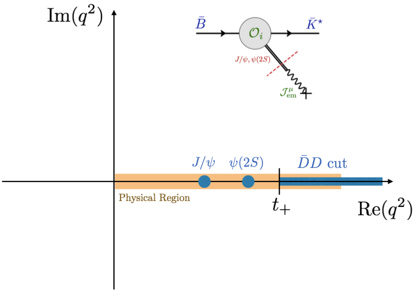
$\Delta C9(q^2)$		KMPW2010	GvDV2019
factorizable contr.		0.27	0.27
$B \to Kll$	$\tilde{\mathcal{A}}(q^2=1)$	$-0.09^{+0.06}_{-0.07}$	$(1.9^{+0.6}_{-0.6}) \cdot 10^{-4}$
$B \to K^*ll$	$\tilde{\mathcal{V}}_1(q^2=1)$	$0.6^{+0.7}_{-0.5}$	$(1.2^{+0.4}_{-0.4}) \cdot 10^{-3}$
	$\tilde{\mathcal{V}}_2(q^2=1)$	$0.6^{+0.7}_{-0.5}$	$(2.1^{+0.7}_{-0.7}) \cdot 10^{-3}$
	$\tilde{\mathcal{V}}_3(q^2=1)$	$1.0^{+1.6}_{-0.8}$	$(3.0^{+1.0}_{-1.0}) \cdot 10^{-3}$
$B_s \to \phi ll$	...	-	???

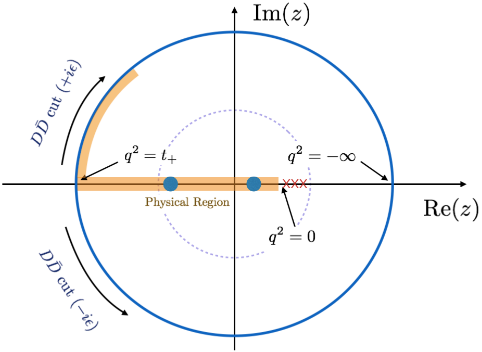
- We reproduce the result of KMPW’2010
- We incude complete set of 3-particle LCDAs Braun,Li,Manashov 2017
- Cancellations + Parametric lead to a reduction of the effect of **two orders of magnitude**

z-parametrisation for $\mathcal{H}_\lambda(q^2)$

Bobeth, Chrzaszcz, van Dyk, JV, 1707.07305

Same strategy as form factors [An. Str. checked directly from 2-loop diags Asatrian, Greub, Virto]





- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$ is analytic in $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{F}_\lambda(z)$
- Expansion needed for $|z| < 0.52$ ($-7\,\mathrm{GeV}^2 \le q^2 \le 14\mathrm{GeV}^2$)

Experimental constraints on z parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017

Experimental constraints :

- The residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

- Angular analyses Belle, Babar, LHCb determine :

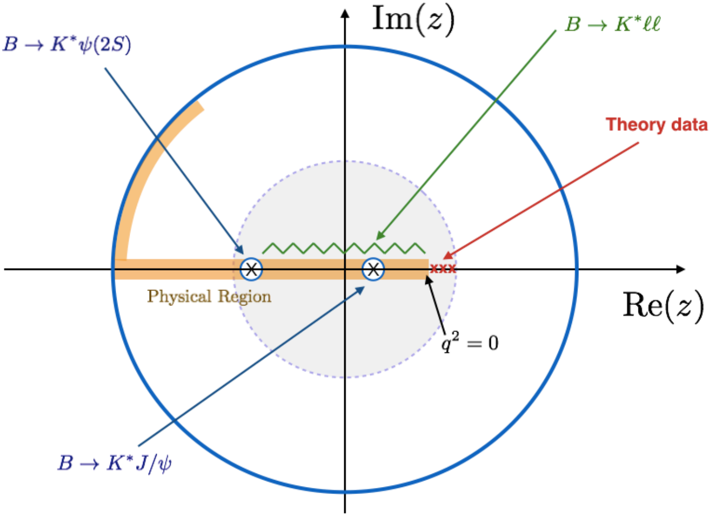
$$|r_\perp^{\psi_n}|,\ |r_\|^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_\|^{\psi_n} r_0^{\psi_n *}\},$$

