BES III Inputs for $(g - 2)_\mu$

September 26, 2019 | Christoph Florian Redmer
for the BESIII Collaboration

HC2NP – 2nd Workshop on Hadronic Contributions to New Physics Searches
Puerto de la Cruz, Tenerife
Muon anomaly: \[ a_\mu = \frac{g_\mu - 2}{2} \]

- Known to 0.5 ppm in theory and experiment
  - Standard Model (SM) \( (11659182.04 \pm 3.65) \times 10^{-10} \) Phys. Rev D97 (2018) 114025
  - Experiment (BNL) \( (11659208.9 \pm 6.3) \times 10^{-10} \) Phys. Rev. D73 (2006) 072003

- Discrepancy between SM prediction and measurement!
- New measurements at FermiLab and J-PARC
- Improvement of SM prediction necessary

Uncertainty of SM prediction completely limited by hadronic contributions!

Use input from experiments to improve SM prediction!
BESIII at BEPCII

NIM A614 (2010) 345

Muon Chambers
- 8 – 9 layers of RPC
- $p > 400$ MeV/c
- $\delta R \Phi = 1.4 \sim 1.7 \text{ cm}$

Superconducting Magnet
- 1 T magnetic field

EM Calorimeter (EMC)
- 6240 CsI(Tl) crystals
- $\sigma(E)/E = 2.5\%$
- $\sigma_{z,\phi}(E) = 0.5 \sim 0.7 \text{ cm}$

Time-of-flight system (TOF)
- $\sigma(t) = 90\text{ps}$ (barrel)
- $\sigma(t) = 110\text{ps}$ (endcap)

Drift Chamber (MDC)
- $\sigma(p)/p = 0.5\%$
- $\sigma_{dE/dx} = 6.0\%$
BESIII at BEPCII

- Operated at BEPCII collider
  - $2.0 \leq \sqrt{s} \ [\text{GeV}] \leq 4.6$
  - Design luminosity achieved
    - $\mathcal{L} = 1.0 \times 10^{33} \text{cm}^{-2}\text{s}^{-1}$ at $\psi(3770)$

- Data taking for
  - Charmonium spectroscopy
  - Charm physics
  - Light hadrons
  - $\tau$ and R-scan
Related to hadronic cross sections by optical theorem

\[ a_{\mu}^{hVP, LO} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} K(s)\sigma(e^+e^- \to \text{hadr})ds \]

\[ K(s) \sim \frac{1}{s} \]

\[ \sigma(e^+e^- \to \text{hadr}) \sim \frac{1}{s} \]

Low energy contributions dominate!

\[ a_{\mu}^{HVP} \]

\[ \delta a_{\mu}^{HVP} \]
ISR Measurements at BESIII

- Detect hadronic system

- ISR photon detected
  - Acceptance from $\pi^+\pi^-$ threshold
  - Large background contamination at high $\sqrt{s'}$

- ISR photon undetected
  - High statistics
  - Acceptance for $\sqrt{s'} > 1$ GeV
  - Small background contamination

\[
\sqrt{s'} = \sqrt{s - 2\sqrt{s}E_\gamma}
\]
- Tagged ISR technique
- Based on 2.9 fb\(^{-1}\) at 3.773 GeV
- \(\mu - \pi\) separation with Artificial Neural Network
- Focus on \(0.6 \leq m_{\pi\pi} \leq 0.9\)
- Normalized to integrated luminosity
- Careful evaluation of systematics
  - Total uncertainty of 0.9% achieved
  - Dominated by
    - Luminosity (0.5%)
    - Radiator function (0.5%)

- Deviations to previous measurements observed

- Ongoing activities
  - Extend analysis to new data sets
  - Investigate \(m_{\pi\pi} \geq 1\) GeV
    - Untagged ISR analysis
- Tagged ISR technique
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- Ongoing activities
  - Extend analysis to new data sets
  - Investigate $m_{\pi\pi} \geq 1$ GeV
    - Untagged ISR analysis
- Precision competitive to measurements by BaBar and KLOE
- Good agreement with all KLOE results
- BESIII result confirms $a_{\mu}^{\mathrm{theo,SM}} - a_{\mu}^{\exp} > 3\sigma$
- Reevaluations of $a_{\mu}^{h\nu}$ including BESIII result improve accuracy by 20%
Scheme successfully applied to measure $e^+e^- \rightarrow \pi^+\pi^-\pi^0/\pi^+\pi^-2\pi^0/\pi^+\pi^-3\pi^0$!
Higher pion multiplicities

