Lattice inputs for $B$ flavour physics

Elvira Gámiz

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Elementales

· Hadronic Contributions to New Physics Searches, Tenerife
  27 September 2019 ·
Flavour-violating and CP-violating processes allow us to test high energy scales

- **Unveiling New Physics effects**
- **Constraining NP models.**

**SM** predictions for those observables depend on only a few parameters → can overconstrain the value of those parameters.

**Tests** not **limited** by available energy **by available precision** (both in experiment and theory)

\[
\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times (\text{matrix elements}) \times \text{lattice QCD}
\]

**Lattice QCD**: Numerical evaluation of QCD path integral (rely only on first principles) using Monte Carlo methods.
Lattice QCD can provide precise inputs for quark masses and $\alpha_s$, hadron spectrum, weak decays (decay constants, form factors, bag parameters)...

... and, in particular, to extract of CKM matrix elements from different sources

$$V_{CKM} = \begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
\pi \rightarrow \ell \nu & K \rightarrow \ell \nu & B \rightarrow \tau \nu \\
K \rightarrow \pi \ell \nu & B \rightarrow \pi \tau \nu, B_s \rightarrow K \ell \nu & \Lambda_b \rightarrow p \ell \nu \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
D \rightarrow \ell \nu & D_s \rightarrow \ell \nu & B_{(s)} \rightarrow D_{(s)}, D^*_{(s)} \ell \nu \\
D \rightarrow \pi \ell \nu & D \rightarrow K \ell \nu & \\
|V_{td}| & |V_{ts}| & |V_{tb}| \\
\langle B^0_d | \bar{B}^0_d \rangle & \langle B^0_s | \bar{B}^0_s \rangle & \\
B \rightarrow \pi \ell \ell & B \rightarrow K \ell \ell
\end{pmatrix}$$
Development of new methods is allowing to increase the scope of LQCD calculations:

- **Baryons** Gunnar Bali talk on Saturday
- Inputs for nuclear physics Andre Walker-Loud talk on Friday
- Nonleoptonic decays ($K \to \pi\pi$...) Chris Sachrajda talk on Friday
- Resonances
- Scattering
- Long-distance effects Chris Sachrajda talk on Friday
- QED effects Guido Martinelli talk on Saturday
- BSM physics George Fleming talk on Tuesday
- $g - 2$ (HVP and light-by-light) Thomas Blum and Laurent Lellouch talks on Thursday
- ...
Introduction: Lattice QCD

Combined chiral-continuum extrapolation

Example: MILC $N_f = 2 + 1 + 1$

Many lattice collaborations doing now simulations with physical light-quark masses; PACS-CS, BMW, MILC, RBC/UKQCD, ETM...

ChPT techniques still necessary to reduce errors and/or correct/estimate systematic effects: light and heavy quark discretization, finite volume, isospin-breaking, mass mistunings, ...

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Introduction: Heavy quarks on the lattice

Challenging description of $b$ quarks on the lattice: $(am_h)^n$ errors.

- **Effective theory description:** *Fermilab interpretation, RHQ, OK, NRQCD...*
  - Cheaper simulations but requires matching and can still have $ap$ artifacts.

- **Improved relativistic description:** *ETM, HPQCD, FNAL/MILC, RBC/UKQCD*
  - Reduce errors (potentially results as accurate as for light quarks) and simplified renormalization or not renormalization at all.
  - Numerically more expensive.

**For bottom**

- Need lattice spacing small enough to safely simulate $b$ ($am_b \leq 0.9$)
- Need HQ inspired parametrization to extrapolate (or interpolate) to the physical $b$
$B$ and $D$ mesons decay constants
### FNAL/MILC, 1712.09262: Errors $\sim 2.5$ smaller than previous calculations

<table>
<thead>
<tr>
<th>$f_{D^+}$ (MeV)</th>
<th>$f_{D_s}$ (MeV)</th>
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<tbody>
<tr>
<td>205</td>
<td>215</td>
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<td>265</td>
<td>275</td>
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</table>

- FNAL/MILC 18
- ETM 14
- Fermilab/MILC 14
- RBC/UKQCD 17
- $\chi$QCD 14
- HPQCD 12
- Fermilab/MILC 11 (Clover c)
- HPQCD 10