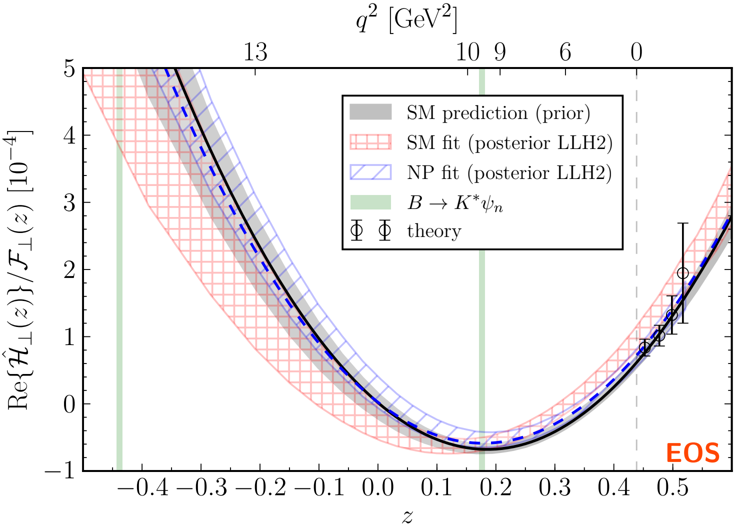
where $\quad r_\lambda^{\psi_n} \equiv \underset{q^2 \to M_{\psi_n}^2}{\mathrm{Res}} \dfrac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \dfrac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 \mathcal{F}_\lambda(M_{\psi_n}^2)}$

- We produce correlated pseudo-observables from a fit (5+5).

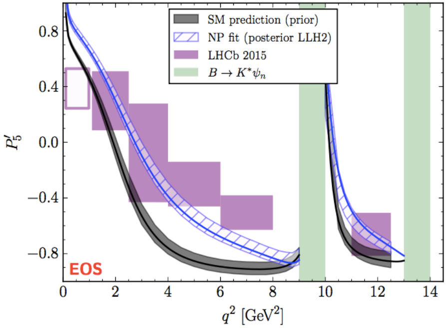
Fit to z-parametrisation

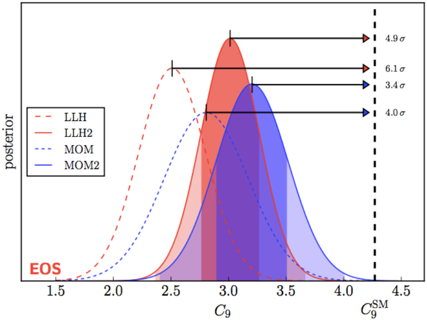
Bobeth, Chrzaszcz, van Dyk, JV, 1707.07305




q^2 [GeV^2]
13
10
9
6
0
5
4
3
2
1
0
−1
$\mathrm{Re}\{\hat{\mathcal{H}}_\perp(z)\}/\mathcal{F}_\perp(z)$ $[10^{-4}]$
SM prediction (prior)
SM fit (posterior LLH2)
NP fit (posterior LLH2)
$B \to K^*\psi_n$
theory
EOS
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
z

SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $\mathcal{C}_9^{\rm NP}$:



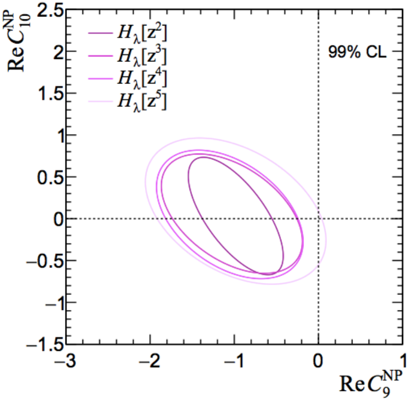


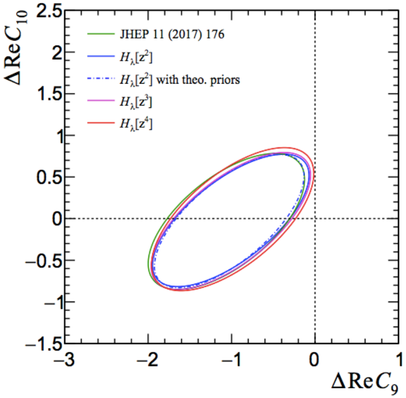
The NP hypothesis with $\mathcal{C}_9^{\mathbf{NP}} \sim -1$ is favored strongly in the global fit