- Error weighted mean of tagged and untagged results
- Improved precision (syst. uncertainties of approx. 3% and 4%, respectively)
- Determination of LO contribution to $a_\mu^{HVP}$
- Measurement of branching fractions of $J/\psi$
Hadronic Light-by-Light Scattering

Counting scheme for contributions to $a_{\mu}^{\text{HLLl}}$

Chiral counting $N_{C}$-counting:

Dominating contributions:
- PS meson exchange
- Pion loops

Dispersive Approaches
- Bern
- Colangelo et al.
- Mainz
- Vanderhaeghen et al.

3D integral representation for PS-pole contribution:

$\begin{align*}
a_{\mu}^{\text{HLLl};\pi(1)} &= \int_{0}^{\infty} \int_{0}^{\infty} dQ_1 \int_{-1}^{1} d\tau \ w_1(Q_1, Q_2, \tau) F_{\pi^0\gamma^*\gamma^*}(\text{continuum}) F_{\pi^0\gamma^*\gamma^*}(\text{continuum}) \\
a_{\mu}^{\text{HLLl};\pi(2)} &= \int_{0}^{\infty} \int_{0}^{\infty} dQ_1 \int_{-1}^{1} d\tau \ w_2(Q_1, Q_2, \tau) F_{\pi^0\gamma^*\gamma^*}(\text{continuum}) F_{\pi^0\gamma^*\gamma^*}(\text{continuum})
\end{align*}$

Universal weight functions $w_1, w_2$

Form factor dependence $F$

Relevant momentum region: 0.25 – 1.25 GeV

Exchange of other resonances $(f_0, a_1, f_2 \ldots)$

$(\text{de Rafael, Phys.Lett.B322, 239, 1994})$
Two-photon Physics

- Cross section of $\gamma \gamma$ processes proportional to square of TFF
- Single-tagged measurements to study momentum dependence of TFF

Reconstruct:
- only one scattered lepton
- produced system
- unmeasured lepton from momentum conservation

Require scattering angle of missing momentum to be small
- Small virtuality of exchanged photon
- $F(Q_1^2, Q_2^2) \rightarrow F(Q_1^2, 0)$

Reject events with $q_{\text{tag}} \cdot \cos(\theta_{\text{miss}}) > -0.99$

$q_{\text{tag}}$: Charge of tagged lepton in units of $[e]$
Based on 2.9 fb⁻¹ at 3.773 GeV

Select:
- Exactly one lepton
- At least two photons

Apply:
- Single-tag condition
- Helicity angle of photons

\[ R_\gamma = \frac{\sqrt{s} - E_{e^\pm \pi^0}^{\text{CMS}} - P_{e^\pm \pi^0}^{\text{CMS}}}{\sqrt{s}} > 0.05 \]

Clear signals of \( \pi^0 \) and \( \eta \)

Incomplete MC description
- Data-driven background subtraction

Divide out point-like cross section using MC distributions
Space-like $\pi^0$ Transition Form Factor

Systematic Uncertainties of $|F(Q^2)|$

Error propagation: $\Delta |F(Q^2)|_i = \frac{1}{2} \frac{1}{\sqrt{|F(Q^2)|^2}} \Delta (|F(Q^2)|^2)_i$

