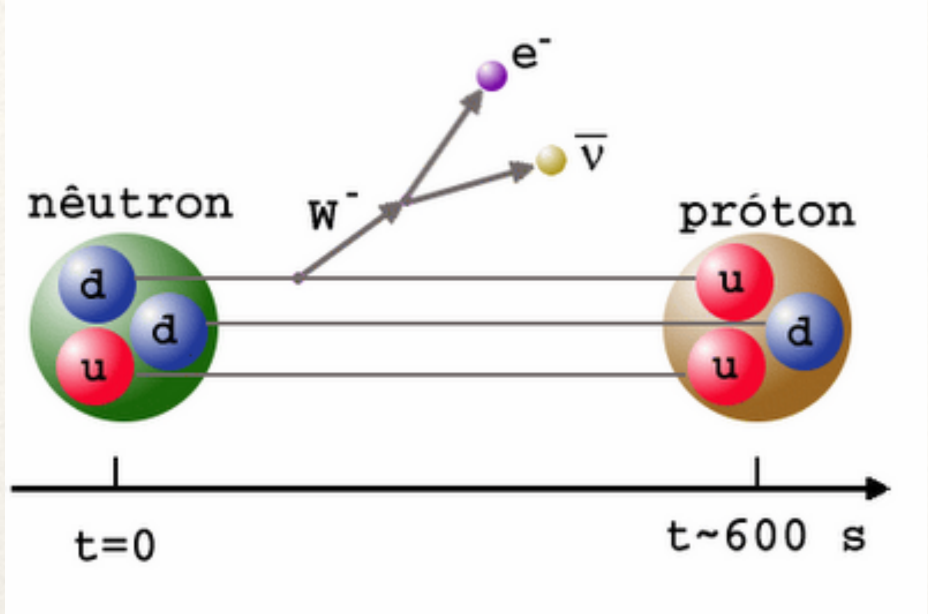
Status of Lattice Calculations of g_A

André Walker-Loud

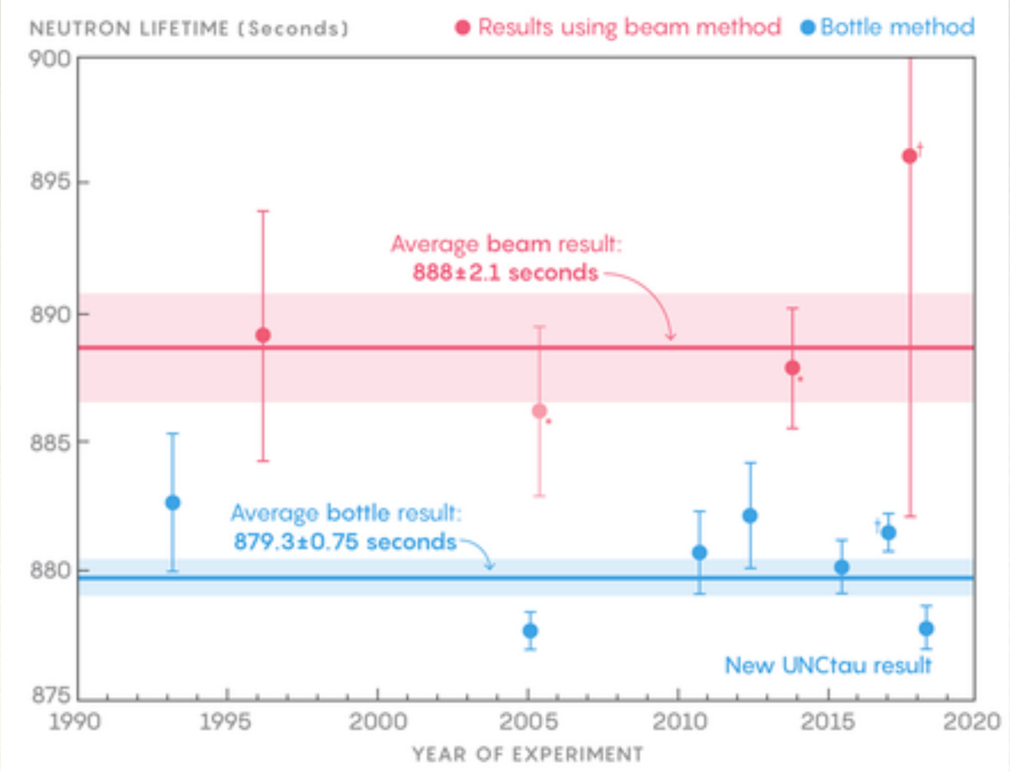
BERKELEY LAB

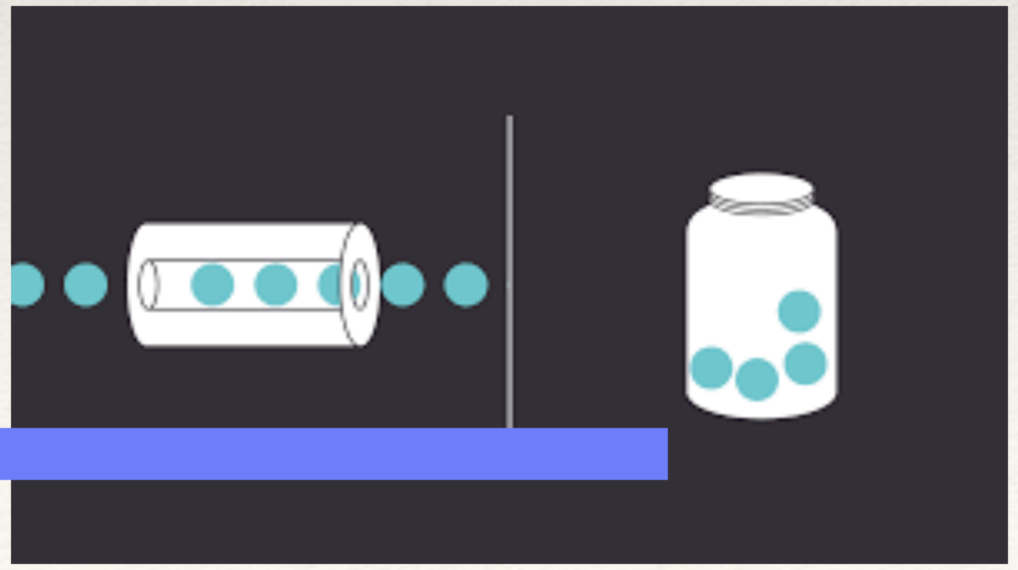
Neutron lifetime and the axial coupling



$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{ud}|^2}{2\pi^3} m_e^5 (1 + 3g_A^2)(1 + RC) f_{V,A}$$

- The neutron lifetime and g_A (neutron decay) are used to probe the limits of the Standard Model
- We should have a (meaningful) Standard Model prediction for g_A - LQCD (lattice QCD)
- To gain confidence in the application of LQCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest, such as g_A
- In order for the theoretical uncertainty on g_A to match the larger uncertainty in the neutron lifetime measurements, we must determine g_A with < 0.2% uncertainty - **is this crazy?**





$$\tau_n^{\text{beam}} = 888.0(2.0)s$$

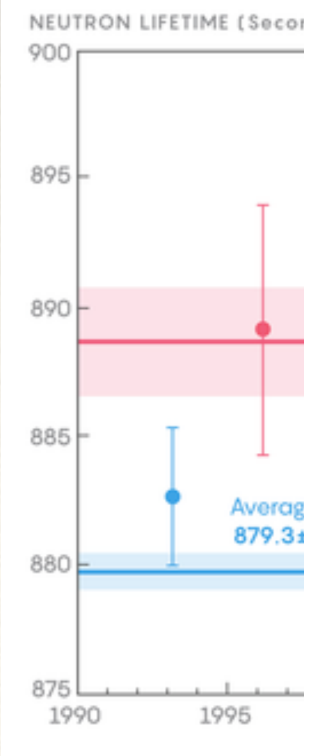
$$\tau_n^{\text{bottle}} = 879.4(0.6)s$$

I learned recently in Steven Clayton's (LANL) talk at MENU on their measurement of the neutron lifetime, that the PDG no longer considers this to be a discrepancy - they've dropped the beam measurements 😱
http://pdg.lbl.gov/2019/listings/rpp2019-list-n.pdf

In the words of my high school physics teacher - "when you do a measurement, and it disagrees with your prediction, just fudge the measurement because you know you are right!"

VALUE (s)	DOCUMENT ID	TECN	COMMENT
879.4 ± 0.6 OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.		
878.3 ± 1.6 ± 1.0	EZHOV 18	CNTR	UCN magneto-gravit. trap
$877.7 \pm 0.7^{+0.4}_{-0.2}$	[1] PATTIE 18	CNTR	UCN asym. magnetic trap
881.5 ± 0.7 ± 0.6	SEREBROV 18	CNTR	UCN gravitational trap
880.2 ± 1.2	[2] ARZUMANOV 15	CNTR	UCN double bottle
882.5 ± 1.4 ± 1.5	[3] STEYERL 12	CNTR	UCN material bottle
880.7 ± 1.3 ± 1.2	PICHLMAIER 10	CNTR	UCN material bottle
878.5 ± 0.7 ± 0.3	SEREBROV 05	CNTR	UCN gravitational trap
• • • We do not use the following data for averages, fits, limits, etc. • • •			
887.7 ± 1.2 ± 1.9	[4] YUE 13	CNTR	In-beam *n*, trapped *p*
881.6 ± 0.8 ± 1.9	[5] ARZUMANOV 12	CNTR	See ARZUMANOV 15
886.3 ± 1.2 ± 3.2	NICO 05	CNTR	See YUE 13
886.8 ± 1.2 ± 3.2	DEWEY 03	CNTR	See NICO 05
885.4 ± 0.9 ± 0.4	ARZUMANOV 00	CNTR	See ARZUMANOV 12
889.2 ± 3.0 ± 3.8	BYRNE 96	CNTR	Penning trap

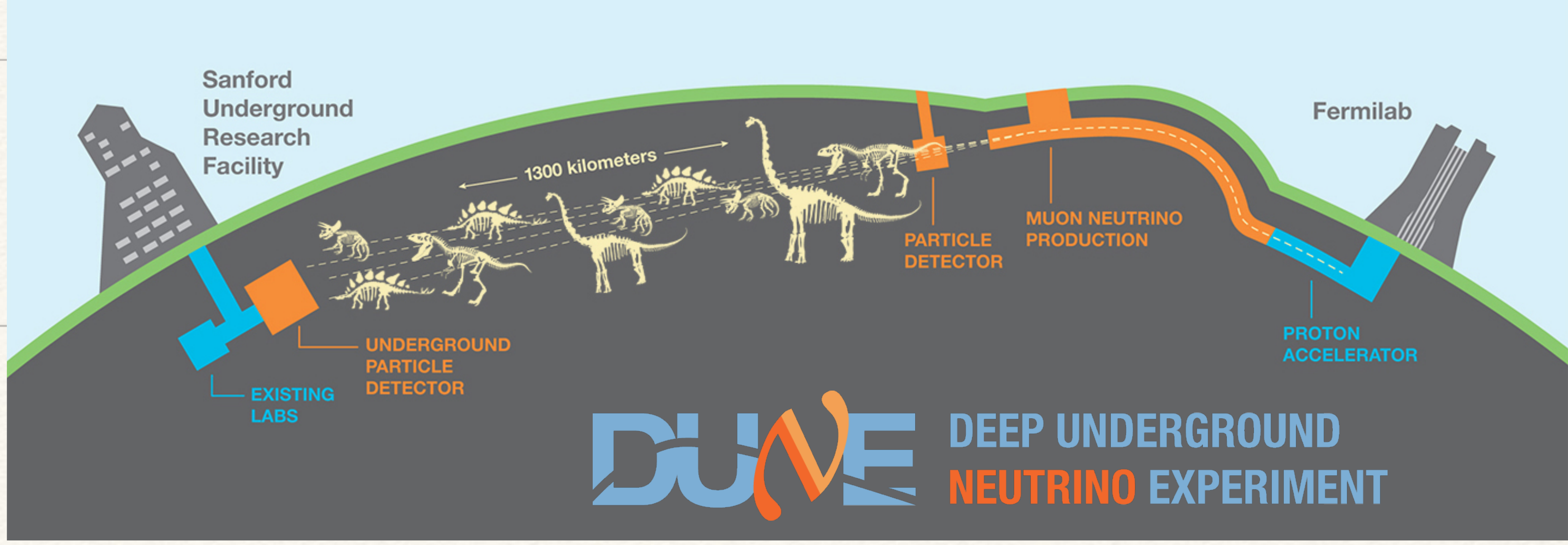
- We should have ...
- To gain confide... k (calibrate) our calculations aga...
- In order for the ... utron lifetime measurements, ...



s

s

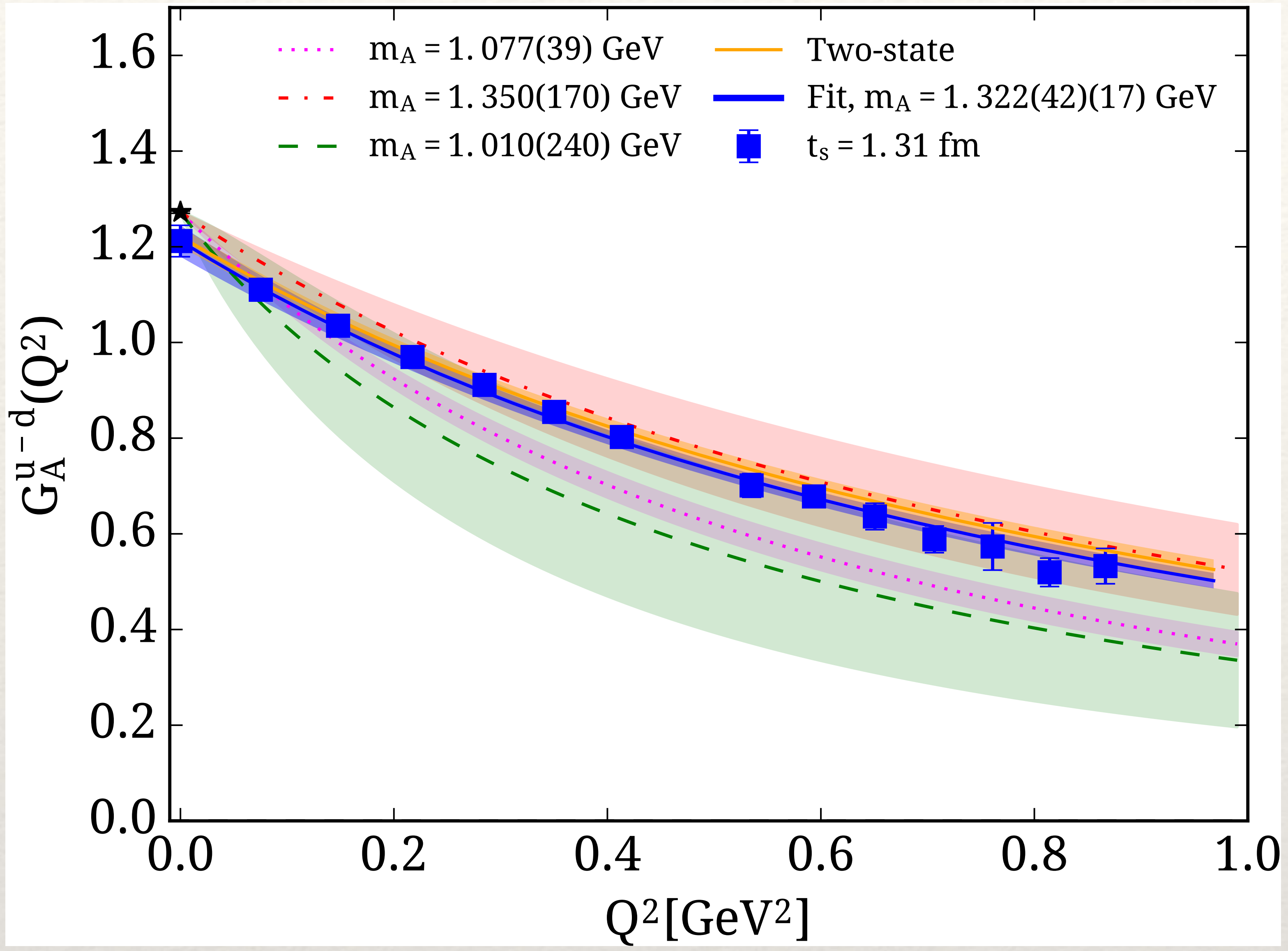
Nucleon Axial FormFactor



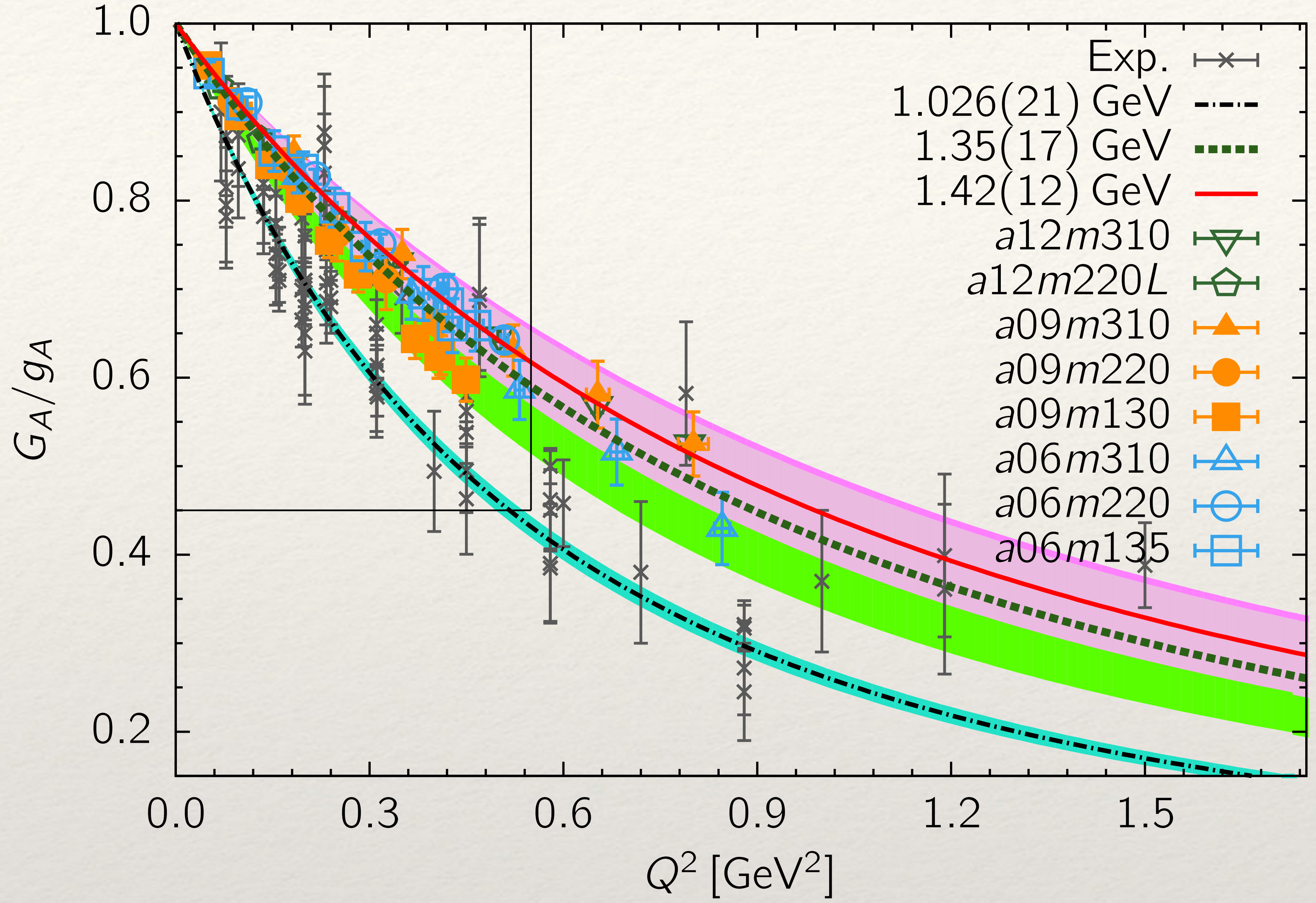
- DUNE is a future neutrino oscillation experiment that will fire a beam of neutrinos from FNAL into an Argonne target in South Dakota.
  - A determination of the CP-violating phase in the neutrino-mixing (PMNS) matrix is one of the goals
  - enough CP violation could explain the matter/anti-matter asymmetry of the universe through Leptogenesis
- The T2K and NOVA experiments are also conducting oscillation experiments
  - "A determination of the nucleon axial form factor at the 5% level would be very helpful, possibly allowing for the isolation of nuclear effects" [private communications with T2K members, Y. Hayato and K. Mcfarland]
- Ultimately, we need to understand neutrino-NUCLEUS cross sections which begins with neutrino-nucleon cross sections
  - The experimental data on $g_A(Q^2)$ is sufficiently limited that a simple dipole-formfactor is assumed
  - The dipole model is too simplistic and overly constraining (the quoted uncertainties do not reflect the true uncertainty of our understanding)

Nucleon Axial FormFactor

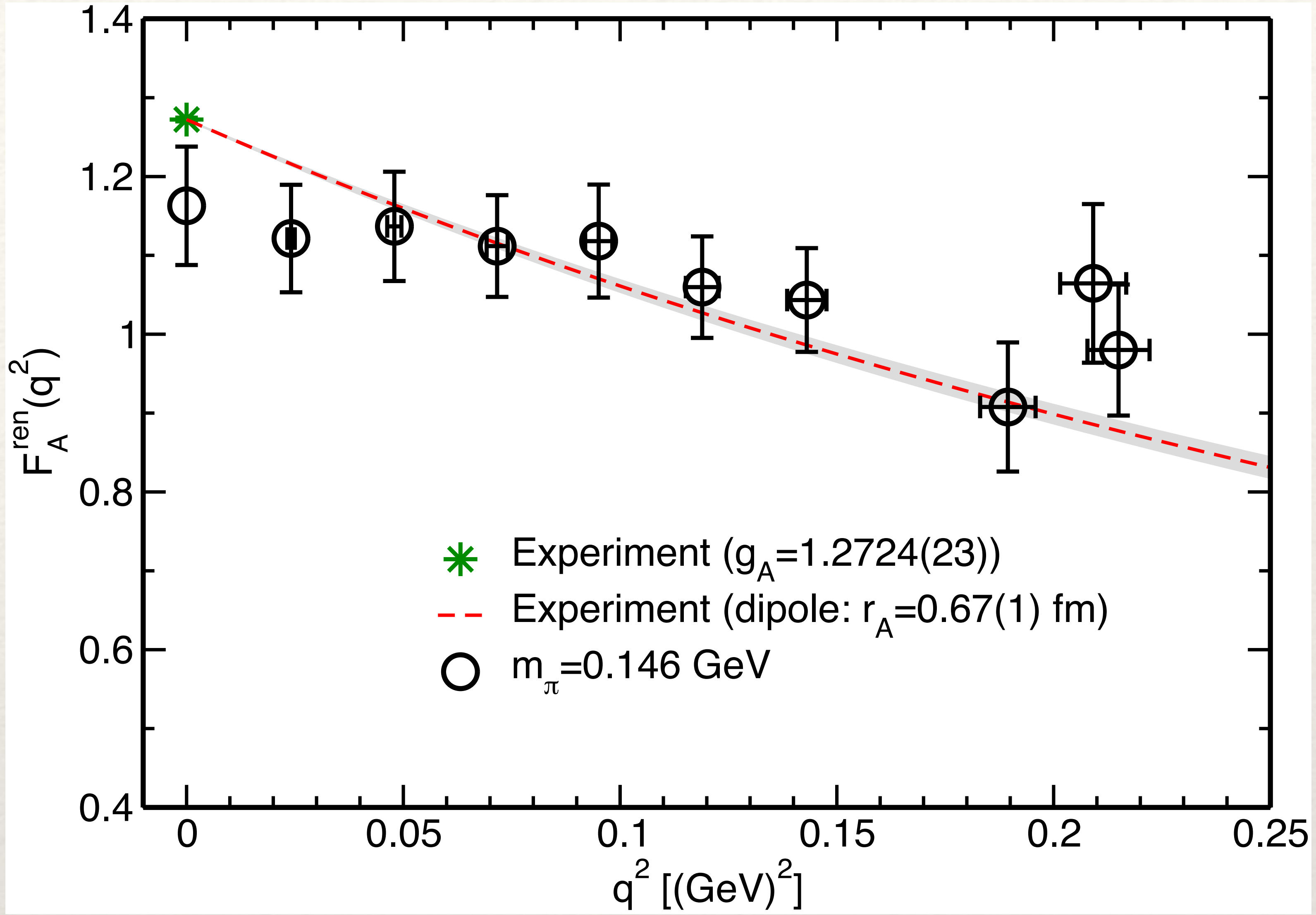
Alexandrou et al. PRD96 (2017) [1705.03399]



Gupta et al. PRD96 (2017) [1705.06834]

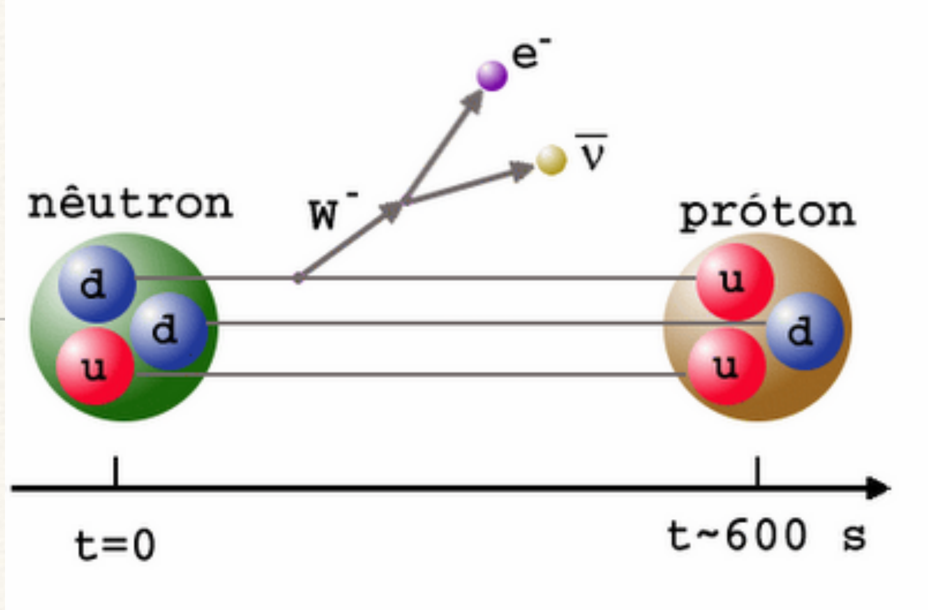


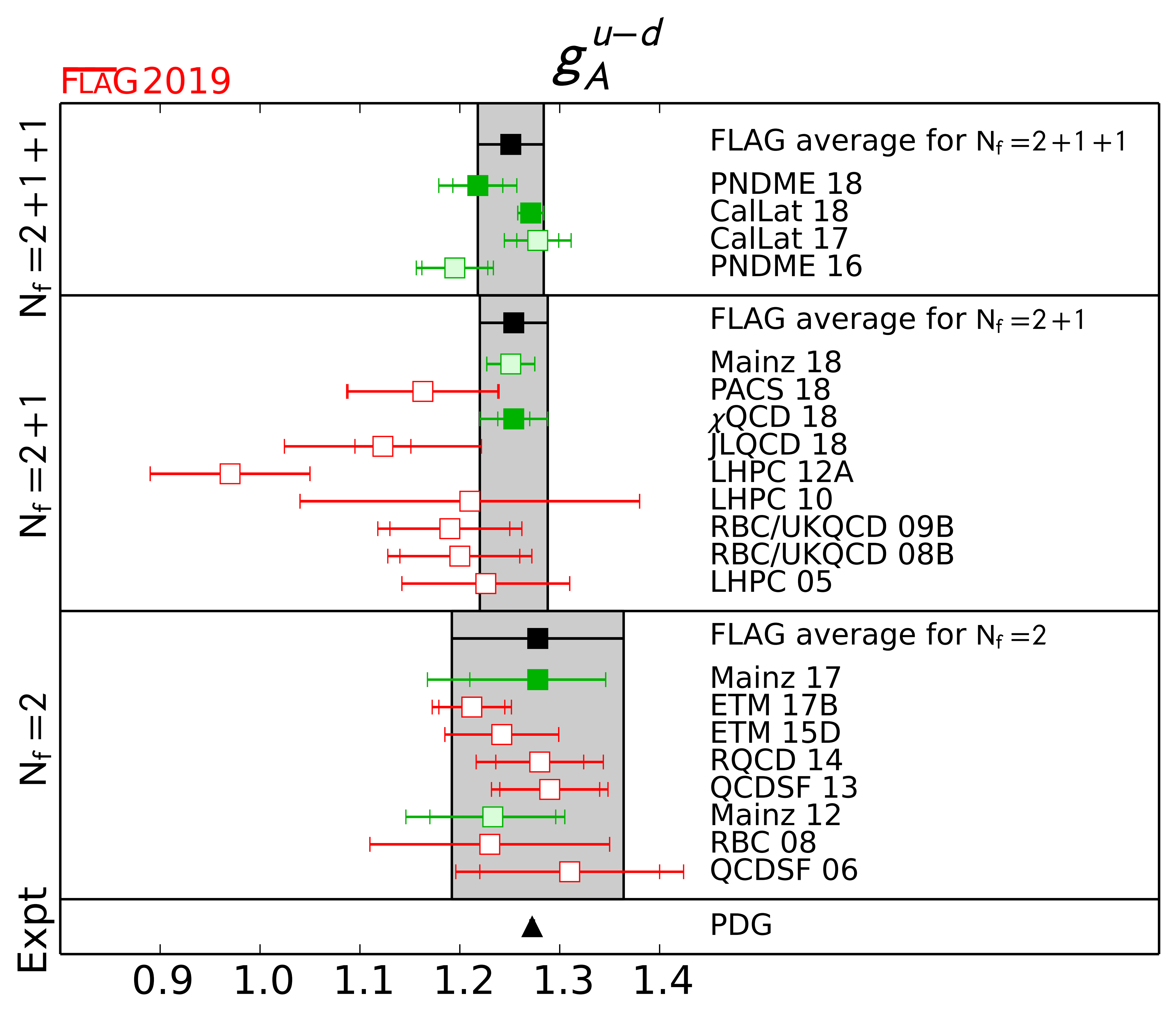
Ishikawa et al. PRD98 (2018) [1807.03974]



- All lattice QCD results determine an axial form factor with a significantly different slope (30%) than that determined from the phenomenological determination - two recent examples here
- Examining the LQCD results, it is difficult to understand/guess where this discrepancy is coming from
- A few years ago - this was the same situation with g_A (no one understood why gA results were consistently low compared to the experimental value)
- For g_A, we made progress by pushing to the extreme the LQCD calculations - a similar strategy here seems warranted

