Status of the theory prediction for g-2

- Introduction
- $a_\mu^{\text{SM}}$: status report/overview of the SM contributions
- $a_\mu^{\text{had VP}}$: status, updates and prospects
- Summary/Outlook
Introduction

- Dirac equation (1928): $g$ is 2 for fundamental fermions
  \[ \vec{\mu} = g \frac{Qe}{2m} \bar{s} \]
- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with $g_s = 2.00229 \pm 0.00008$
  
- **1948: Schwinger** calculates the famous radiative correction:
  that $g = 2 (1 + a)$, with
  \[ a = (g - 2)/2 = \alpha/(2\pi) = 0.001161 \]
  
  This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

  "If you can’t join ‘em, beat ‘em"

- The anomaly $a$ (Anomalous Magnetic Moment) is from the Pauli term:
  \[ \delta \mathcal{L}_{\text{eff}}^{\text{AMM}} = -\frac{Qe}{4m} a \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) F_{\mu\nu}(x) \]
  
  This is a dimension 5 operator, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But it occurs through radiative corrections and is calculable in perturbation theory.
\( a_\mu \): back to the future

- CERN started it nearly 40 years ago
- Brookhaven delivered 0.5ppm precision
- E989 at FNAL and J-PARC’s g-2/EDM experiments are happening and should give us certainty

\[ (a_\mu - 11659000) \times 10^{-10} \]

\( g-2 \) history plot and motto from Fred Jegerlehner’s book:

‘The closer you look the more there is to see’
\[ a_\mu = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{hadronic}} + a_{\mu}^{\text{NP}}? \]

- if mean values stay and with no \( a_{\mu}^{\text{SM}} \) improvement: 5σ discrepancy

- if also EXP+TH can improve \( a_{\mu}^{\text{SM}} \) `as expected’ (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8σ

- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...

“Muon g-2 Theory Initiative” formed in June 2017

Group photo from the Seattle workshop in September 2019, https://indico.fnal.gov/event/21626/

“map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental results”
Kinoshita et al.: $g-2$ at 1, 2, 3, 4 & 5-loop order

A triumph for perturbative QFT and computing!

T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio (PRLs, 2012)

- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

$10^\text{th}$

12672

diagrams
\( a_\mu \) QED

- **Schwinger 1948**: 1-loop \( a = (g-2)/2 = \alpha/(2\pi) = 116\ 140\ 970 \times 10^{-11} \)

- 2-loop graphs:

- 72 3-loop and 891 4-loop diagrams ...

- **Kinoshita et al. 2012**: 5-loop completed numerically (12672 diagrams):

\[
a_\mu^{\text{QED}} = 116\ 584\ 718.951 (0.009) (0.019) (0.007) (0.077) \times 10^{-11}
\]

errors from: lepton masses, 4-loop, 5-loop, \( \alpha \) from \(^{87}\text{Rb}\)

- QED extremely accurate, and the series is stable:

\[
a_\mu^{\text{QED}} = C_\mu^{2n} \sum_n \left( \frac{\alpha}{\pi} \right)^n
\]

\( C_\mu^{2,4,6,8,10} = 0.5, 0.765857425(17), 24.05050996(32), 130.8796(63), 753.29(1.04) \)

contr. to \( a_\mu \approx 1 \times 10^{-3}, \quad 4 \times 10^{-6}, \quad 3 \times 10^{-7}, \quad 4 \times 10^{-9}, \quad 5 \times 10^{-11} \)

- Could \( a_\mu^{\text{QED}} \) still be wrong?

Some classes of graphs known analytically (Laporta; Aguilar, Greynat, deRafael),

- but 4-loop and 5-loop rely heavily on numerical integrations

Recently several independent checks of 4-loop and 5-loop diagrams:


- all 4-loop graphs with internal lepton loops now calculated independently, e.g.

- 4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, PLB772(2017)232, also recent numerical results by Volkov, PRD96(2017)096018

- all agree with Kinoshita et al.’s results, recent changes in 5-loop numerics irrelevant for μ, [PRD97 (2018) 036001]

so QED is on safe ground ✓
Electro-Weak

- Electro-Weak 1-loop diagrams:

  \[
  a_{\mu}^{\text{EW}(1)} = 195 \times 10^{-11}
  \]

- known to 2-loop (1650 diagrams, the first full EW 2-loop calculation): Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

  - agreement, \( a_{\mu}^{\text{EW}} \) relatively small, 2-loop relevant: \( a_{\mu}^{\text{EW}(1+2 \text{loop})} = (154 \pm 2) \times 10^{-11} \)
  - with Higgs mass now known, updated by Gnendiger, Stoeckinger, S-Kim, PRD 88 (2013) 053005

  \[
  a_{\mu}^{\text{EW}(1+2 \text{loop})} = (153.6 \pm 1.0) \times 10^{-11}
  \]

- Recently new numerical 2-loop EW result, based on GRACE-FORM packages, avoiding the heavy mass expansion used previously: Ishikawa, Nakazawa, Yasui, PRD 99 (2019) 073004

\[ \leftrightarrow \text{weak 2-loop: } -41.2 \ (1.0) \Rightarrow (-38.6 \pm 1.0) \times 10^{-11} \text{, i.e. shift up of EW by < 2%} \]

Compare with \( a_{\mu}^{\text{QED}} = 116 \ 584 \ 718.951 \ (80) \times 10^{-11} \)
\( a_\mu \text{ hadronic} \)

- **Hadronic:** non-perturbative, the limiting factor of the SM prediction?  \( X \rightarrow \checkmark \)

\[
a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}
\]

\( \mu \) \( \text{had.} \) \( \text{LO} \) \( \text{NLO} \) \( \text{L-by-L} \)
\( a_\mu \text{ had, L-by-L: Light-by-Light} \)