Prospects: LHC Run-2 unbinned fits to z-parametrization

Chrzaszcz, Mauri, Serra, Coutinho, van Dyk 1805.06378

Mauri, Serra, Coutinho 1805.06401

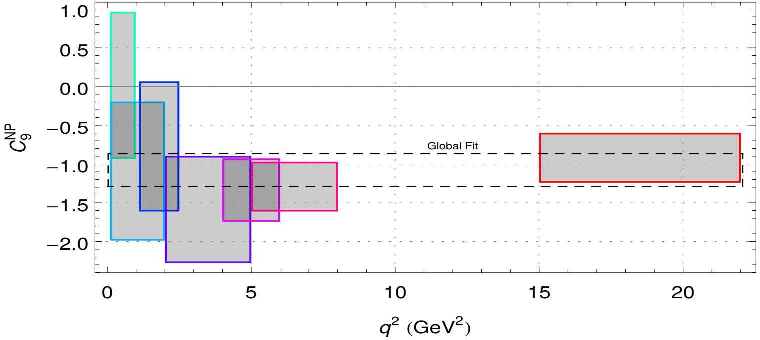




Unbinned fits to $B \to K^* \mu\mu$ (Left) and $B \to K^* \ell\ell$ (Right)

'A posteriori' test of non-local effect

☐ Testing the data : q^2-dependence Descotes-Genon, Hofer, Matias, Virto 1510.04239



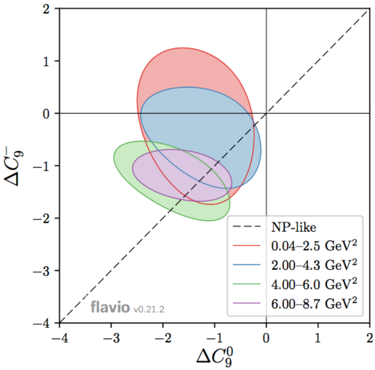
▶ Tiny uncertainites will allow to test hadronic contributions precisely

See also Altmannshofer, Straub 1503.06199, Ciuchini et al 1512.07157, Chobanova et al 1702.02234

'A posteriori' test of non-local effect

□ Testing the data : K^*-helicity dependence

Altmannshofer, Niehoff, Stangl, Straub 1703.09189



▶ Tiny uncertainites will allow to test hadronic contributions precisely

Summary

Local:

- Local Form factors in the narrow-width limit well under control. LCSRs at the limit.
- Absolutely no excuse to do the transition $K^* \to K\pi$ in your life
- Recalculation of all $B \to K^*$ form factors from LCSRs with B-DAs with twist-4 accuracy
- Finite-Width effects are 20% at the level of BRs, universal and q^2-independent
- Higher resonance effects can have dramatic effect on $B \to K^*$.
  High mass BRs and Moments very efficient to bound this possibility

Non-Local:

- Strategy very similar to Local FFs.
- LCOPE calculation of PS contribution redone and verified. Might be much smaller.
- z-expansion has potential. More studies in progress.
- A posteriori consistency checks very useful.

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- Local Form factors in the narrow-width limit well under control. LCSRs at the limit.
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Thank You

Extra

$B \to K\pi$ Form factors

Definition of Lorentz-Invariant Form Factors:

$$
\begin{aligned}
i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu b|\bar{B}^0(q+k)\rangle &= F_\perp\, k_\perp^\mu \\
-i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}^0(q+k)\rangle &= F_t\, k_t^\mu + F_0\, k_0^\mu + F_\parallel\, k_\parallel^\mu \\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu} q_\nu b|\bar{B}^0(q+k)\rangle &= F_\perp^T\, k_\perp^\mu \\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu} q_\nu\gamma_5 b|\bar{B}^0(q+k)\rangle &= F_0^T\, k_0^\mu + F_\parallel^T\, k_\parallel^\mu
\end{aligned}
$$

Functions $F_i^{(T)}(k^2, q^2, q\cdot\bar{k})$. Partial-wave expansion:

$$
\begin{aligned}
F_{0,t}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=0}^{\infty} \sqrt{2\ell+1}\, F_{0,t}^{(\ell)}(k^2, q^2)\; P_\ell^{(0)}(\cos\theta_K) \\
F_{\perp,\parallel}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2, q^2)\; \frac{P_\ell^{(1)}(\cos\theta_K)}{\sin\theta_K}
\end{aligned}
$$