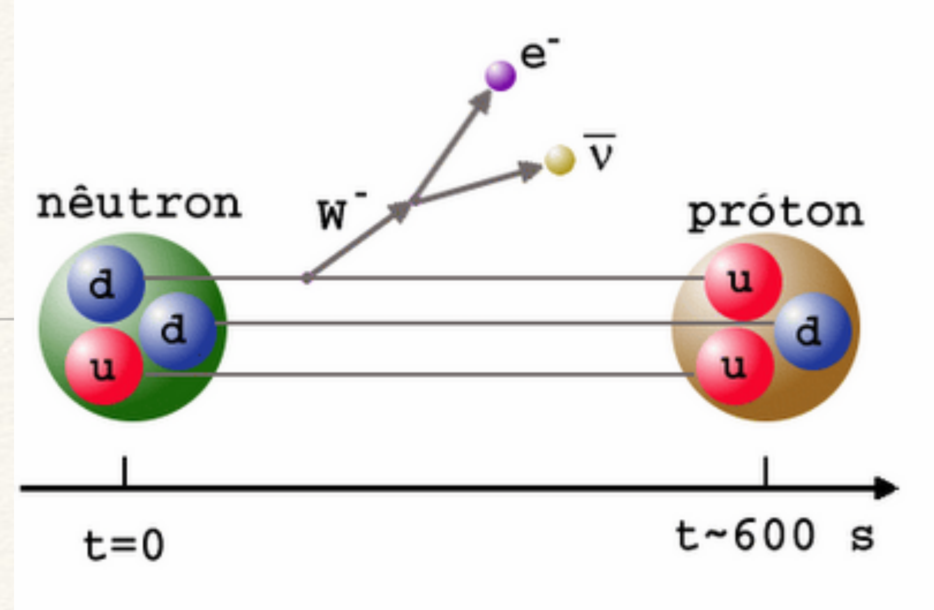
nucleon axial coupling from LQCD

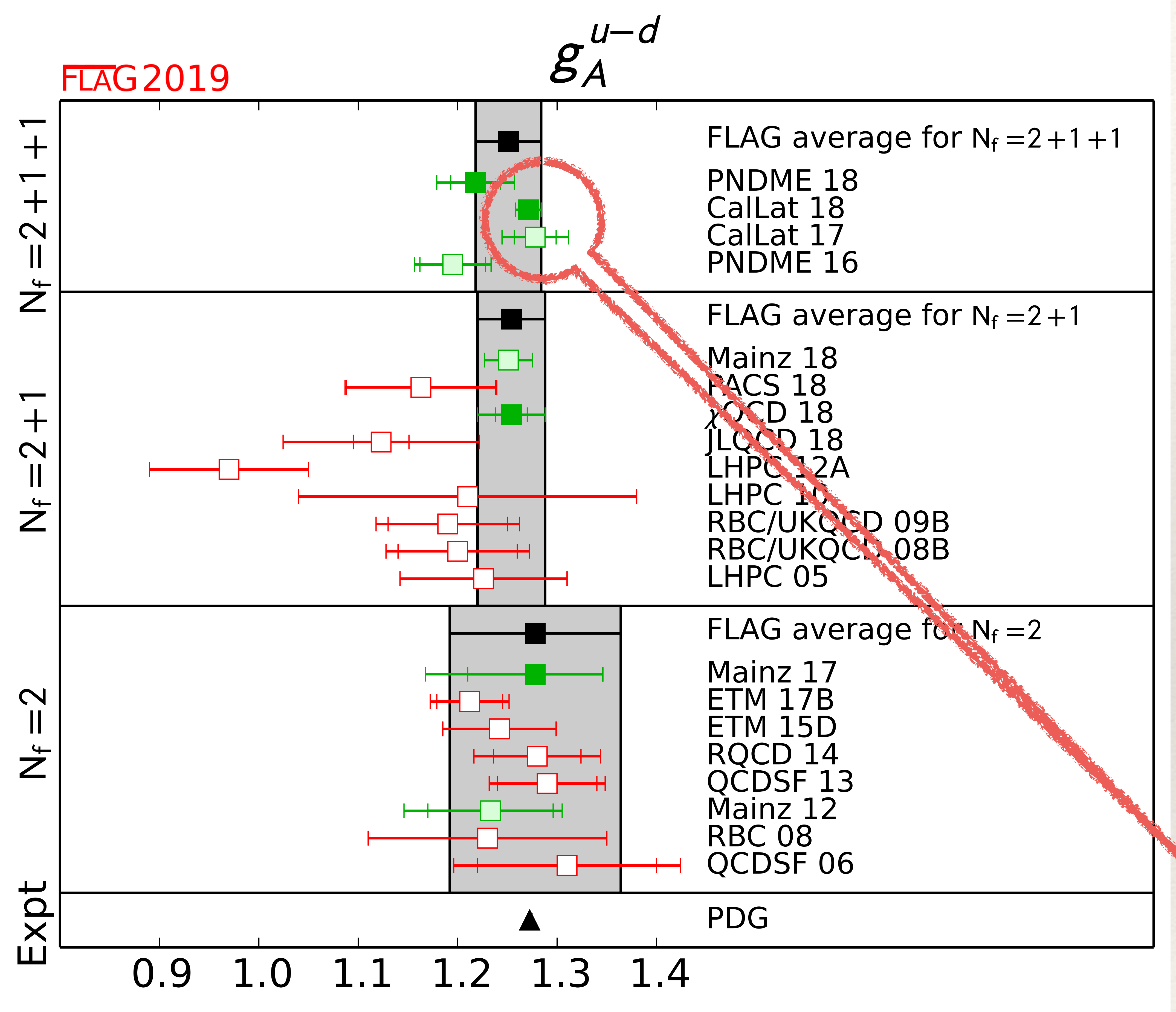




- To gain confidence in the application of Lattice QCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest
- g_A was supposed to be a good benchmark calculation for single nucleon structure - but it proved to have significant systematic challenges, preventing results with the precision anticipated
- FLAG 2019 has included single nucleon quantities in their averaging for the first time

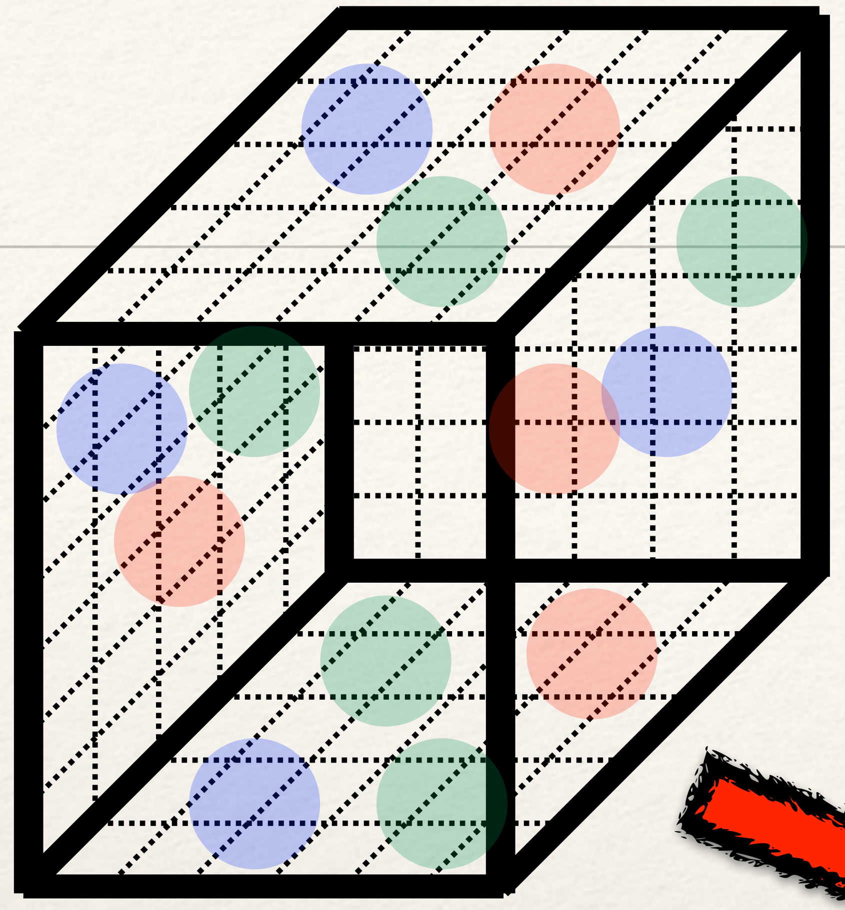
nucleon axial coupling from LQCD





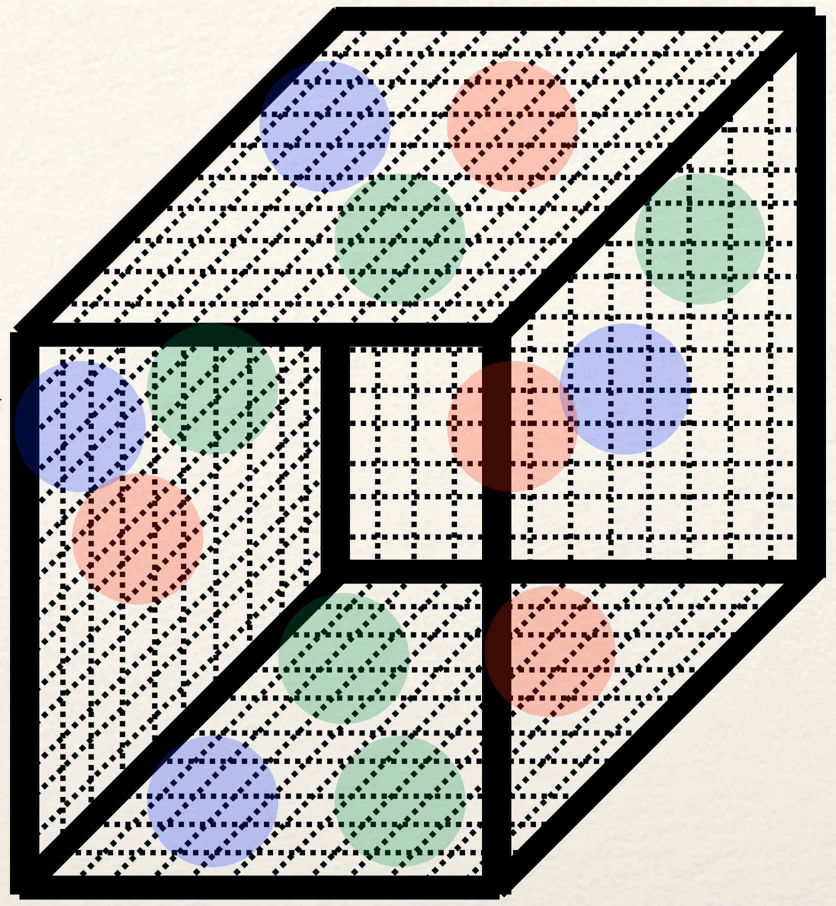
- To gain confidence in the application of Lattice QCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest
- g_A was supposed to be a good benchmark calculation for single nucleon structure - but it proved to have significant systematic challenges, preventing results with the precision anticipated
- FLAG 2019 has included single nucleon quantities in their averaging for the first time
- Notice one result is significantly more precise than the others

Standard Systematics



continuum limit
need 3 or more
lattice spacings

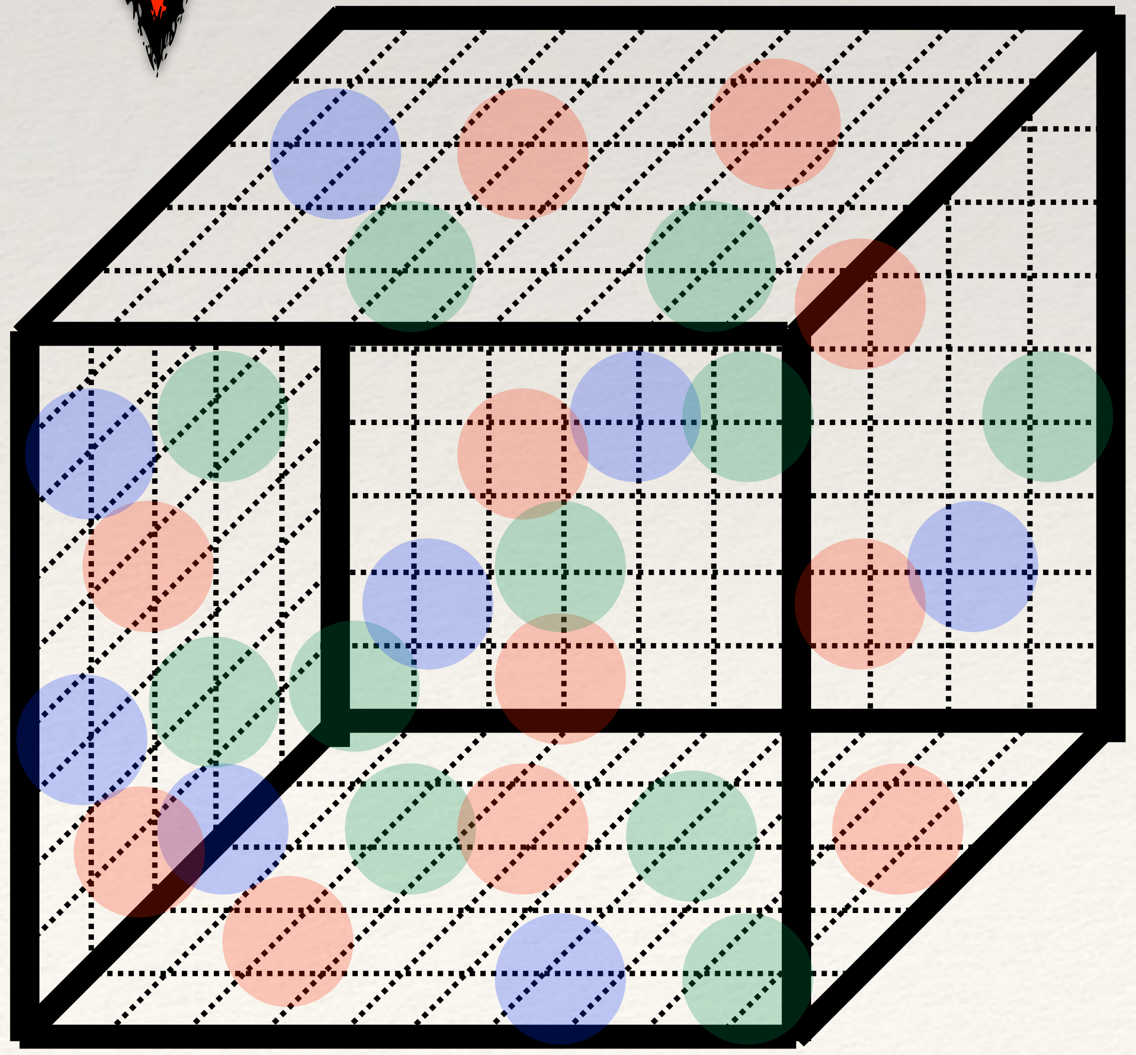
$$t_{comp} \propto \frac{1}{a^6}$$



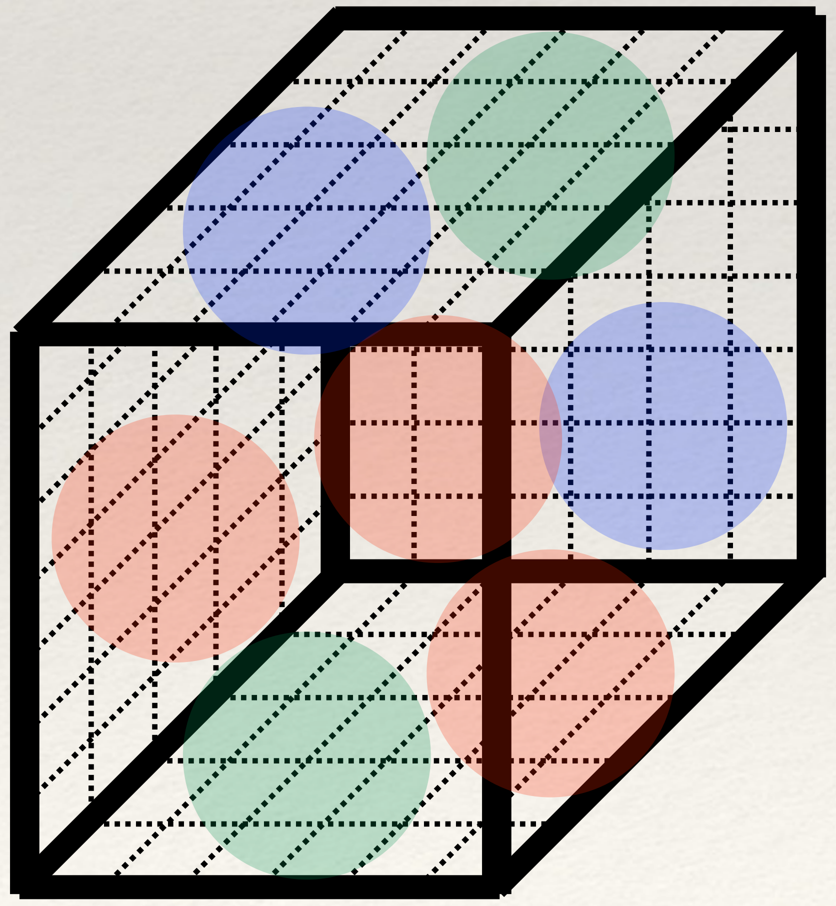
infinite volume limit

$$t_{comp} \propto V^{5/4}$$

$$V = N_L^3 \times N_T$$



physical pion masses
exponentially bad
signal-to-noise problem

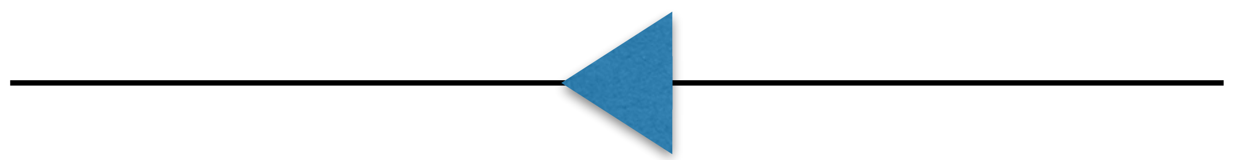


Slide adapted from E. Berkowitz

LQCD challenges for NP

Most difficult challenge: an exponentially bad signal-to-noise problem

Parisi, Phys. Rep. 103 (1984) 203



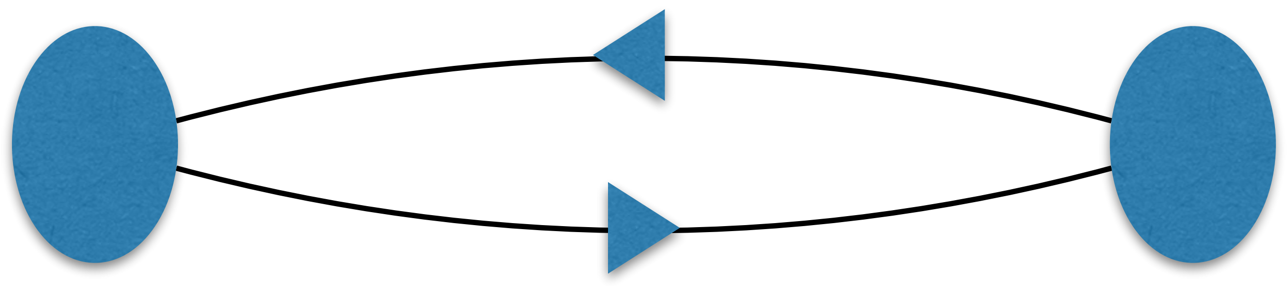
$$\sim e^{-\frac{1}{2} m_\pi t} + e^{-\frac{1}{3} m_N t} + \cdots$$

Lepage, TASI 1989

Each **quark propagator** carries information about pions and nucleons (conversations with David Kaplan)

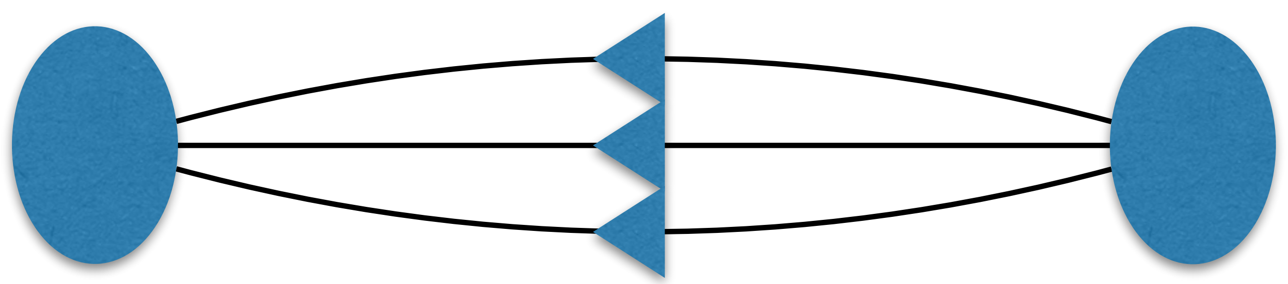
$$\lambda_\pi(t) \gg \lambda_N(t)$$

$$\lambda_i(t) \sim e^{-E_i t}$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

Large pion eigenvalues must cancel to expose small nucleon eigenvalues



$$(u^T C \gamma_5 d) u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N} \exp\left[-A \left(m_N - \frac{3}{2} m_\pi \right) t \right]$$

exponential noise
power-law statistics

LQCD challenges for NP

2-point correlation function

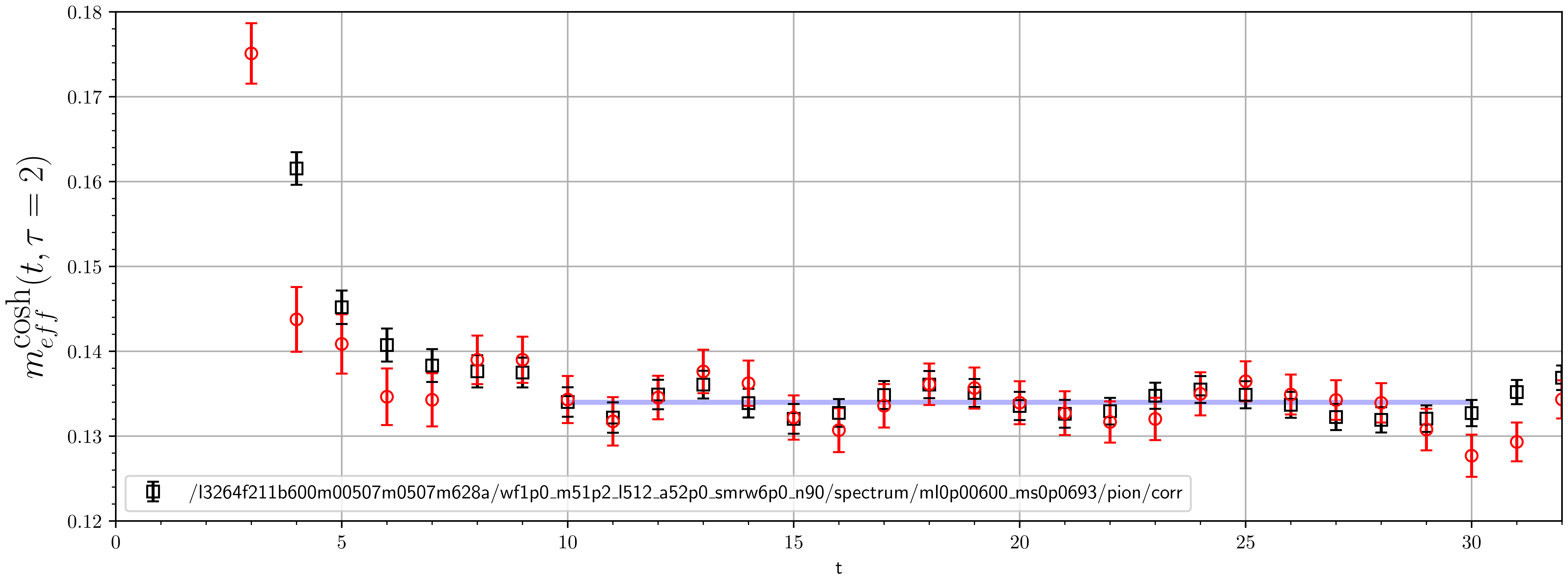
$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

$$m_{eff}(t) = \frac{1}{\tau} \ln\left(\frac{C(t)}{C(t+\tau)}\right)$$

For pions, need to consider leading finite temperature effects

$$C(t) = \sum_n z_n z_n^\dagger \left(e^{-E_n t} + e^{-E_n (T-t)}\right)$$

$$m_{eff}^{\cosh}(t,\tau) = \frac{1}{\tau}\cosh^{-1}\left(\frac{C(t+\tau)+C(t-\tau)}{2C(t)}\right)$$

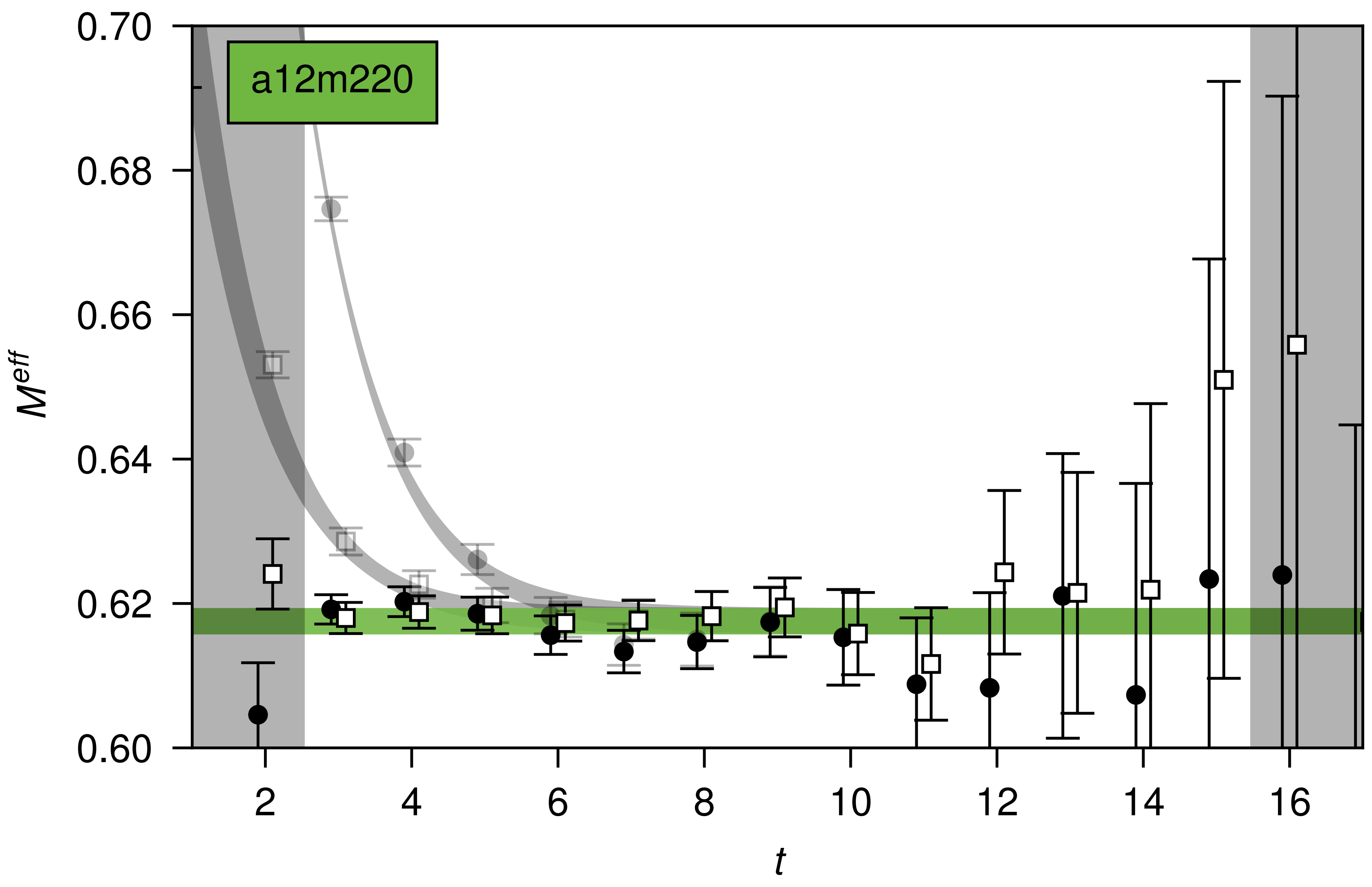


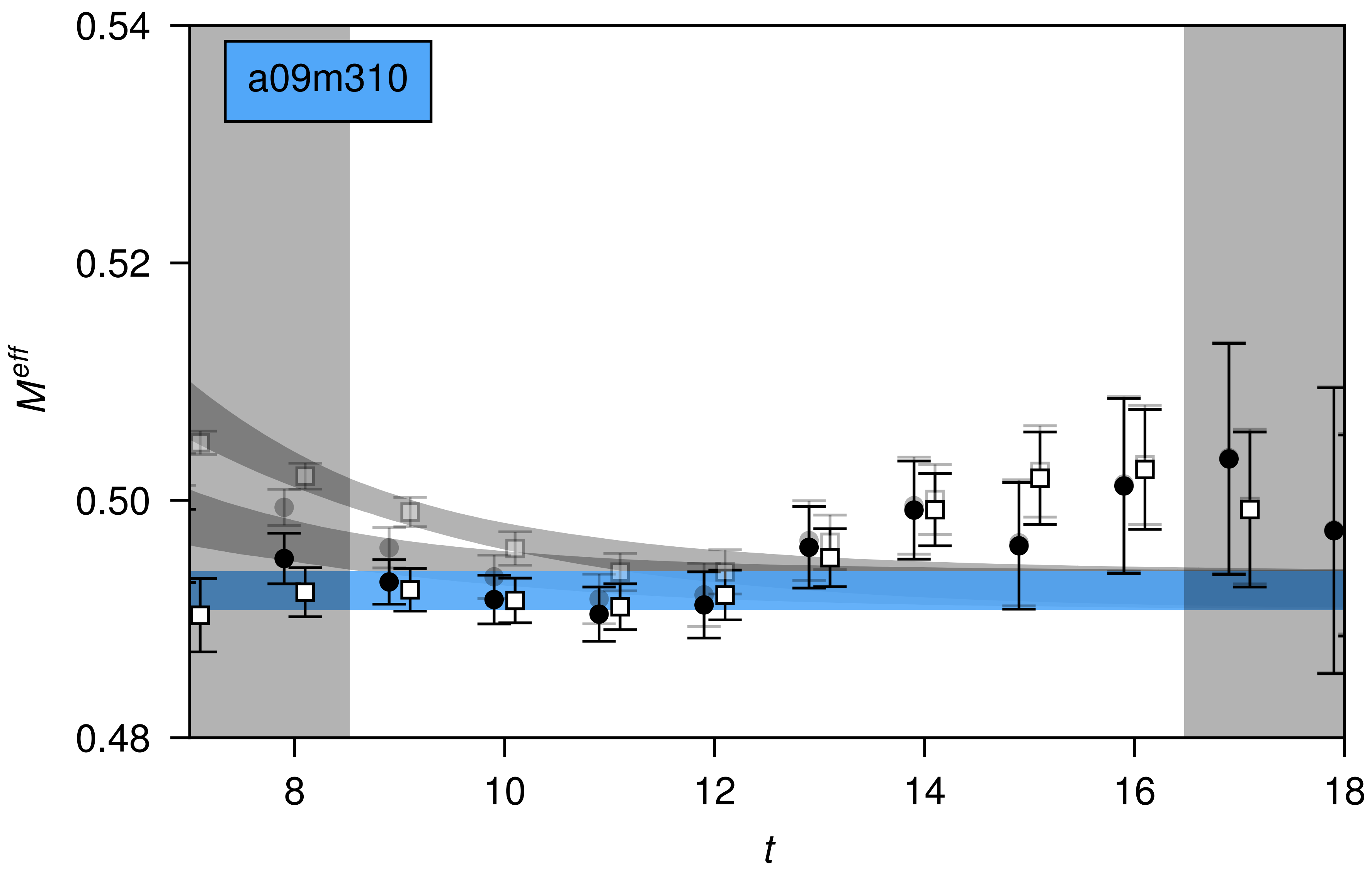
Effective mass of Pion 2-point correlation function
red and black “data” are from different choices of *interpolating* operators

Noise is constant in time - can determine very clean ground state (blue band)

LQCD challenges for NP

2-point correlation function





Two examples of nucleon effective mass

Correlated late-time fluctuations… what is the ground state?