- **L-by-L**: \( \gamma \rightarrow \text{hadrons} \rightarrow \gamma^* \gamma^* \gamma^* \) non-perturbative, impossible to fully measure \( \times \)

- so far use of *model calculations*, based on large \( N_c \) limit, Chiral Perturbation Theory, plus *short distance constraints* from OPE and pQCD

- *meson exchanges* and *loops* modified by form factor suppression, but with limited experimental information:
  - in principle off-shell form-factors (\( \pi^0, \eta, \eta', 2\pi \rightarrow \gamma^* \gamma^* \)) needed
  - at most possible, directly experimentally: \( \pi^0, \eta, \eta', 2\pi \rightarrow \gamma \gamma^* \)

- additional quark loop, pQCD matching; theory not fully satisfying conceptually \( \odot \)

- several independent evaluations, different in details, but *good agreement for the leading \( N_c \) (\( \pi^0 \) exchange) contribution*, differences in sub-leading bits

- mostly used so far:
  - `Glasgow consensus’ (>10 years old) by Prades+deRafael+Vainshtein:
    \[
    a_\mu^{\text{had, L-by-L}} = (105 \pm 26) \times 10^{-11}
    \]
  - compatible with A. Nyffeler’s
    \[
    a_\mu^{\text{had, L-by-L}} = (116 \pm 39) \times 10^{-11}
    \]
\( a_\mu \) had, L-by-L: **Light-by-Light Prospects**

- **Transition FFs** can be measured by KLOE-2 and BESIII using small angle taggers:
  \[ e^+e^- \rightarrow e^+e^-\gamma\gamma^* \rightarrow \pi^0, \eta, \eta', 2\pi \]
  \( \) expected to constrain leading pole contributions from \( \pi, \eta, \eta' \) to ~ 15% Nyffeler, PRD94, 053006
  or calculate on the lattice: \( \pi^0 \rightarrow \gamma^*\gamma^* \) Gerardin, Meyer, Nyffeler, PRD94, 074507

- 1. **Breakthrough with new dispersive approaches**
  - dispersion relations formulated for the general HLbL tensor or for \( a_\mu \) directly
  - allowing to constrain/calculate the HLbL contributions from data
  - e.g. Colangelo et al. have precise results for the \( \pi \)-box contribution from data for \( F^\pi_\gamma (q^2) \),
  - and now for the most important pion-pole contribution, Hoferichter et al., JHEP 1704 (2017) 161

- 2. Ultimately ‘first principles’ full prediction from lattice QCD+QED
  - several groups: Mainz, RBC-UKQCD, ... much increased effort
  - within few years a 10% estimate probably possible, 30% already useful

  *First results very encouraging,* now hunt down errors/systematics:
  need to extrapolate to continuum and infinite volume,
  need to fully take into account disconnected contributions
Hadronic Light-by-Light: dispersive approach

For HVP \[ \Rightarrow 2 \text{Im} \pi_{\text{had}} \equiv \sum_{\text{had.}} \int d\Phi \left| \pi_{\text{had}} \right|^2 \Rightarrow \text{Im}\Pi_{\text{had}}(s) = \left( \frac{s}{4\pi\alpha} \right)\sigma_{\text{had}}(s) \]

For HLbL \[ \Rightarrow \Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\text{pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma} + \ldots \]

\[ \Rightarrow \text{Dominated by pole (pseudoscalar exchange) contributions} \]

\[ \Rightarrow \Pi_{\mu\nu\lambda\sigma}^{\text{pole}} = \]

\[ \Rightarrow \text{Sum all possible diagrams to get } a_{\mu}^{\text{HLbL}} \]

- Recent review by Danilkin+Redmer+Vanderhaeghen using dispersive techniques estimates \((8.7 \pm 1.3) \times 10^{-10}\) [Prog. Part. Nucl. Phys. 107 (2019) 20]

- With new results & progress, message to sceptics/politicians: L-by-L _can_ be reliably predicted! ✓
Summary of HLbL (as of May ’19, very preliminary!)

Contributions to $10^{11} \cdot a^\text{HLbL}_\mu$

- Pseudoscalar poles $= 93.8^{+4.0}_{-3.6}$
- Pion box $(\text{kaon box} \sim -0.5) = -15.9(2)$
- S-wave $\pi\pi$ rescattering $= -8(1)$
- Scalars and tensors with $M_R > 1$ GeV $\sim -2(3)$
- Axial vectors $\sim 8(3)$
- Short-distance contribution $\sim 10(10)$

Central value: $85 \pm XX$
Uncertainties added in quadrature: $XX = 12$
Uncertainties added linearly: $XX = 21$
\[ a_\mu^{\text{had, VP}} = a_\mu^{\text{had, VP LO}} + a_\mu^{\text{had, VP NLO}} + a_\mu^{\text{had, Light–by–Light}} \]

**HVP:** - most precise prediction by using \( e^+e^- \) hadronic cross section (+ tau) data and well known dispersion integrals

- done at LO and NLO (see graphs)

- and also at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]
  \[ a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10} \] not so small, from e.g.:

  Lots of activity by several groups, errors coming down, see lattice talks.
Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:

\[
\text{pQCD not useful. Use the dispersion relation and the optical theorem.}
\]

\[
\sim = \int \frac{ds}{\pi(s-q^2)} \text{Im} \sim \sim \quad \text{had.}
\]

2 \text{Im} \sim \sim = \sum \text{had.} \int d\Phi | \sim \sim |^2

\[
a_{\mu}^{\text{had},\text{LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s)\sigma_{\text{had}}(s)
\]

- Weight function \( \hat{K}(s)/s = O(1)/s \)
  \( \implies \) Lower energies more important
  \( \implies \pi^+\pi^- \) channel: 73% of total \( a_{\mu}^{\text{had},\text{LO}} \)

