Short-distance constraints and resonance exchange in HLbL

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Status of the muon $(g-2)_\mu$

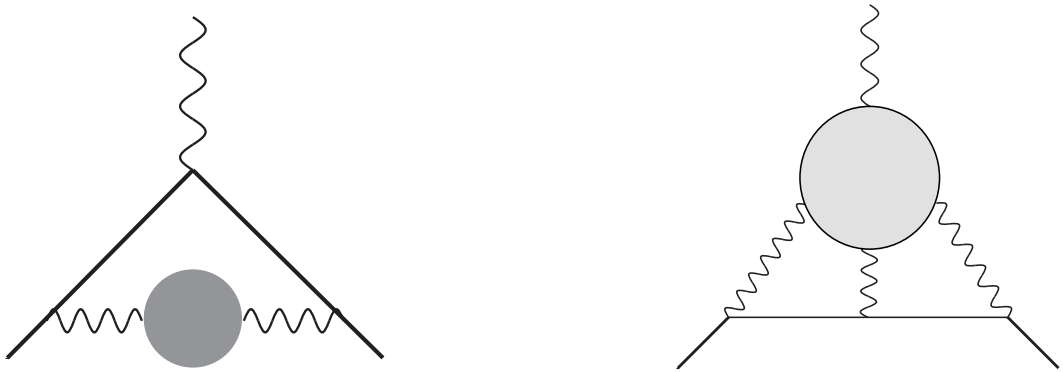
- While awaiting for the Fermilab number:

$$a_\mu^{\rm exp} = 116592091(54)(33) \times 10^{-11}$$

- Long-standing discrepancy[a] with the SM estimate (3 to 4σ):

$$a_\mu^{\rm SM} = 116591823(1)(34)(26) \times 10^{-11}$$

- Excellent control over the dominant EW and EM corrections. Hadronic contributions small but dominate the uncertainty.



[a] Only one experiment, not yet challenged...

Hadronic contributions

- HVP leading effect ($\sim 700 \times 10^{-10}$). Uncertainties can be reduced with e^+e^--based and/or τ-based analyses. [Davier et al, Teubner et al]

  Lattice QCD at a really advanced stage. [Mainz, BMWc, RBC/UKQCD]

- HLbL much harder to estimate. Connection to experiment more convoluted, albeit dispersion analyses promising. [Bern, Mainz]

  Lattice QCD catching up fast. [Mainz, RBC/UKQCD]

- Experimentally, we have a much improved projected uncertainty, 16×10^{-11}.

- Hadronic contributions cannot account for the present discrepancy, but we need better control of theoretical uncertainties to claim NP interpretations, when(if) the time comes.

HLbL estimates

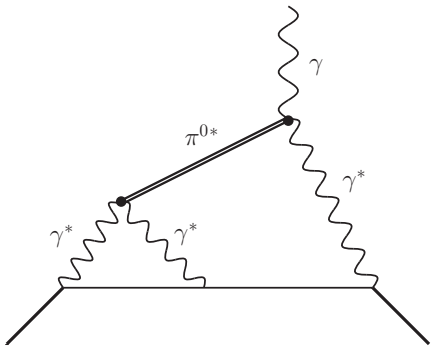
- Three main routes: form factor ansatz, lattice QCD, dispersion relations.
- Main contributions from form factor analyses:

Contribution	BPP	HKS,HK	KN	MV	PdRV	N,JN
π^0, η, η'	$85(13)$	$82.7(6.4)$	$83(12)$	$114(10)$	$114(13)$	$99(16)$
axial vectors	$2.5(1.0)$	$1.7(1.7)$	$-$	$22(5)$	$15(10)$	$22(5)$
scalars	$-6.8(2.0)$	$-$	$-$	$-$	$-7(7)$	$-7(2)$
π, K loops	$-19(13)$	$-4.5(8.1)$	$-$	$-$	$-19(19)$	$-19(13)$
$_{\pi,K}$ loops $_{+}$subl. N_C	$-$	$-$	$-$	$0(10)$	$-$	$-$
quark loops	$21(3)$	$9.7(11.1)$	$-$	$-$	2.3	$21(3)$
Total	$83(32)$	$89.6(15.4)$	$80(40)$	$136(25)$	$105(26)$	$116(39)$

- Overall agreement with the 'pion-pole' contribution, main discrepancies in other contributions.
- Used PDG average not really coming from a calculation.
- A number of theoretical issues still open.