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{Signal}{Noise} \rightarrow \sqrt{N_{state}}e^{-(m_N-\frac{3}{2}m_\pi)t}$$

Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

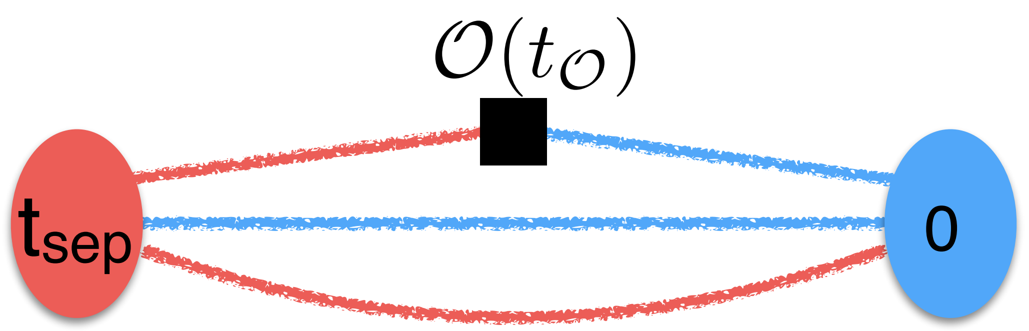
This problem is exacerbated with form-factor calculations (g_A) and 2+ nucleons

- quark contraction cost becomes dominant
- density of excited states grows significantly and gap becomes small (nuclear interaction energies instead of pion mass gap)

LQCD challenges for NP

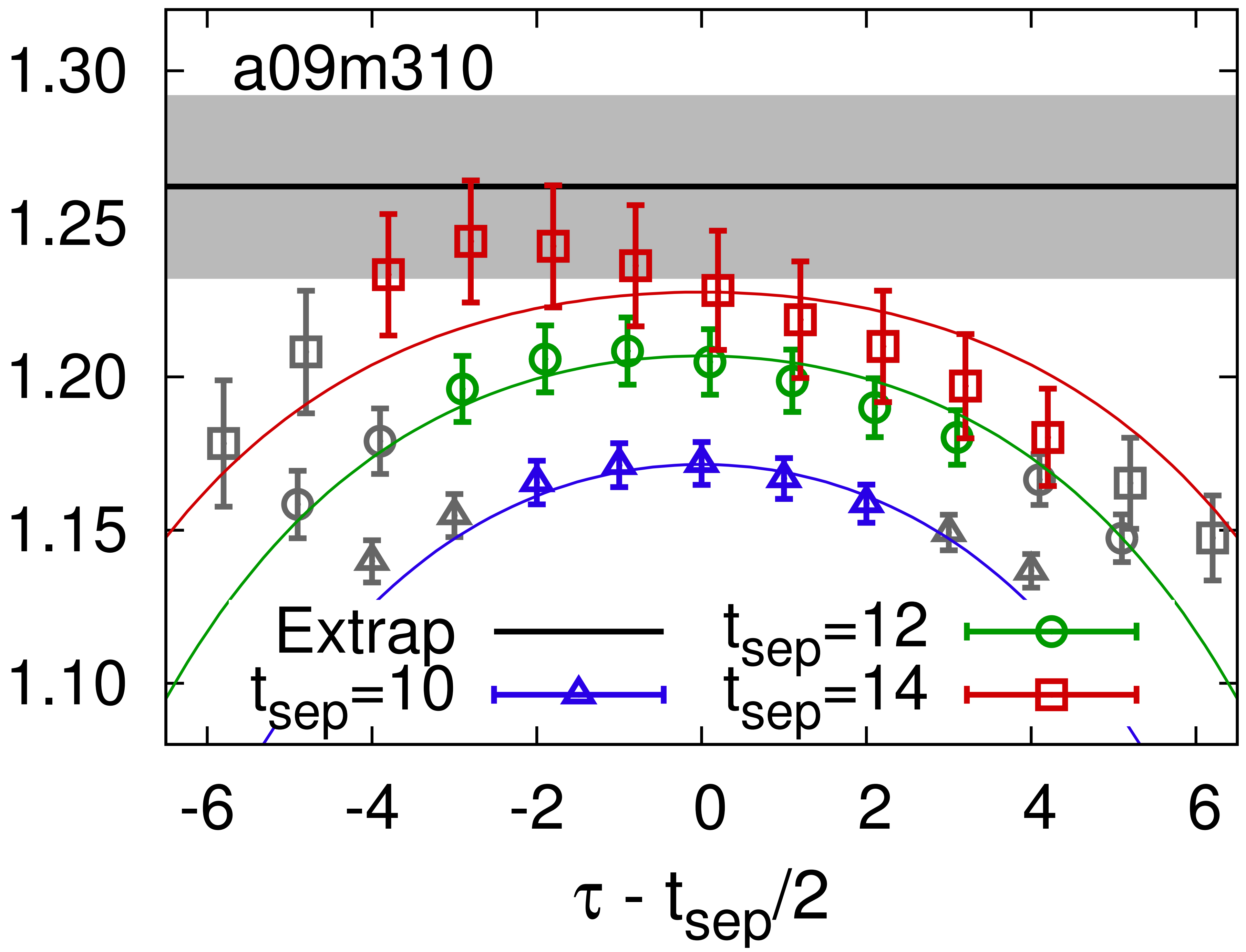
Nucleon axial charge calculation

fixed source-sink separation time, t_{sep}



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

in long-time (**t_{sep}**) limit - should be flat



Repeat for multiple values of **t_{sep}**

Extrapolate to **$t_{sep} \to \infty$**

t_{sep}	**fm**
14	**1.22**
12	**1.05**
10	**0.875**

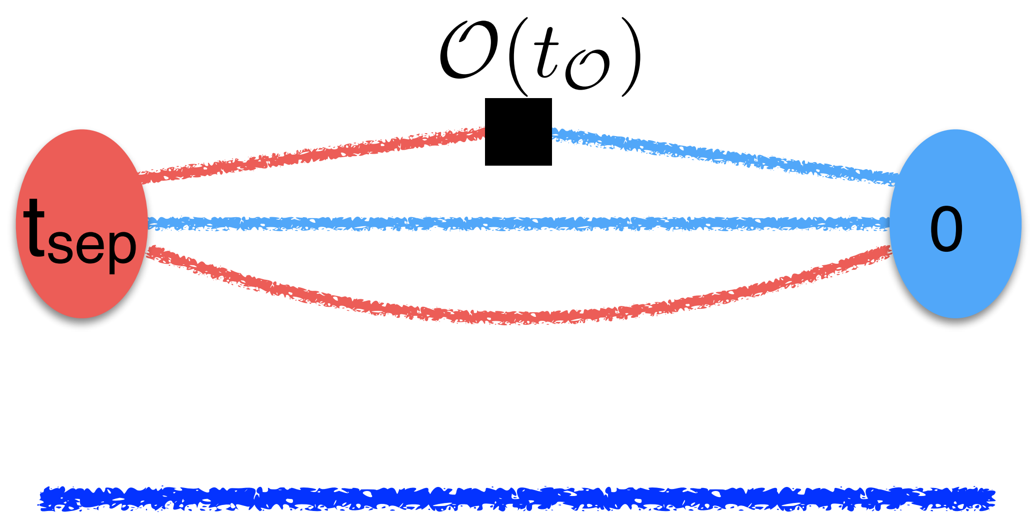
typical calculation

Bhattacharya, Cirigliano, Cohen, Gupta, Lin, Yoon Phys.Rev.D 94 (2016)

LQCD challenges for NP

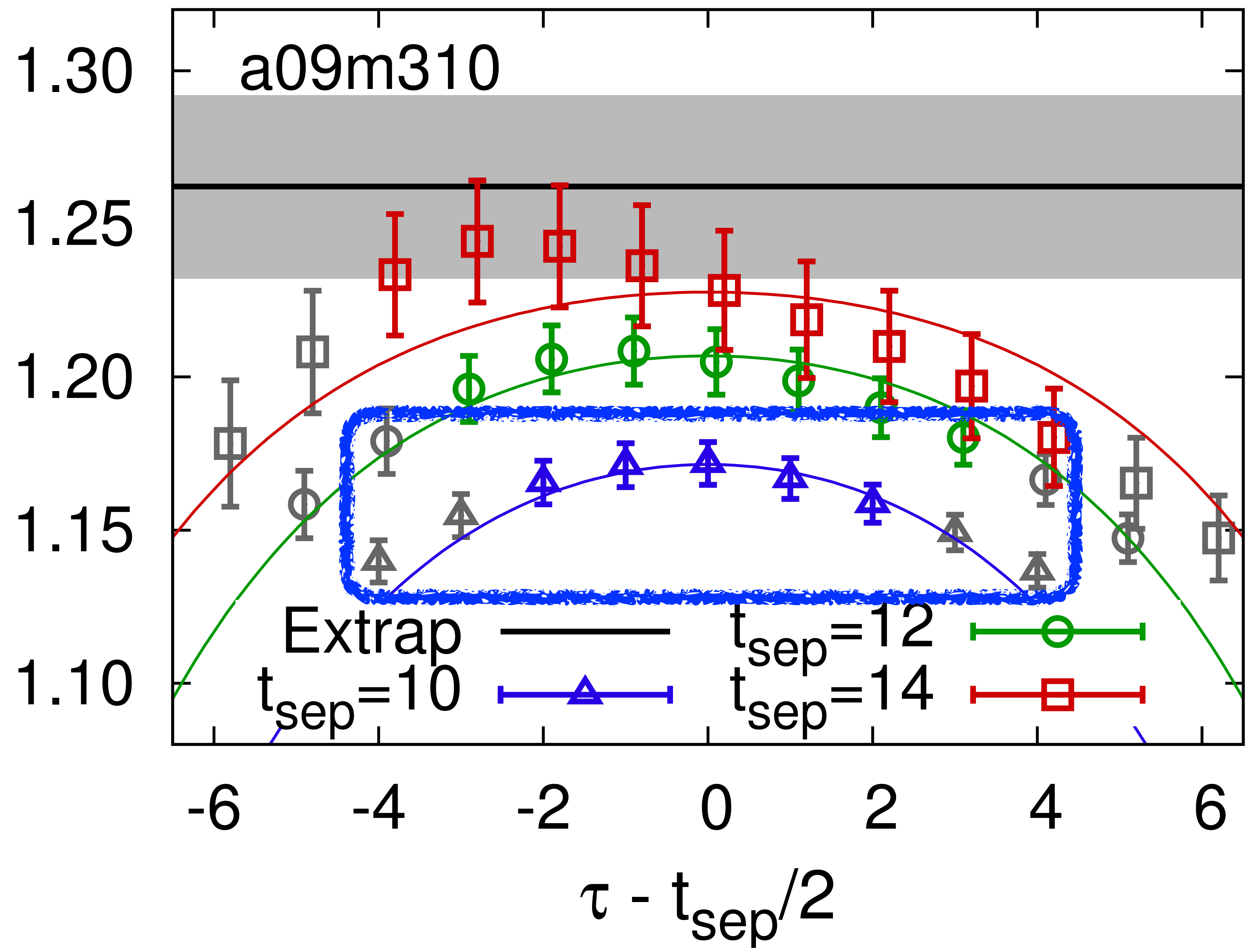
Nucleon axial charge calculation

fixed source-sink separation time, t_{sep}



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

in long-time (**t_{sep}**) limit - should be flat



Repeat for multiple values of **t_{sep}**

Extrapolate to **t_{sep}**$\rightarrow \infty$

t_{sep}	fm
14	1.22
12	1.05
10	0.875

typical calculation

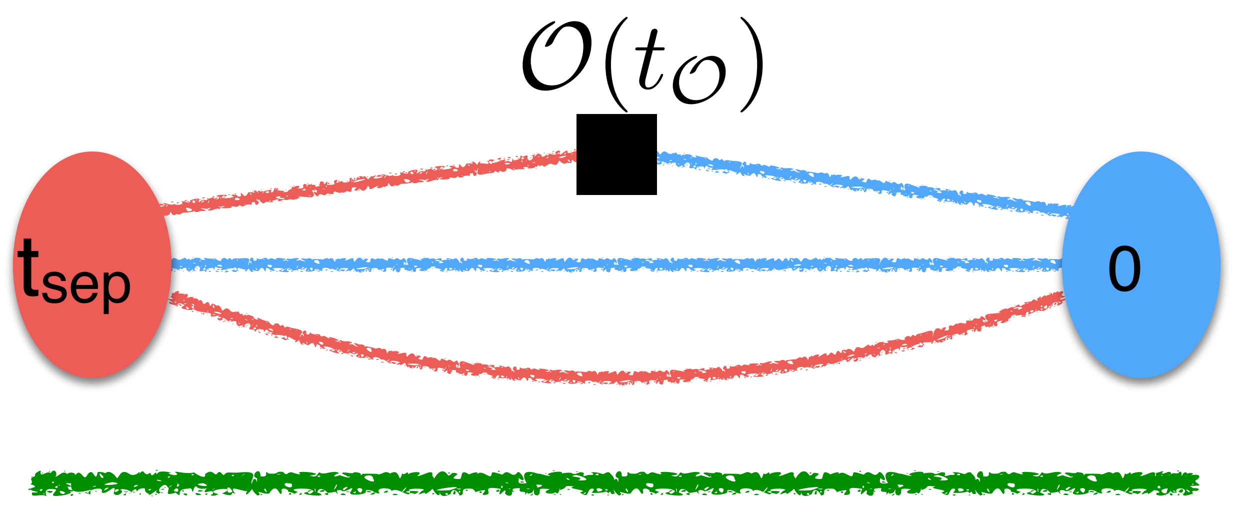
Bhattacharya, Cirigliano, Cohen, Gupta, Lin, Yoon

Phys.Rev.D 94 (2016)

LQCD challenges for NP

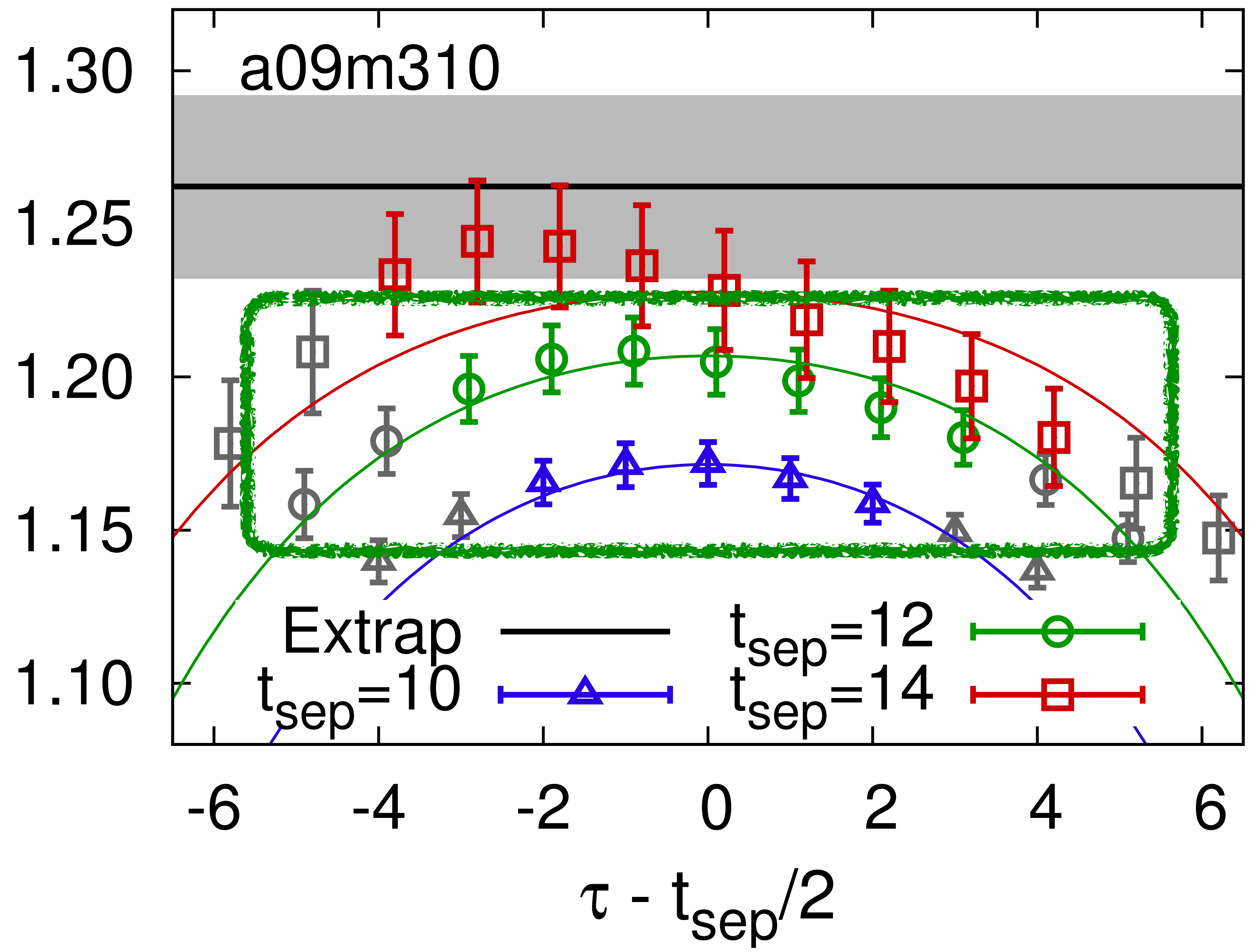
Nucleon axial charge calculation

fixed source-sink separation time, t_{sep}



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

in long-time (**t_{sep}**) limit - should be flat



Repeat for multiple values of **t_{sep}**
Extrapolate to **$t_{sep} \to \infty$**

t_{sep}	fm
14	1.22
12	1.05
10	0.875

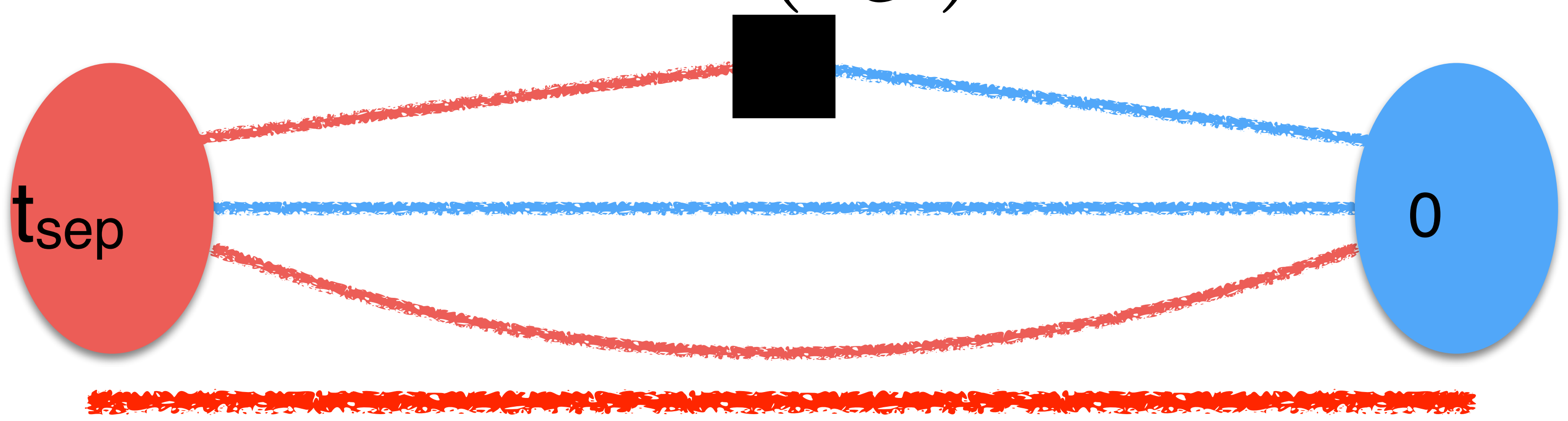
typical calculation
Bhattacharya, Cirigliano, Cohen, Gupta, Lin, Yoon
Phys.Rev.D 94 (2016)

LQCD challenges for NP

Nucleon axial charge calculation

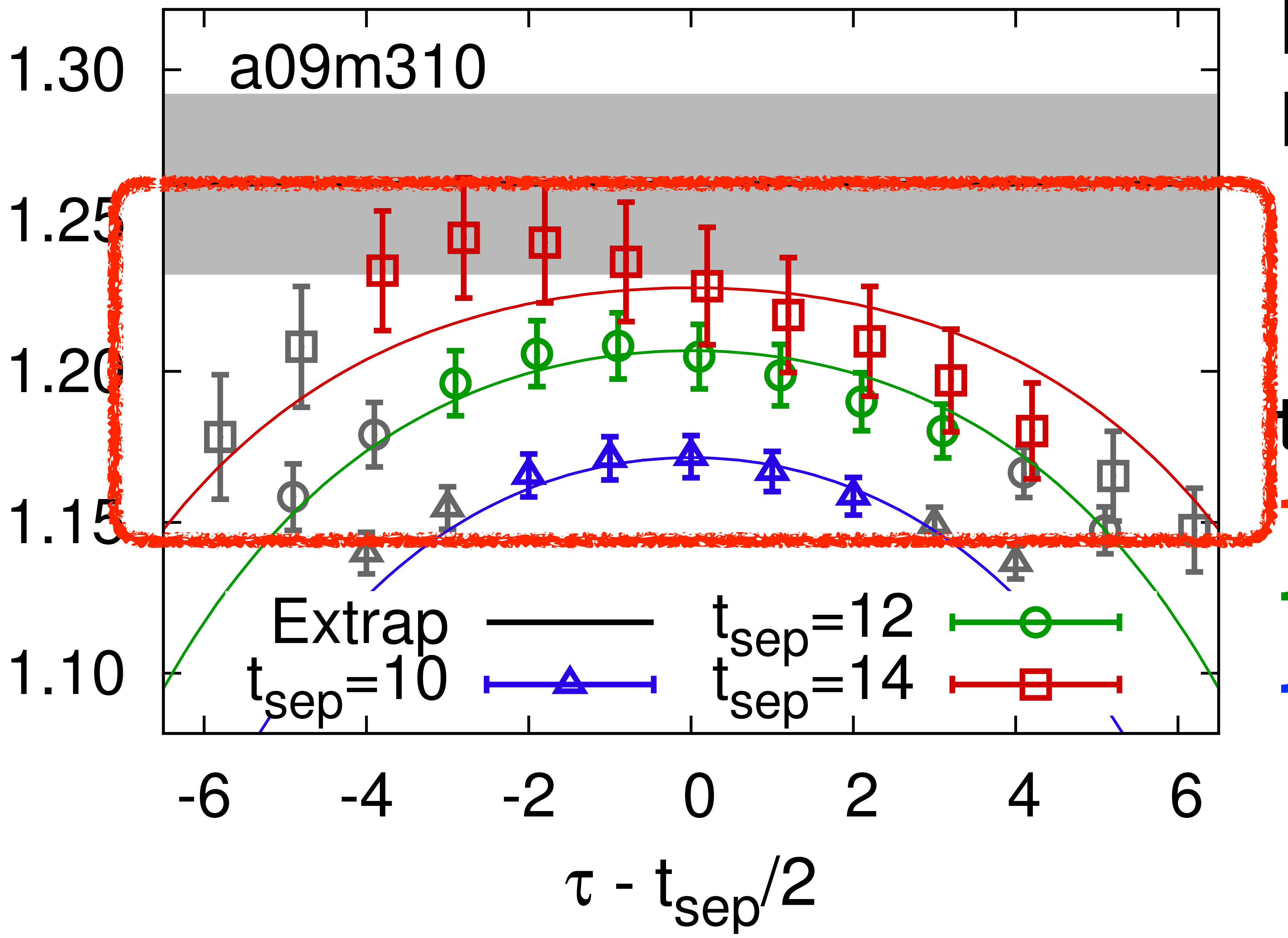
fixed source-sink separation time, t_{sep}

$\mathcal{O}(t_{\mathcal{O}})$



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

in long-time (**t_{sep}**) limit - should be flat



Repeat for multiple values of **t_{sep}**

Extrapolate to **t_{sep}**$\rightarrow\infty$

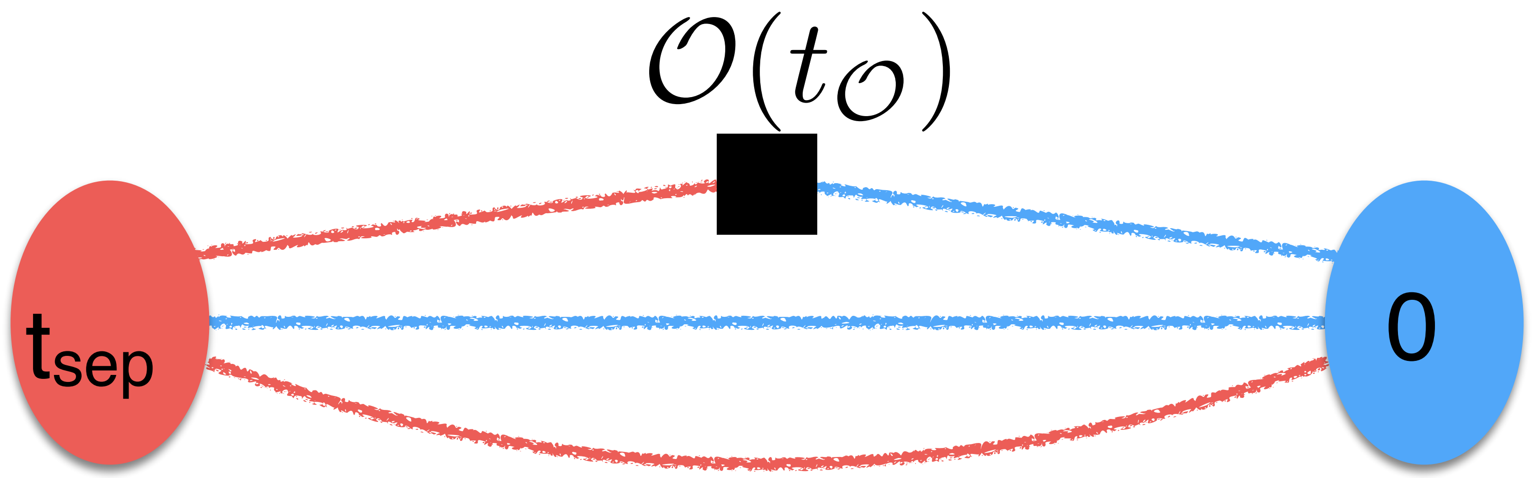
t_{sep}	fm
14	1.22
12	1.05
10	0.875

typical calculation

Bhattacharya, Cirigliano, Cohen, Gupta, Lin, Yoon Phys.Rev.D 94 (2016)

On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

Phys. Rev. D96 (2017)
arXiv:1612.06963

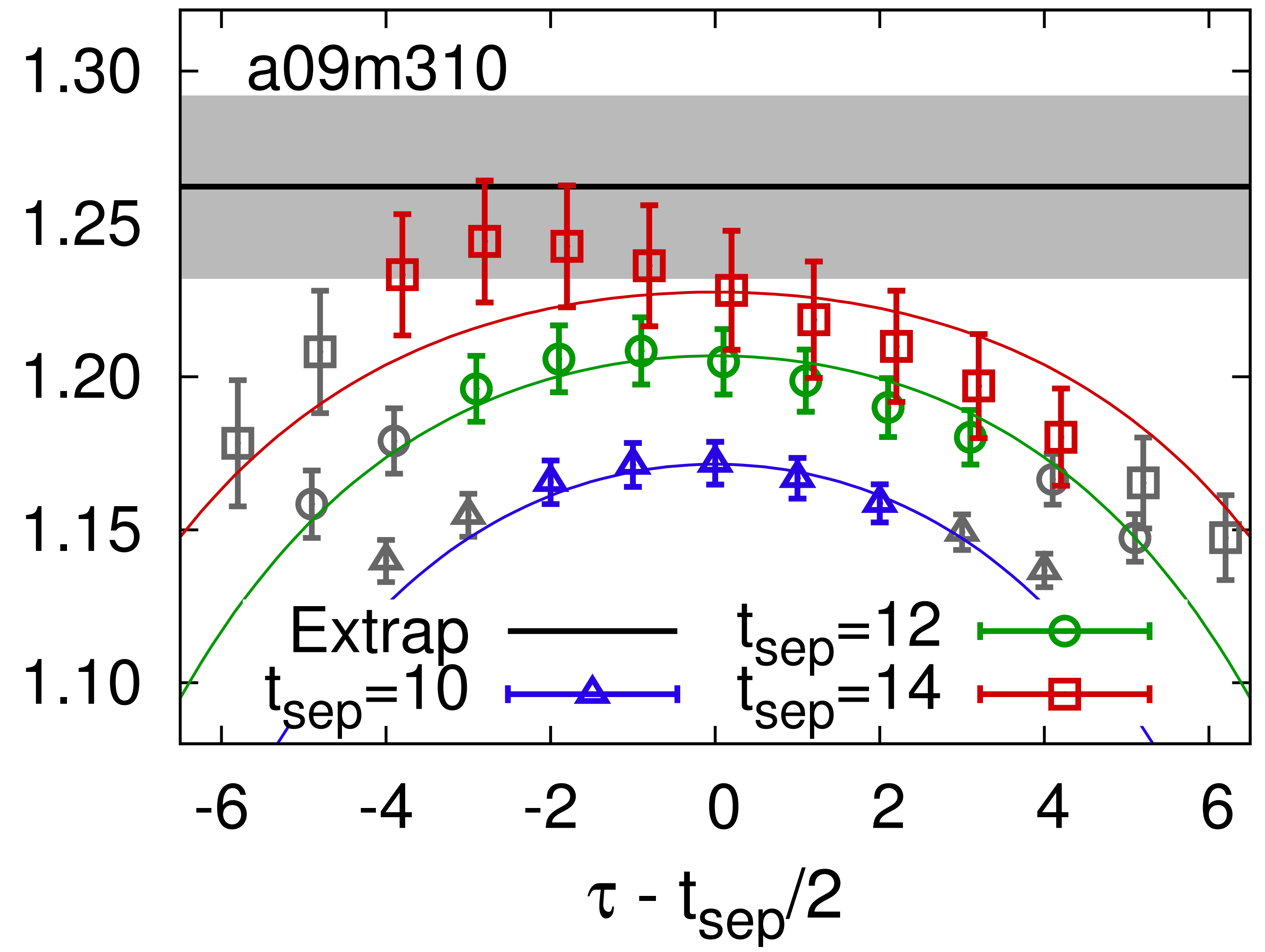


fixed source-sink separation time, $\mathbf{t_{sep}}$
repeat for a few different $\mathbf{t_{sep}}$

standard method

- two time variables to extrapolate in, $\mathbf{t_{sep}}$, τ
- late $\mathbf{t_{sep}}$ typically used to control excited states, but noisy and susceptible to correlated fluctuations