How to get the most precise \( \sigma_{\text{had}}^0 \) e^+e^- data:

- Low energies: sum \( \sim 35 \) exclusive channels, \( 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, KK, KK\pi, KK\pi\pi, \eta\pi, \ldots \), [use iso-spin relations for missing channels]

- Above \( \sim 1.8 \) GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of data combination (locally in \( \sqrt{s} \)): many experiments, different energy bins, stat+sys errors from different sources, correlations; must avoid inconsistencies/bias

- traditional \`direct scan\' (tunable e^+e^- beams) vs. \`Radiative Return\' [+ τ spectral functions]

- \( \sigma_{\text{had}}^0 \) means \`bare\' \( \sigma \), but WITH FSR: RadCorrs

[ KNT 18: \( \delta a_{\mu}^{\text{had}, \text{RadCor VP+FSR}} \sim 0.8 \times 10^{-10} \) ]
<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy range [GeV]</th>
<th>$\omega^{\text{hadLOVP}}_{\gamma} \times 10^{10}$</th>
<th>$\Delta_{\text{had}}^{5}(M_{T}^{2}) \times 10^{4}$</th>
<th>New data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{0}\gamma$</td>
<td>$m_{\pi} \leq \sqrt{s} \leq 0.600$</td>
<td>0.12 ± 0.01</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\pi^{0}\pi^{-}$</td>
<td>$2m_{\pi} \leq \sqrt{s} \leq 0.305$</td>
<td>0.87 ± 0.02</td>
<td>0.01 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\pi^{+}\pi^{-}\pi^{0}$</td>
<td>$3m_{\pi} \leq \sqrt{s} \leq 0.660$</td>
<td>0.01 ± 0.00</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\eta\gamma$</td>
<td>$m_{\eta} \leq \sqrt{s} \leq 0.660$</td>
<td>0.00 ± 0.00</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
</tbody>
</table>

Chiral perturbation theory (ChPT) threshold contributions

Data based channels ($\sqrt{s} \leq 1.937$ GeV)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy range [GeV]</th>
<th>$\omega^{\text{hadLOVP}}_{\gamma} \times 10^{10}$</th>
<th>$\Delta_{\text{had}}^{5}(M_{T}^{2}) \times 10^{4}$</th>
<th>New data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{0}\gamma$</td>
<td>$0.600 \leq \sqrt{s} \leq 1.350$</td>
<td>4.46 ± 0.10</td>
<td>0.36 ± 0.01</td>
<td>[65]</td>
</tr>
<tr>
<td>$\pi^{0}\pi^{-}$</td>
<td>$0.305 \leq \sqrt{s} \leq 1.937$</td>
<td>502.97 ± 1.97</td>
<td>34.26 ± 0.12</td>
<td>[34,35]</td>
</tr>
<tr>
<td>$\pi^{+}\pi^{-}\pi^{0}$</td>
<td>$0.660 \leq \sqrt{s} \leq 1.937$</td>
<td>47.79 ± 0.89</td>
<td>4.77 ± 0.08</td>
<td>[36]</td>
</tr>
<tr>
<td>$\pi^{+}\pi^{-}\pi^{0}$</td>
<td>$0.613 \leq \sqrt{s} \leq 1.937$</td>
<td>14.87 ± 0.20</td>
<td>4.02 ± 0.05</td>
<td>[40,42]</td>
</tr>
<tr>
<td>$\pi^{+}\pi^{-}\pi^{0}$</td>
<td>$0.850 \leq \sqrt{s} \leq 1.937$</td>
<td>19.39 ± 0.78</td>
<td>5.00 ± 0.20</td>
<td>[44]</td>
</tr>
<tr>
<td>$(2\pi^{+}\pi^{-}\pi^{0})_{\text{non}}$</td>
<td>$1.013 \leq \sqrt{s} \leq 1.937$</td>
<td>0.99 ± 0.09</td>
<td>0.33 ± 0.03</td>
<td>...</td>
</tr>
<tr>
<td>$3\pi^{+}3\pi^{-}$</td>
<td>$1.131 \leq \sqrt{s} \leq 1.937$</td>
<td>0.23 ± 0.01</td>
<td>0.09 ± 0.01</td>
<td>[66]</td>
</tr>
<tr>
<td>$(2\pi^{+}2\pi^{-}\pi^{0})_{\text{non}}$</td>
<td>$1.322 \leq \sqrt{s} \leq 1.937$</td>
<td>1.35 ± 0.17</td>
<td>0.51 ± 0.06</td>
<td>...</td>
</tr>
<tr>
<td>$K^{+}K^{-}$</td>
<td>$0.988 \leq \sqrt{s} \leq 1.937$</td>
<td>23.03 ± 0.22</td>
<td>3.37 ± 0.03</td>
<td>[45,46,49]</td>
</tr>
<tr>
<td>$K_{L}^{0}K_{L}^{0}$</td>
<td>$1.004 \leq \sqrt{s} \leq 1.937$</td>
<td>13.04 ± 0.19</td>
<td>1.77 ± 0.03</td>
<td>[50,51]</td>
</tr>
<tr>
<td>$KK\pi$</td>
<td>$1.260 \leq \sqrt{s} \leq 1.937$</td>
<td>2.71 ± 0.12</td>
<td>0.89 ± 0.04</td>
<td>[53,54]</td>
</tr>
<tr>
<td>$KK\pi\pi$</td>
<td>$1.350 \leq \sqrt{s} \leq 1.937$</td>
<td>1.93 ± 0.08</td>
<td>0.75 ± 0.03</td>
<td>[50,53,55]</td>
</tr>
<tr>
<td>$\eta\gamma$</td>
<td>$m_{\eta} \leq \sqrt{s} \leq 1.937$</td>
<td>0.70 ± 0.02</td>
<td>0.09 ± 0.00</td>
<td>[67]</td>
</tr>
<tr>
<td>$\eta\pi^{+}\pi^{-}$</td>
<td>$1.091 \leq \sqrt{s} \leq 1.937$</td>
<td>1.29 ± 0.06</td>
<td>0.39 ± 0.02</td>
<td>[68,69]</td>
</tr>
<tr>
<td>$(\eta\pi^{+}\pi^{-}\pi^{0})_{\text{non}}$</td>
<td>$1.333 \leq \sqrt{s} \leq 1.937$</td>
<td>0.60 ± 0.15</td>
<td>0.21 ± 0.05</td>
<td>[70]</td>
</tr>
<tr>
<td>$\eta\pi^{0}$</td>
<td>$1.333 \leq \sqrt{s} \leq 1.937$</td>
<td>0.31 ± 0.03</td>
<td>0.10 ± 0.01</td>
<td>[70,71]</td>
</tr>
<tr>
<td>$\omega(\rightarrow \pi^{0}\gamma)\pi^{0}$</td>
<td>$0.920 \leq \sqrt{s} \leq 1.937$</td>
<td>0.88 ± 0.02</td>
<td>0.19 ± 0.00</td>
<td>[72,73]</td>
</tr>
<tr>
<td>$\eta\phi$</td>
<td>$1.569 \leq \sqrt{s} \leq 1.937$</td>
<td>0.42 ± 0.03</td>
<td>0.15 ± 0.01</td>
<td>...</td>
</tr>
<tr>
<td>$\eta(\rightarrow \text{npp})K_{\text{non}}\phi\rightarrow KK$</td>
<td>$1.569 \leq \sqrt{s} \leq 1.937$</td>
<td>0.01 ± 0.02</td>
<td>0.00 ± 0.01</td>
<td>[53,75]</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>$1.890 \leq \sqrt{s} \leq 1.937$</td>
<td>0.03 ± 0.00</td>
<td>0.01 ± 0.00</td>
<td>[76]</td>
</tr>
<tr>
<td>$n\bar{n}$</td>
<td>$1.912 \leq \sqrt{s} \leq 1.937$</td>
<td>0.03 ± 0.01</td>
<td>0.01 ± 0.00</td>
<td>[77]</td>
</tr>
</tbody>
</table>