<table>
<thead>
<tr>
<th>Source</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>External</td>
<td></td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.25%</td>
</tr>
<tr>
<td>Photon detection efficiency</td>
<td>1%</td>
</tr>
<tr>
<td>Luminosity</td>
<td>0.25%</td>
</tr>
<tr>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td>$q_{tag} \cdot \cos \theta_{miss} &lt; -0.99$</td>
<td>0.1% – 3.1%</td>
</tr>
<tr>
<td>$\cos \theta_H &lt; 0.8$</td>
<td>0.2% – 4.5%</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \phi_{\gamma\gamma}</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \theta_{\gamma\gamma} - 0.01q_{tag}</td>
</tr>
<tr>
<td>$R_\gamma &lt; 0.05$</td>
<td>1.0% – 7.7%</td>
</tr>
<tr>
<td>Background subtraction</td>
<td></td>
</tr>
<tr>
<td>Reconstruction efficiency</td>
<td>1.6% – 17.2%</td>
</tr>
<tr>
<td>Signal shape</td>
<td>0.1% – 1.9%</td>
</tr>
<tr>
<td>Event counting</td>
<td>0.1% – 11.1%</td>
</tr>
<tr>
<td>Background shape</td>
<td>0.2% – 21.0%</td>
</tr>
<tr>
<td>Total</td>
<td>3.9% – 30.0%</td>
</tr>
</tbody>
</table>

- Contributions added in quadrature
- Full correlation between contributions of analysis conditions and background subtraction assumed
- Error estimate does not consider radiative effects
  - To be evaluated with recently released Ekhara 3.0

Space-like $\pi^0$ Transition Form Factor

Comparison with previous measurements

- Competitive accuracy up to $3.1 \text{ GeV}^2$
- Unprecedented accuracy below $Q^2 = 1.5 \text{ GeV}^2$
- First measurement below $0.5 \text{ GeV}^2$
- Limited by acceptance for $\pi^0 \rightarrow \gamma\gamma$

Construction of space-like TFF using time-like experimental results in dispersive calculations

“Mainz approach”
- \( N_f = 2+1 \) Wilson fermions
Single-Tag measurement

- Combine 7.5 fb\(^{-1}\) from 3.773 GeV to 4.6 GeV
- Event selection analogous to single pseudoscalar analysis
- Machine learning tools to suppress \(e^+e^- \rightarrow e^+e^-\mu^+\mu^-\)
- Subtraction of \(\rho\) contribution in \(e^+e^- \rightarrow e^+e^-\pi^+\pi^-\)
  - Fit peak in data using shape from theory

- Study \(\pi^+\pi^-\) invariant mass in bins of \(Q^2\) and \(\cos\theta^*\)

- First single-tag measurement of \(\pi^+\pi^-\)!
- Access to:
  - low momentum transfers \(0.2 < Q^2 \text{[GeV}^2\text{]} < 2.0\)
  - low invariant masses \(m_{\pi^+\pi^-} < M \text{[GeV]} < 2.0\)
  - full coverage of \(\cos\theta^*\)
Outlook

- **Single-tagged measurements**
  - Complete TFF studies of single mesons \((\eta, \eta')\)
  - Extend two-meson studies to neutral channels \((\pi^0\pi^0, \pi^0\eta, \eta\eta)\)
  - Investigate higher multiplicity final states \((3\pi, 4\pi, \ldots)\)
    - Axial and tensor contributions to \(a_\mu\)
    - Exploit additional data sets to increase \(Q^2\) range

- **Double-tagged measurements**
  - Complementary to BaBar measurement of \(\eta'\) TFF
  - Cover all single pseudoscalar states for \(Q^2 < 2\ \text{GeV}^2\)
  - Feasibility studies successful
  - Development and installation of dedicated taggers
BESIII provides important input to \((g-2)\mu\) calculations!

- Hadronic cross section measurements with scans and ISR
  - Information from hadronic threshold to tau-charm region
  - Competitive accuracy
  - \(\pi^+\pi^-\) result confirms \(a^{\text{theo, SM}}_{\mu} - a^{\text{exp}}_{\mu} > 3\sigma\) \(,\) extending investigations to \(m_{\pi\pi} \geq 1\) GeV
  - Preliminary results on \(e^+e^- \rightarrow \pi^+\pi^-\pi^0, e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\)

- Two-photon physics program
  - Single-tag measurements
    - \(\pi^0, \eta, \) and \(\eta'\) transition form factors with unprecedented accuracy \((Q^2 < 1.5 \text{ GeV}^2)\)
    - \(\pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, \eta\eta\)
      - First measurement at low \(Q^2\)
      - Covers masses from threshold and the full helicity angle
  - First double-tagged measurement \(\gamma^*\gamma^* \rightarrow \pi^0\) started
\[ e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \]