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

Consider a correlation function of the type:

$$\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$$

which obeys a dispersion relation:

$$\mathcal{P}_{ab}^{\mathsf{OPE}}(k^2,q^2) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{P}_{ab}(s,q^2)}{s-k^2}$$

Duality + Borel transformation:

$$\frac{1}{\pi}\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\, \mathrm{Im}\mathcal{P}_{ab}(s,q^2) = \mathcal{P}_{ab}^{\mathsf{OPE}}(q^2,\sigma_0,M^2)\,,$$

Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

From Unitarity:

$$2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) = \sum_h \int d\tau_h\, \langle 0|j_a\,|h(k)\rangle \underbrace{\langle h(k)|j_b|\bar{B}^0(q+k)\rangle}_{\text{form factor}}$$

- Traditionally,

$$h(k) = K^* + continuum \quad \Rightarrow \quad 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\, F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$$

- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017

$$h(k) = K\pi + \cdots$$

LCSRs with B-meson DAs, natural for this generalization.

Light-Cone Sum Rules for P-wave $B \to K\pi$ Form Factors

$$\boxed{\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\;\; \omega_i(s, q^2)\;\; f_+^\star(s)\;\; F_i^{(T)(\ell=1)}(s, q^2) = \mathcal{P}_i^{(T),\text{OPE}}(q^2, \sigma_0, M^2)}$$

- s_0 – Effective threshold
- $\omega_i(s, q^2)$ – (known) kinematic factors
- $\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma_\mu d|0\rangle = f_+(k^2)\;\bar{k}_\mu + \frac{m_K^2 - m_\pi^2}{k^2} f_0(k^2)\; k_\mu$
- $\mathcal{P}_i^{(T),\text{OPE}}$ – OPE result for the correlation function

What have we done? arXiv:1908.02267 [hep-ph]

$$\boxed{\int_{s_{\rm th}}^{s_0} ds\ e^{-s/M^2}\ \ \omega_i(s,q^2)\ \ f_+^\star(s)\ \ F_i^{(T)(\ell=1)}(s,q^2) = \mathcal{P}_i^{(T),\rm OPE}(q^2,\sigma_0,M^2)}$$

- Generalize LCSRs in Khodjamirian, Mannel, Offen 2006 beyond the K^* case, including LCSRs for A_0, $T_{2,3}$
- Recalculate $\mathcal{P}_i^{(T),\rm OPE}$ including 3-particle contributions, and extended consistently to twist-4 accuracy. Full (numerical) agreement with Gubernari,Kokulu,van Dyk 2018 (not input parameters)
- Revisit s_0 $\Rightarrow$ significantly lower value!! – f_{K^*} is derived quantity
- Study of Narrow-width limit, Finite-Width effects, and effects beyond the K^*
- Applications to $B \to K\pi\ell\ell$

Differential decay rate of $\tau \to K\pi\nu_\tau$:

$$\frac{d\Gamma}{ds} = \frac{N_\tau}{s^3}\left(1 - \frac{s}{m_\tau^2}\right)^2\left(1 + 2\frac{s}{m_\tau^2}\right)\lambda_{K\pi}^{3/2}\,|\widetilde{f}_+(s)|^2\left\{1 + \frac{3(\Delta m^2)^2}{(1+2s/m_\tau^2)\,\lambda_{K\pi}}\,|\widetilde{f}_0(s)|^2\right\}$$

with the normalization [Total BR will give $|f_+(0)|^2 = 0.99$, consistent with $f_+^{LQCD}(0) = 0.97$]

$$N_\tau = \frac{G_F^2\,|V_{us}|^2\,|f_+(0)|^2\,m_\tau^3}{1536\pi^3}\,S_{EW}^{\rm had}$$

Belle fits to models: [This gives $f_{K^*} \simeq 205$ MeV, compared to $f_{K^*} = 217(5)$ MeV (NWL)]