Estimated contributions ($\sqrt{s} \leq 1.937$ GeV)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy range [GeV]</th>
<th>$\omega^{\text{hadLOVP}}_{\gamma} \times 10^{10}$</th>
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<th>New data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\pi^{+}\pi^{-}3\pi^{0})_{\text{non}}$</td>
<td>$1.013 \leq \sqrt{s} \leq 1.937$</td>
<td>0.50 ± 0.04</td>
<td>0.16 ± 0.01</td>
<td>...</td>
</tr>
<tr>
<td>$(\pi^{+}\pi^{-}4\pi^{0})_{\text{non}}$</td>
<td>$1.313 \leq \sqrt{s} \leq 1.937$</td>
<td>0.21 ± 0.21</td>
<td>0.08 ± 0.08</td>
<td>...</td>
</tr>
<tr>
<td>$KK3\pi$</td>
<td>$1.569 \leq \sqrt{s} \leq 1.937$</td>
<td>0.03 ± 0.02</td>
<td>0.02 ± 0.01</td>
<td>...</td>
</tr>
<tr>
<td>$\omega(\rightarrow \text{nn})2\pi$</td>
<td>$1.285 \leq \sqrt{s} \leq 1.937$</td>
<td>0.10 ± 0.02</td>
<td>0.03 ± 0.01</td>
<td>...</td>
</tr>
<tr>
<td>$\omega(\rightarrow \text{nn})3\pi$</td>
<td>$1.322 \leq \sqrt{s} \leq 1.937$</td>
<td>0.17 ± 0.03</td>
<td>0.06 ± 0.01</td>
<td>...</td>
</tr>
<tr>
<td>$\omega(\rightarrow \text{nn})KK$</td>
<td>$1.569 \leq \sqrt{s} \leq 1.937$</td>
<td>0.00 ± 0.00</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\eta\pi^{+}\pi^{-}2\pi^{0}$</td>
<td>$1.338 \leq \sqrt{s} \leq 1.937$</td>
<td>0.08 ± 0.04</td>
<td>0.03 ± 0.02</td>
<td>...</td>
</tr>
</tbody>
</table>

Other contributions ($\sqrt{s} > 1.937$ GeV)

| Inclusive channel                  | $m_{\pi} \leq \sqrt{s} \leq 11.999$ | 43.67 ± 0.67                     | 82.82 ± 1.05                     | [56,62,63]|
| $J/\psi$                            | ...                                  | 6.26 ± 0.19                     | 7.07 ± 0.22                     | ...       |
| $\psi'$                             | ...                                  | 1.58 ± 0.04                     | 2.51 ± 0.06                     | ...       |
| $\Upsilon(1S - 4S)$                  | ...                                  | 0.09 ± 0.00                     | 1.06 ± 0.02                     | ...       |
| pQCD                                | $11.199 \leq \sqrt{s} \leq \infty$  | 2.07 ± 0.00                     | 124.79 ± 1.10                   | ...       |
| Total                               | $m_{\pi} \leq \sqrt{s} \leq \infty$ | 693.26 ± 2.46                   | 276.11 ± 1.11                   | ...       |
HVP cross section input

\[ a_{\mu, \text{LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} \frac{R(s)K(s)}{s}, \text{ where } R(s) = \frac{\sigma_{\text{had}, \gamma}^0(s)}{4\pi\alpha^2/3s} \]

Non-perturbative
(Experimental data, isopsin, ChPT...)