- $N_f = 2 + 1 + 1$ FLAG-2019 averages for isospin-averaged decay constants have errors

\[
\sim 0.33 - 0.14\%
\]

- $f_D = 212.0(0.7)\text{MeV}$
- $f_{D_s} = 249.9(0.5)\text{MeV}$
- $f_{D_s}/f_D = 1.1783(0.0016)$

- Using Rosner, Stone & Van de Water exp. averages for the PDG and FNAL/MILC17 decay constants + known short- and long-distance EW corrections + uncertainty from unknown meson-structure-dependent EM corrections:

\[
|V_{cs}|_{SM}^{lep} = 1.000(2)_{f_{D_s}}^{(16)_{expt}^{(3)_{EM}}} , \quad |V_{cd}|_{SM}^{lep} = 0.2151(6)_{f_{D}}^{(49)_{expt}^{(6)_{EM}}}
\]

**EM error** comes from the unknown structure-dependent corrections and it is based on analogous corrections for pions and kaons: **need a direct calculation**
Second row CKM unitarity

$$\Delta_c \equiv |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.049(2) |V_{cd}|(32) |V_{cs}|(0) |V_{cb}|$$

compatible with three-generation CKM unitarity within 1.5σ

- Precision limited by exp. error on leptonic $D_{s}^+$ decay widths: BES-III, Belle II

Green squares: Leptonic determinations

Green triangles: Semilept. determinations

Meinel 16: From $\Lambda_c \rightarrow \Lambda\ell\nu$

Grey bands: FLAG2019 averages including semileptonic and leptonic

- $|V_{cd}|_{\text{FLAG}}^{N_f=2+1+1} = 0.2219(43)$
- $|V_{cd}|_{\text{FLAG}}^{N_f=2+1+1} = 1.002(14)$

* With these averages and

$$|V_{cb}|_{\text{incl+excl}} = 41.40(77) \cdot 10^{-3}$$

from FNAL/MILC 1712.09262
B and Bs decay constants

FNAL/MILC, 1712.09262: Errors $\sim 3$ smaller than previous calculations

<table>
<thead>
<tr>
<th>$f_{B+}$ (MeV)</th>
<th>$f_{Bs}$ (MeV)</th>
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<tbody>
<tr>
<td>175 185 195 205 215 225 235 245 255</td>
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</tbody>
</table>

Fermilab/MILC 17
HPQCD 17 (pseudoscalar current)
ETM 16
HPQCD 13 (NRQCD b)
RBC/UKQCD 14
HPQCD 12 (NRQCD b)
HPQCD 11 (HISQ b)
Fermilab/MILC 11 (Clover b)

$N_f = 2 + 1 + 1$ FLAG-2019 averages for isospin-averaged decay constants have errors

$\sim 0.68 - 0.4\%$

$f_B = 190.0(1.3)\text{MeV}$
$f_{Bs} = 230.3(1.3)\text{MeV}$
$f_{Bs}/f_B = 1.209(0.005)$

With FNAL/MILC17 $f_{B+}$ and exp. average in Rosner, Stone, van de Water, 1509.02220

$|V_{ub}| = 4.07(3)f_{B+}(37)_{\text{expt}} \cdot 10^{-3}$
**B** and **B_s** decay constants

**FNAL/MILC, 1712.09262**: Errors $\sim 3$ smaller than previous calculations

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<tr>
<td>u, d, s, c sea</td>
<td>$f_B$</td>
<td>$f_{B_s}$</td>
<td>$f_B$</td>
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<td>$f_B$</td>
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$N_f = 2 + 1 + 1$ \textbf{FLAG-2019} averages for isospin-averaged decay constants have errors $\sim 0.68 - 0.4\%$

- $f_B = 190.0(1.3)\text{MeV}$
- $f_{B_s} = 230.3(1.3)\text{MeV}$
- $f_{B_s}/f_B = 1.209(0.005)$

- With **FNAL/MILC17** $f_{B^+}$ and exp. average in \textit{Rosner, Stone, van de Water, 1509.02220}

  $$|V_{ub}| = 4.07(3)f_{B^+}(64)_{\text{expt}} \cdot 10^{-3}$$

  with the large uncertainty agrees with both inclusive and exclusive determinations.