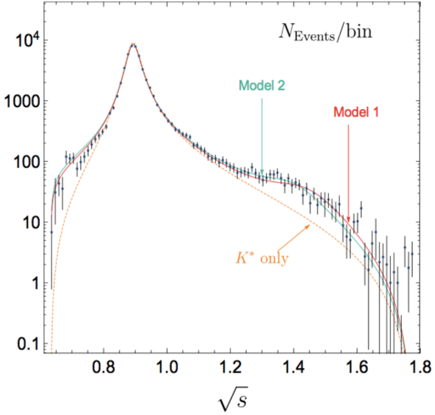
$$\widetilde{f}_+(s) = \sum_R \frac{\xi_R\, m_R^2}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}\ , \qquad f_0(s) = f_+(0)\cdot\sum_{R_0}\frac{\xi_{R_0}\, s}{m_{R_0}^2 - s - i\sqrt{s}\,\Gamma_{R_0}(s)}\ ,$$

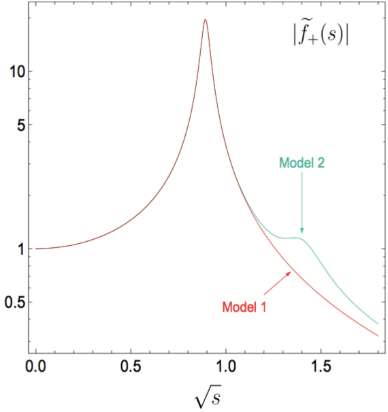
Model 1 : $\xi_{K^*(892)} = 1\ ,\ \xi_{K_0^*(800)} = 1.27\ ,\ \xi_{K_0^*(1430)} = 0.954\, e^{i\,0.62}$

Model 2 : $\xi_{K^*(892)} = 0.988\, e^{-i\,0.07}\ ,\ \xi_{K^*(1410)} = 0.074\, e^{i\,1.37}\ ,\ \xi_{K_0^*(800)} = 1.57$

$K\pi$ form factor $f_+(s)$ from $\tau \to K\pi\nu_\tau$

Data from Belle, arXiv:0706.2231 [hep-ex]





Effective threshold: 2-point SVZ sum rule

Knowing $|f_+(s)|$ we can extract s_0 from a QCD sum rule:

$$\begin{aligned}\Pi_{\mu\nu}(k) &= i\int d^4x e^{ikx}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu s(x), \bar{s}(0)\gamma_\nu d(0)|0\rangle \\ &= (k_\mu k_\nu - k^2 g_{\mu\nu})\,\Pi(k^2) + k_\mu k_\nu\,\widetilde{\Pi}(k^2)\end{aligned}$$

$$\Pi(M^2, s_0) \equiv \frac{1}{\pi}\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\Pi(s) = \int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\,\frac{\lambda_{K\pi}^{3/2}(s)}{32\pi^2 s^3}\,|f_+(s)|^2$$

$$\begin{aligned}\Pi^{\rm OPE}(M^2, s_0) &= \frac{1}{8\pi^2}\int_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\frac{(s-m_s^2)^2(2s+m_s^2)}{s^3} \\ &\quad + \frac{\alpha_s(M)}{\pi}\,\frac{M^2}{4\pi^2}\left(1 - e^{-s_0/M^2}\right) + \frac{v_4}{M^2} + \frac{v_6}{2M^4}\end{aligned}$$

Borel parameter M^2	Effective threshold s_0	
$1.00\ \mathrm{GeV}^2$	$1.28 \pm 0.18\ \mathrm{GeV}^2$ (Model 1) $1.25 \pm 0.18\ \mathrm{GeV}^2$ (Model 2)	$1.26 \pm 0.18\ \mathrm{GeV}^2$ (Average)
$1.25\ \mathrm{GeV}^2$	$1.33 \pm 0.12\ \mathrm{GeV}^2$ (Model 1) $1.31 \pm 0.12\ \mathrm{GeV}^2$ (Model 2)	$1.31 \pm 0.12\ \mathrm{GeV}^2$ (Average)
$1.50\ \mathrm{GeV}^2$	$1.36 \pm 0.09\ \mathrm{GeV}^2$ (Model 1) $1.34 \pm 0.09\ \mathrm{GeV}^2$ (Model 2)	$1.35 \pm 0.09\ \mathrm{GeV}^2$ (Average)

Table 3: *Values for the effective threshold s_0 extracted from the SVZ sum rules.*