Non-perturbative/perturbative
(Experimental data, pQCD, Breit-Wigner...)

Perturbative
(pQCD)

Must build full hadronic cross section/\( R \)-ratio...
HVP: $\pi^+\pi^-$ channel [KNT18, PRD97, 114025]

$\pi^+\pi^-$ accounts for over 70% of $a^\text{had, LO VP}_\mu$

→ Combines 30 measurements totalling nearly 1000 data points

$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$a^+_{\mu} [0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 502.97 \pm 1.14_{\text{stat}} \pm 1.59_{\text{sys}} \pm 0.06_{\text{vp}} \pm 0.14_{\text{fsr}}$

$= 502.97 \pm 1.97_{\text{tot}}$ HLMNT11: 505.77 ± 3.09

$\Rightarrow$ 15% local $\chi^2_{\text{min}}$/d.o.f. error inflation due to tensions in clustered data
\( \Rightarrow \) Tension exists between BaBar data and all other data in the dominant \( \rho \) region.

\( \rightarrow \) Agreement between other radiative return measurements and direct scan data largely compensates this.

**BaBar data alone** \( \Rightarrow a_{\mu}^{\pi^+\pi^-} \) (BaBar data only) \( = 513.2 \pm 3.8 \).

Simple weighted average of all data \( \Rightarrow a_{\mu}^{\pi^+\pi^-} \) (Weighted average) \( = 509.1 \pm 2.9 \).

(i.e. - no correlations in determination of mean value)

**BaBar data dominate when no correlations are taken into account** for the mean value

**Highlights** importance of fully incorporating all available correlated uncertainties.
HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006,
Hoferichter+Hoid+Kubis, arXiv:1907.01556

- Comprehensive dispersive study of the $2\pi$ vector form factor, including space-like data and phase shift analysis, also for $\pi^+\pi^-\pi^0$ channel
- leading to stronger constraints compared to pure direct data fit and integration.
- For good fit quality, energy calibration for narrow resonances crucial.

**Figure 9:** Fit result for the pion VFF in the space-like region, together with the NA7 data.

![Fit result for the VFF $|F^V_\pi(s)|^2$](image)
HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006

- Detailed analysis and comparison with other work on a region-by-region basis,
  -> allows improved understanding of differences between different groups
  -> important input, also for Theory Initiative White Paper

**Figure 11:** Fit result for the pion VFF in the $\pi^-\pi^-$ interference region, together with the $e^+e^-$ data sets. The curve is the result of the VFF fit to the data points including energy rescaling as shown in Fig. 10, but with an $\pi^-$ mass reset to the PDG value and compared to the original data points without energy rescaling.

**Figure 12:** Relative difference between the data points (including the energy rescaling (4.10)) and the fit result for $|F_\pi^\gamma(s)|^2$. As in all plots, we show fit errors and total uncertainties as two separate error bands. The total uncertainty is given by the fit error and the systematic uncertainty, added in quadrature.
DHMZ: arXiv:1908.00921

- Add latest data. Use fit, based on analyticity & unitarity, similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraint/lower errors at low energies.
- For $2\pi$, based on difference between result w/out KLOE and BaBar, sizeable additional sys. error is applied and mean value adjusted.
**HVP: KK channels**  
[KNT18, PRD97, 114025]

### Results

Results from individual channels

- $K^+ K^-$
  - [KNT18: arXiv:1802.02995]
- $K_S K_L$

New data:


\[
a_{\mu}^{K^+ K^-} = 23.03 \pm 0.22_{\text{tot}}
\]

HLMNT11: $22.15 \pm 0.46_{\text{tot}}$

Large increase in mean value

\[
a_{\mu}^{K_S K_L} = 13.04 \pm 0.19_{\text{tot}}
\]

HLMNT11: $13.33 \pm 0.16_{\text{tot}}$

Large changes due to new precise measurements on $\phi$
HVP: $\sigma_{\text{had}}$ channels below 2 GeV [KNT18, PRD97, 114025]

$\rightarrow$ Dominance of $2\pi$ below 0.9 GeV evident for both cross section and uncertainty

$\rightarrow$ Large improvement to cross section and uncertainty from new $4\pi$ data
**HVP: $\sigma_{\text{had}}$ inclusive region [KNT18]**


**KEDR data improves the inclusive data combination below $c\bar{c}$ threshold**

$\Rightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$\alpha^\text{Inclusive}_\mu = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$
**HVP: KNT18 total and comparison w. other work**

### Results

HLMNT(11): $694.91 \pm 4.27$

This work: \( a^\text{had, LO VP}_\mu = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}} \)

\( = 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}} \)

\( = 693.27 \pm 2.46_{\text{tot}} \)

\( a^\text{had, NLO VP}_\mu = -9.82 \pm 0.04_{\text{tot}} \)

\( \Rightarrow \text{Accuracy better then } 0.4\% \)

(uncertainties include all available correlations)

\[ \begin{array}{cccc}
\text{DEHZ03:} & 696.3 \pm 7.2 \\
\text{HMNT03:} & 692.4 \pm 6.4 \\
\text{DEHZ06:} & 690.9 \pm 4.4 \\
\text{HMNT06:} & 689.4 \pm 4.6 \\
\text{FJ06:} & 692.1 \pm 5.6 \\
\text{DHMZ10:} & 692.3 \pm 4.2 \\
\text{JS11:} & 690.8 \pm 4.7 \\
\text{HLMNT11:} & 694.9 \pm 4.3 \\
\text{FJ17:} & 688.1 \pm 4.1 \\
\text{DHMZ17:} & 693.1 \pm 3.4 \\
\text{KNT18:} & 693.3 \pm 2.5 \\
\end{array} \]