Form factor analysis

Pion-pole contribution:



- Vertices given by the $\pi\gamma\gamma$ form factor,

$$\int d^4x\; e^{iq_1\cdot x}\langle 0|T\left\{J^\mu_{\rm EM}(x)\,J^\nu_{\rm EM}(0)\right\}|\pi^0(p)\rangle = \epsilon^{\mu\nu\alpha\beta}q_{1\,\alpha}q_{2\,\beta}\,F_{\gamma^*\gamma^*\pi^0}\left(Q_1^2,Q_2^2\right)$$

- $F_{\pi^0\gamma\gamma}$ not known from first principles. Information only on certain kinematical limits:

(a) $$F_{\gamma^*\gamma^*\pi^0}(0,0) = -\frac{N_C}{12\pi^2 f_\pi}K(0,0), \quad K(0,0)=1 \qquad \text{(Anomaly)}$$

(b) $$\lim_{Q^2\to\infty} K(Q^2,Q^2) = \frac{8\pi^2 f_\pi^2}{N_c}\frac{1}{Q^2} \qquad \text{(OPE)}$$

(c) $$\lim_{Q^2\to\infty} K(0,Q^2) \sim \frac{1}{Q^2} \qquad \text{(Brodsky-Lepage)}$$

Form factor analysis

- Ansätze with different short and long-distance constraints:

$$K(q_1^2, q_2^2) = 1; \qquad K(q_1^2, q_2^2) = \frac{m_V^4}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}$$

$$K(q_1^2, q_2^2) = \frac{m_V^2}{m_V^2 - q_1^2 - q_2^2}; \qquad K(q_1^2, q_2^2) = \frac{m_V^4 - \frac{4\pi^2 f_\pi^2}{N_c}(q_1^2 + q_2^2)}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}$$

- In principle, the more constraints the better (closer to QCD). However, interesting to play with them to test which ones are numerically important.
- The same strategy can be repeated for the other contributions.

MAIN HURDLES:

- Hard to pin down the discrepancies: different interpolators for different channels, subject to different constraints.
- Not always clear how the short distances can be incorporated into form factors.
- Related to discussions on 'on-shellness' vs 'off-shellness' of the pion.

Correlators vs form factors

- In general,

$$\textbf{Correlator} \neq \sum (\text{particle exchange})$$

- The presence of contact terms is important to fulfill general properties e.g. gauge invariance or anomaly matching.
- Simple example: AA correlator at low energies.

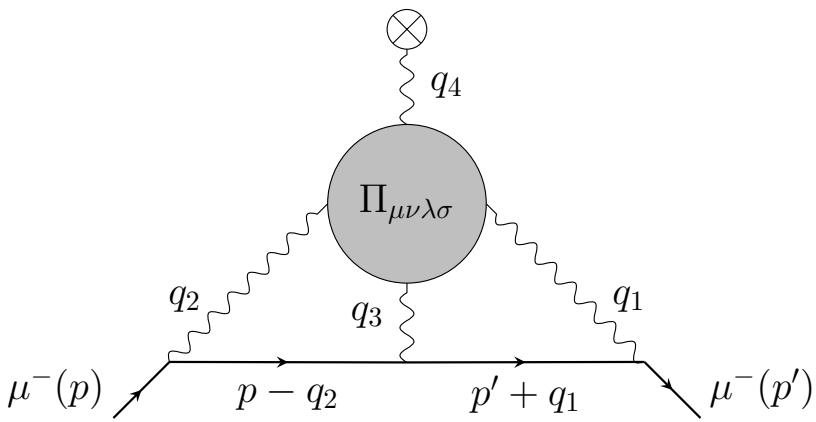
$$\Pi^{\mu\nu}_{AA}(q) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) f_\pi^2$$

- Pion propagation is not enough. Contact terms are fundamental.
- How can one implement the correct contact terms? External sources in Lagrangian formulations, e.g. ChPT.
- Problem: Lagrangians only at specific kinematical regimes (pQCD, ChPT).