Significantly low value compared to the usual $s_0^{K^*} \simeq 1.7$ GeV^2

Models for $B \to K\pi$ form factors

Assume that the P-wave $K\pi$ state couples to its interpolating current $\bar{s}\Gamma d$ resonantly, through a set of Breit-Wigner-type vector resonances:

$$\langle K(k_1)\pi(k_2)|\bar{s}\gamma^\mu d|X\rangle = \sum_{R,\eta} BW_R(k^2)\langle K(k_1)\pi(k_2)|R(k,\eta)\rangle\langle R(k,\eta)|\bar{s}\gamma^\mu d|X\rangle$$

$$f_+(s) \quad = \quad -\sum_R \frac{m_R\, f_R\, g_{RK\pi}\ \ e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

$$F_i^{(T),(\ell=1)}(s,q^2) \quad = \quad \sum_R \frac{Y_{R,i}^{(T)}(s,q^2)\, g_{RK\pi}\, \mathcal{F}_{R,i}^{(T)}(q^2)\ e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

This model is totally equivalent to the model fitted by Belle for $f_+(s)$.

Generalized Light-Cone Sum Rule + BW model

$$\boxed{\sum_R \mathcal{F}_{R,i}^{(T)}(q^2)\, d_{R,i}^{(T)}\, I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)}$$

with

$$I_R(s_0, M^2) = \frac{m_R}{16\,\pi^2} \int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\, \frac{g_{RK\pi}\, \lambda_{K\pi}^{3/2}(s)\, |f_+(s)|}{s^{5/2}\sqrt{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}}$$

and

$$d_{R,\perp} = -d_{R,-} = (m_B + m_R)^{-1}\,, \quad d_{R,\parallel} = \frac{(m_B + m_R)}{2}\,, \quad d_{R,t} = -m_R\,,$$

$$d_{R,\perp}^T = -d_{R,-}^T = 1\,, \quad d_{R,\parallel}^T = \frac{(m_B^2 - m_R^2)}{2}\,.$$

Narrow-width limit

Consider the sum rule with a single resonance R:

$$\boxed{\mathcal{F}^{(T)}_{R,i}(q^2)\ d^{(T)}_{R,i}\ I_R(s_0,M^2) = \mathcal{P}^{(T),\mathrm{OPE}}_i(q^2,\sigma_0,M^2)}$$

$$\begin{aligned} I_R(s_0,M^2) \quad &= \quad 3\,m_R\,f_R\,\mathcal{B}(R\to K^+\pi^-)\int_{s_{\rm th}}^{s_0} ds\ e^{-s/M^2}\ \frac{m_R}{\sqrt{s}}\left[\frac{1}{\pi}\,\frac{\sqrt{s}\,\Gamma_R(s)}{(m_R^2-s)^2+s\,\Gamma_R^2(s)}\right] \\ &\xrightarrow{\Gamma_R^{\rm tot}\to 0}\ 3\,m_R\,f_R\,\mathcal{B}(R\to K^+\pi^-)\ e^{-m_R^2/M^2} \end{aligned}$$

$$\Rightarrow \quad 3\,m_R f_R\, d^{(T)}_{R,i}\,\mathcal{F}^{(T)}_{R,i}(q^2)\ e^{-m_R^2/M^2}\ \mathcal{B}(R\to K^+\pi^-) = \mathcal{P}^{(T),\mathrm{OPE}}_i(q^2,\sigma_0,M^2)$$

This agrees with Khodjamirian, Mannel, Offen 2006

Finite-width effects

Consider the sum rule with a single K^*:

$$\boxed{\mathcal{F}^{(T)}_{K^*,i}(q^2)\; d^{(T)}_{K^*,i}\; I_{K^*}(s_0,M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2,\sigma_0,M^2)}$$

Define the "Width ratio" $\mathcal{W}_{K^*}$:

$$\mathcal{W}_{K^*} \equiv \frac{\mathcal{F}^{(T)}_{K^*,i}(q^2)}{\mathcal{F}^{(T)}_{K^*,i}(q^2)_{\mathrm{NWL}}} = \frac{I_{K^*}(s_0,M^2)\big|_{\Gamma_{K^*}\to 0}}{I_{K^*}(s_0,M^2)} = \frac{2m_{K^*}f_{K^*}\,e^{-m^2_{K^*}/M^2}}{I_{K^*}(s_0,M^2)}$$

- $\mathcal{W}_{K^*}$ is independent of the form factor type
- $\mathcal{W}_{K^*}$ is independent of q^2

$\Rightarrow$ BRs are corrected by $|\mathcal{W}_{K^*}|^2$, ratios are uncorrected! + true in q^2 bins.