\( \Rightarrow 2\pi \text{ dominance} \)
KNT18 $\alpha^\text{SM}_\mu$ update

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th></th>
<th>2017</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>QED</strong></td>
<td>11658471.81 (0.02)</td>
<td>$\rightarrow$</td>
<td>11658471.90 (0.01)</td>
<td>[arXiv:1712.06060]</td>
</tr>
<tr>
<td><strong>EW</strong></td>
<td>15.40 (0.20)</td>
<td>$\rightarrow$</td>
<td>15.36 (0.10)</td>
<td>[Phys. Rev. D 88 (2013) 053005]</td>
</tr>
<tr>
<td><strong>LO HLbL</strong></td>
<td>10.50 (2.60)</td>
<td>$\rightarrow$</td>
<td>9.80 (2.60)</td>
<td>[EPJ Web Conf. 118 (2016) 01016]</td>
</tr>
<tr>
<td><strong>NLO HLbL</strong></td>
<td>0.30 (0.20)</td>
<td>$\rightarrow$</td>
<td></td>
<td>[Phys. Lett. B 735 (2014) 90]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>HLMNT11</strong></th>
<th></th>
<th></th>
<th><strong>KNT18</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LO HVP</strong></td>
<td>694.91 (4.27)</td>
<td>$\rightarrow$</td>
<td>693.27 (2.46)</td>
<td>this work</td>
</tr>
<tr>
<td><strong>NLO HVP</strong></td>
<td>-9.84 (0.07)</td>
<td>$\rightarrow$</td>
<td>-9.82 (0.04)</td>
<td>this work</td>
</tr>
<tr>
<td><strong>NNLO HVP</strong></td>
<td></td>
<td></td>
<td>1.24 (0.01)</td>
<td>[Phys. Lett. B 734 (2014) 144]</td>
</tr>
</tbody>
</table>

| **Theory total**    | 11659182.80 (4.94)    | $\rightarrow$ | 11659182.05 (3.56) | this work             |
| **Experiment**      |                       |       | 11659209.10 (6.33) | world avg             |
| **Exp - Theory**    | 26.1 (8.0)            | $\rightarrow$ | 27.1 (7.3) | this work             |

| $\Delta \alpha_\mu$ | 3.3$\sigma$ | $\rightarrow$ | 3.7$\sigma$ | this work             |
Results

KNT18 update

KNT18

\[\mu\]

[arXiv:1802.02995]

Alex Keshavarzi

BNL (x4 accuracy)

3.7\(\sigma\)

7.0\(\sigma\)

7\(\sigma\) if E989 obtains same mean value with projected improvement in error
HVP: new developments from other groups

DHMZ: arXiv:1908.00921

• The resulting mean value is similar, for total LOHVP in \((10^{-10})\): \(693.9\pm4.0\) [vs. KNT’s \(693.3\pm2.5\)] but this description inflated the error beyond what a local error inflation of a combined fit does.
• Adding all contributions, they then quote \(3.3\sigma\).
Summary/Outlook:

• The still unresolved muon $g$-2 discrepancy is consolidated at about 3 - 4 $\sigma$ and has triggered new experiments and a lot of theory activities

• More hadronic data expected;
  - in the $2\pi$ channel from BaBar, CMD-3, SND,
  - in subleading channels, $3\pi$, $4\pi$, KK
  - in the inclusive region from BES III and KEDR,
  - and BELLE II will be able to contribute with many ISR measurements

• Longer term: lattice + a direct measurement in the space-like: MUonE

• The dominant uncertainties from the hadronic contributions will be further squeezed, and L-by-L is becoming the bottleneck, but a lot of progress (new data driven approaches + lattice) is expected for the next few years

• The Muon g-2 Theory Initiative is planning to publish a White Paper by end of this year, aiming at presenting the best (`agreed’, conservative) SM prediction of $g$-2
  $\rightarrow$ SM theory is (nearly) ready for the next round of experimental results
Thank you.
HVP: new developments from KNT

KNT re-analysis 2019: $\pi^+ \pi^-$ channel

$\alpha_{\mu}^{\pi^+\pi^-}(\sqrt{s} \leq 1.937 \text{ GeV}) = (503.46 \pm 1.90) \times 10^{-10}$

Global $\chi^2_{\text{min}}/\text{d.o.f} = 1.26$


KNT18: $\alpha_{\mu}(\pi^+\pi^-) = (503.11 \pm 1.97) \times 10^{-10}$

KNT19 (preliminary): $\alpha_{\mu}(\pi^+\pi^-) = (503.46 \pm 1.90) \times 10^{-10}$
HVP: new developments from KNT

KNT re-analysis 2019: $\pi^+ \pi^-$ channel

- Fit of all $\pi^+ \pi^-$ data: $368.84 \pm 1.30$
- Direct scan only: $370.77 \pm 2.61$
- KLOE combination: $366.88 \pm 2.15$
- BaBar (09): $376.71 \pm 2.72$
- BESIII (15): $368.15 \pm 4.22$
- CLEO-c (17): $376.69 \pm 7.05$

$\alpha_{\mu}^{\pi^+ \pi^-}(0.6 \leq \sqrt{s} \leq 0.9 \text{ GeV}) \times 10^{-10}$
Adding up all the channels, pQCD & narrow resonances contributions, we get

\[
\begin{align*}
\alpha_{\mu,\text{LO VP}}^{\text{had}} \text{(KNT19 prelim.)} &= (693.8 \pm 2.4) \times 10^{-10} \\
\alpha_{\mu,\text{NLO VP}}^{\text{had}} \text{(KNT19 prelim.)} &= (-9.83 \pm 0.04) \times 10^{-10}
\end{align*}
\]

(KNT18: \((693.3 \pm 2.5) \times 10^{-10}\))

(KNT18: \((-9.82 \pm 0.04) \times 10^{-10}\))
Outlook: prel. news from SND (1) [~15 procs. under analysis]

V. Druzhinin, EPS 2019

\[ e^+ e^- \rightarrow \pi^+ \pi^- \]

Systematic uncertainty on the cross section (%)

<table>
<thead>
<tr>
<th>Source</th>
<th>&lt; 0.6 GeV</th>
<th>0.6 - 0.9 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Selection criteria</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>(e/\pi) separation</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Nucl. interaction</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Theory</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.9</strong></td>
<td><strong>0.8</strong></td>
</tr>
</tbody>
</table>

The analysis is based on 4.7 pb\(^{-1}\) data recorded in 2013, ~1/10 full SND data set.