Q1: Is there a way to make progress (apart from lattice QCD)?

Q2: Is this an issue for a_μ?

The Melnikov-Vainshtein limit



- In the limit $Q_2^2 \simeq Q_3^2 \gg Q_1^2 \gtrsim \Lambda_{\rm QCD}$ an OPE links VVVV to the (anomalous) VVA.
- The resulting short-distance constraint leads to a (sizeable) increase

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- Attempts to implement it with form factors not entirely successful. E.g., [Melnikov, Vainshtein]

$$\mathcal{A}_{\pi^0} = F_{\pi\gamma\gamma}(q_2, q_3) \frac{1}{q_1^2 - m_\pi^2} F_{\pi\gamma\gamma}(q_1, 0)$$

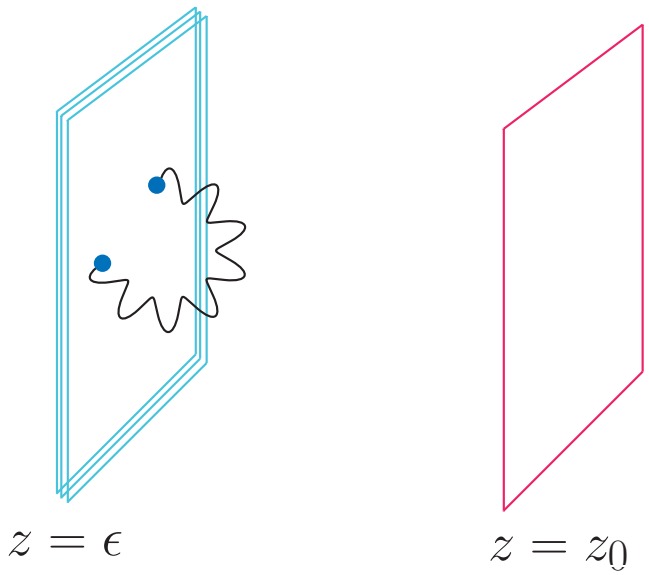
consistent only if $F_{\pi\gamma\gamma}(q_1, 0) = 1$. Hard to argue phenomenologically...

A toy model

- 5-dimensional model:

$$S_5 = \int d^4x \int_0^{z_0} dz \Big\{ -\lambda\sqrt{-g}\,\mathrm{tr}\big[F_{(L)}^{MN}F_{(L)MN} + F_{(R)}^{MN}F_{(R)MN}\big] + c\,\mathrm{tr}\big[\omega_5(L_M) - \omega_5(R_M)\big]\Big\}$$

with $\omega_5(L) = \mathrm{tr}\left[LF_{(L)}^2 + \frac{i}{2}L^3F_{(L)} - \frac{1}{10}L^5\right]$



- Choose AdS$_5$ space, $ds^2 = g_{MN}dx^M dx^N = \frac{1}{z^2}(-dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$, with $\eta_{\mu\nu}$ mostly negative.

A toy model

- UV boundary conditions (AdS/CFT prescription): fields on the boundary are sources of the 4d theory, *i.e.*,

$$L_\mu(x,0) = l_\mu(x) \qquad R_\mu(x,0) = r_\mu(x)$$

coupled to $J^a_{L\,\mu} = \overline{q}_L \gamma^\mu t^a q_L$ and $J^a_{R\,\mu} = \overline{q}_R \gamma^\mu t^a q_R$.

- IR boundary conditions:

$$L_\mu(x,z_0) - R_\mu(x,z_0) = 0 \qquad F_L^{z\mu}(x,z_0) + F_R^{z\mu}(x,z_0) = 0$$

such that chiral symmetry is (spontaneously) broken to $SU(3)_L \times SU(3)_R \to SU(3)_V$.

