Hunting $\tau$ loops in $B^+ \rightarrow K^+\mu^+\mu^-$

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based on ongoing work with G.Isidori, M.König, S. Liechti, P. Owen, N.Serra
Flavour anomalies in semileptonic B-decays:

\[ b \rightarrow c \tau \nu \]
\[ \tau/\mu \text{ universality} \]

\[ b \rightarrow s \ell \ell \]
\[ \mu/e \text{ universality} \]
Flavour anomalies in semileptonic B-decays:

Combined explanation calls for NP coupled dominantly to 3rd generation
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Combined explanation calls for NP coupled dominantly to 3rd generation

General prediction: huge enhancement of \( b \rightarrow s \tau \tau \) transitions!
Probing $b \rightarrow s\tau\tau$ directly is experimentally very challenging:

\[
\begin{align*}
B^+ &\rightarrow K^+\tau^+\tau^- \\
B_s &\rightarrow \tau^+\tau^- \\
\mathcal{B}_{\text{exp}} &< 2.25 \cdot 10^{-3} \quad \text{[BaBar]} \\
\mathcal{B}_{\text{SM}} & = 1.2 \cdot 10^{-7} \\
\mathcal{B}_{\text{exp}} &< 6.8 \cdot 10^{-3} \quad \text{[LHCb]} \\
\mathcal{B}_{\text{SM}} & = 7.73 \cdot 10^{-7}
\end{align*}
\]
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Lots of data available in $b \to s\mu\mu$. 
Constraining NP in taus...from muons?

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Can we probe \( b \to s\tau\tau \) via its imprint on the \( B^+ \to K^+\mu^+\mu^- \) dimuon spectrum?

...a solid **description** of SM spectrum shape in the full \( q^2 \) range is needed!
EFT description of $b \rightarrow s\ell\ell$

Weak effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i,$$

$$O_9^\ell = \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell) \quad O_{10}^\ell = \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

SM:

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}$$

$$O_1^q = (\bar{s} \gamma_\mu P_L q)(\bar{q} \gamma^\mu P_L b) \quad O_2^q = (\bar{s}^\alpha \gamma_\mu P_L q^\beta)(\bar{q}^\beta \gamma^\mu P_L b^\alpha)$$

NP: $C_i^{\text{SM}} \rightarrow C_i^{\text{SM}} + \delta C_i^{\text{NP}}$ and/or new operators
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Local (short distance)
Weak effective Lagrangian:

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Two ingredients needed:
EFT description of $b \rightarrow s\ell\ell$

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**SM:**

- $C_i^{SM}$

**NP:**

- $C_i^{SM} \rightarrow C_i^{SM} + \delta C_i^{NP}$ and/or new operators

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- $C_i^{SM}(\mu)$
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Local (short distance)

Two ingredients needed:

- $C_i^{SM}(\mu)$
- form factors $f_i(q^2)$ for $B \rightarrow K$
Non-local effects: the charm loop

**Non-local** (long distance) effects arise via 4-quark + chromomagnetic operator. Included via

\[ C_9 \rightarrow C_9^{\text{eff}}(q^2) = C_9 + Y(q^2) \]
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[Semi]perturbative approach valid at low \( q^2 \):

Pert. contribution + expansion in \( \frac{\Lambda_{QCD}^2}{q^2 - 4m_c^2} \)

[Khodjamirian et al., 1212.0234]

cannot be applied in the full kinematical range:

\[ q^2 > m_{J/\psi}^2 \]

\[ q^2 > 4m_D^2 \]
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...intrinsically non perturbative objects!

Goal: model long-distance effects at experiments, in the entire spectrum.
Long-distance effects at experiments
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- Standard approach: *exclude* events close to resonances  [Babar, Belle, CDF, CMS, LHCb…]
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  [Babar, Belle, CDF, CMS, LHCb…]

- LHCb [2016] first fit to full spectrum, including resonances:  
  [Lyon, Zwicky 1406.0566]  
  [LHCb 1612.06764]

\[
Y(q^2) = \sum_{V} \eta_V e^{i\delta_V} A_{V}^{\text{res}}(q^2)
\]

fit parameters  Breit Wigner
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fit parameters Breit Wigner

Why working towards a better parametrisation?
Long-distance effects at experiments

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- access long-distance info unaccessible from first principles [e.g. phases]
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Why working towards a better parametrisation?