Beyond the $K^*(892)$

Consider the sum rule with $R = \{K^*(892), K^*(1410)\}$:

$$\boxed{\sum_R \mathcal{F}^{(T)}_{R,i}(q^2)\; d^{(T)}_{R,i}\; I_R(s_0, M^2) = \mathcal{P}^{(T),\mathrm{OPE}}_i(q^2, \sigma_0, M^2)}$$

		$M^2 = 1.00\,\mathrm{GeV}^2$	$M^2 = 1.25\,\mathrm{GeV}^2$	$M^2 = 1.50\,\mathrm{GeV}^2$
Model 1	$I_{K^*(892)}$	0.1506(23)	0.1781(16)	0.1992(13)
	$I_{K^*(1410)}$	0.0050(07)	0.0062(07)	0.0072(06)
Model 2	$I_{K^*(892)}$	0.1491(22)	0.1766(20)	0.1975(16)
	$I_{K^*(1410)}$	0.0048(07)	0.0061(06)	0.0070(06)

Table 8: *Values for the quantities I_R for $R = \{K^*(892), K^*(1410)\}$ for the different values of the Borel parameter M^2 and for the two models for the $K\pi$ form factor. The $K^*(1410)$ contribution is very suppressed in the sum rules, with $I_{K^*(1410)}/I_{K^*(892)} \simeq 0.03$ in all cases.*

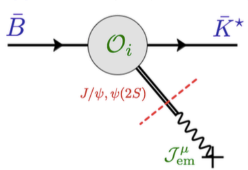
Form Factor	This work	Ref. [12]	Ref. [24]	Ref. [15]	Ref. [17]
$\mathcal{F}_{K^*,\perp}(0) = V^{BK^*}(0)$	0.26(15)	0.39(11)	0.36(18)	0.32(11)	0.34(4)
$\mathcal{F}_{K^*,\parallel}(0) = A_1^{BK^*}(0)$	0.20(12)	0.30(8)	0.25(13)	0.26(8)	0.27(3)
$\mathcal{F}_{K^*,-}(0) = A_2^{BK^*}(0)$	0.14(13)	0.26(8)	0.23(15)	0.24(9)	0.23(5)
$\mathcal{F}_{K^*,t}(0) = A_0^{BK^*}(0)$	0.30(7)	–	0.29(8)	0.31(7)	0.36(5)
$\mathcal{F}^T_{K^*,\perp}(0) = T_1^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,\parallel}(0) = T_2^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,-}(0) = T_3^{BK^*}(0)$	0.13(12)	–	0.22(14)	0.20(8)	0.18(3)

Table 6: *Results for the form factors at $q^2 = 0$ in the narrow-width limit, compared to corresponding results in the literature. The approach in Ref. [17] is a completely different LCSR approach, in terms of K^* DAs.*

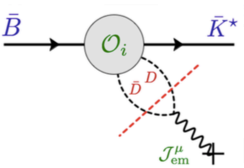
$\mathcal{F}^{BK^*}(q^2=0)$	V^{BK^*}	$A_1^{BK^*}$	$A_2^{BK^*}$	$A_0^{BK^*}$	$T_{1,2}^{BK^*}$	$T_3^{BK^*}$
Ref. [12]	0.39	0.30	0.26	–	0.33	–
Inputs [12], no g_+	0.38	0.29	0.26	0.31	0.33	0.25
Inputs [12], with g_+	0.27	0.21	0.14	0.31	0.24	0.14
Our inputs, but $s_0 = 1.7\,\mathrm{GeV}^2$	0.33	0.26	0.17	0.38	0.29	0.17
Our inputs, our s_0, no g_+	0.36	0.28	0.25	0.30	0.31	0.23
Our inputs, our s_0, with g_+	0.26	0.20	0.14	0.30	0.22	0.13

Table 7: *Deconstruction of the different effects explaining the difference between our results for the form factors at $q^2 = 0$ and those in Ref. [12]. The difference stems mainly from the inclusion of the twist-four two-particle contributions. See the text for more details.*

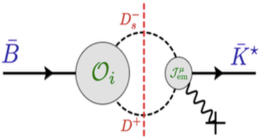
Analytic structure of Non-Local form factor