- Pion multiplet related to $A_5(x,z)$. Defining the Wilson line

$$\xi_L(x,z) = P \exp\left\{ -i \int_z^{z_0} dz'\, L_z(x,z') \right\}$$

the change of variables

$$L_M^\xi(x,z) = \xi_L^\dagger(x,z)\left[L_M(x,z) + i\partial_M\right]\xi_L(x,z),$$

replaces A_5 by

$$U(x) \equiv \xi_L(x)\xi_R^\dagger(x) = \exp\left[\frac{2i\pi^a(x)t^a}{f_\pi}\right]$$

A toy model

- Important: the change of variables does not leave the CS term invariant, but induces a shift

$$\omega_5(L^\xi) = \omega_5(L) + \omega_5(\Sigma_L) + d\alpha_4(L, \Sigma_L)$$

where

$$\alpha_4(L, \Sigma_L) = \frac{1}{2}\mathrm{tr}\left[\Sigma_L(LF_{(L)} + F_{(L)}L) + i\Sigma_L L^3 - \frac{1}{2}\Sigma_L L \Sigma_L L - i\Sigma_L^3 L\right] \quad , \quad \Sigma_L = d\xi_L \xi_L^\dagger$$

- The 3 free parameters of the model can be fixed to

$$\lambda = \frac{N_c}{48\pi^2}; \qquad z_0^2 = \frac{N_c}{6\pi^2 f_\pi^2}; \qquad c = \frac{N_c}{24\pi^2}$$

from the pQCD quark loop in Π_{AA}, f_π and the chiral anomaly.

HOLOGRAPHIC RECIPE: Given an action $S_5(A_M)$,

(1) Split the fields as $A_\mu(x,z) = a(x,z)\,\hat{a}_\mu^\perp(x) + \bar{a}(x,z)\,\hat{a}_\mu^\parallel(x) + \dfrac{\alpha(z)}{f_\pi}\partial_\mu \pi(x)$

(2) Solve the EoM and plug them back in. This defines $S_{\text{eff}}(\hat{a}_\mu(x))$.

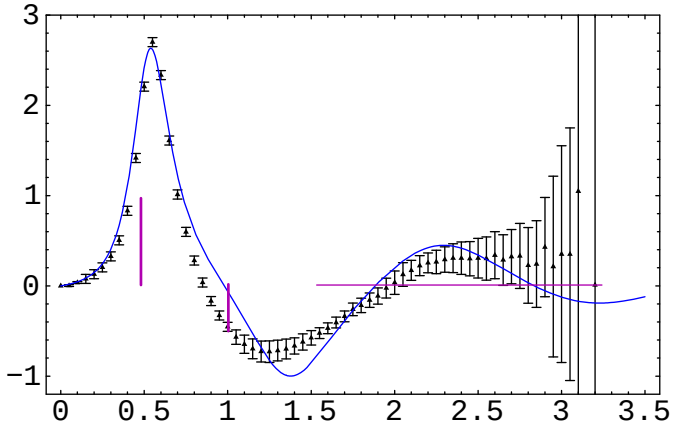
(3) Correlators fully analytical:

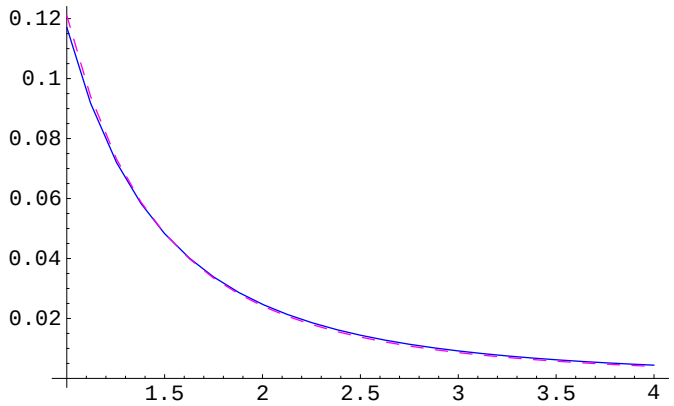
$$\Pi_{AA}^{\mu\nu} = \frac{\delta^2 S_{\text{eff}}}{\delta \hat{a}_\mu \delta \hat{a}_\nu}$$

How far can we go with the toy model?