- Could be competitive once Belle-II results are available.
Given the current and projected experimental uncertainties on the $D$ and $B$ meson leptonic decay rates, better lattice-QCD calculations of the QCD decay constants are not needed in the near future, but ... ... we need to include strong isospin breaking and **QED corrections** to leptonic (and semileptonic) decay rates.

See **G. Martinelli** talk on Saturday

- Pioneering work by RM123, [N. Carrasco et al 1502.00257](https://arxiv.org/abs/1502.00257)
- Preliminary results for radiative leptonic decays of light and $D$—mesons from several groups.
- Work towards developing the framework to calculate radiative corrections to semileptonic decays.
Neutral $B$-meson mixing

See Alexander Lenz talk on Tuesday
Neutral meson mixing BSM

In the Standard Model and beyond, short-distance contributions to the mixing can be described via a $\mathcal{H}_{eff}^{\Delta F=2}$.

In general: $\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i O_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{O}_i$

SM:

\[
O_1 = (\bar{f}^\alpha \gamma_\mu L q^\alpha) (\bar{f}^\beta \gamma^\mu L q^\beta) \\
O_2 = (\bar{f}^\alpha L q^\alpha) (\bar{f}^\beta L q^\beta) \\
O_3 = (\bar{f}^\alpha L q^\beta) (\bar{f}^\beta L q^\alpha)
\]

BSM:

\[
O_4 = (\bar{f}^\alpha L q^\alpha) (\bar{f}^\beta R q^\beta) \\
O_5 = (\bar{f}^\alpha L q^\beta) (\bar{f}^\beta R q^\alpha)
\]

\[
\tilde{O}_{1,2,3} = O_{1,2,3} \text{ with the replacement } L(R) \rightarrow R(L)
\]

Recent and on-going lattice calculations of $K$, $D$, and $B$ mixing matrix elements for all five operators → constraints on BSM physics
**New**: $N_f = 2 + 1 + 1$ HPQCD 1907.01025: complete set of bag parameters.

- NRQCD for $b$, HISQ (relativistic description) for $l, s, c$ (plots from HPQCD 1907.01025)

- Red circles: HPQCD19
- Blue crosses: FNAL/MILC16
- Green squares: HPQCD09
- Purple diamonds: ETMC13

Not in plot: RBC/UKQCD

- $B_{B_s} / B_{B_d} = 1.0002^{+73}_{-93}$

* Averages:
  - HPQCD19+FNAL/MILC16

Apparent disagreement between $N_f = 2$ and $N_f = 2 + 1, 2 + 1 + 1$ for $B^{(4,5)}$ may be caused by the use of different intermediate renormalization schemes.

(N_f = 2 + 1 + 1 HPQCD 1907.01025, N_f = 2 + 1 FNAL/MILC 1602.03560, HPQCD 0902.1815, N_f = 2 ETMC 1308.1851)
For the mass differences, in the **Standard Model**

\[
\Delta M_q \propto \left| V_{tq}^* V_{tb} \right|^2 f_{B_q}^2 \hat{B}_{B_q}^{(1)}, \text{ where } \frac{8}{3} f_{B_q}^2 B_{B_q}^{(1)}(\mu) M_{B_q}^2 = \langle \bar{B}^0 | O_1^q | B^0 \rangle(\mu)
\]

- Using tree-level inputs for the CKM matrix elements **CKMfitter 2018**

### Table

<table>
<thead>
<tr>
<th></th>
<th>(\Delta M_s) (ps(^{-1}))</th>
<th>(\Delta M_d) (ps(^{-1}))</th>
<th>(\Delta M_d/\Delta M_s)</th>
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<tbody>
<tr>
<td>HPQCD 19</td>
<td><img src="#" alt="Graph" /></td>
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<tr>
<td>RBC/UKQCD 18</td>
<td><img src="#" alt="Graph" /></td>
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<tr>
<td>FNAL/MILC 16</td>
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<tr>
<td>HQET SR 19</td>
<td><img src="#" alt="Graph" /></td>
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<tr>
<td>exp. average PDG18</td>
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(HQET SR 19: King, Lenz, Rauh, 1904.00940 use lattice (FLAG19 average) decay constants)
$B$-mixing: $V_{td,ts}$

- $B$-mixing results: HPQCD 1907.01025, RBC/UKQCD 1812.08791, FNAL/MILC 1602.03560
- $B \to K(\pi)\mu^+\mu^-$ results from D. Du et al, 1510.02349
- Full/tree CKM unitarity results come from CKMfitter's fit 2018 using all inputs/only observable mediated at tree level of weak interactions.