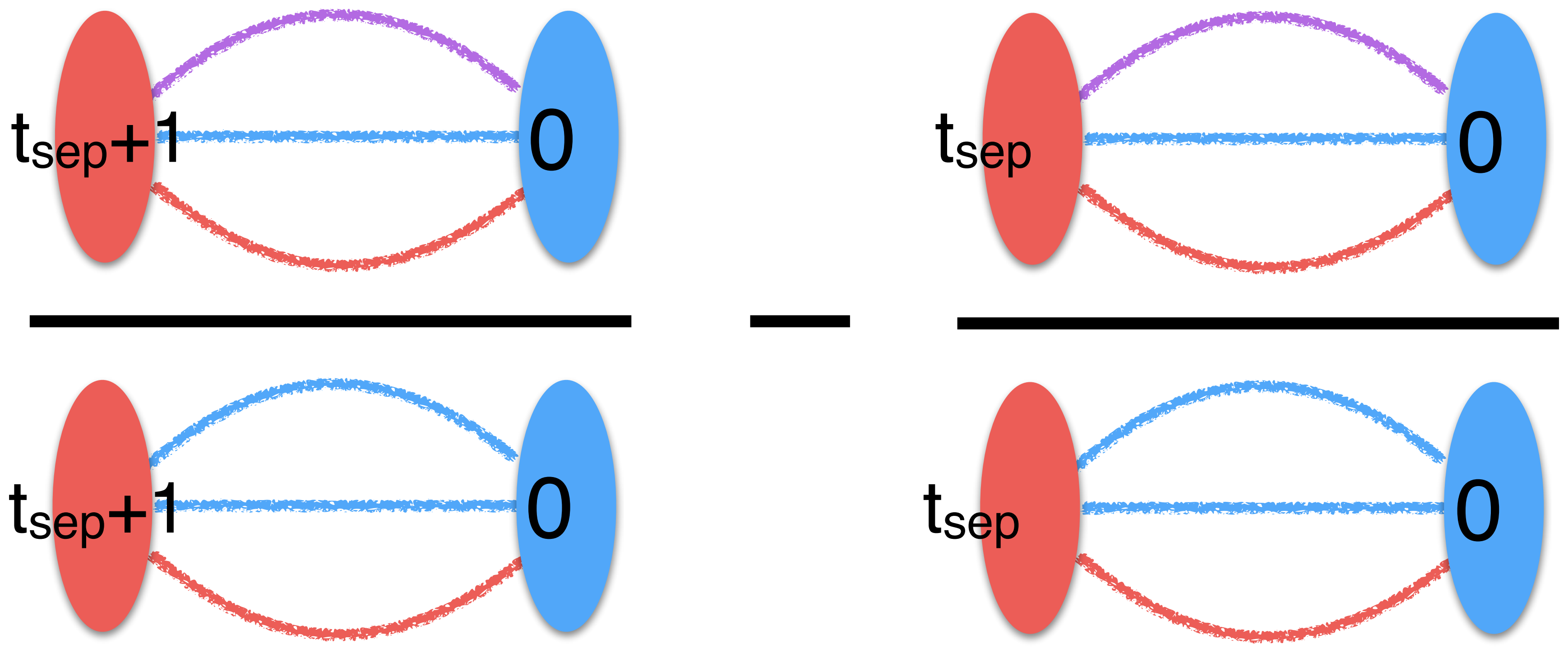
PNDME arXiv:1606.07049



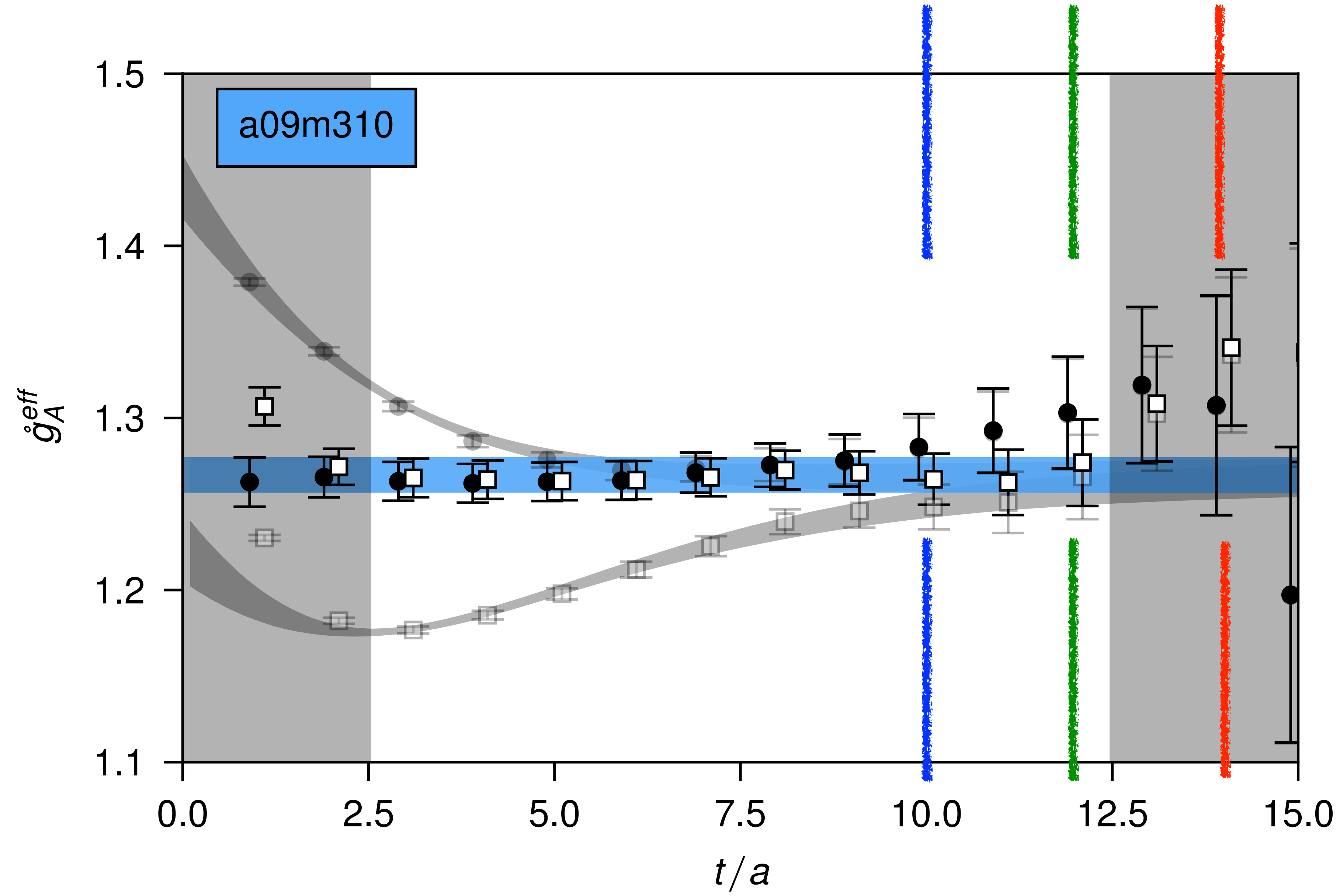
$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \dots$$

our unconventional method - sum over all current insertion time - subtract neighboring times - can be derived from invoking the Feynman-Hellmann Theorem

idea traced back to Maiani, Martinelli, Paciello and Taglienti **Nucl. Phys. B293 (1987)**



- one time variable, $\mathbf{t_{sep}}$, (sum τ)
- excited states more strongly suppressed
- Many $\mathbf{t_{sep}}$ available for extrapolation including precise results at early $\mathbf{t_{sep}}$



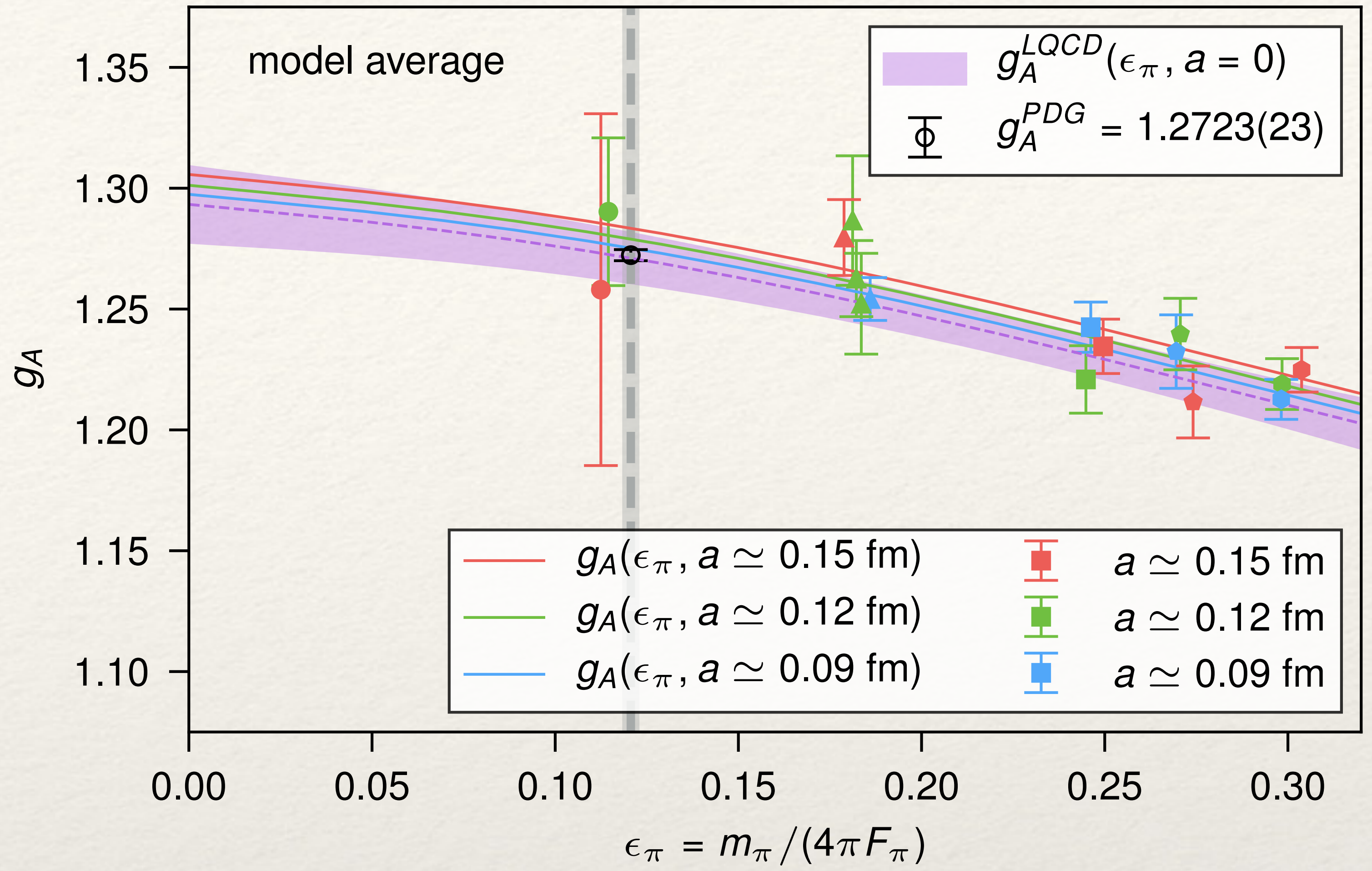
$$\partial_\lambda m_\lambda \Big|_{\lambda=0} = g_\lambda + z\left(e^{-(t_{sep}+1)\Delta_{10}} - e^{-t_{sep}\Delta_{10}}\right) + \cdots$$

improving the determination of g_A

Nature 558 (2018) no.7708, 91-94
Chang et al.
[arXiv:1805.12130]

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



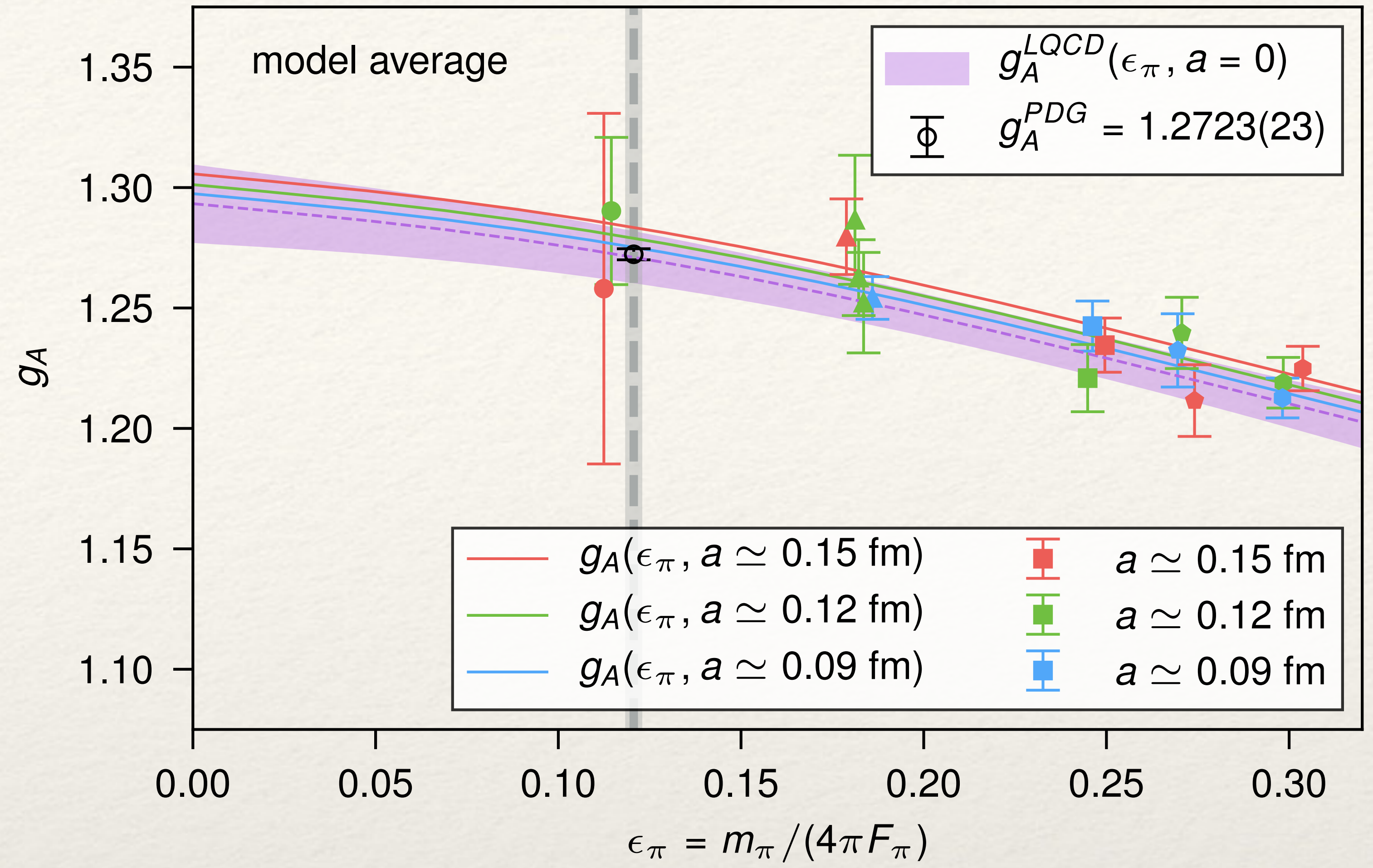
$$g_A^{\mathrm{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- More precise results at the physical pion mass will improve the three largest uncertainties:
  - statistical (s), extrapolation (χ) and model selection (M) **NOTE**, **a12m130** has 2.3% uncertainty
- Following our existing strategy, we anticipate getting to 0.5% by the end of this year
- Getting below (or maybe to 0.5%) will require a 4th lattice spacing as well (~0.06fm)
- Adding a FV study on additional pion mass points will improve the FV uncertainty
- The isospin uncertainty seems unnecessary…

improving the determination of g_A

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\mathrm{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

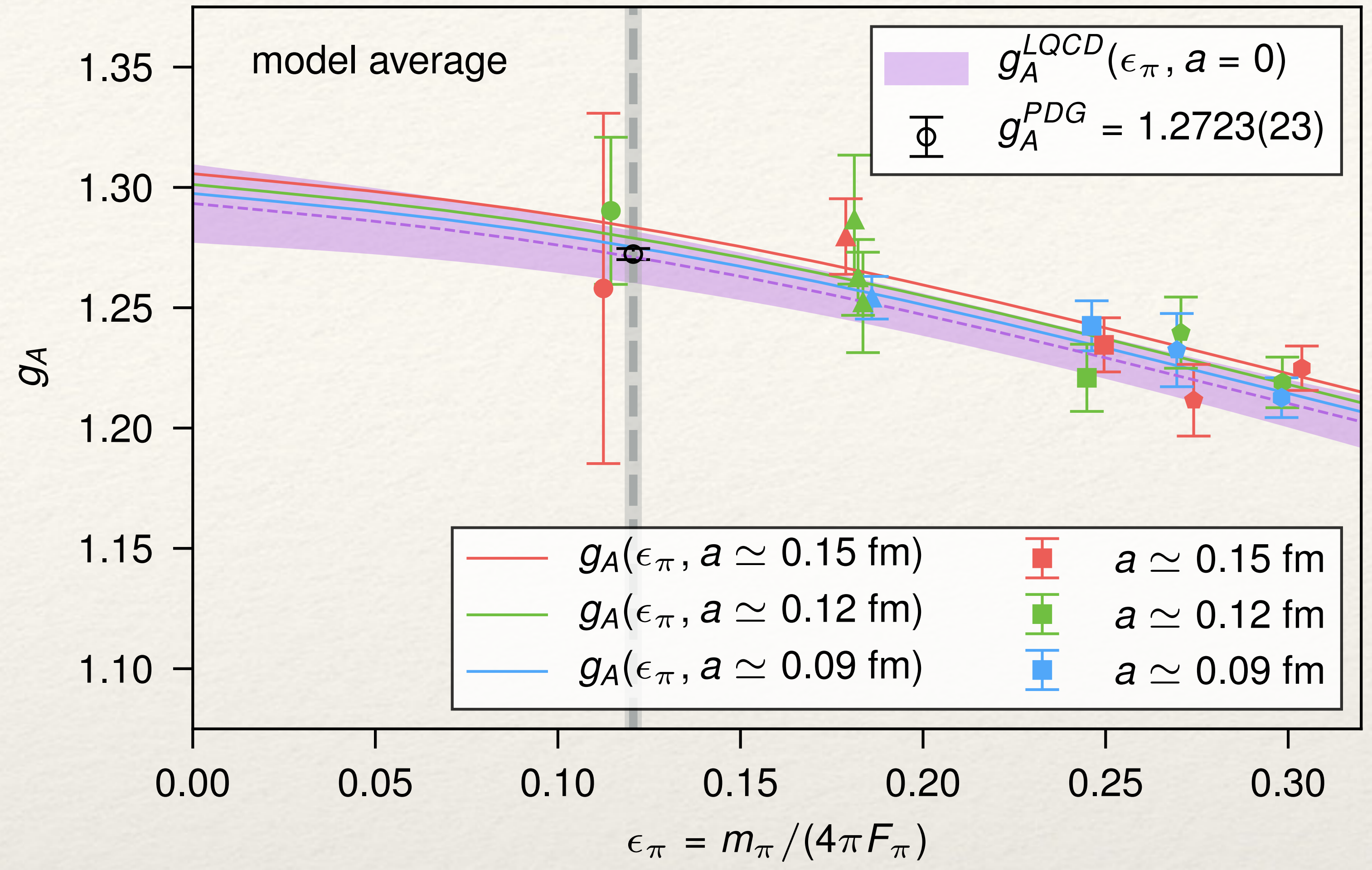
Isospin corrections **(my understanding prior to May 18 this year - see ACFI workshop)**

- The leading radiative corrections are subtracted from the experimental measurement leaving corrections of $\mathcal{O}\left(\frac{\alpha_{EM}^2}{\pi^2}\right) \sim 0.0005\%$
- There are $(m_d\text{-}m_u)^2$ corrections $\mathcal{O}\left(\frac{(m_d-m_u)^2}{(m_d+m_u)^2}\epsilon_\pi^4\right) \sim 0.002\%$
- There are mixed corrections of $\mathcal{O}\left(\alpha_{EM}\frac{m_d-m_u}{m_d+m_u}\epsilon_\pi^2\right) \sim 0.004\%$
- The largest isospin correction comes from the extrapolation to $\epsilon_{\pi^-} = \frac{m_{\pi^-}}{4\pi F_{\pi^-}}$ $\epsilon_{\pi^0} = \frac{m_{\pi^0}}{4\pi F_{\pi^0}}$

improving the determination of g_A

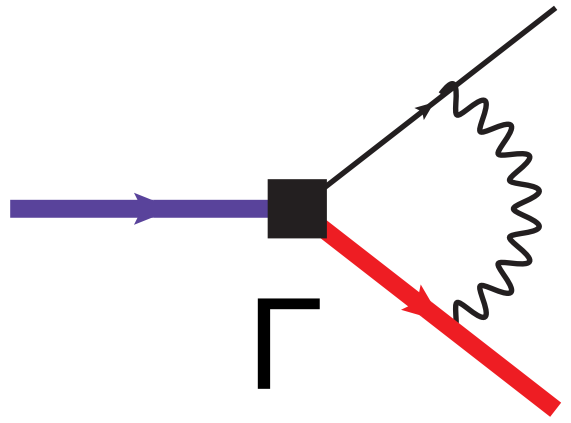
Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\mathrm{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- There is a radiative correction - not previously computed - that leads to a 0.4% correction - **See talk of Leendert Hayen** at ACFI workshop and arXiv:1906.09870



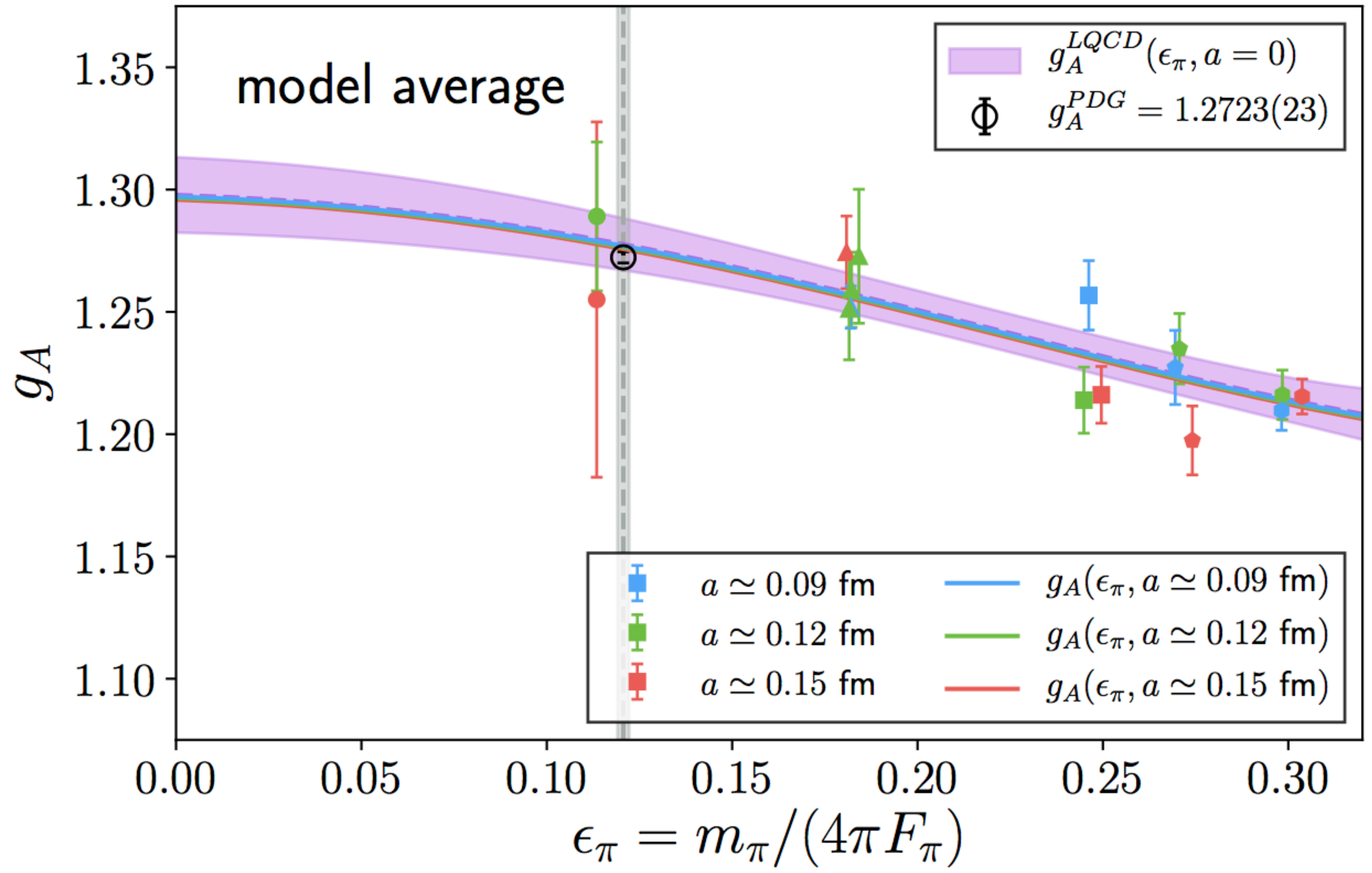
$\Gamma = \gamma_\mu$ or $\gamma_\mu\gamma_5$ $\qquad |V_{ud}|^2\tau_n\left(f_V + 3f_A\lambda^2\right) = \dfrac{2\pi^3}{G_F^2 m_e^5 g_V^2}\dfrac{1}{1+RC}$

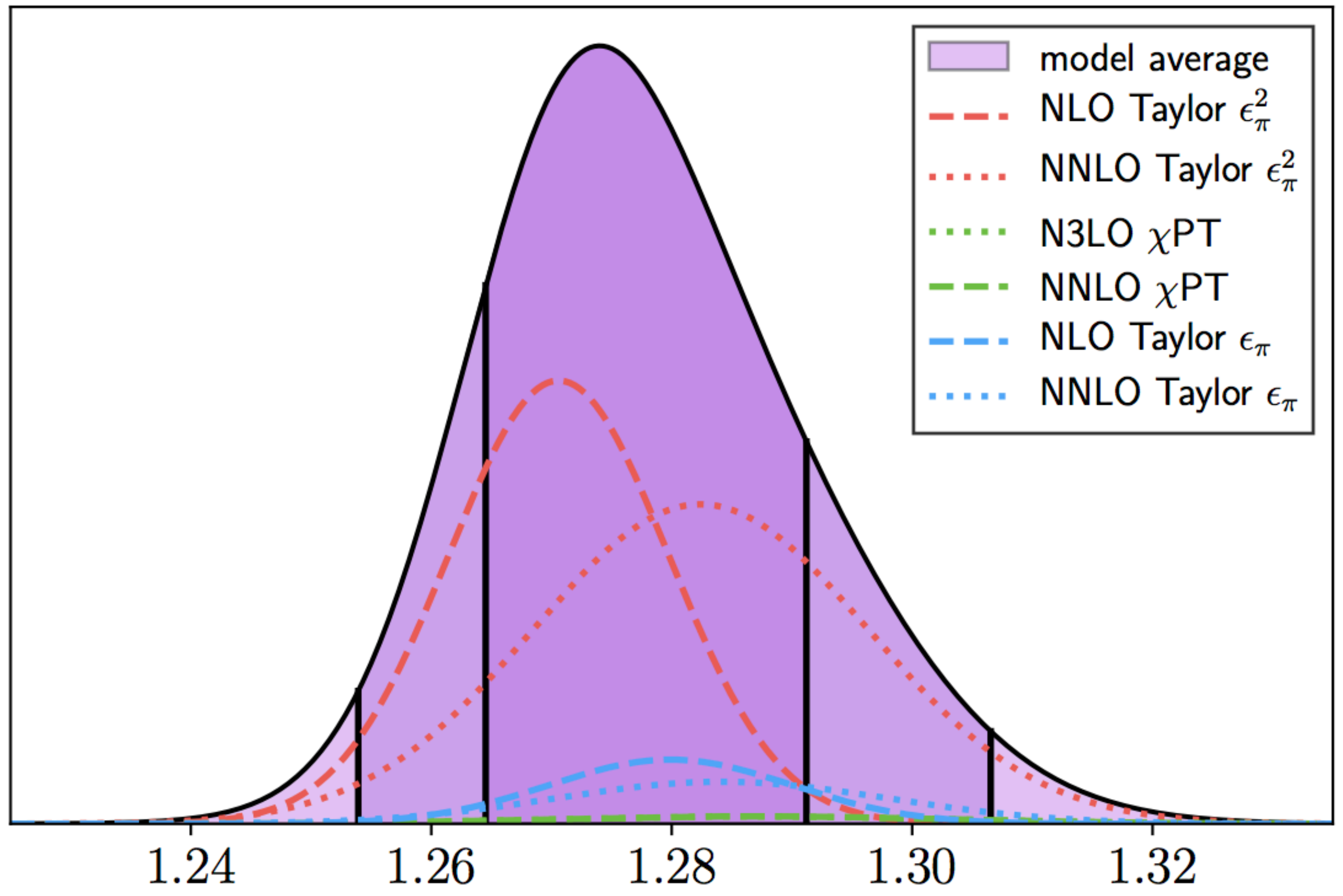
previously, assumed $f_A = f_V$ to 10^{-6}, but 0.4% correction. Does not effect V_{ud} but does impact BSM constraints

- At 0.5%, we should incorporate isospin breaking corrections and verify this result
  - if we agree, we can return to isospin-symmetric to push down to 0.2%