Not QCD but interesting features:

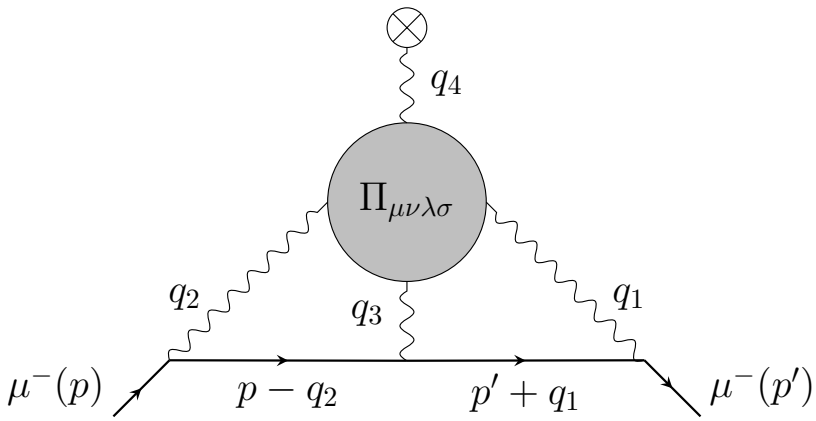
- Via Kaluza-Klein reduction, the 4d theory is a full-fledged realization of large-N_c QCD.
- Lagrangian approach: guarantees not just unitarity, but allows to compute correlators.
- The topological term ensures that the chiral anomaly is correctly implemented.
- With the AdS metric, one reproduces all the short-distance constraints we tested so far.
- The spectrum of V and A is not accurately reproduced. However, on the Euclidean this has a tiny impact. Consider e.g. Π_{LR},





Excellent laboratory to explore QFT issues in HLbL.

The HLbL tensor



Using the effective action, one finds a close expression for it:

$$\Pi_{\mu\nu\lambda\rho} = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\lambda\rho\alpha'\beta'}\left[\frac{c^2}{\lambda}\int dz \int dz' T^{\beta}_{12}(z) G^{\alpha\alpha'}_{A}(z,z';p) T^{\beta'}_{34}(z') + F^{(12)}_{\pi\gamma\gamma}\frac{q_1^{\alpha} q_2^{\beta} q_3^{\alpha'} q_4^{\beta'}}{p^2 - m_\pi^2} F^{(34)}_{\pi\gamma\gamma}\right]$$

where

$$F_{\pi\gamma\gamma}(Q_1^2, Q_2^2) = \frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} dz\ \alpha'(z)\ v(z, Q_1^2) v(z, Q_2^2)$$

- This represents the full contribution of the Goldstone modes and axial excitations.
- The (inclusive) calculation of the HLbL in the large-N_c limit is straightforward.

Pion dominance?

- The longitudinal piece of the HLbL tensor can be projected via

$$G_A^{\mu\nu} = \frac{p^\mu p^\nu}{p^2} G_A^{\parallel}$$

The result is simplified to

$$\begin{aligned}
\mathcal{W}_L &= \frac{N_c^2}{144\pi^4 f_\pi^2 p^2}\left[\int_0^{z_0} dz\alpha'(z)v_1(z)v_2(z)v_3(z)v_4(z)\right] \\
&+ \frac{N_c^2}{144\pi^4 f_\pi^2 p^2}\int_0^{z_0} dz'\alpha'(z)v_1(z)v_2(z)\int_0^{z_0} dz'\alpha'(z')v_3(z')v_4(z') - F_{\pi\gamma\gamma}^{(12)}\frac{1}{p^2-m_\pi^2}F_{\pi\gamma\gamma}^{(34)} \\
&= \frac{N_c^2}{144\pi^4 f_\pi^2 p^2}\left[\int_0^{z_0} dz\alpha'(z)v_1(z)v_2(z)v_3(z)v_4(z)\right] + \mathcal{O}(m_\pi^2)
\end{aligned}$$

i.e., there is a cancellation and one is left (in the chiral limit) with a contact piece only!

- This piece is responsible also for the correct implementation of the MV short-distance constraint, i.e., the MV constraint is not saturated by propagating terms.