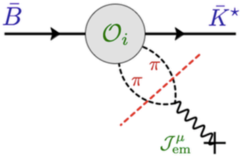


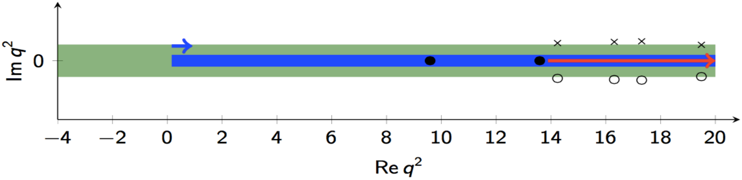
(a)



(b)

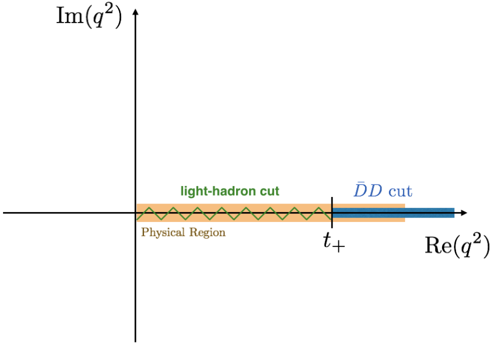






Light-hadron cut

The non-local ME of $O^c_{1,2}$ also contains a cut at low q^2 from intermediate "light-hadron" states:



$$\mathrm{Disc}[\mathcal{H}_\lambda(q^2 > t_+)] \sim \sum_X \langle 0|j_{\mathrm{em}}|X^{1^{--}}_{cc}\rangle\langle X^{1^{--}}_{cc} K^*|(\bar{s}c)(\bar{c}b)|\bar{B}\rangle$$

$$\mathrm{Disc}[\mathcal{H}_\lambda(0 < q^2 < t_+)] \sim \sum_X \langle 0|j_{\mathrm{em}}|X^{1^{--}}\rangle\langle X^{1^{--}} K^*|(\bar{s}c)(\bar{c}b)|\bar{B}\rangle$$

Light-hadron cut

$\star$ Support for $\langle X^{1^{--}} K^* | (\bar{s}c)(\bar{c}b) | \bar{B} \rangle \ll \langle X_{cc}^{1^{--}} K^* | (\bar{s}c)(\bar{c}b) | \bar{B} \rangle$:

- OZI rule.
- $\mathcal{B}(B \to K^{(*)}\omega) \approx 2 \text{ - } 5 \cdot 10^{-6}$ (in agreement with QCDF from $[\bar{s}q][\bar{q}b]$)

$$\Rightarrow \langle K^* \omega | (\bar{s}c)(\bar{c}b) | \bar{B} \rangle \lesssim \underbrace{C_q/C_c}_{few\,\%} \langle K^* \omega | (\bar{s}q)(\bar{q}b) | \bar{B} \rangle$$

- Same argument for $\mathcal{B}(B \to K^{(*)}\phi)$
- In absence of OZI, the natural size of these BRs is 10^{-3} not 10^{-6} :

$$\mathcal{B}(B \to K J/\psi) = 9 \times 10^{-4} \qquad \mathcal{B}(B \to K^* J/\psi) = 1.3 \times 10^{-3}$$
$$\mathcal{B}(B \to K[D^* \bar{D}]) = 6 \times 10^{-3} \qquad \mathcal{B}(B \to K[D^* \bar{D}^*]) = 8 \times 10^{-3}$$
$$\mathcal{B}(B \to K[D\bar{D}]) = 5 \times 10^{-4}$$

- Note also the **total** BR: $\mathcal{B}(B \to K^{(*)}[\bar{K}K]) \sim 10^{-5} \ll 10^{-3}$
- Test: $\mathcal{B}(B \to K^{(*)} X^{1^{--}}(\text{high mass})) \ll 10^{-3}$

Light-hadron cut

Conclusion: OZI $\rightarrow$ Two order of magnitude suppression.

- But CKM- and penguin-suppressed light-quark loops are there.
- Not OZI suppressed.
- Must be constrained if precision is sought (but rough estimate might suffice).
- Can do dispersive analysis Khodjamirian, Mannel, Wang 2012 ...
- Could use $b \rightarrow d$ analogues + U-spin.