Analysis Details

Model average extrapolation



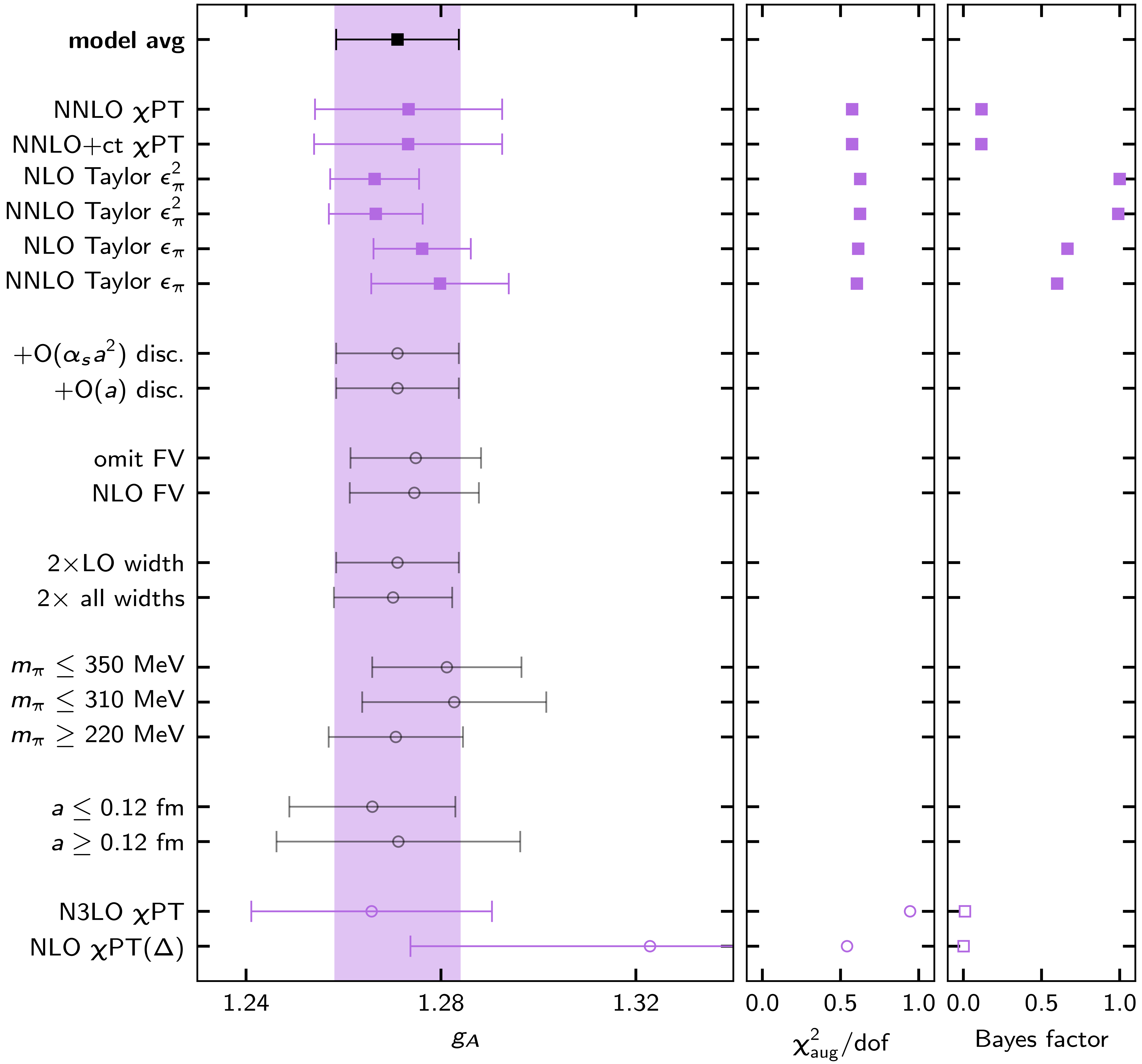


Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

$$\begin{aligned}
\text{NNLO } \chi\text{PT}: &\quad \text{Eq. (S8)} + \delta_a + \delta_L \\
\text{NNLO+ct } \chi\text{PT}: &\quad \text{Eq. (S8)} + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L
\end{aligned}$$

Analysis Details

Stability of Extrapolation Analysis



Final result

6 fits included in the model average and their relative weights

discretization corrections

finite volume corrections

sensitivity to prior width

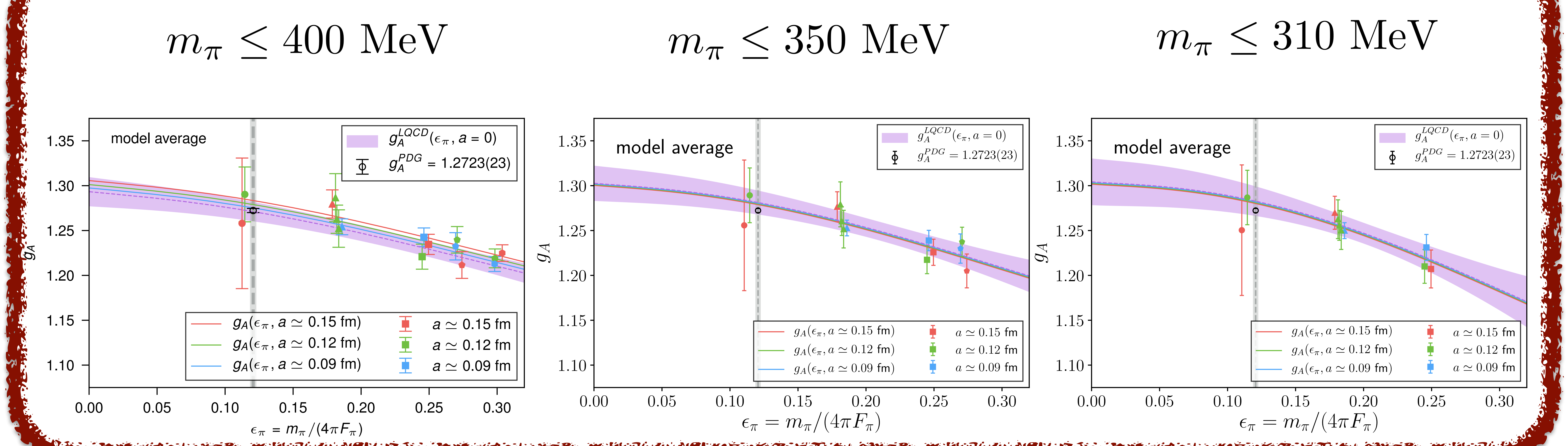
sensitivity to pion mass cuts

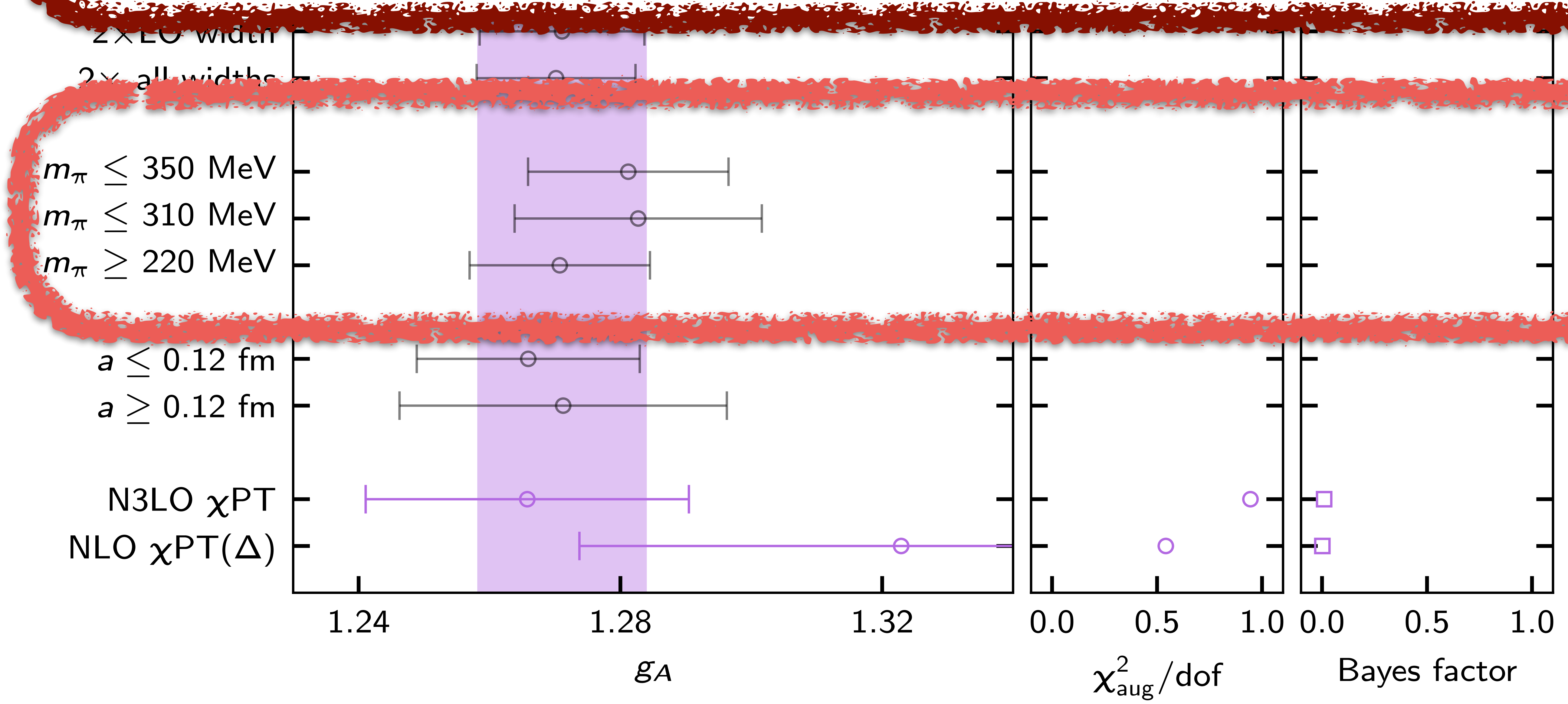
sensitivity to lattice spacing cuts

additional XPT analysis

Analysis Details

Stability of Extrapolation Analysis





sensitivity to prior width

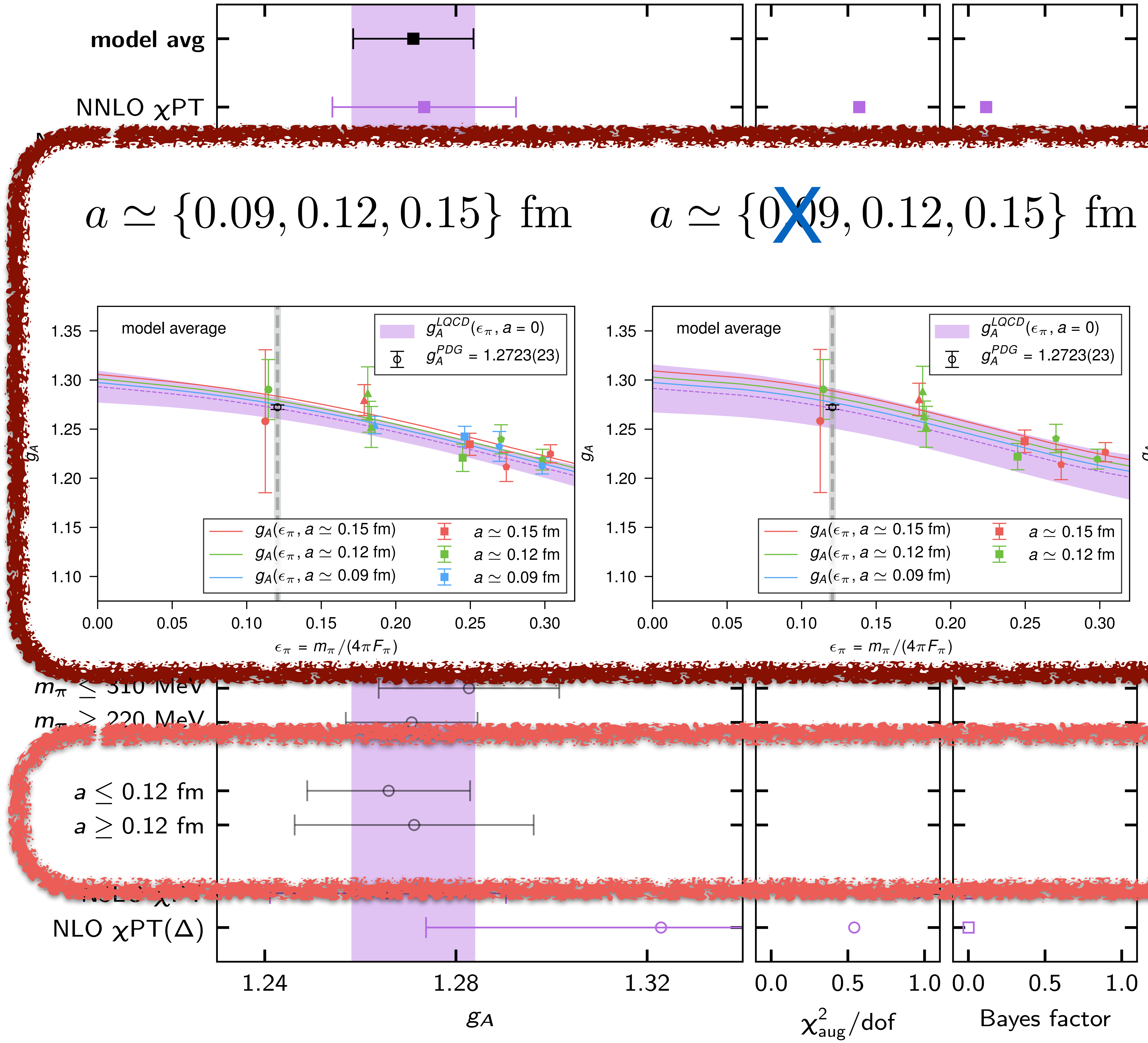
sensitivity to pion mass cuts

sensitivity to lattice spacing cuts

additional XPT analysis

Analysis Details

Stability of Extrapolation Analysis



Final result

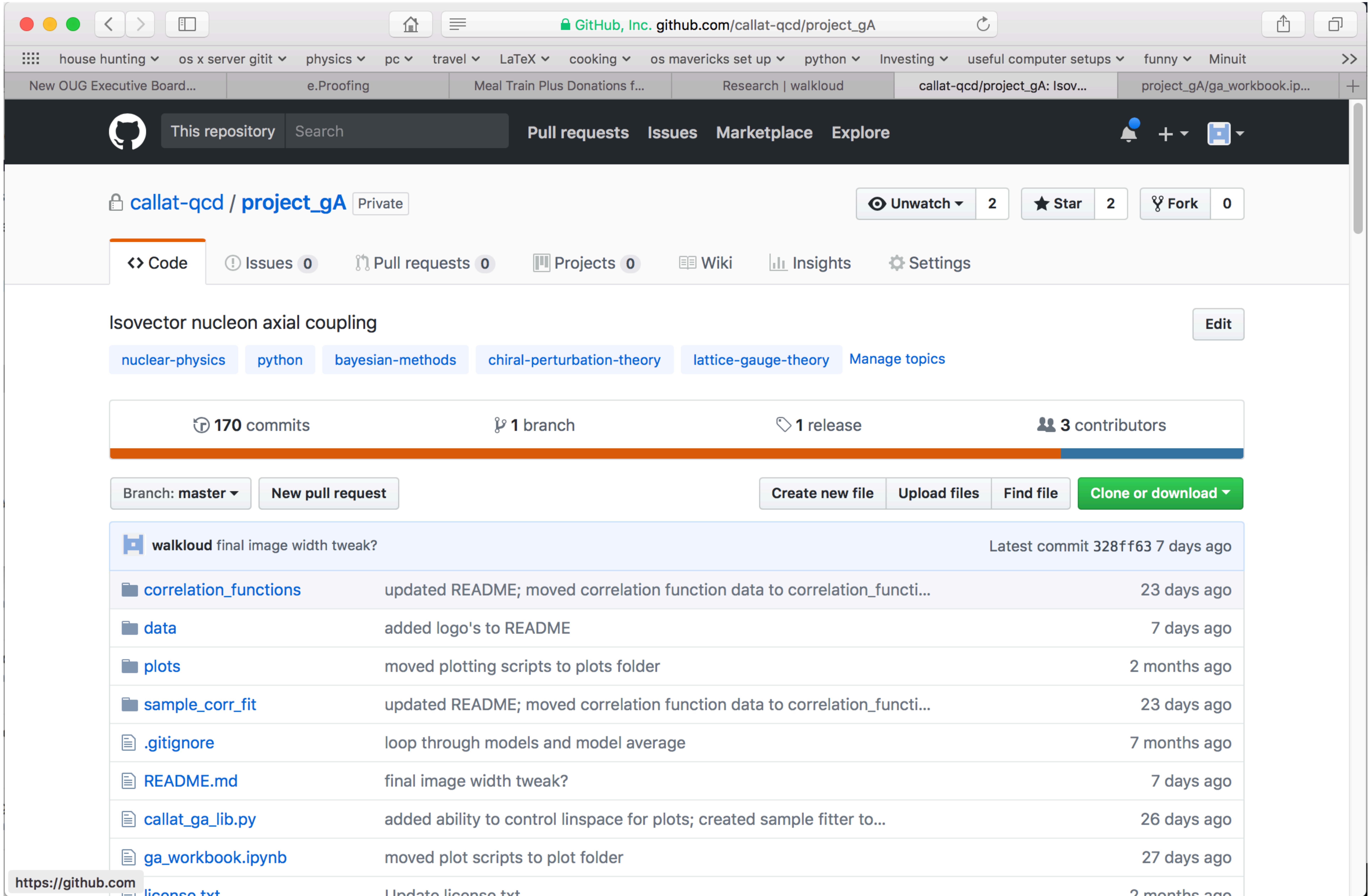
cuts

sensitivity to lattice spacing cuts

additional XPT analysis

https://github.com/callat-qcd/project_gA

raw correlation functions, correlation function analysis results, extrapolation analysis

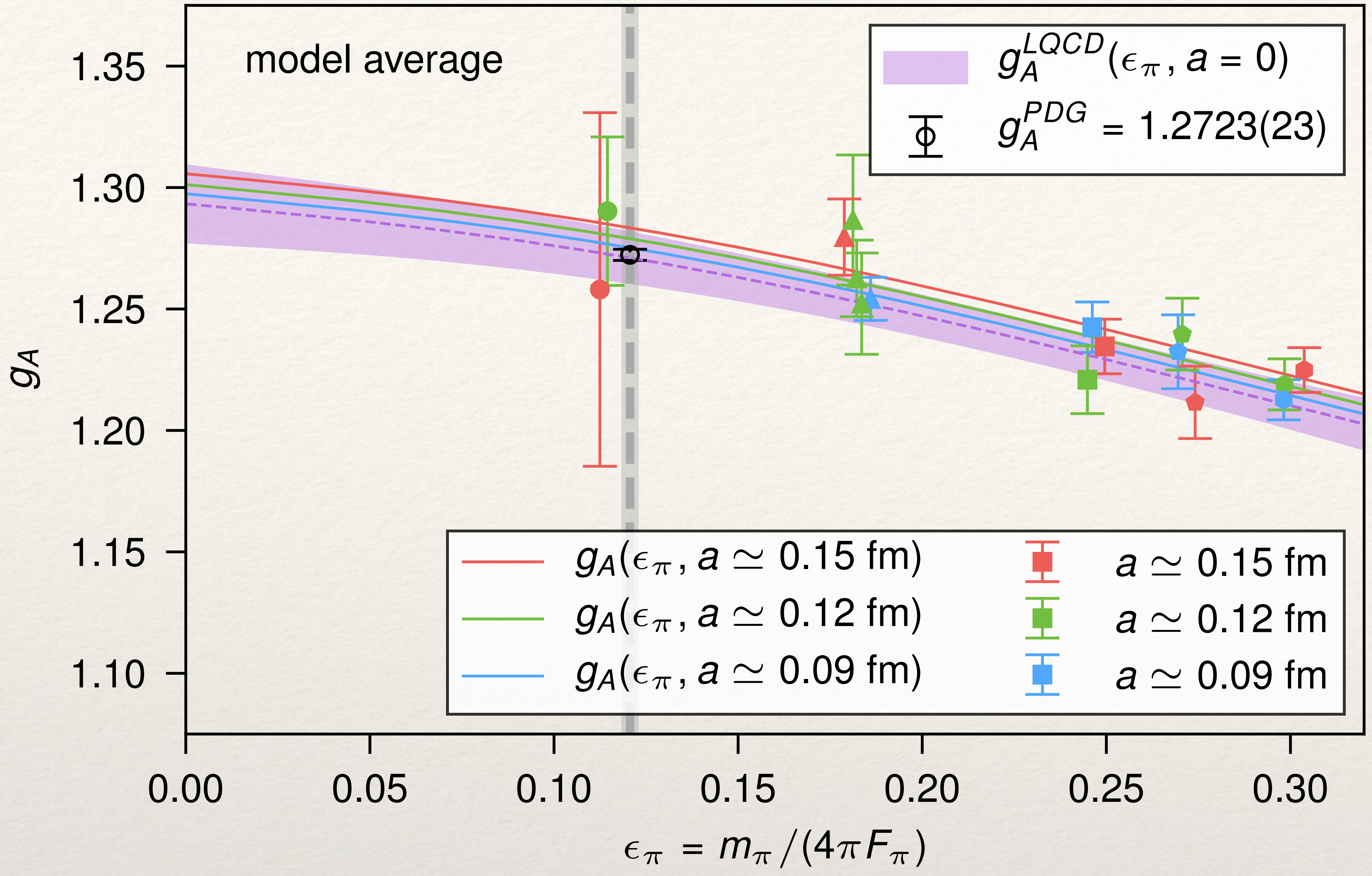


Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

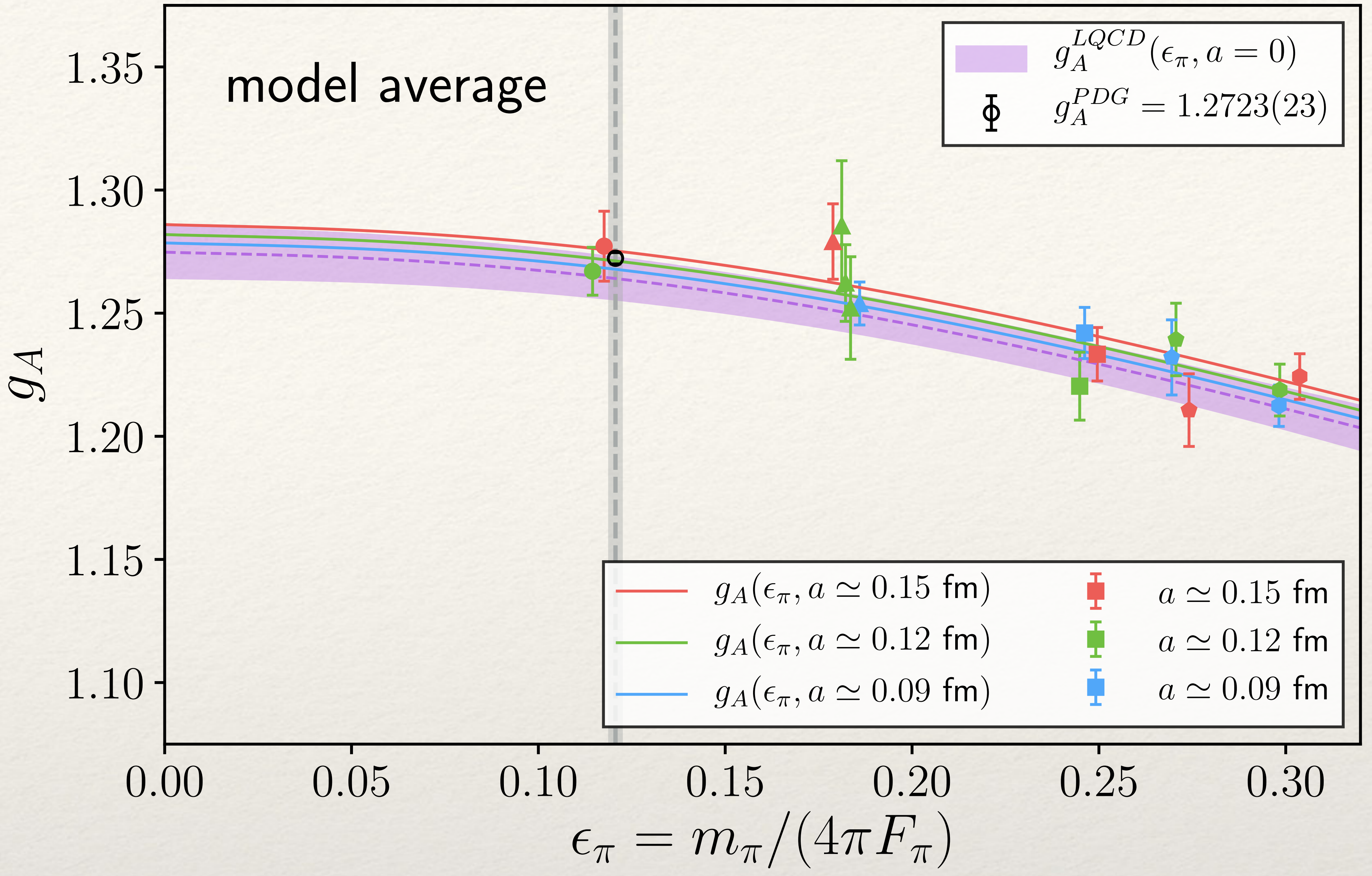
1 year on Titan (ORNL) + 2 years on GPU machines at LLNL

Sierra Early Science

PRELIMINARY



California Lattice Collaboration

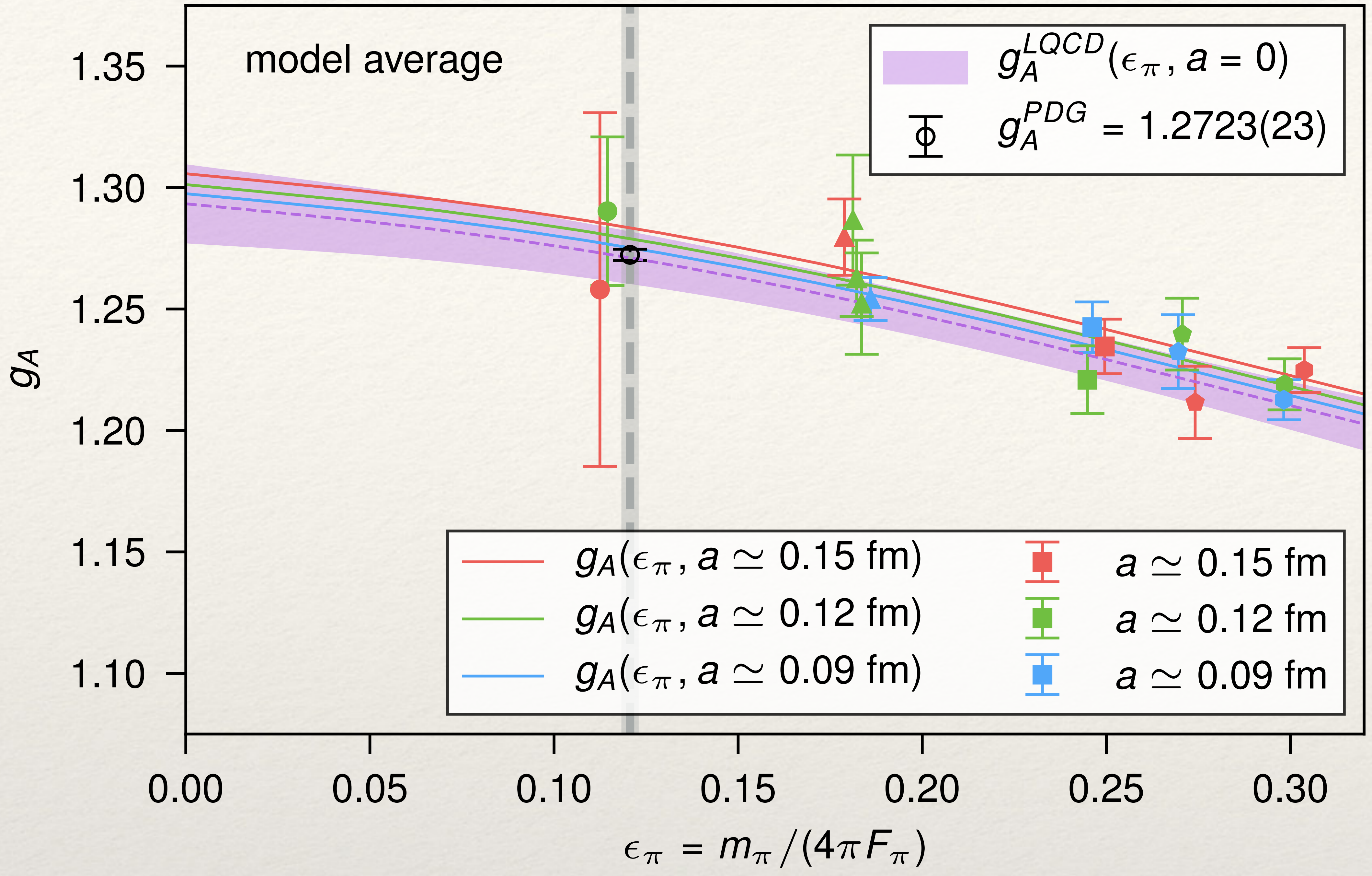


- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$
- We are running $g_A(Q^2)$ on Summit this year (DOE INCITE)
  - We anticipate improving g_A to ~0.5%

$$\boxed{g_A = 1.2711(125) \rightarrow 1.2641(93) \ \ [0.74\%]}$$

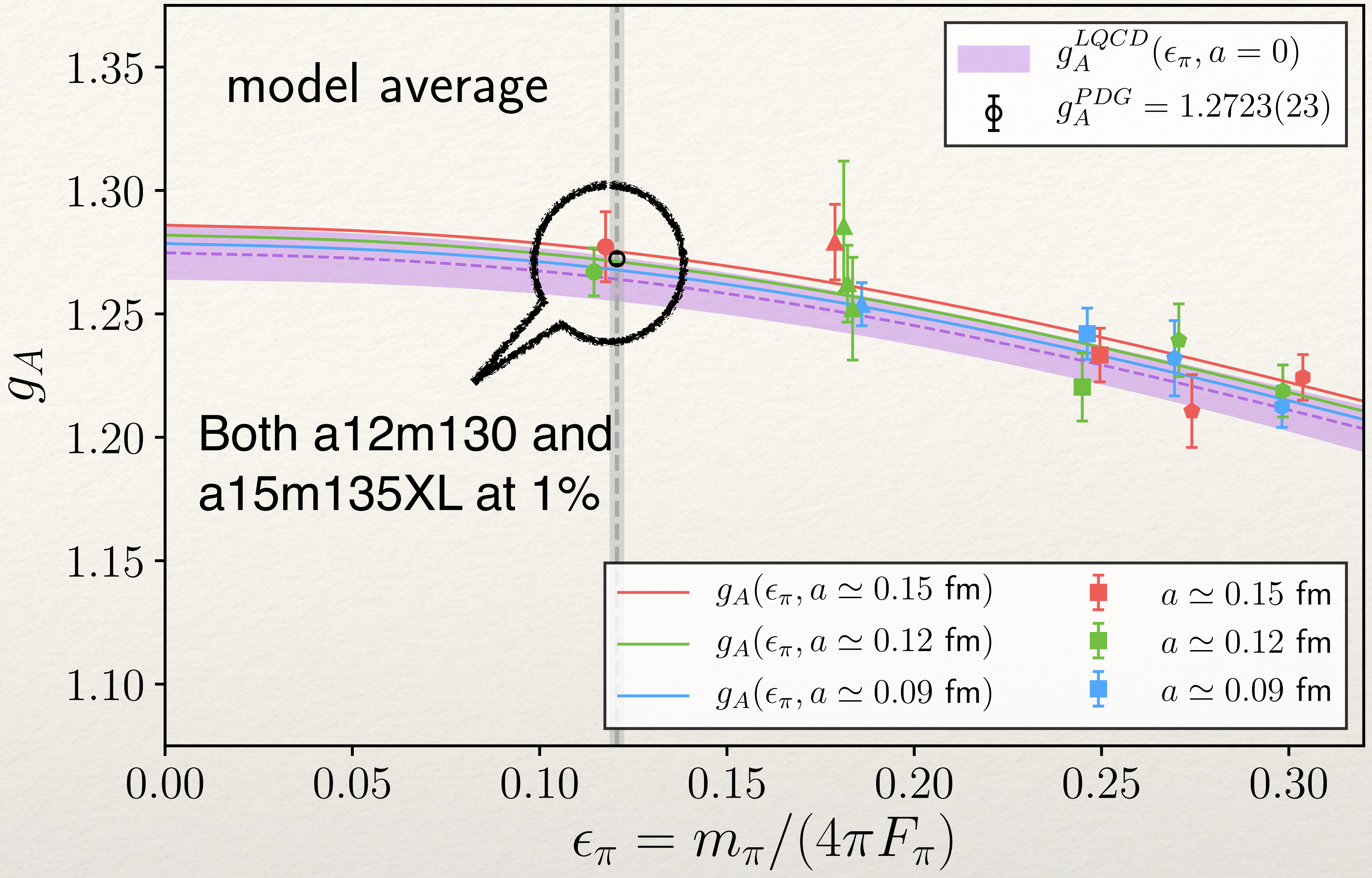
Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