- access **long-distance** info unaccessible from first principles [e.g. phases]
- extract reliable **short-distance** info [hence NP!]

Breit Wigner
For the charm we employ a dispersive approach, with subtraction in $q^2 = 0$:

$$\Delta Y_{c\bar{c}}(q^2) = \frac{q^2}{\pi} \int_{s_0}^{\infty} \frac{ds}{s} \frac{\rho_{c\bar{c}}(s)}{(s - q^2)}$$
Charm loops: resonances

For the **charm** we employ a dispersive approach, with subtraction in $q^2 = 0$:

$$\Delta Y_{cc}(q^2) = \frac{q^2}{\pi} \int_{s_0}^{\infty} \frac{ds}{s} \frac{\rho_{cc}(s)}{(s - q^2)}$$

We include single- and two-particle contributions:

$$\rho_{cc}(s) \approx \rho_{cc}^{1P}(s) + \rho_{cc}^{2P}(s)$$
Charm loops: resonances

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$$
\Delta Y_{c\bar{c}}(q^2) = \frac{q^2}{\pi} \int_{s_0}^{\infty} ds \frac{\rho_{c\bar{c}}(s)}{s (s - q^2)}
$$

We include single- and two-particle contributions:

$$
\rho_{c\bar{c}}(s) \approx \rho_{c\bar{c}}^{1P}(s) + \rho_{c\bar{c}}^{2P}(s)
$$

Charmonium resonances:

$$
\Delta Y_{c\bar{c}}^{1P}(q^2) = \sum \eta_V e^{i\delta_V} \frac{q^2}{m_V^2} A_V^{\text{res}}(q^2)
$$

$V = J/\psi, \psi(2S), \psi(3770), \psi(4040), \psi(4160), \psi(4415)$
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Charmonium resonances:

BW, subtracted in $q^2 = 0!$

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Charm loops: two-particle states

Two-particle $\bar{c}c$ states:

$$\Delta Y_{cc}^{2P}(q^2) = \sum_{VV'} \eta_{V'Ve^{i\delta_{V'V}}} A_{VV'}^{2P}(q^2)$$

$$A_{VV'}^{2P}(s) = \frac{s}{\pi} \int_{s_0}^{\infty} \frac{d\tilde{s}}{\tilde{s}} \frac{\rho_{VV}(\tilde{s})}{\tilde{s}(\tilde{s} - s)} ,$$

$$VV' = DD, D^*D^*, DD^*$$
Two-particle $\bar{c}c$ states:

$$\Delta Y^{2P}_{\bar{c}c}(q^2) = \sum_{VV'} \eta_{V'V} e^{i\delta_{V'V}} A^{2P}_{V'V}(q^2)$$

$$A^{2P}_{V'V}(s) = \frac{s}{\pi} \int_{s_0}^{\infty} d\tilde{s} \frac{\rho_{V'V} (\tilde{s})}{\tilde{s} (\tilde{s} - s)} ,$$

$$VV' = DD, D*D*, DD*$$

$$\rho_{V'V}(s) = \text{Im} \left\{ \begin{array}{c} b \\ \mu \end{array} \begin{array}{c} \kappa \\ \mu \end{array} \begin{array}{c} D^{(\omega)} \\ D^{(\omega)} \end{array} \right\} = ?$$
Charm loops: two-particle states