Room for improvement in lattice calculations.

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Lattice inputs for $B$ flavour physics
Rare decays $\mathcal{B}(B_s(d) \to \mu^+\mu^-)$

$B$ mixing bag parameters in the SM can be used for an indirect prediction of $\mathcal{B}(B \to \mu^+\mu^-)$ Buras, hep-ph/0303060

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q) \frac{6\pi \eta_y}{\eta_B} \left( \frac{\alpha}{4\pi M_W \sin^2 \theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

SM branching fractions vs exper. measurements (Plot from HPQCD 1907.01025)

- Taking the most accurate lattice inputs:
  - HPQCD'19: Indirect determination using HPQCD B-mixing.
    (EM uncertainties are not included, see C. Bobeth talk)
  - FNAL/MILC'17: Direct determination using FNAL/MILC decay constants.
  - PDG'19: experim. averages/upper bounds

Further reduction of the errors will start giving strong constraints.
$B$ semileptonic decays
Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements \(|V_{cb}|\) and \(|V_{ub}|\) at the \(\sim 3\sigma\) level.

**LQCD inputs**

* \(B \to \pi \ell \nu\): \(f_+(q^2) (f_0(q^2))\)

* \(\Lambda_b \to p \mu \nu / \Lambda_b \to \Lambda_c \mu \nu\):
  Six form factors for each channel

* \(B \to D^* \ell \nu\): \(F(\omega = 1)\)

* \(B \to D \ell \nu\): \(G(\omega)\) (related to \(f_+(q^2)\))

\(w = v_B \cdot v_D\) velocity transfer to the leptonic pair

\[
\frac{d\Gamma(B \to D^* \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |F(w)|^2
\]

\[
\frac{d\Gamma(B \to D \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |G(w)|^2
\]
Status exclusive $|V_{ub}|$ extraction

$|V_{ub}|$ from $B \rightarrow \pi l \nu$

Combined BCL fit to experim. and $N_f = 2 + 1$ lattice data on different $q^2$ regions

RBC/UKQCD, 1501.05373
FNAL/MILC, 1503.07839
HPQCD, hep-lat/0601021

$|V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3}$

Good consistency between lattice and experimental shapes and commensurate errors

$|V_{ub}|^{inclusive,HFLAV2017} = (4.52 \pm 0.15^{+0.11}_{-0.14}) \cdot 10^{-3} \sim 3\sigma$ disagreement.

Leptonic determinations: Less precise (dominated by exp. errors on $B(B \rightarrow \tau \nu)$) and BaBar and Belle results don’t agree very well.

Important role for Belle II for both leptonic and semileptonic
Status exclusive $|V_{ub}|$ extraction

$|V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3}$

**Challenge:** Extend lattice calculations to larger momenta.

**Preliminary $N_f = 2 + 1$ JLQCD:**

- Möbius Domain Wall (relativistic): use multiple $m_h > m_c$ and **extrapolate to** $m_b$

**Preliminary $N_f = 2 + 1 + 1$ HPQCD:**

- NRQCD heavy quarks, also $f_T$
- Several points with $q^2 \sim 6\text{GeV}^2$

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**J. Koponen** talk at Lattice 2019

**C. Bouchard** talk at Lattice 2019

Lattice inputs for $B$ flavour physics
**Status exclusive $|V_{ub}|$ extraction**

\[ |V_{ub}|^{FLAG2019} = 3.73(14) \cdot 10^{-3} \]

**Preliminary $N_f = 2 + 1$ JLQCD:**

J. Koponen talk at Lattice 2019

Möbius Domain Wall (relativistic): use multiple $m_h > m_c$ and **extrapolate to** $m_b$

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C. Bouchard talk at Lattice 2019

NRQCD heavy quarks.

**Several points with** $q^2 \sim 6 \text{GeV}^2$

**Other On-going:** $N_f = 2 + 1 + 1$ HPQCD (fully relativistic), FNAL/MILC (also $f_T$) for $B \to \pi(K)$ and $B \to K$ (close to finalized). $N_f = 2 + 1$ RBC/UKQCD.
Status exclusive $|V_{ub}|$ extraction

Alternative way of getting $|V_{ub}|$: $B_s \to K\ell\nu$.