Sierra Early Science

PRELIMINARY



California Lattice Collaboration

- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$
- We are running $g_A(Q^2)$ on Summit this year (DOE INCITE)
  - We anticipate improving g_A to ~0.5%

$$g_A = 1.2711(125) \to 1.2641(93) \quad [0.74\%]$$

Nature 558 (2018) no. 7708, 91-94 1 year on Titan (ORNL) + 2 years

PRELIMINARY

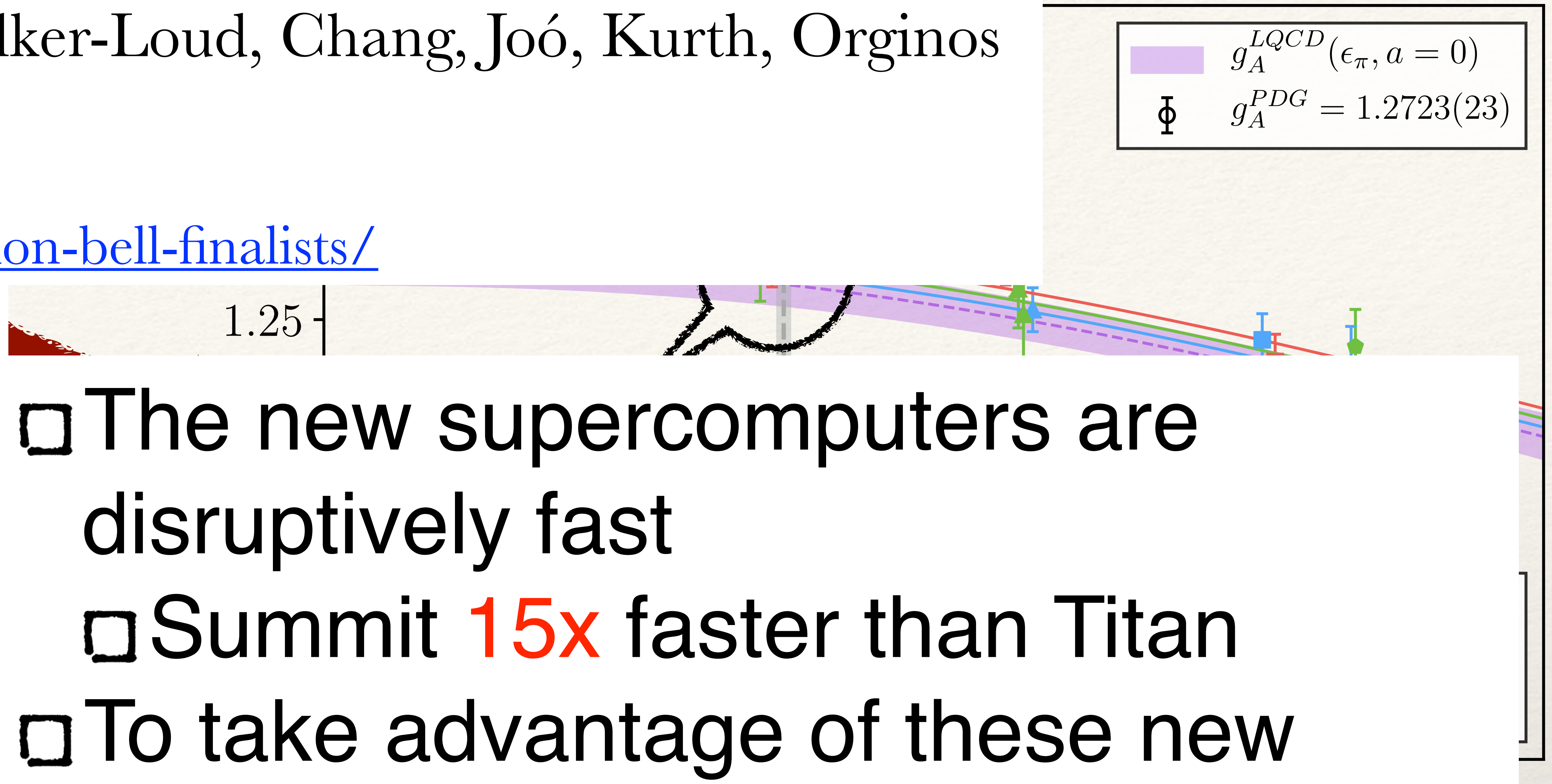
Simulating the *weak* death of the neutron in a femtoscale universe with near-Exascale computing

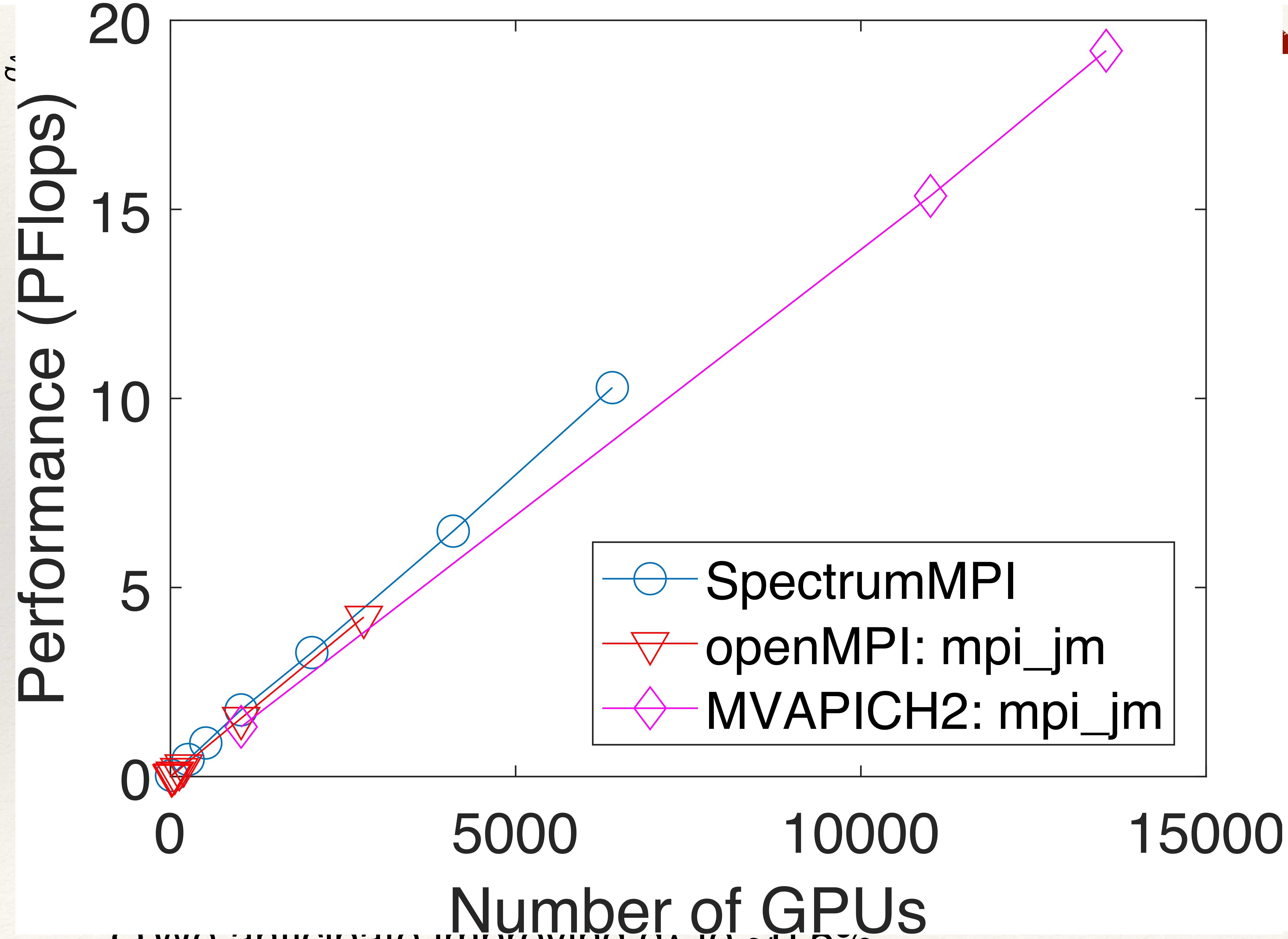
Berkowitz, Clark, Gambhir, McElvain, Nicholson, Rinaldi, Vranas, Walker-Loud, Chang, Joó, Kurth, Orginos

SuperComputing 2018 [arXiv:1810.01609]

https://www.olcf.ornl.gov/2018/09/17/uncharted-territory/

https://blogs.nvidia.com/blog/2018/09/17/nvidia-volta-tensor-core-gpus-gordon-bell-finalists/

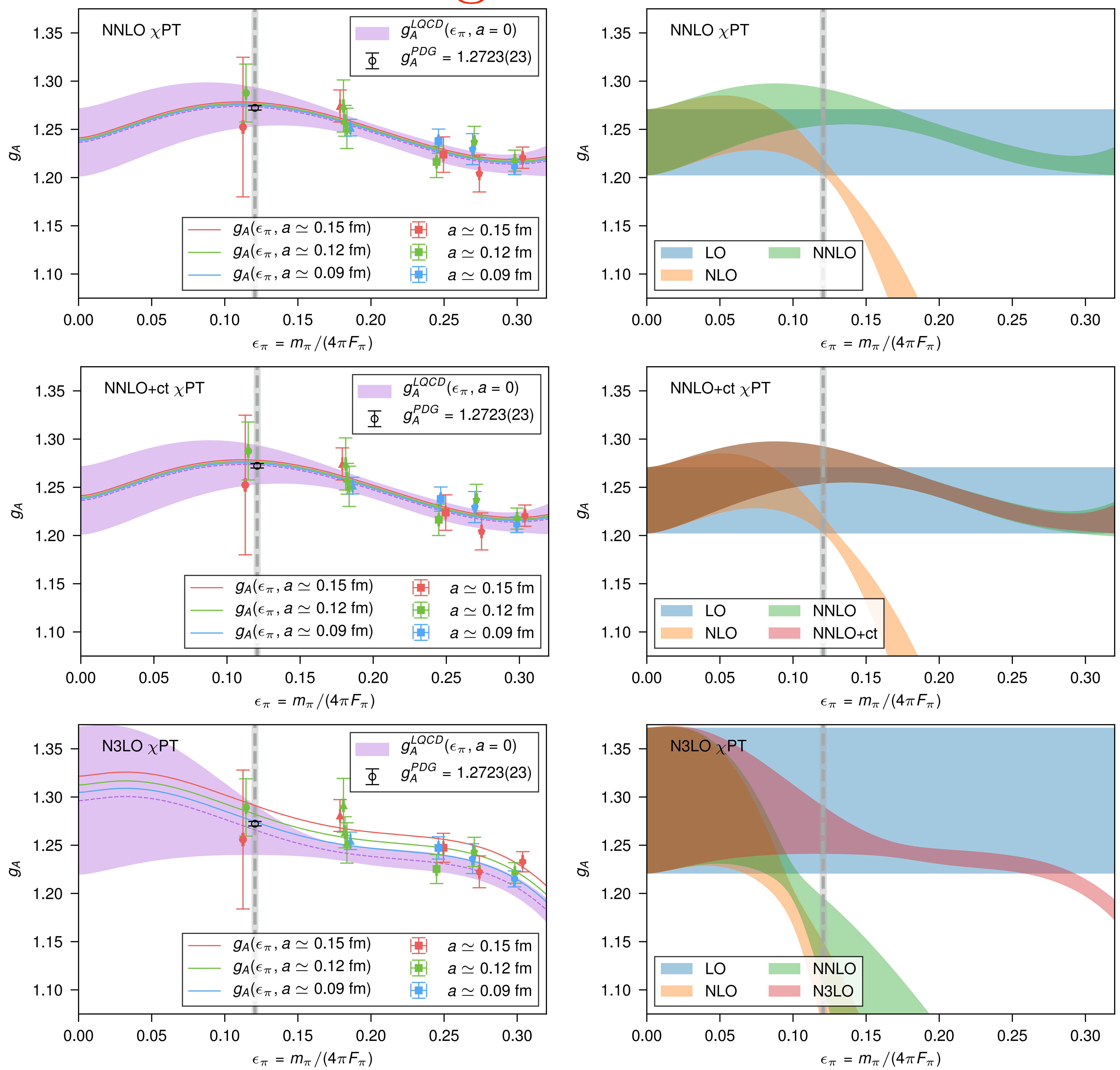




- The new supercomputers are disruptively fast
  - Summit 15x faster than Titan
- To take advantage of these new machines - need
  - Optimized GPU code (QUDA)
  - Sophisticated, user friendly job-management software
    - METAQ (bash) [arXiv:1702.06122]
    - MPI_JM (C++) [coming soon]

- We anticipate improving g_A to ~0.5%

convergence of the chiral expansion…



$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

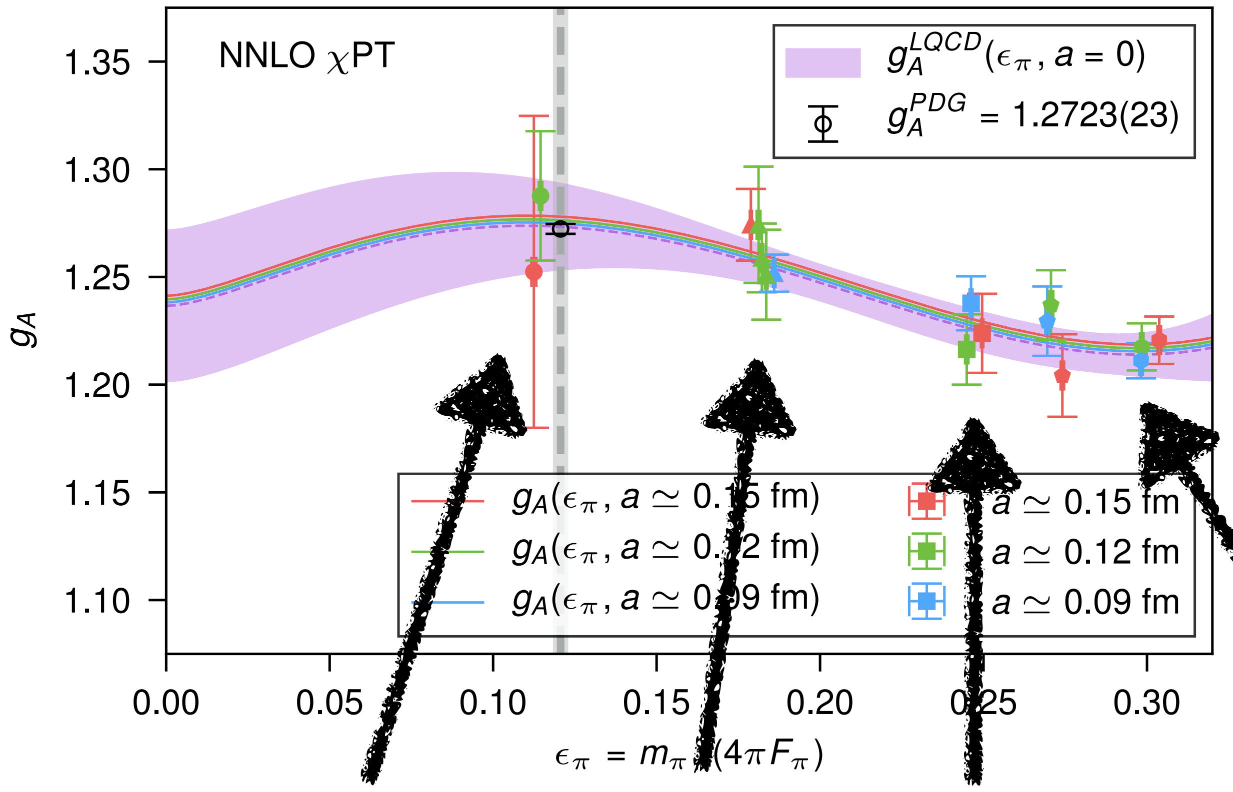
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

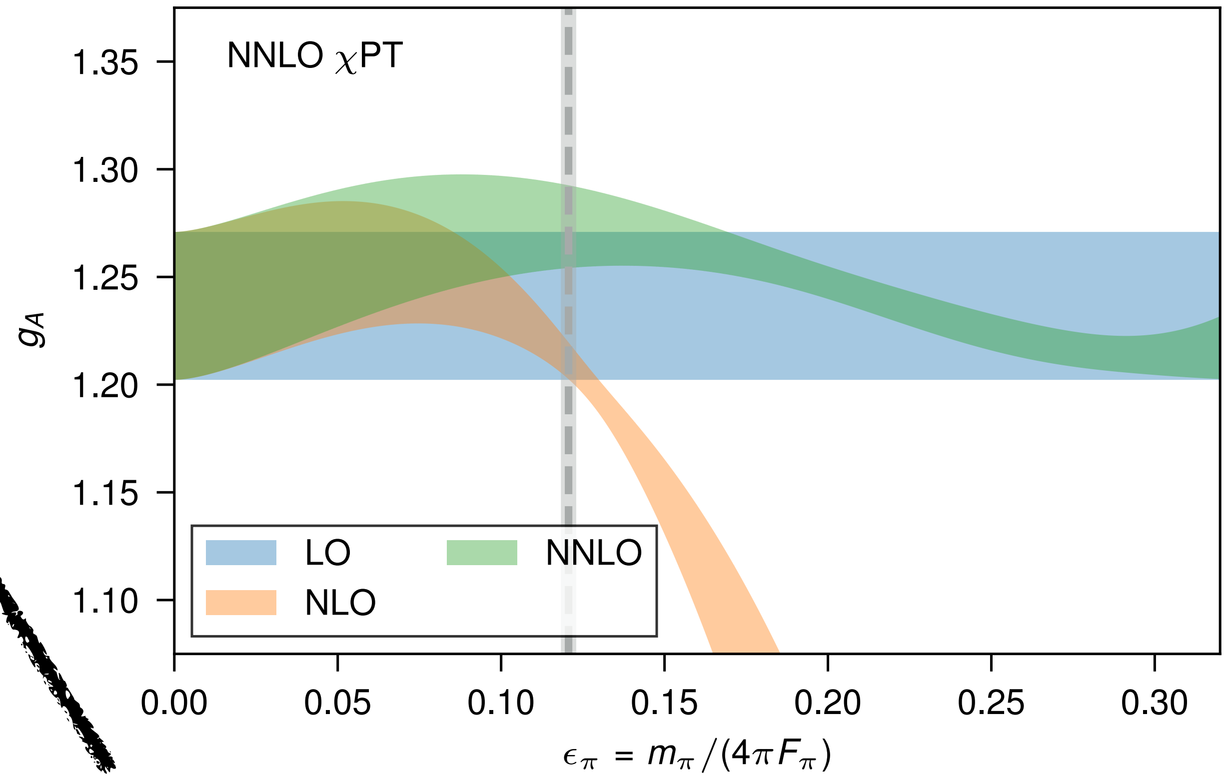
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4 \epsilon_\pi^4$$

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4 \left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3} g_0 + \frac{37}{12} g_0^3 + 4 g_0^5 \right) \ln^2(\epsilon_\pi^2) \right]$$

Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

convergence of the chiral expansion…





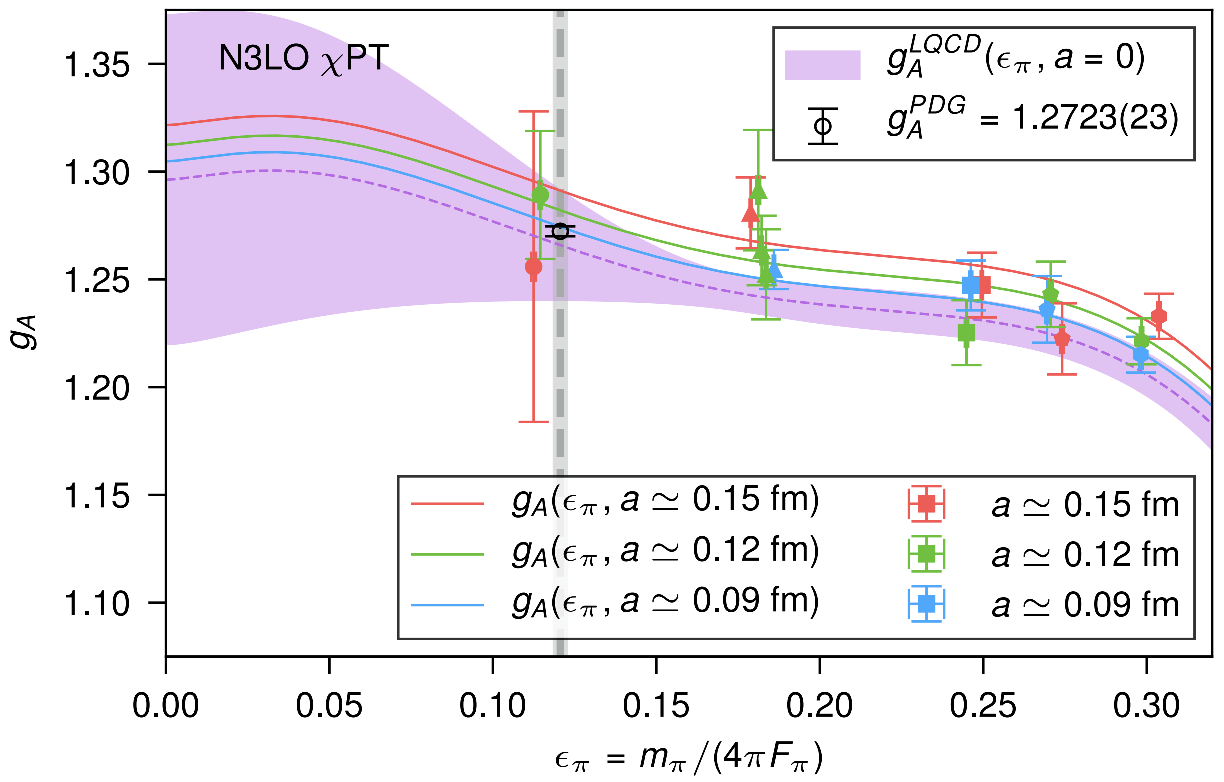
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

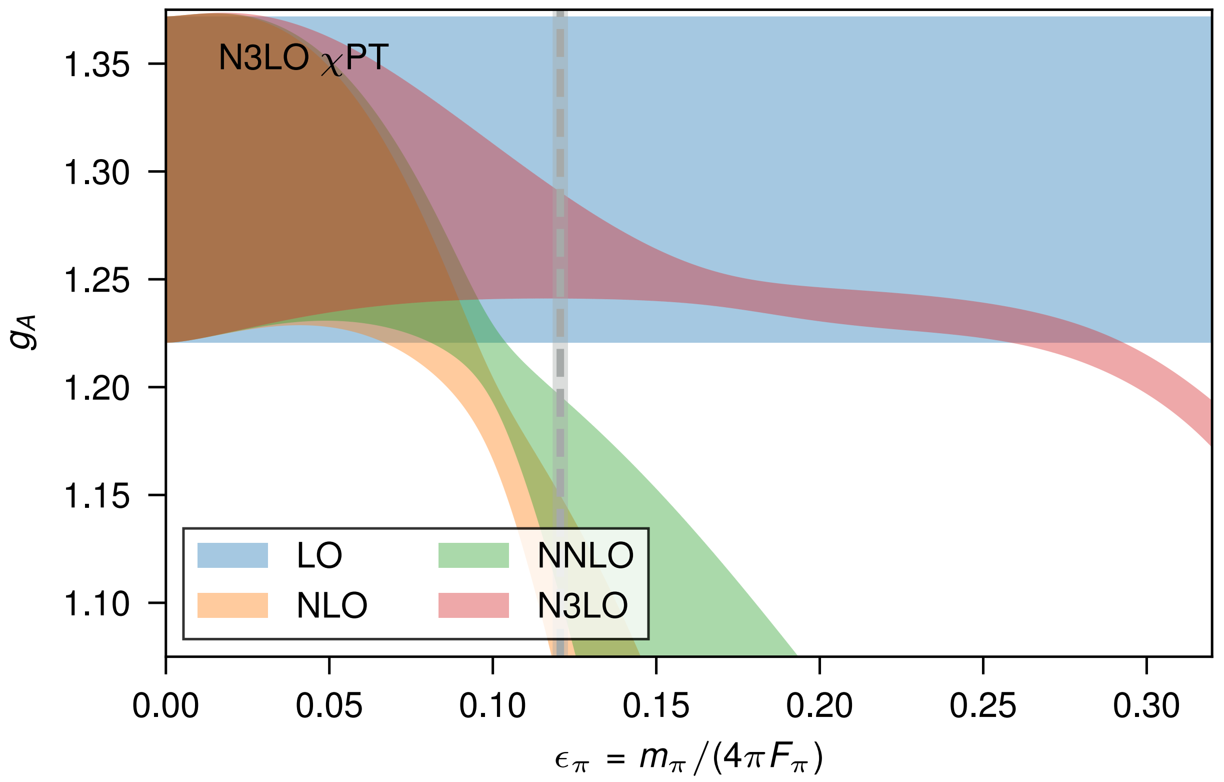
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130$ MeV 220 310 400

can we trust extrapolation of quantities with chiraly-enhanced behavior?

if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?





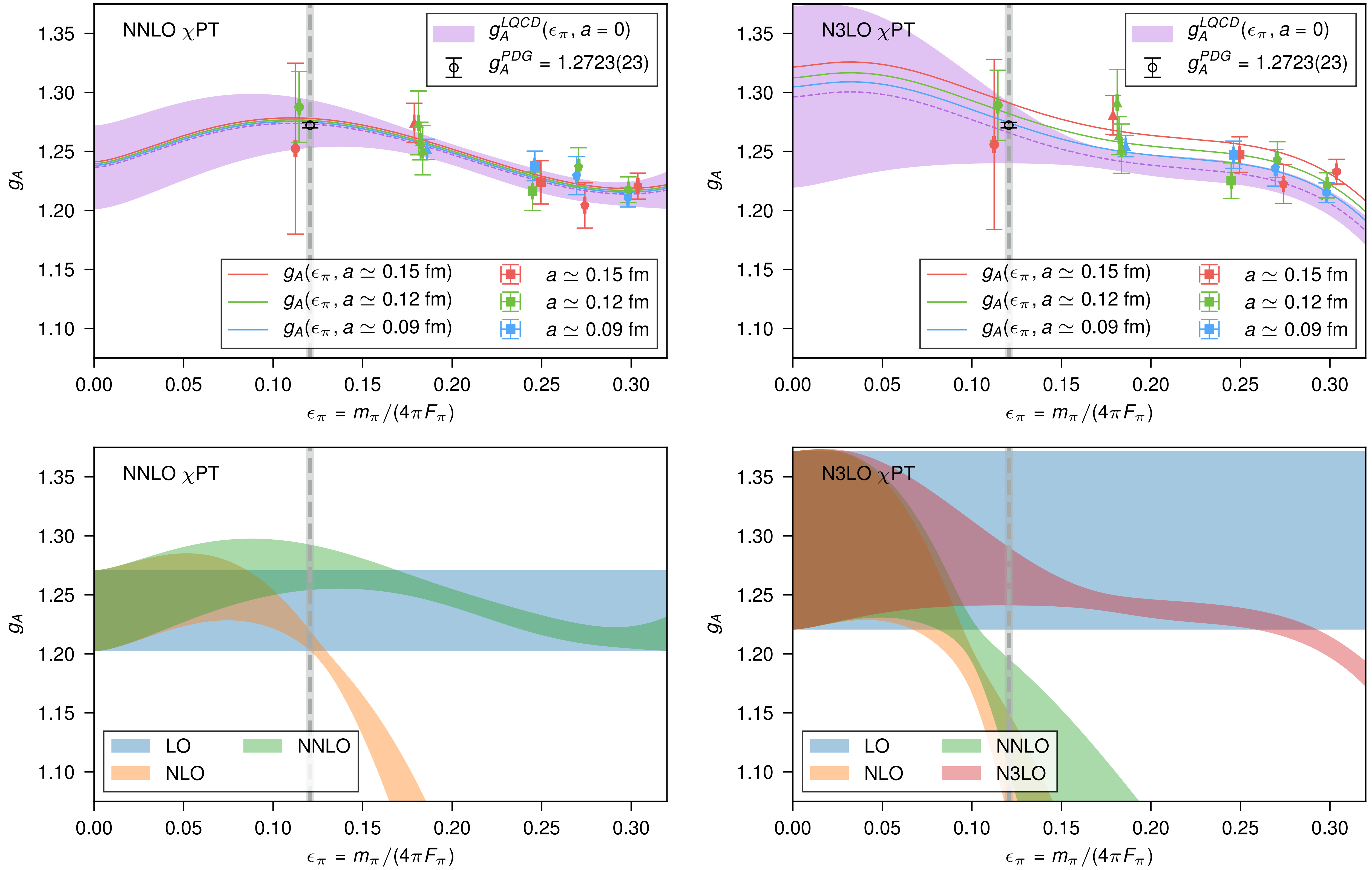
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4 \left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3} g_0 + \frac{37}{12} g_0^3 + 4 g_0^5 \right) \ln^2(\epsilon_\pi^2) \right]$$

Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

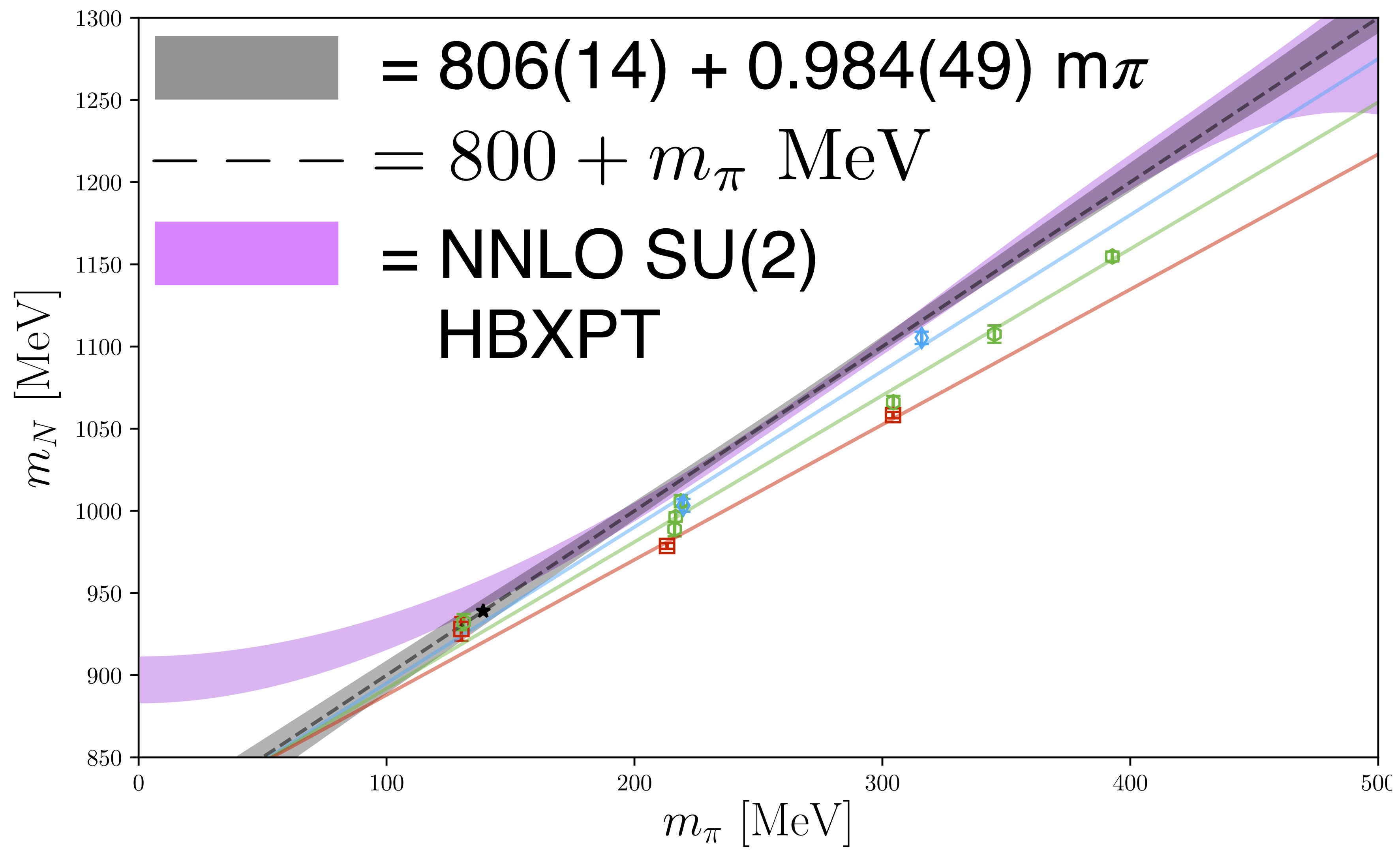
convergence of the chiral expansion…

- The point is not - will this ruler approximation hold to arbitrary precision?
  - Already, it holds to sufficient precision that it demonstrates a strong cancellation between different orders in the expansion
  - Is it sufficiently accurate that one can extract the pion-nucleon sigma term? $\sigma_{\pi N} = 67 \pm 4$ MeV
  - Laurent (BMW) find a much smaller sigma term, 38(3)(3) PRL 116 (2016) [arXiv:1510.08013]
  - Allmost all other lattice groups find similarly small values - in significant tension with the phenomenological determinations [Hoferichter, Ruiz de Elvira, Kubis, Meissner PhysRept 625 (2016)]
  - as well as Martin Camalich, Geng, Vincente Vacas PRD 82 (2010) fitting LQCD spectrum data

convergence of the chiral expansion…



PRELIMINARY 2019



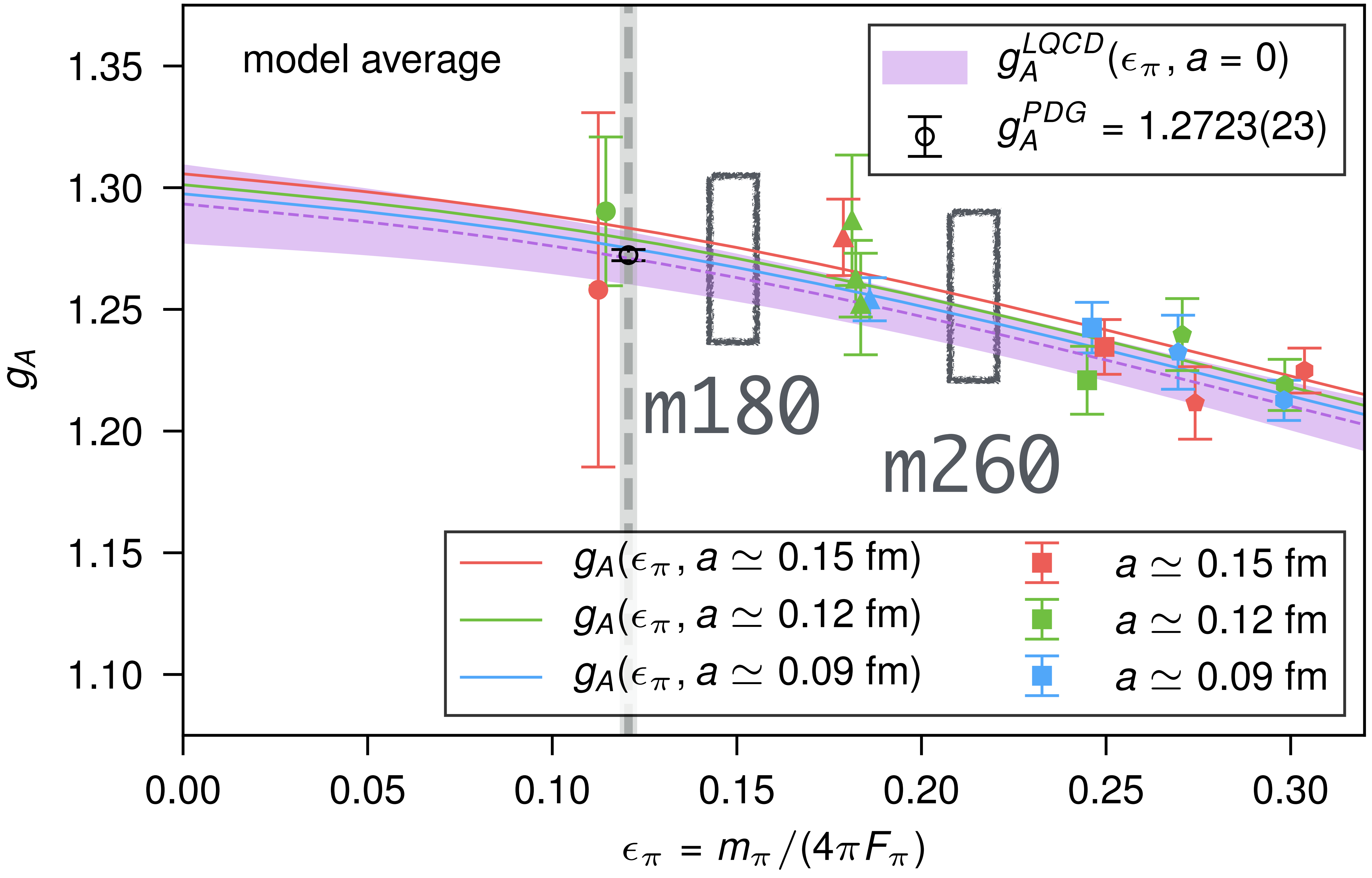
- Chiral corrections to g_A from $SU(2)$ HBχPT($\not\Delta$) at the physical pion mass

N^{n}LO	LO	NLO	N^{2}LO	N^{3}LO
N^{2}LO	1.237(34)	-0.026(30)	0.062(14)	–
N^{3}LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

- $SU(2)$ HBχPT($\not\Delta$) is a failed expansion…

- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
- $SU(2)$ HBχPT(Δ) has a chance of being a converging expansion - but it won't be pretty

Understanding the quark mass dependence



- To make a definitive statement on the death of $SU(2)$ HBχPT(Δ)
  - We are running on new ensembles at $M_\pi \sim 180$, 260 MeV
  - We plan to run on a~0.06fm as well (probably necessary to stabilize 0.5% precision result)

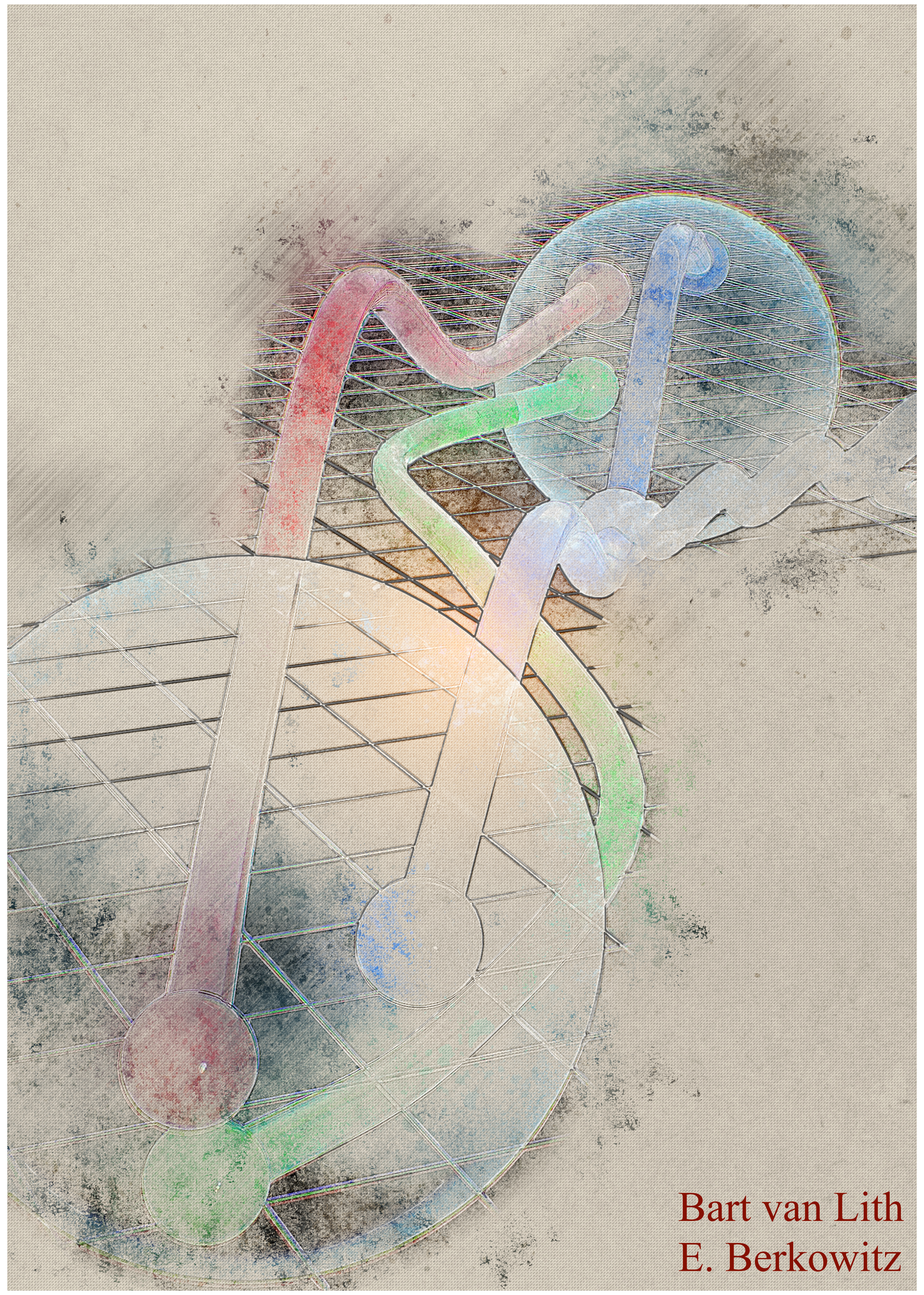
arXiv:1805.12130
https://github.com/callat-qcd/project_gA

Lattice QCD Team

(postdoc, grad student)

California Lattice Collaboration

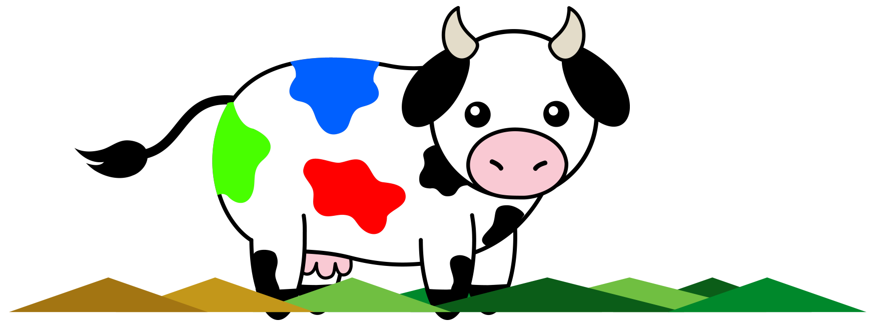
plus a few friends



Chia Cheng (Jason) Chang	LBNL, RIKEN-iTHEMS
Amy Nicholson	Berkeley —> UNC, Chapel Hill
Enrico Rinaldi	RIKEN-BNL
Evan Berkowitz	Forschungszentrum Jülich
Nicolas Garron	Liverpool —> Liverpool Hope U.
David Brantley	W&M, LBNL —> LLNL
Henry Monge-Camacho	W&M, LBNL —> UNC
Chris Monahan	INT —> W&M
Chris Bouchard	Glasgow
Kate Clark	NVIDIA
Bálint Joó	JLab
Thorsten Kurth	NERSC, LBNL
Kostas Orginos	W&M, JLab
Pavlos Vranas	LLNL
André Walker-Loud	LBNL

New characters
- Christian Drischler
- Chris Koerber
- Ben Hörz
- Dean Howarth
- Arjun Gambhir
- Ken McElvain

gauge configs provided free of constraints from MILC



unofficial MILC cow

MILC = MIMD Lattice Computation

(the acronym has an acronym in the acronym)

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING | OAK RIDGE National Laboratory | LEADERSHIP COMPUTING FACILITY | Lawrence Livermore National Laboratory | SciDAC Scientific Discovery through Advanced Computing

DOE Topical Collaboration
Double Beta Decay

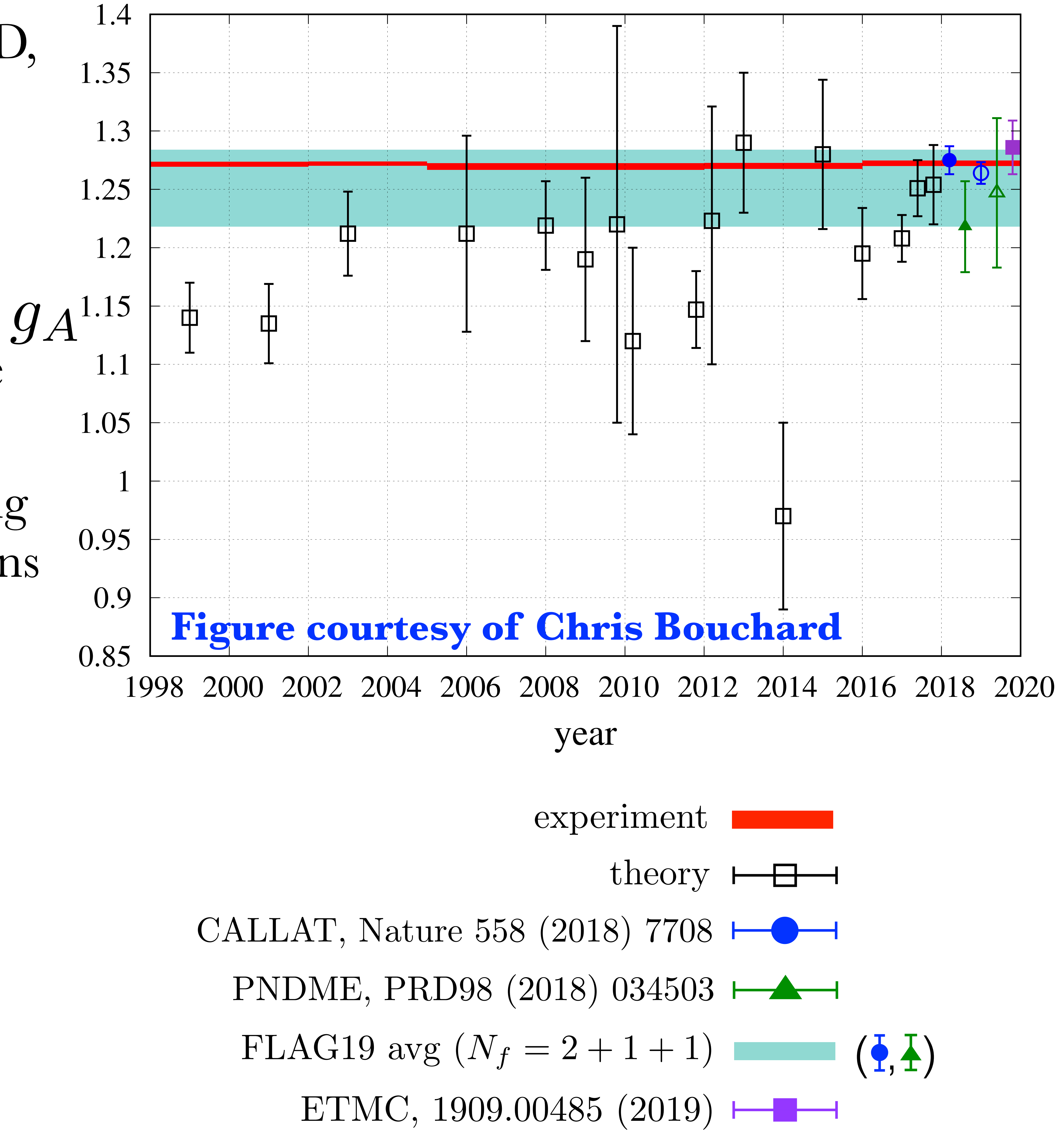
U.S. DEPARTMENT OF ENERGY | Office of Science | NNSA National Nuclear Security Administration

Status of Lattice Calculations of g_A

- We finally have a robust, precise determination of g_A from LQCD, and there are now O(5) groups with few percent precision and physical pion mass calculations
- All groups find the discretization effects in g_A are at the order of the stochastic uncertainties or smaller
- Achieving a 0.5% uncertainty is anticipated to only require more statistics (there are not any foreseen issues)
- LQCD confirmation of the new 0.4% QED correction: achieving 0.2% could be realized with isospin symmetric LQCD calculations
- LQCD has no hope to catch the experimental precision

$$g_A^{PDG} = 1.2723(23) \qquad 0.18\%$$
$$g_A^{\text{PERKEO III}} = 1.27641(45)(33) \quad 0.04\%$$

- Combined with lifetime measurements, LQCD can be used to constrain BSM right-handed currents
- We do not have a dedicated plan to increase precision on g_A
  - We are interested in computing thee QED corrections
  - We are computing $g_A(Q^2)$ (INCITE 2019), which will lead to improved g_A determination



Status of Lattice Calculations of g_A

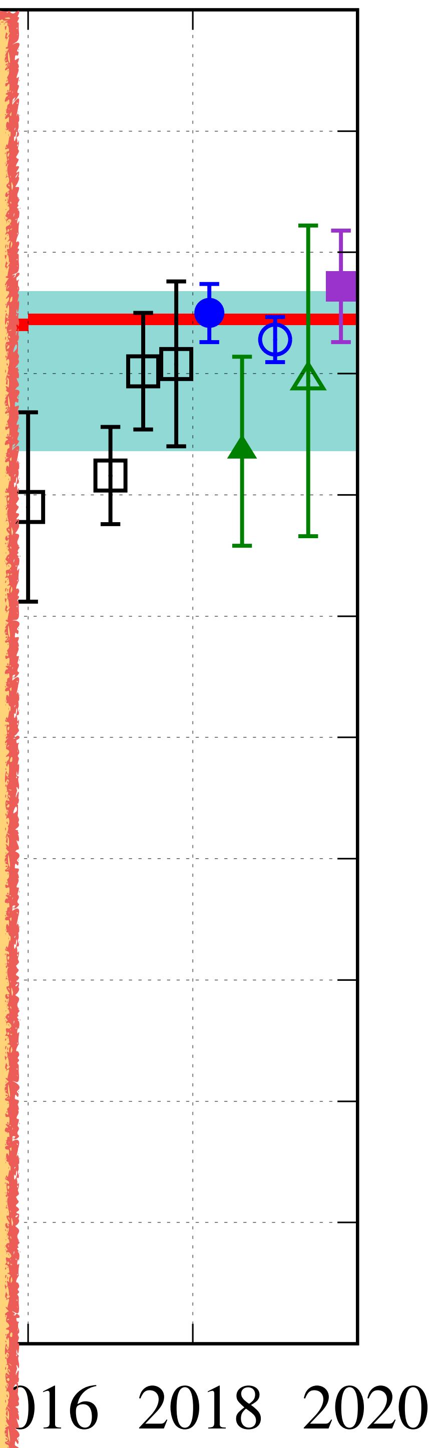
- $SU(2)\ \mathrm{HB}\chi\mathrm{PT}(\not\Delta)$ is a failed EFT at the physical pion mass
  - A conclusive paper will include 7 pion mass points to allow for a real fit to N4LO (g_A) and N3LO (M_N)
  - $SU(2)\ \mathrm{HB}\chi\mathrm{PT}(\Delta)$ *might* save the EFT, but this will require more LQCD results (N→Δ, Δ→Δ matrix elements)

- This is a very exciting time for LQCD as the new supercomputers (nearExascalee) are *disruptively* faster
  - The Exascale machines have been commissioned (in the US at least)
  - we have entered the era where LQCD can be used to determine few-nucleon physics observables
  - Coupled with effective EFTs of nuclear physics - we can build a quantitative understanding of nuclear physics
    a predictive theory of nuclear structure and reactions from the SM

- We fin… and th… physic…
- All gro… the sto…
- Achiev… statisti…
- LQCD… 0.2% c…
- LQCD…

g…

- Comb… constr…
- We do…
  - We …
  - We are computing $g_A(Q^2)$ (INCITE 2019), which will lead to improved g_A determination



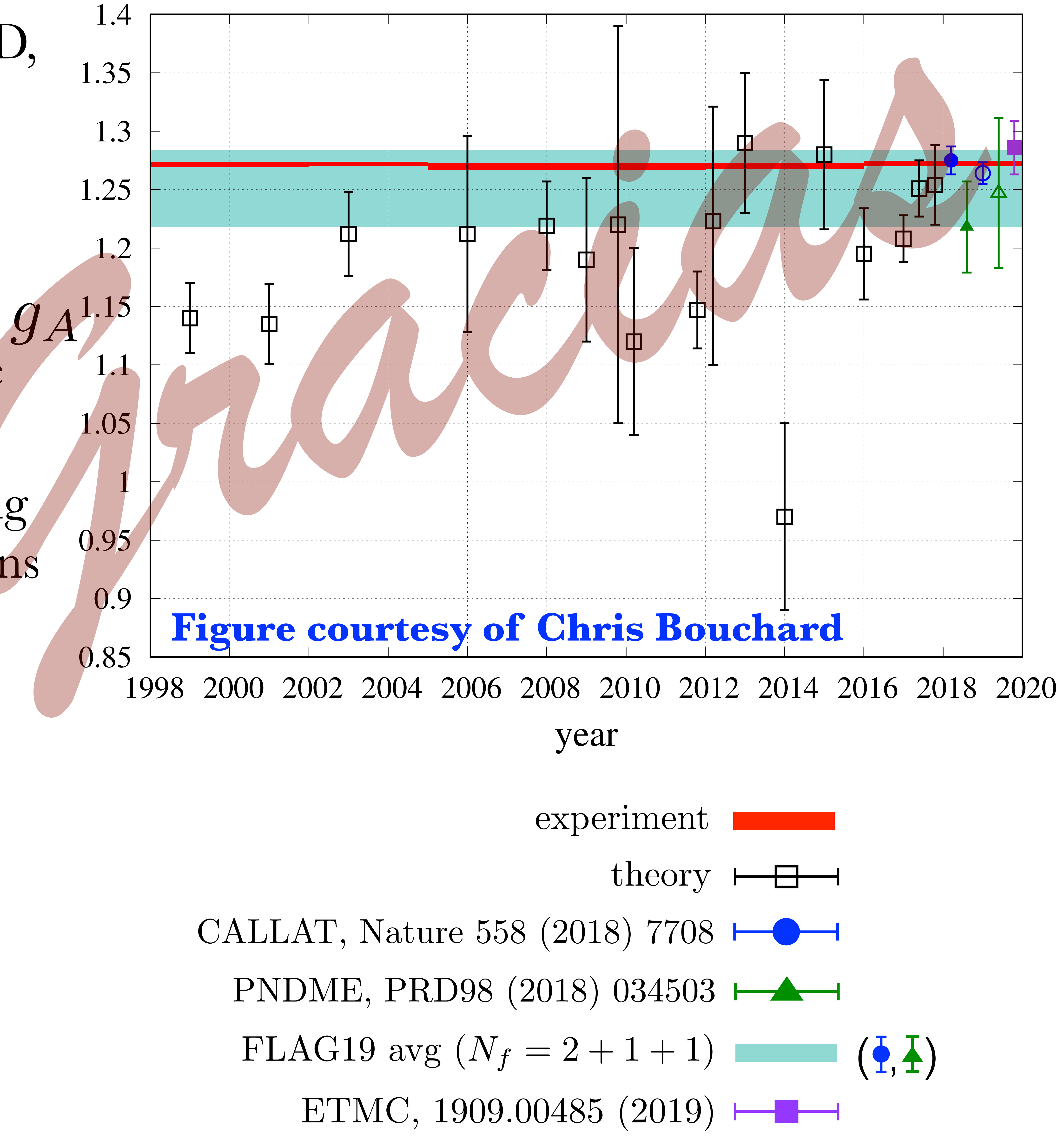
ETMC, 1909.00485 (2019)

Status of Lattice Calculations of g_A

- We finally have a robust, precise determination of g_A from LQCD, and there are now O(5) groups with few percent precision and physical pion mass calculations
- All groups find the discretization effects in g_A are at the order of the stochastic uncertainties or smaller
- Achieving a 0.5% uncertainty is anticipated to only require more statistics (there are not any foreseen issues)
- LQCD confirmation of the new 0.4% QED correction: achieving 0.2% could be realized with isospin symmetric LQCD calculations
- LQCD has no hope to catch the experimental precision

$$g_A^{PDG} = 1.2723(23) \qquad 0.18\%$$
$$g_A^{\mathrm{PERKEO\ III}} = 1.27641(45)(33) \quad 0.04\%$$

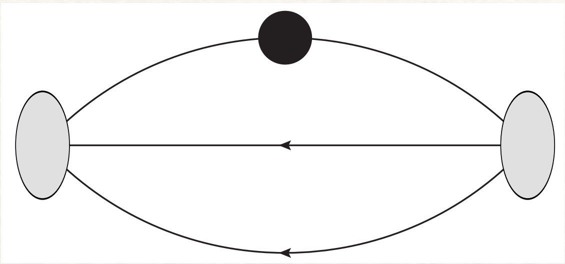
- Combined with lifetime measurements, LQCD can be used to constrain BSM right-handed currents
- We do not have a dedicated plan to increase precision on g_A
  - We are interested in computing thee QED corrections
  - We are computing $g_A(Q^2)$ (INCITE 2019), which will lead to improved g_A determination

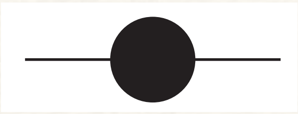


Muchas Gracias

Nucleon Axial FormFactor

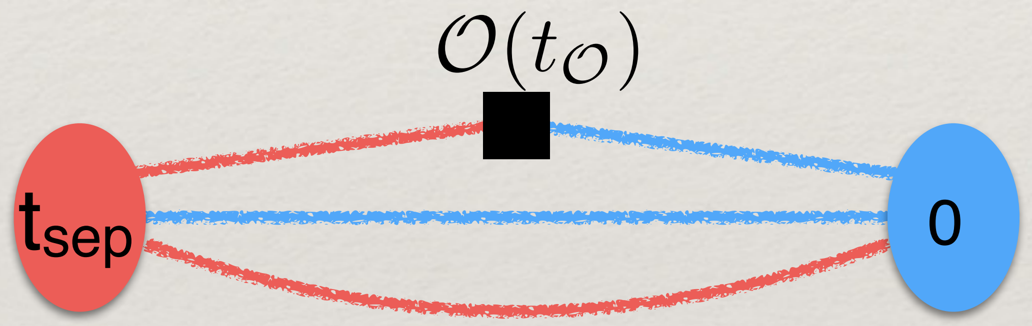
- Inherent to our gA calculation was the “Feynman-Hellmann” Propagator





$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

- For each choice of current and momentum, a new FH propagator is required
- We have tried several variants of stochastic methods to relax this constraint, but the noise is too large
- We have resorted to the standard fixed source-sink separation method (with our tail between our legs a little)



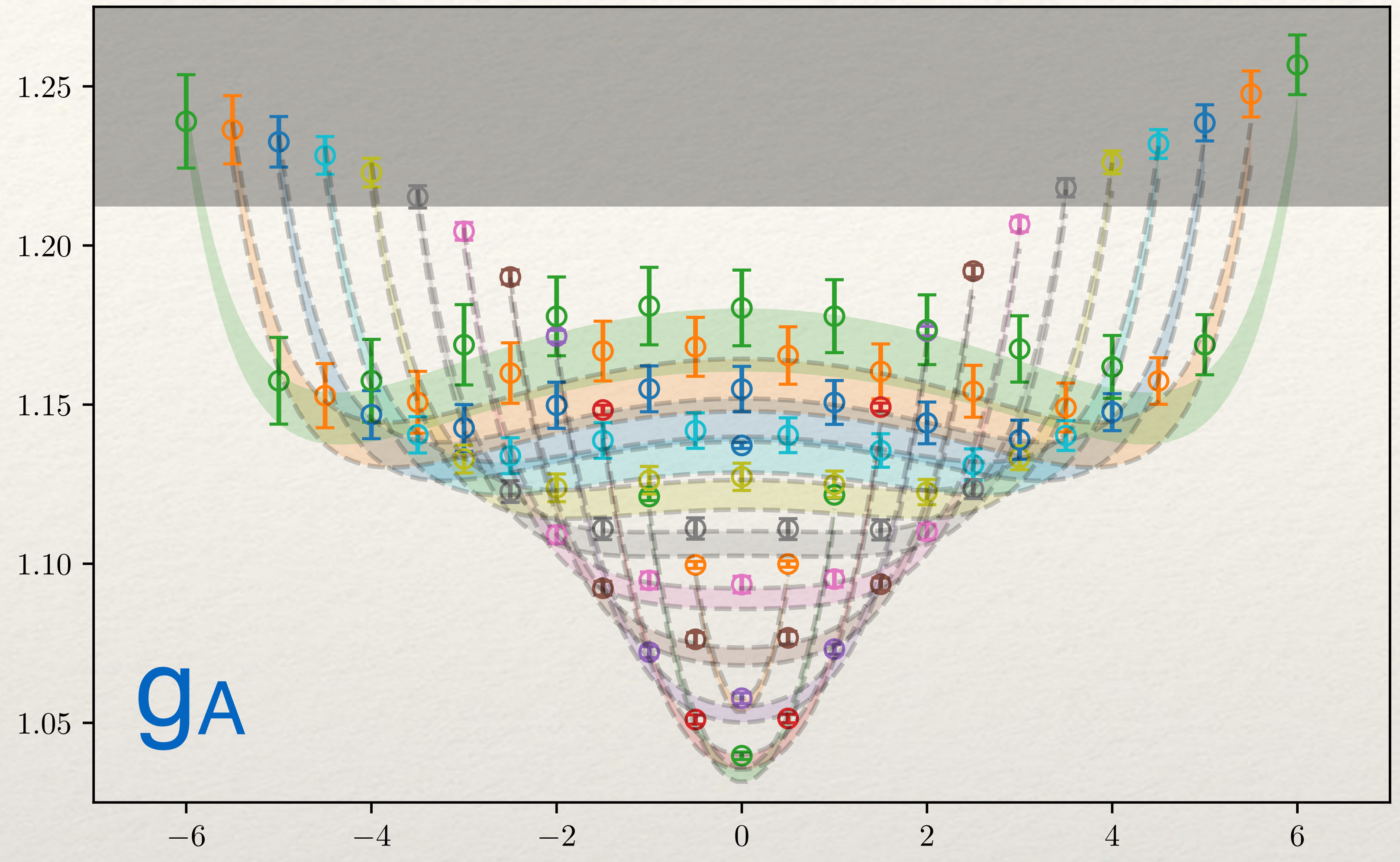
repeat for multiple values of $\mathbf{t_{sep}}$