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$VV' = DD, D^*D^*, DD^*$

...estimate from helicity arguments!
Charm loops: two-particle states

Two-particle $\bar{c}c$ states:

$$\Delta Y^{2P}_{\bar{c}c}(q^2) = \sum_{VV'} \eta_{VV'} e^{i\delta_{VV'}} A^{2P}_{VV'}(q^2)$$

$$A^{2P}_{VV'}(s) = \frac{s}{\pi} \int_{s_0}^{\infty} \frac{d\tilde{s}}{\tilde{s}} \frac{\rho_{VV}(\tilde{s})}{\tilde{s} - s},$$

where $VV' = DD, D^*D^*, DD^*$

$$\rho_{VV}(s) = \Im \left\{ \frac{\beta}{\bar{\kappa}} \overline{D}^{(\omega)} \, \overline{D}^{(\omega)} \right\} = \sum_n c^n_{VV'} \beta^n(4m^2_{VV'}/s) \quad \beta(\tau) = \sqrt{1 - \tau}$$

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$$
V V' = DD, D^* D^*, DD^*
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$$

Keeping leading partial wave only: $\rho_{DD} \sim \beta^3$, $\rho_{D^* D^*} \sim \beta^3$, $\rho_{DD^*} \sim \beta$
Charm loops: two-particle states

Two-particle $\bar{c}c$ states:

$$\Delta Y^{2\mathbf{P}}_{c\bar{c}}(q^2) = \sum_{VV'} \eta_{VV'} e^{i\delta_{VV'}} A^{2\mathbf{P}}_{VV'}(q^2) \quad A^{2\mathbf{P}}_{VV'}(s) = \frac{s}{\pi} \int_{s_0}^{\infty} \frac{d\tilde{s}}{\tilde{s}} \frac{\rho_{VV'}(\tilde{s})}{(\tilde{s} - s)} ,$$

$$VV' = DD, D^*D^*, DD^*$$

...estimate from helicity arguments!

$$\rho_{VV}(s) = \text{Im} \left\{ \frac{B}{D^{\omega} \bar{D}^{\omega}} \int_{\mu^2}^{K^2} \frac{dK^2}{K^2} \right\} = \sum_n c^{VV'}_n \beta^n (4m_{VV'}^2/s) \quad \beta(\tau) = \sqrt{1 - \tau}$$

Keeping leading partial wave only: \( \rho_{DD} \sim \beta^3 \), \( \rho_{D^*D^*} \sim \beta^3 \), \( \rho_{DD^*} \sim \beta \)

Constrain fit using perturbative charm loop:

$$Y^{1\mathbf{P}}_{c\bar{c}}(q^2) + Y^{2\mathbf{P}}_{c\bar{c}}(q^2) \approx Y_{c\bar{c}}^{\text{pert}}(q^2) \quad q^2 \ll 4m_c^2$$
Two-particle $\bar{c}c$ states:

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\[V V' = DD, D*D*, DD^*\]

Keeping leading partial wave only: \[\rho_{DD} \sim \beta^3, \rho_{D*D*} \sim \beta^3, \rho_{DD^*} \sim \beta\]

Constrain fit using perturbative charm loop:

\[Y_{c\bar{c}}^{1P}(q^2) + Y_{c\bar{c}}^{2P}(q^2) \approx Y_{c\bar{c}}^{\text{pert}}(q^2) \quad q^2 \ll 4m_c^2\]

Up contribution is CKM suppressed: only resonances included.
Charm loops: two-particle states

\[ \rho_{DD} = \left( 1 - \frac{4m_D^2}{s} \right)^{3/2} \]

\[ \rho_{D^*D} = \left( 1 - \frac{4m_{D^*}^2}{s} \right)^{1/2} \]

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We parametrise hadronic long-distance contributions as:

\[ Y(q^2) = Y_0 + Y_{\text{light}}^{1P}(q^2) + \Delta Y_{c\bar{c}}^{1P}(q^2) + \Delta Y_{c\bar{c}}^{2P}(q^2) \]
Our proposal

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$$Y(q^2) = Y_0 + Y_{\text{light}}^{1P}(q^2) + \Delta Y_{c\bar{c}}^{1P}(q^2) + \Delta Y_{c\bar{c}}^{2P}(q^2)$$

The $q^2$-dependence is fixed by the position of one- and two-particle thresholds.
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The \( q^2 \)-dependence is fixed by the position of one- and two-particle thresholds. Magnitudes and phases are fit parameters (12)!
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What is new?