- The origin of this contact term is closely related to the chiral anomaly of the VVA triangle.

The VVA triangle

- Consider the correlator

$$\Gamma_{\mu\nu\lambda}(q_3) = i \int d^4x d^4y \; e^{iq_3\cdot(x-y)} \langle 0|T\left\{j^{\rm em}_\mu(x) j^{\rm em}_\nu(y) j^5_\lambda(0)\right\}|0\rangle$$
$$= \frac{1}{24\pi^2}\left[\omega_L(q_3^2) t^{\parallel}_{\mu\nu\lambda} + \omega_T(q_3^2) t^{\perp}_{\mu\nu\lambda}\right]$$

- It is known that [Vainshtein; Knecht et al]

$$\omega_L(q) = -\frac{2N_c}{q^2}$$

to all orders in pQCD. Corrections are $\mathcal{O}(m_\pi^2)$.

- In the model VVA can be computed from the effective action:

$$(S^{(3)}_{\rm CS})^{\perp} = \frac{2c}{3}\varepsilon^{\mu\nu\lambda\rho}\int d^4x \left[1 + 3\int_0^{z_0} dz a(z,k_1) v(z,k_2) v'(z,k_3)\right] \hat{a}^{\perp}_\mu(k_1)\partial_\nu \hat{v}_\lambda(k_2)\hat{v}_\rho(k_3)$$

$$(S^{(3)}_{\rm CS})^{\parallel} = \frac{c}{3}\varepsilon^{\mu\nu\lambda\rho}\int d^4x \left[1 + 3\int_0^{z_0} dz \alpha'(z) v(z,k_2) v(z,k_3)\right] \frac{\partial^\alpha \hat{a}^{\parallel}_\alpha(k_1)}{\Box}\partial_\nu \hat{v}_\lambda(k_2)\partial_\mu \hat{v}_\rho(k_3)$$

together with the pion propagation.

The VVA triangle

- Longitudinal component: the pion cancels against the energy-dependent part of the 3-point vertex.
- Only the boundary piece survives (in the chiral limit) and $\omega_L(q)$ is recovered. In general,

$$\omega_L(q) = -\frac{2N_c}{q^2}\left[1 - \frac{12\pi^2 f_\pi}{N_c}\frac{m_\pi^2}{q^2 - m_\pi^2}F_{\pi\gamma\gamma}(0,q)\right]$$

- Compare with the continuation proposed in [Melnikov, Vainshtein]

$$\omega_L(q) = -\frac{2N_c}{q^2 - m_\pi^2}$$

  Different chiral continuations, which could be numerically important.
- If one takes only the pion piece at low energies one finds

$$\lim_{q\to 0}\omega_L^{(\pi)}(q) = -\frac{2N_c}{q^2}$$

  Pion dominance cannot be such a bad approximation after all.
- Subtlety: the action is in the LR prescription, so one should actually shift to the Adler-Bardeen one with

$$\Gamma_{\mu\nu\lambda} = \hat{\Gamma}_{\mu\nu\lambda} + \frac{N_c}{12\pi^2}\varepsilon_{\mu\nu\lambda\alpha}(q_1 - q_2)^\alpha$$

Conclusions

- A Lagrangian approach to HLbL provides an inclusive analysis of the leading large-N_c effects. One has access to full correlators and can clarify unresolved issues of form factor analyses. Nice perks: unitarity, anomaly and short-distance constraints correctly implemented.

- Pion dominance is not compatible with the correct implementation of the chiral anomaly, but numerically it gives an excellent estimate.

- The anomaly is saturated by a contact term. This is the main contribution to HLbL. The uncertainty on HLbL must be reassessed: presumably smaller than previously estimated.

- The contact term is crucial to fulfill the MV short-distance constraint, i.e., it is not saturated with a form factor. The infamous structureless form factor is just a manifestation of this.

- The previous results are generic QFT consequences for the leading large-N_c contributions to HLbL. Specific numbers will of course depend on the model, but even part of the result should be model-independent. Preliminary numbers seem to indicate so.

- Lattice simulations should be able to (hopefully) confirm these features.