Also,

$$f_0^+, (B_s \to K\ell\nu)/f_0^+ (B_s \to D_s\ell\nu)$$

to get $|V_{ub}/V_{cb}|$

- Three LQCD calculations of the relevant form factors: HPQCD 1406.2279, RBC/UKQCD 1501.05373, FNAL/MILC 1901.02561
- Many more on-going
- LQCD error smaller than for $B \to \pi$ form factors

**Experimentally:** Under investigación by LHCb, expected to be measured at the $\Upsilon(5S)$ run at Belle-II

(maybe 5-10% precision for the decay rate at Belle-II)
Form factors for $B \to D$ decays

$|V_{cb}|$ from $B \to D$ in acceptable agreement with either $B \to D^*$ exclusive and inclusive

$|V_{cb}|$ from $B \to D^*$ relies on form factor parametrizations to extrapolate experimental data to zero recoil (lattice data).

Many LQCD collaborations working on these decays (expect new results soon): HPQCD, FNAL/MILC, RBC/UKQCD, LANL-SWME...

- Preliminary results discussed by A. Lytle talk at Lattice19

Include latest Belle and BaBar $B \to D^*$ data
$B \to D^{(*)}\ell\nu$: Lepton Flavor Universality tests

\[ R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)} \]

\begin{align*}
\text{Belle} & \quad \text{SM prediction} \\
& \quad \text{contours}
\end{align*}

\begin{align*}
\text{Plot from G. Caria 1904.08794} \\
\text{Belle 2019:} & \quad R(D) = 0.307 \pm 0.037 \pm 0.016, \quad R(D^*) = 0.283 \pm 0.018 \pm 0.014 \\
& \quad \text{Belle 2019 consistent with SM, but world average at } \sim 3\sigma \text{ from SM}
\end{align*}

\textbf{Needs reliable LQCD form factors for } B \to D^* \text{ at non-zero recoil}
Use **BGL** parametrization to describe form factors shape. Currently subject to intensive cross-checks.

**Preliminary estimate of errors:** For $h_{A_1}(\omega = 1.1) \sim 1\%$
**$B \rightarrow D^*\ell\nu$ beyond zero recoil**

**Preliminary:** $N_f = 2 + 1$ JLQCD (Plots courtesy T. Kaneko)

- Only statistical errors included so far
- Different colors are different $a$ (Red = smallest $a$)

$$R_1 = h_V / h_{A_1}$$

Consistency among LQCD, BGL, CLN, HQET

- **New:** $B_s \rightarrow D_s^*\ell\nu$ at zero recoil E. McLean et al 1904.02046 (relativistic $b$)
More $b \to c\ell\nu$ LFU tests

Can help to shed light on existing tensions

$$R(X \to Y) = \frac{\mathcal{B}(X \to Y\tau\nu)}{\mathcal{B}(X \to Y\mu\nu)}$$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Theoretical (Lattice) Prediction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(B_s \to D_s)$</td>
<td>$0.2987(46)$, HPQCD 1906.00701 New [HPQCD 1703.09728, FNAL/MILC 1202.6346, ETMC 1310.5238]</td>
<td>LHCb</td>
</tr>
<tr>
<td>$f^+<em>{B_s \to K(0)}/f^+</em>{B_s \to D_s(0)}$ for $</td>
<td>V_{ub}</td>
<td>/</td>
</tr>
<tr>
<td>$R(B_s \to D^*_s)$</td>
<td>in progress (longer term): similar (easier) to $R(D^*)$</td>
<td>LHCb</td>
</tr>
<tr>
<td>$R(B_c \to J/\psi)$</td>
<td>in progress: A. Lytle Lattice19 <strong>Preliminary 0.290(7)</strong></td>
<td>0.71 ± 0.25 LHCb 1711.05623</td>
</tr>
<tr>
<td>$R(\Lambda_b \to \Lambda_c)$</td>
<td>0.333 ± 0.010 **</td>
<td>LHCb</td>
</tr>
<tr>
<td>$R(\Lambda_b \to \Lambda_c^*)$</td>
<td>in progress: S. Meinel, G. Rendón</td>
<td>LHCb</td>
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</table>