\(\pi^+ \pi^- \pi^0\) invariant mass spectra

- **Below 1.8 GeV:** Fit
  \[
  \frac{dN}{dm} = \sigma(m) \cdot \frac{dL}{dm} \cdot \varepsilon
  \]
  \[
  \sigma(m) = \frac{12\pi}{m^3} F_{\rho\pi}(m) \sum_{\nu=\omega,\phi,\rho',\rho''} \frac{\Gamma_{\nu m_{3\nu}^{3/2}} \sqrt{\Gamma_{V \rightarrow 3\pi}^{ee} B(\nu \rightarrow 3\pi)}}{D_V(m)} e^{i\varphi_V} \sqrt{F_{\rho\pi}(m_V)}
  \]

- **Above 3 GeV:** Determine \(B(J/\psi \rightarrow \pi^+ \pi^- \pi^0)\)

- **Tagged**
  - \(M_{3\pi}(\text{GeV}/c^2)\)
  - BESIII Preliminary

- **Untagged**
  - \(M_{3\pi}(\text{GeV}/c^2)\)
  - BESIII Preliminary
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$

- Good agreement with previous measurements
- Improved precision
  - $\sim 3\%$ syst. uncertainty in full mass range
  - $< 2\%$ at narrow resonances
- Confirms BaBar result at $\omega''$
- To be used to evaluate $a_{\mu}^{hVLP}$
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$

Strategy similar to $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0\gamma$ analysis

- Error weighted mean of tagged and untagged results
- Good agreement with previous measurements
- Improved precision (approx. 3% syst. uncertainty)

$$a_\mu^{\pi^+\pi^-2\pi^0,LO} = \frac{1}{4\pi^3} \int \frac{(1.8 \text{ GeV})^2}{(4m_\pi)^2} ds K(s) \sigma_{\pi^+\pi^-2\pi^0}(s)$$

<table>
<thead>
<tr>
<th>BESIII (preliminary)</th>
<th>$a_\mu^{\pi^+\pi^-2\pi^0,LO}/10^{-10}$</th>
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<tbody>
<tr>
<td></td>
<td>$18.63 \pm 0.27 \pm 0.57$</td>
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</table>
Fit $\omega$ signal on smooth background in every bin of $M_{\pi^+ \pi^- \pi^0 \pi^0}$

- Approx. 4% syst. uncertainty

- Good agreement with previous measurements

$e^+ e^- \rightarrow \omega \pi^0$
From background evaluation of $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$

Good agreement calculations using isospin relations
Space-like $\pi^0$ Transition Form Factor

**Comparison to Theory (I)**

**Models:**

- This work, stat. err. only
- This work, total error
- VMD
- LMD+V
- 1-Octet
- 2-Octet
- 3-Octet

![Graph showing comparison of models](graph.png)

**Equations:**

\[
F_{VMD}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} \frac{M_V^2}{M_V^2 + Q^2}
\]

\[
F_{LMD+V}(Q^2) = -\frac{F_\pi}{3} \frac{h_1 Q^4 - h_5 Q^2 + h_7}{(M_{V1}^2 + Q^2)(M_{V1}^2 + Q^2)M_{V1}^2 M_{V2}^2}
\]

\[
F_{n=1,2-\text{Octet}}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} + \sum_{i=1}^{n} \frac{4\sqrt{2}h_{V_i}f_{V_i}}{3F_\pi} Q^2 (D_{\rho_i} - D_{\omega_i})
\]

\[
F_{3-\text{Octet}}(Q^2) = -\frac{N_c}{12\pi^2 F_\pi} + \sum_{i=1}^{3} \frac{4\sqrt{2}h_{V_i}f_{V_i}}{3F_\pi} Q^2 (D_{\rho_i} + F_{\omega_i}H_{\omega_i}D_{\omega_i} + A_{i}^{\pi 0} F_{\phi_i}D_{\phi_i})
\]

Parameters of models fixed according to publications