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Our proposal

We parametrise hadronic long-distance contributions as:

\[ Y(q^2) = Y_0 + Y_{\text{light}}^1(q^2) + \Delta Y_{c\bar{c}}^1(q^2) + \Delta Y_{c\bar{c}}^2(q^2) \]

The \( q^2 \)-dependence is fixed by the position of one- and two-particle thresholds. Magnitudes and phases are fit parameters (12)!

What is new?

- inclusion of two-particle intermediate \( \bar{c}c \) states
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The \( q^2 \)-dependence is fixed by the position of one- and two-particle thresholds. Magnitudes and phases are fit parameters (12)

What is new?

- inclusion of two-particle intermediate \( \bar{c}c \) states
- charm contribution subtracted in \( q^2 = 0 \): \( \Delta Y_{c\bar{c}}^n(0) = 0 \), remainder in \( Y_0 \)
We parametrise hadronic long-distance contributions as:

\[ Y(q^2) = Y_0 + Y_{\text{light}}^{1\text{P}}(q^2) + \Delta Y_{c\bar{c}}^{1\text{P}}(q^2) + \Delta Y_{c\bar{c}}^{2\text{P}}(q^2) \]

The \( q^2 \)-dependence is fixed by the position of one- and two-particle thresholds. Magnitudes and phases are fit parameters (12)!

What is new?

- inclusion of two-particle intermediate \( \bar{c}c \) states
- charm contribution subtracted in \( q^2 = 0: \Delta Y_{c\bar{c}}^{\text{nP}}(0) = 0 \), remainder in \( Y_0 \)
- theory constraints from perturbative results
The tau loop also enters as a $q^2$-dependent shift in $C_9^{\text{eff}}(q^2)$:

$$Y_{\tau\tau}^{2p}(q^2) = -\frac{\alpha}{2\pi} C_9^{\tau} \left[ h_s(m_{\tau}^2, q^2) - \frac{1}{3} h_p(m_{\tau}^2, q^2) \right]$$
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Non-local effect, distinct from mixing between $O_{9}^{\mu}$ and $O_{9}^{\tau}$.

Allows for model independent extraction of $C_{9}^{\tau}$!
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\]

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Characteristic imprint on $B^+ \to K^+\mu^+\mu^-$ spectrum:

- $s$-wave, hence cusp at $q^2 = 4m_\tau^2$
- alter $q^2$ dependence above/below threshold
Tau effects in the spectrum
Tau effects in the spectrum

$$\mathcal{B}(B^+ \rightarrow K^+ \tau^+ \tau^-) = \text{BaBar upper limit}$$

\[ d\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2 \]

- $C_9^\tau = C_9^{\text{SM}}$
- $C_9^\tau = 580$

$q^2 = m_{\mu\mu}^2$
Tau effects in the spectrum

$$\mathcal{B}(B^+ \to K^+ \tau^+ \tau^-) = \text{BaBar upper limit}$$

cusp at $\tau\tau$ threshold
Tau effects in the spectrum

\[ \mathcal{B}(B^+ \rightarrow K^+ \tau^+ \tau^-) = \text{BaBar upper limit} \]

cusp at \( \tau \tau \) threshold

distortion above and below threshold
Preliminary sensitivity and prospects

**Preliminary** sensitivity @ LHCb:
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\[ \mathcal{B}(B^+ \to K^+ \tau^+ \tau^-) \lesssim 8.1 \cdot \mathcal{O}(10^{-4}) \quad @ 95 \% \text{ C.L.} \]

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Future projections, assuming FF uncertainty reduced to 30 %:

\[ \mathcal{B}(B^+ \to K^+\tau^+\tau^-) \lesssim 7.6 \cdot \mathcal{O}(10^{-4}) \quad @ 95 \% \, C.L. \]
Conclusions and outlook

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Coming next: Full fledged fit,
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Coming next: Full fledged fit, possible extension to $B \to K^*\mu^+\mu^-$. 
Thank you!