Neutral-current $b$ decays

Flavor-changing neutral currents $b \to q$ transitions are potentially sensitive to NP effects $B \to K^*\gamma$, $B \to K(\ast)\ell^+\ell^-$, $B \to \pi\ell^+\ell^-$

Sets of tensions between SM predictions and experimentally measured $b \to s\ell^+\ell^-$ observables

**Branching fraction measurements:** $B^0 \to K^{*0}\mu^+\mu^-$, $B^+ \to K^{(*)+}\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$

**Angular analyses:** $B^+ \to K^{(*)+}\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$

**Tests of Lepton Flavour Universality ($\mu/e$):**
$B^0 \to K^{*0}\mu^+\mu^-$, $B^+ \to K^{(*)+}\mu^+\mu^-$

Very small sensitivity to hadronic form factors $\sim 10^{-4}$

$$R_{K^{(*)}}(q^2_{\text{min}}, q^2_{\text{max}}) \equiv \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 dB(B \to K^{(*)}\mu^+\mu^-)}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 dB(B \to K^{(*)}e^+e^-)}$$

See talks at **Tuesday afternoon session**
Neutral-current $b$ decays: Lattice inputs

For $B \rightarrow P \ell \ell$, hadronic contributions are parametrized in terms of matrix elements of current (vector, axial and tensor) operators through three form factors $f_+ , f_0$ (for $m_\ell \neq 0$) and $f_T$ + non-factorizable contributions

Allow the calculation of branching fractions, angular observables and LFUV quantities

(Tests against experiment, extract CKM matrix elements $|V_{td,ts}|$, constrain Wilson coefficients $C_9$ and $C_{10}$)

- Non-factorizable contributions under control? New physics or charm-loops? See K. Nakayama (JLQCD), talk at Lattice 2019

Elvira Gámiz Lattice inputs for $B$ flavour physics
Current status: Form factors for $B \rightarrow K\ell^+\ell^-$

$B \rightarrow K\ell^+\ell^-$: HPQCD 1306.0434, 1306.2384, FNAL/MILC, 1509.06235

Statistically correlated, different systematics.

- HPQCD: NRQCD $b +$ HISQ $u, d, s$
- FNAL/MILC: Fermilab $b +$ asqtad $u, d, s$

Consistent results for $f_0,+,T$, and with LCSR

Khodjamarian et al 1006.4945
Form factors for $B \to K \ell^+\ell^-$

From D. Du et al 1510.02349, FNAL/MILC 1509.06235 (non-factorizable contributions under control?)

$1 - 2\sigma$ experiment-SM tensions.

Focus on large bins above and below charmonium resonances.
Form factors for $B \to \pi \ell^+ \ell^-$

**FNAL/MILC, 1507.01618, D. Du et al. 1510.02349** Take $f_+$ and $f_0$ from combined fit of lattice + experimental data for $B \to \pi \ell \nu$ (assume not significant NP effects at tree level).

The largest error is the one from the form factors.

- **D. Du et al. 1510.02349** $R^{SM}_\pi = \frac{B(B \to \pi \tau \nu_{\tau})}{B(B \to \pi \ell \nu)} = 0.641(17)$.
- Expected to be measured at Belle-II, possible to determine at LHCb

Elvira Gámiz  Lattice inputs for $B$ flavour physics
Rare semileptonic $B$ decays to $\nu\bar{\nu}$ states

D. Du et al. 1510.02349 with FNAL/MILC form factors

**Predictions** for both neutral and charged channels: complementary information (also $|V_{td,ts}|$)

- Theoretically clean (no problem with charm LD contributions)
- Difficult to measure experimentally. **Belle-II** expected precision $\sim 10\%$ for $B \rightarrow K$

\[
\mathcal{B}(B^0 \rightarrow \pi^0\nu\bar{\nu}) \cdot 10^7 = 0.668(41)(49)(16); \quad \mathcal{B}(B^0 \rightarrow K^0\nu\bar{\nu}) \cdot 10^7 = 40.1(2.2)(4.3)(0.9)
\]

\[
\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu}) \cdot 10^6 = 9.62(1)(92); \quad \mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu}) \cdot 10^6 = 4.94(52)(6)
\]
Future lattice inputs for $B$ semileptonic