V. Druzhinin, EPS HEP 2019
Outlook: prel. news from SND (2) Will this become a mediator?

\[ e^+ e^- \rightarrow \pi^+ \pi^- \]

V. Druzhinin, EPS 2019

0.53 < \sqrt{s} < 0.88 \text{ GeV}

<table>
<thead>
<tr>
<th></th>
<th>( a_\mu(\pi^+ \pi^-) \times 10^{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SND &amp; VEPP-2000</td>
<td>411.8 \pm 1.0 \pm 3.7</td>
</tr>
<tr>
<td>SND &amp; VEPP-2M</td>
<td>408.9 \pm 1.3 \pm 5.3</td>
</tr>
<tr>
<td>BABAR</td>
<td>414.9 \pm 0.3 \pm 2.1</td>
</tr>
</tbody>
</table>

V. Druzhinin EPS HEP 2019
Comparison with other similar works

<table>
<thead>
<tr>
<th>Channel</th>
<th>This work (KNT18)</th>
<th>DHMZ17</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>503.74 ± 1.96</td>
<td>507.14 ± 2.58</td>
<td>-3.40</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>47.70 ± 0.89</td>
<td>46.20 ± 1.45</td>
<td>1.50</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>13.99 ± 0.19</td>
<td>13.68 ± 0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0\pi^0$</td>
<td>18.15 ± 0.74</td>
<td>18.03 ± 0.54</td>
<td>0.12</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>23.00 ± 0.22</td>
<td>22.81 ± 0.41</td>
<td>0.19</td>
</tr>
<tr>
<td>$K^0_SK^0_L$</td>
<td>13.04 ± 0.19</td>
<td>12.82 ± 0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>$1.8 \leq \sqrt{s} \leq 3.7$ GeV</td>
<td>34.54 ± 0.56 (data)</td>
<td>33.45 ± 0.65 (pQCD)</td>
<td>1.09</td>
</tr>
<tr>
<td>Total</td>
<td>693.3 ± 2.5</td>
<td>693.1 ± 3.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

⇒ Total estimates from two analyses in very good agreement
⇒ Masks much larger differences in the estimates from individual channels
⇒ Unexpected tension for $2\pi$ considering the data input likely to be similar
  → Points to marked differences in way data are combined
  → From $2\pi$ discussion: $a_{\mu}^{\pi^+\pi^-} (\text{Weighted average}) = 509.1 \pm 2.9$
⇒ Compensated by lower estimates in other channels
  → For example, the choice to use pQCD instead of data above 1.8 GeV
⇒ FJ17: $a_{\mu}^{\text{had},\text{LO VP}, FJ17} = 688.07 \pm 41.4$
  → Much lower mean value, but in agreement within errors
Benayoun+DelBuono+Jegerlehner: arXiv:1903.11034

- New analysis using effective theory based on (broken) Hidden Local Symmetry

<table>
<thead>
<tr>
<th>Channel</th>
<th>BHLS</th>
<th>BLHS$_2$ (BS) excl. $\tau$</th>
<th>BLHS$_2$ (RS) incl. $\tau$</th>
<th>BLHS$_2$ (RS) excl. $\tau$</th>
<th>Exp. Value.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>493.73 ± 0.70</td>
<td>494.52 ± 0.92</td>
<td>494.51 ± 0.83</td>
<td>494.50 ± 1.04</td>
<td>497.82 ± 2.80</td>
</tr>
<tr>
<td>$\pi^0\gamma$</td>
<td>4.42 ± 0.03</td>
<td>4.48 ± 0.03</td>
<td>4.42 ± 0.03</td>
<td>4.42 ± 0.03</td>
<td>3.47 ± 0.11</td>
</tr>
<tr>
<td>$\eta\gamma$</td>
<td>0.63 ± 0.01</td>
<td>0.63 ± 0.01</td>
<td>0.64 ± 0.01</td>
<td>0.64 ± 0.01</td>
<td>0.55 ± 0.02</td>
</tr>
<tr>
<td>$\pi^+\pi^0$</td>
<td>42.56 ± 0.54</td>
<td>43.03 ± 0.55</td>
<td>42.97 ± 0.55</td>
<td>43.12 ± 0.50</td>
<td>41.38 ± 1.28</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>18.10 ± 0.14</td>
<td>18.05 ± 0.13</td>
<td>18.14 ± 0.16</td>
<td>18.11 ± 0.14</td>
<td>17.37 ± 0.55</td>
</tr>
<tr>
<td>$K_LK_S$</td>
<td>11.53 ± 0.08</td>
<td>11.70 ± 0.08</td>
<td>11.65 ± 0.09</td>
<td>11.65 ± 0.10</td>
<td>11.98 ± 0.36</td>
</tr>
<tr>
<td>HLS Sum</td>
<td>570.97 ± 0.92</td>
<td>572.42 ± 1.08</td>
<td>572.32 ± 1.03</td>
<td>572.44 ± 1.20</td>
<td>572.57 ± 3.15</td>
</tr>
<tr>
<td>$\chi^2/N_{pts}$</td>
<td>949.1/1056</td>
<td>1062.2/1152</td>
<td>1128.0/1237</td>
<td>1038.2/1152</td>
<td>×</td>
</tr>
<tr>
<td>Probability</td>
<td>96.7%</td>
<td>91.6%</td>
<td>94.6%</td>
<td>96.7%</td>
<td>×</td>
</tr>
</tbody>
</table>