- Lattice on-going (and planned) calculations should reduce form factors errors considerably: For $B \to K(\pi)\ell\ell$

  **FNAL/MILC, Z. Gelzer** talk at Lattice19, **HPQCD** (both relativistic and NRQCD $b$)

  - Compared to previous FNAL/MILC analyses expect: similar statistics, smaller discret. errors (HISQ) and remove chiral extrapolation errors (phys. masses)

- Theoretical framework for semileptonic $B$ decays to unstable vector meson final states exists **Briceño et al 1406.5965, Agadjanov et al 1605.03386, Hansen, Meyer, Robaina 1704.08993**

  - Pilot studies of form factors for $B_s \to K^*(\to K\pi)\ell\nu$, $B \to K^*(\to K\pi)\ell\ell$,.. underway **Leskovec, Meinel, Rendón**; See **Rendón** talk at Lattice 2019

- Long-distance effects (and, in particular, charm loops) under control?

  - New ideas to study those effects on the lattice **K. Nakayama** talk at Lattice19 (Ishikawa, Hashimoto)

    (similar formulation to LD effects in $K \to \pi\ell\ell$ **C. Sachrajda** talk)

  - Exploratory studies of $B$ inclusive semileptonic decays **JLQCD** (see,
### Summary of lattice inputs

<table>
<thead>
<tr>
<th>Quantity</th>
<th>current error</th>
<th>2025 Projections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{D_s}$</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>$f_{D_s}/f_{D^+}$</td>
<td>0.14%</td>
<td></td>
</tr>
<tr>
<td>$f_{B_s}$</td>
<td>0.56%</td>
<td></td>
</tr>
<tr>
<td>$f_{B_s}/f_{B^+}$</td>
<td>0.4%</td>
<td></td>
</tr>
<tr>
<td>$B_{B_s}$</td>
<td>4%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$B_{B_s}/B_{B_d}$</td>
<td>2.5</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.3%</td>
<td>0.3%</td>
</tr>
<tr>
<td>$\bar{m}_c(m_c)$</td>
<td>0.4%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

| $B \to \pi$ for $|V_{ub}|_{\text{theor}}$ | 2.9% | 1%(1.4%) |
| $B \to D$ for $|V_{cb}|_{\text{theor}}$ | 1.4% | 0.3%(1%) |
| (first param. BCL z-exp.) | 1.5% | 0.5%(1.1%) |
| $B \to D^*$ for $|V_{cb}|_{\text{theor}}$ | 1.4% | 0.4%(0.7%) |
| $h^{B \to D^*}_{A_1}(\omega = 1)$ |                  | 1-1.5% |
| $P^{B \to D^*}_{1}(\omega = 1)$ |                  |        |
| $\Lambda_b \to p(\Lambda_c)$ |                  |        |
| for $|V_{ub}/V_{cb}|_{\text{theor}}$ | 4.9% | 1.2%(1.6%) |

| $B \to K$ (first param. BCL z-exp.) | 2% | 0.7%(1.2%) |
| $B_{s} \to K$ (first param. BCL z-exp.) | 4% | 1.3%(1.7%) |

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**M. Della Morte, E. Lunghi, E. G.** for *Opportunities in Flavour Physics at the HL-LHC and HE-LHC, 1812.07638 (with a few updates)*
Backup slides
Heavy-Light mesons decay constants

Example: **FNAL/MILC, 1712.09262**

- $\Phi_{Hx} \equiv f_{Hx} \sqrt{M_{Hx}}$ with $H_x$ a heavy-light meson $\bar{h}x$.

- $f_{p4s}$ is a reference scale.

- Dashed vertical lines indicate the cut $am_h = 0.9$.

Use several **EFTs** to construct the fit functions

$$\Phi_{Hq} = (1 + \text{SET})(1 + \text{HQET})(1 + \text{HM}r\text{PQASChPT}) \left(\frac{m'_c}{m_c}\right)^{3/27} \Phi_0$$

- **Symanzik Effective Theory (SET):** $c_1\alpha_s(a\Lambda)^2 + ... + c_3\alpha_s(am_h)^2 + ...

(\text{FNAL/MILC small errors due to: highly improved action, physical light quark masses, no renormalization, MILC ensembles with fine lattice spacings (down to 0.042 fm)})