Table 7: HLS contributions to $10^{10} \times a^{HVP-LO}$ integrated up to 1.05 GeV, including FSR. The first data column displays the results using the former BHLS [25, 27] and, the second one, those derived from the Basic Solution for BHLS$_2$, the $\tau$ decay data being discarded. The next two data columns refer to the results obtained using the Reference Solution for BHLS$_2$ using the largest set of data samples, keeping or discarding the $\tau$ data. The last data column refers to the numerical integration for each channel of the same set of data which are used in the BHLS/BHLS$_2$ fits.
Benayoun+DelBuono+Jegerlehner:

arXiv:1903.11034

- Their preferred mean value is for total LOHVP in \((10^{-10})\):
  \[687.1 \pm 3.0 \ (+1.3\,-1.0)_{\text{sys}}\]
  [vs. KNT’s 693.3\pm2.5]

- Adding all contributions, they then >\(\sim\) 4.2\(\sigma\).

Figure 11: Recent evaluations of \(10^{10} \times a^{\text{HVP-LO}}\). On top, the result derived by direct integration of the data combined with perturbative QCD; the next six points display some recent evaluations derived by LQCD methods and reported in resp. [124], [125], [126], [127], [11] and [128] with \(N_f = 2 + 1 + 1\). The second point from [126] displayed has been derived by supplementing lattice data with some phenomenological information. These are followed by the evaluations from [71],[129] and [4]. The value derived using BHLS [27] – updated with the presently available data – and the evaluation from BHLS\(_2\) are given with their full systematic uncertainties (see text).
HVP from the lattice

Christoph Lehner at the recent meeting of the Theory Initiative for g-2, Mainz, June 2018:

`We need to improve the precision of our pure lattice result so that it can distinguish the "no new physics" results from the cluster of precise R-ratio results.'


<table>
<thead>
<tr>
<th>Experiment</th>
<th>( a_{\mu}^{\text{LO-HVP}} \times 10^{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETM-14</td>
<td></td>
</tr>
<tr>
<td>HPQCD-17</td>
<td></td>
</tr>
<tr>
<td>BMW-18</td>
<td></td>
</tr>
<tr>
<td>RBC/UKQCD-18</td>
<td></td>
</tr>
<tr>
<td>ETM-18</td>
<td></td>
</tr>
<tr>
<td>FHM (prelim)</td>
<td></td>
</tr>
<tr>
<td>Mainz (prelim)</td>
<td></td>
</tr>
<tr>
<td>Jegerlehner-17</td>
<td></td>
</tr>
<tr>
<td>DHMZ-17</td>
<td></td>
</tr>
<tr>
<td>KNT-18</td>
<td></td>
</tr>
<tr>
<td>RBC/UKQCD-18</td>
<td></td>
</tr>
<tr>
<td>LQCD</td>
<td></td>
</tr>
<tr>
<td>Pheno.</td>
<td></td>
</tr>
<tr>
<td>Pheno+LQCD</td>
<td></td>
</tr>
</tbody>
</table>

No new physics
Magnetic Moments: $a_e$ vs. $a_\mu$

$a_e = 1\ 159\ 652\ 180.73\ (0.28)\ 10^{-12}$ \begin{footnotesize}[0.24ppb]\end{footnotesize} 
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

$a_\mu = 116\ 592\ 089(63)\ 10^{-11}$ \begin{footnotesize}[0.54ppm]\end{footnotesize} 
Bennet et al., PRD 73(2006)072003

- $a_e^{\text{EXP}}$ more than 2000 times more precise than $a_\mu^{\text{EXP}}$, but for $e^-$ loop contributions come from very small photon virtualities, whereas muon `tests’ higher scales

- dimensional analysis: sensitivity to NP (at high scale $\Lambda_{NP}$): $a_{NP}^{e} \sim C \ m^2_{e} / \Lambda^2_{NP}$

$\mu$ wins by $m^2_{\mu}/m^2_{e} \sim 43000$ for NP, but $a_e$ determines $\alpha$, tests QED & low scales
Magnetic Moments: $a_e^{\text{SM}}$ has changed with recent shift of $\alpha$

- General structure:  
  $$a_e^{\text{SM}} = a_e^{\text{QED}} + a_e^{\text{hadronic}} + a_e^{\text{weak}}$$

- **Was:**  
  $$a_e^{\text{SM}} = 1\,159\,652\,182.03(72) \times 10^{-12}$$  
  [Aoyama+Kinoshita+Nio, PRD 97(2018)036001]  
  including a small shift from $\ldots\,81.78(77)$ after 2018 update of Kinoshita’s 5-loop numerics  
  using $\alpha$ measured with Rubidium atoms [$\alpha$ to 0.66 ppb]

- is, due to a new $\alpha$ measurement with Cs-133 atoms [Parker et al., Science 360 (2018) 191], now more precise [$\alpha$ to 0.2 ppb] and shifted down to  
  $$a_e^{\text{SM}} = 1\,159\,652\,181.61(23) \times 10^{-12}$$

- Comparison with the experimental measurement gives a   
  -2.5 $\sigma$ discrepancy for $a_e$:  
  $$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -0.88(36) \times 10^{-12}$$

- which one may consider together with the muon $g-2$ discrepancy (of about +3.7$\sigma$, see below) when discussing possible New Physics contributions

- but note that for the electron the SM overshoots! Pressure on dark photons...