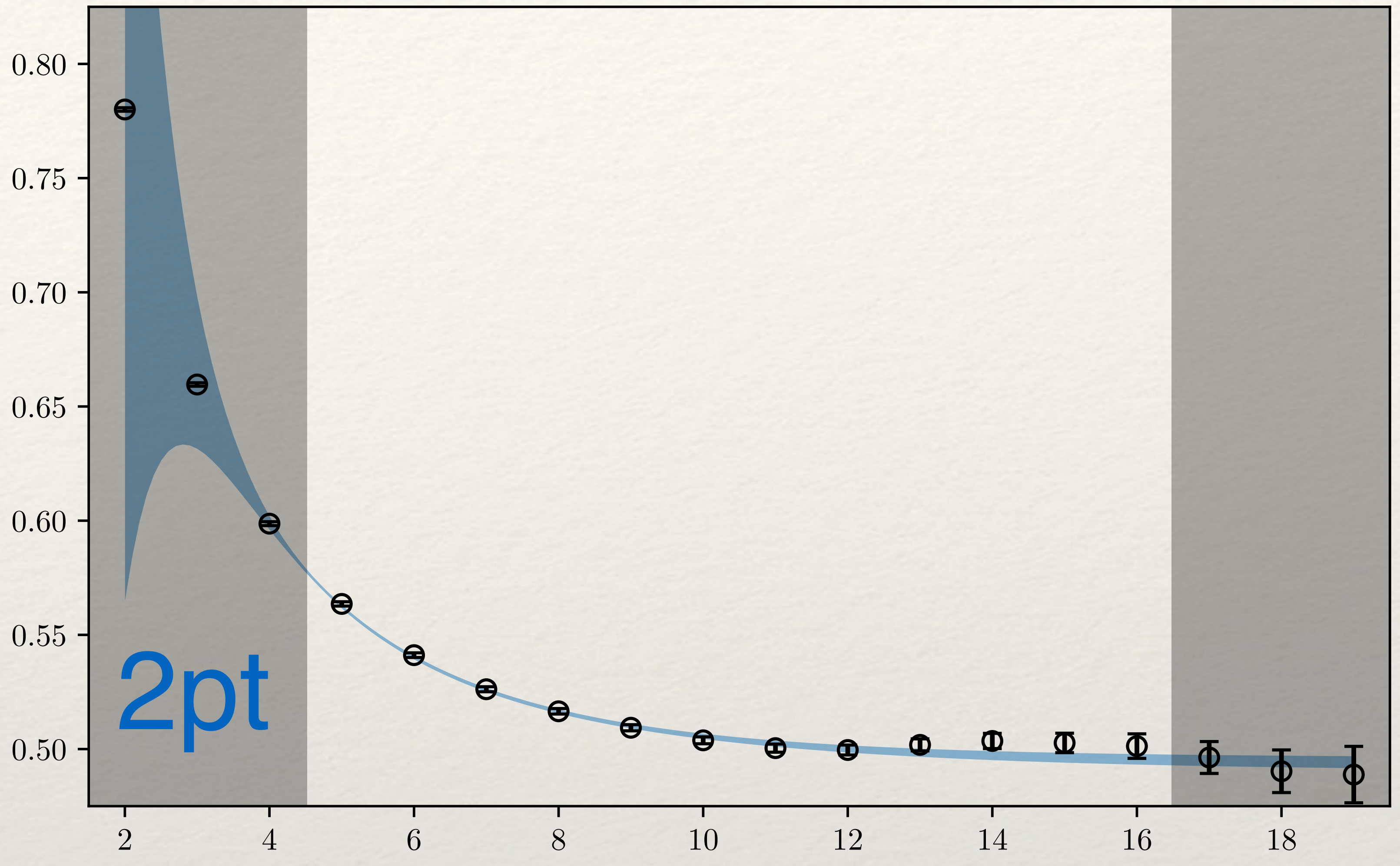
- However, if there was a lesson to be learned from our g_A calculation when applying the fixed source-sink separation method - it is imperative to use many values of $\mathbf{t_{sep}}$ and also small values
  - See also S. Meinel, Chiral Dynamics 2012 and Hasan et al. (LHPC) 1903.06487

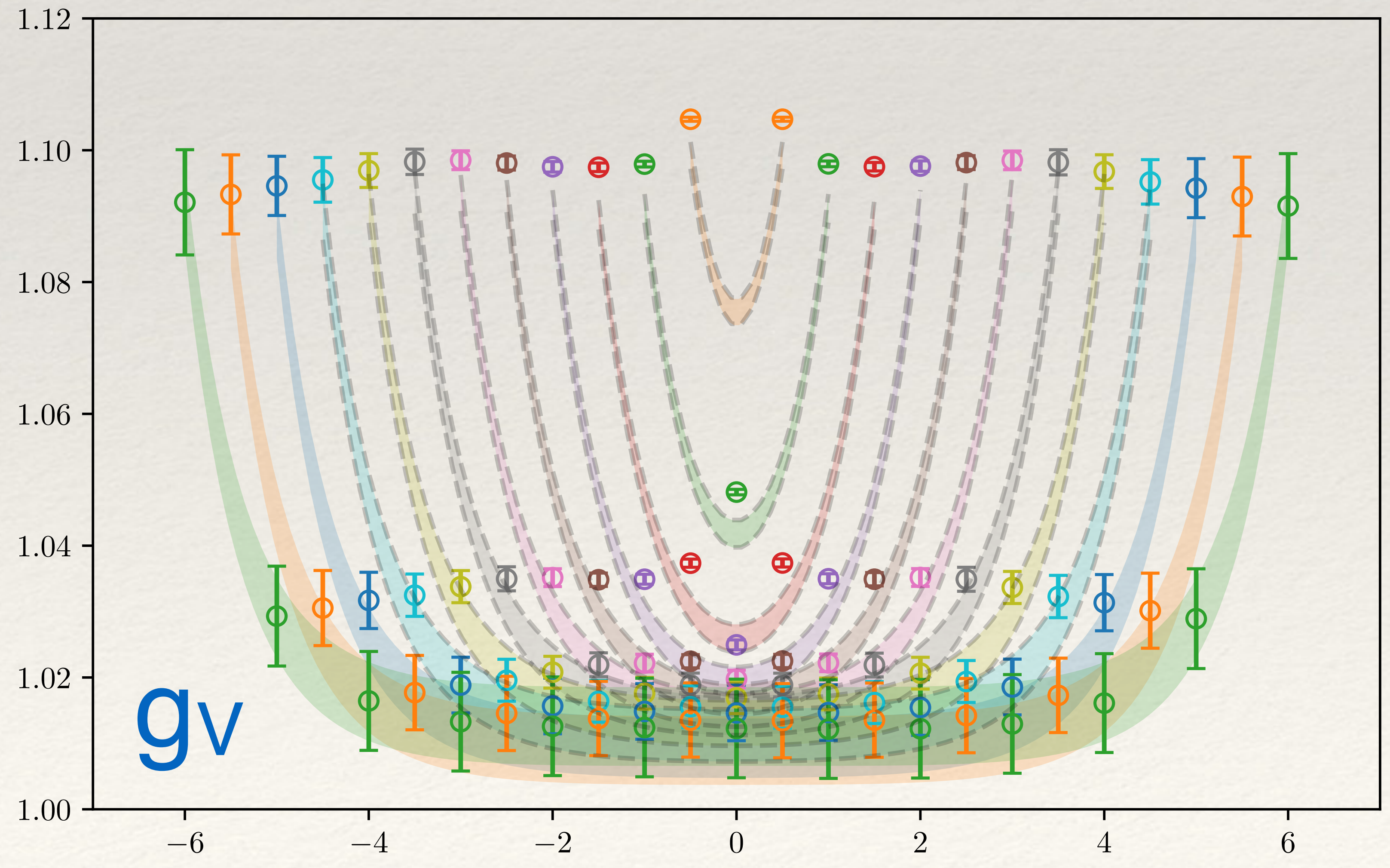
Nucleon Axial FormFactor PRELIMINARY

a09m310

t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]

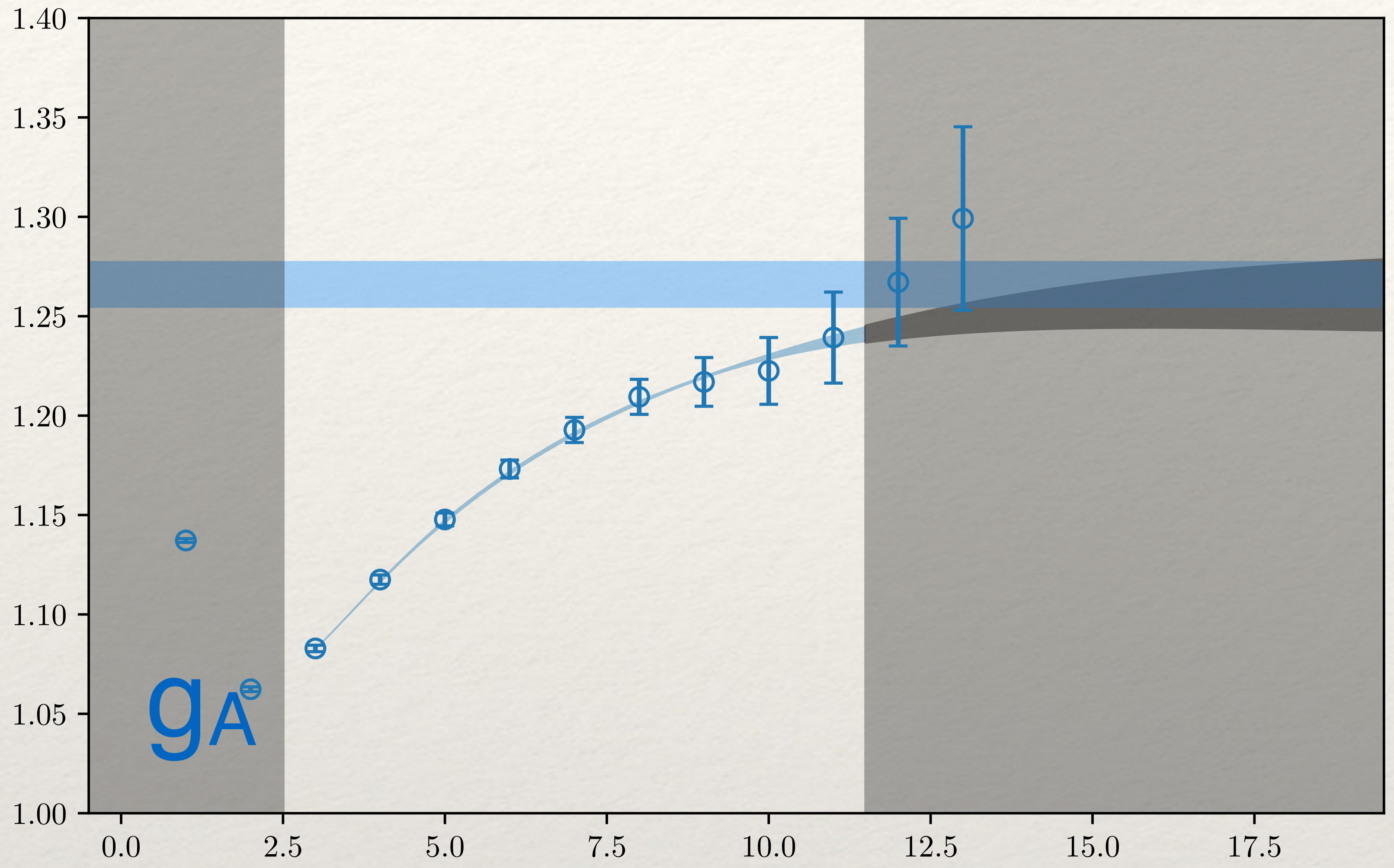


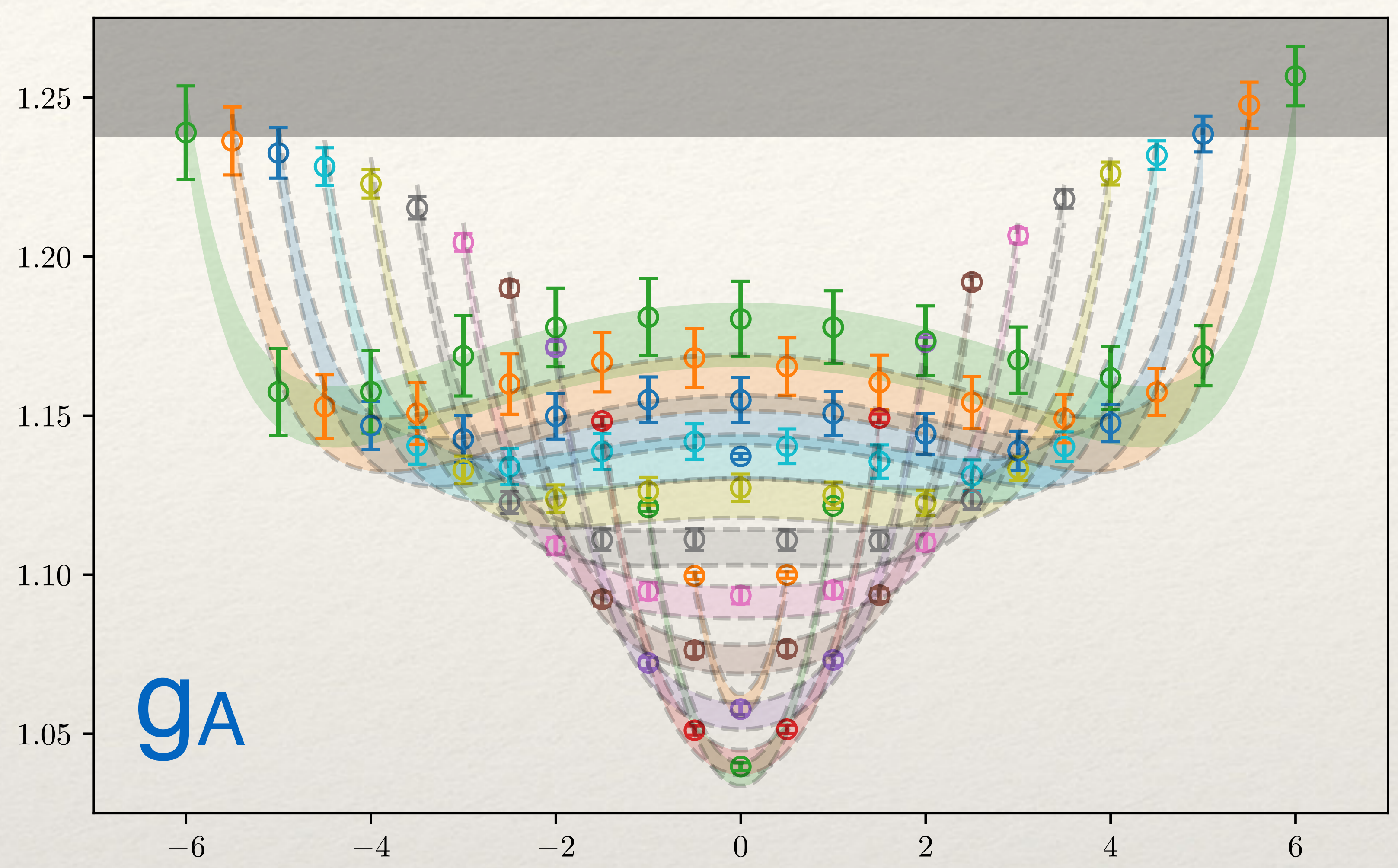


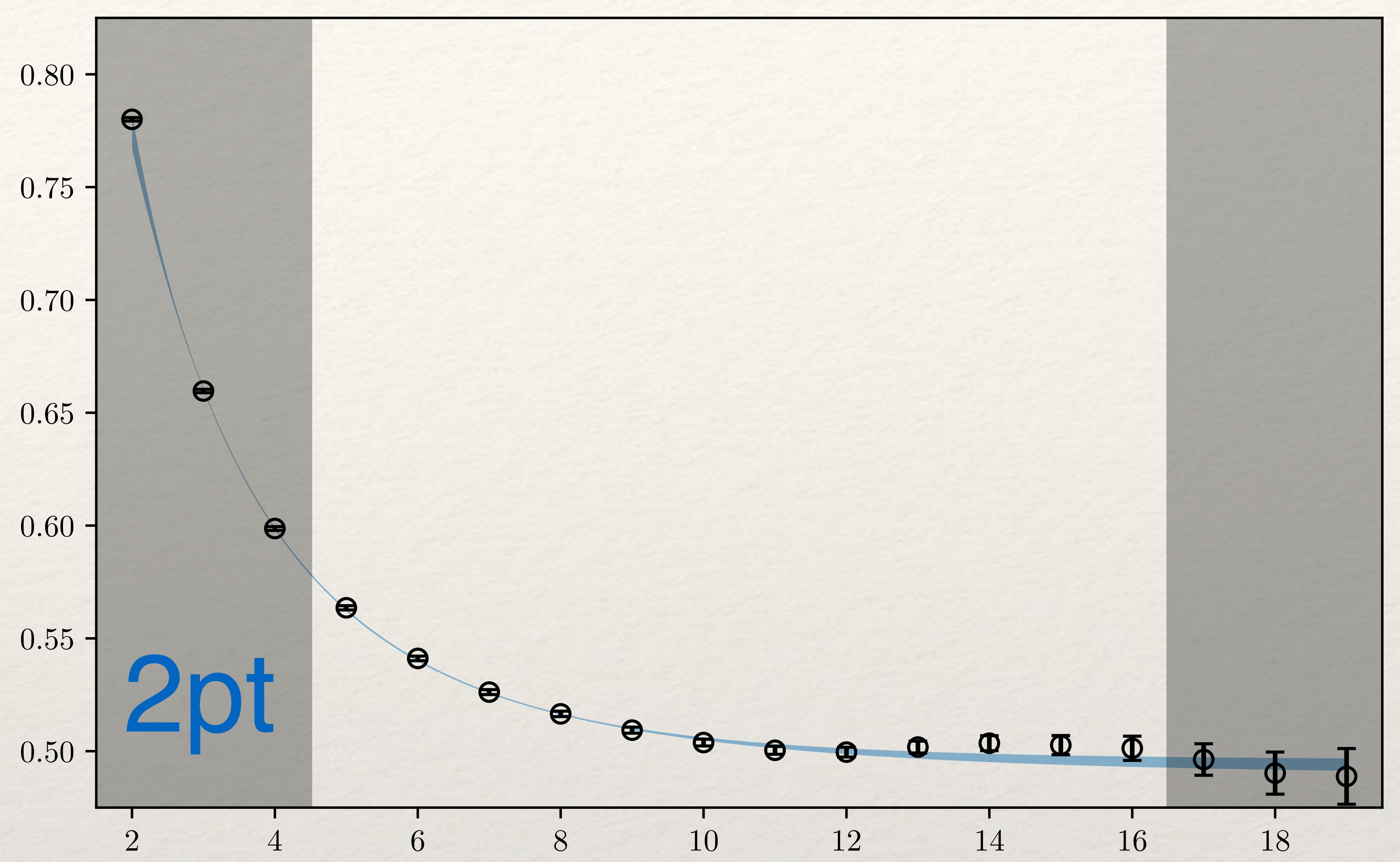


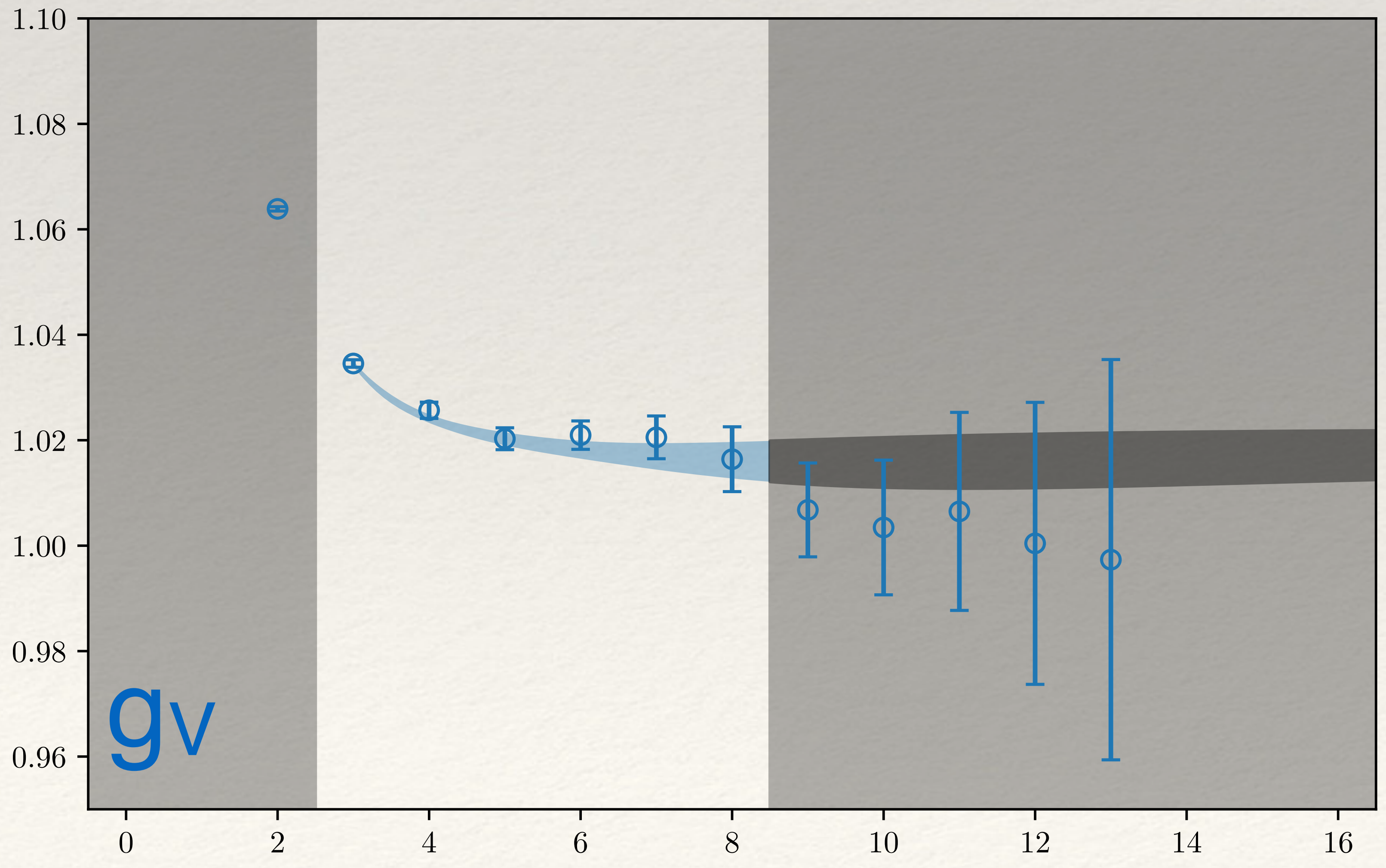
Nucleon Axial FormFactor PRELIMINARY

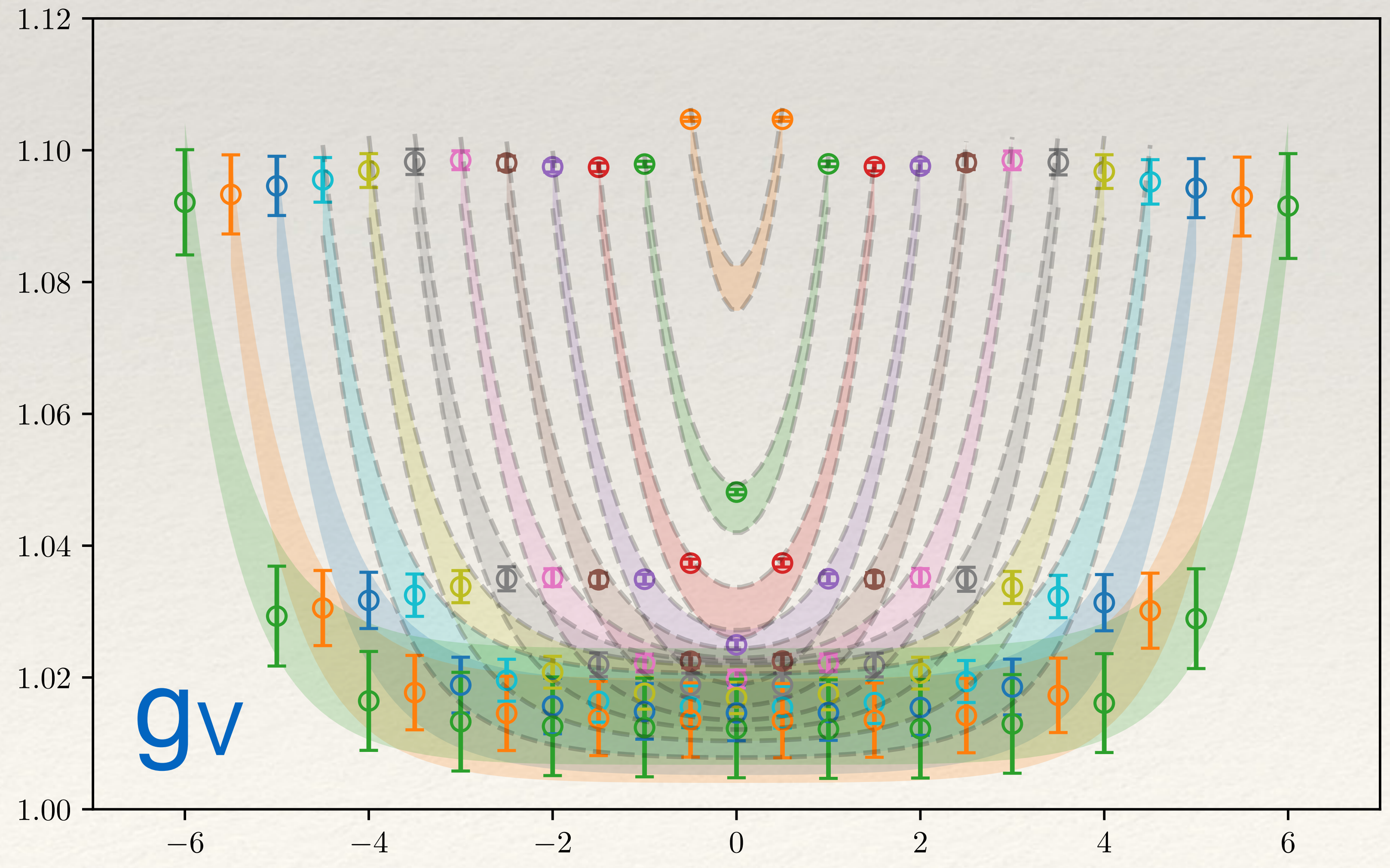
a09m310 - 8 sources - 1 coherent sink t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]











Add summed and then subtracted 3pt data to analysis

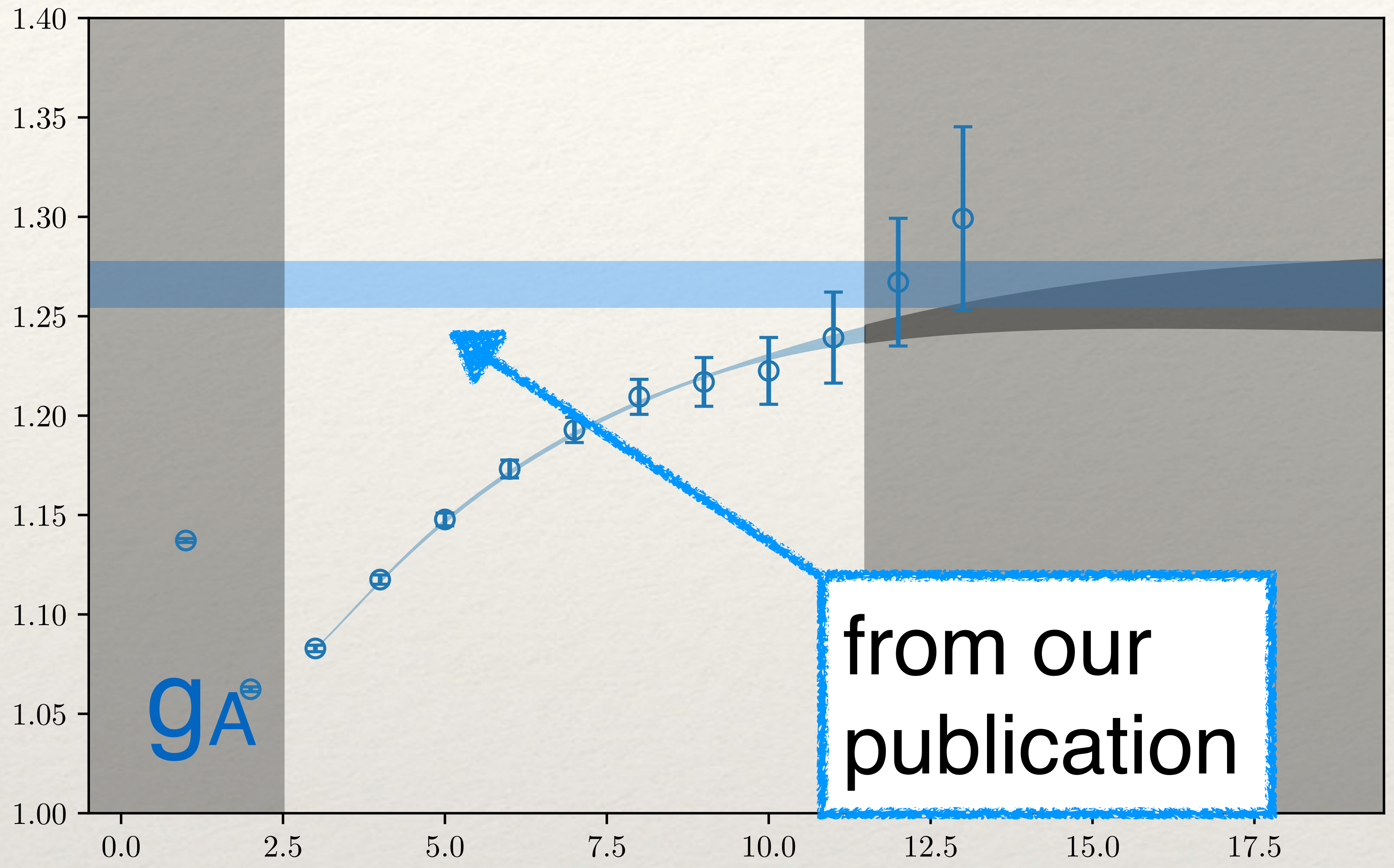
$$C_\Gamma^{sum}(t_{sep}) = \sum_{\tau=t_0+1}^{t_{sep}-1} C_3(t_{sep},\tau_\Gamma)$$

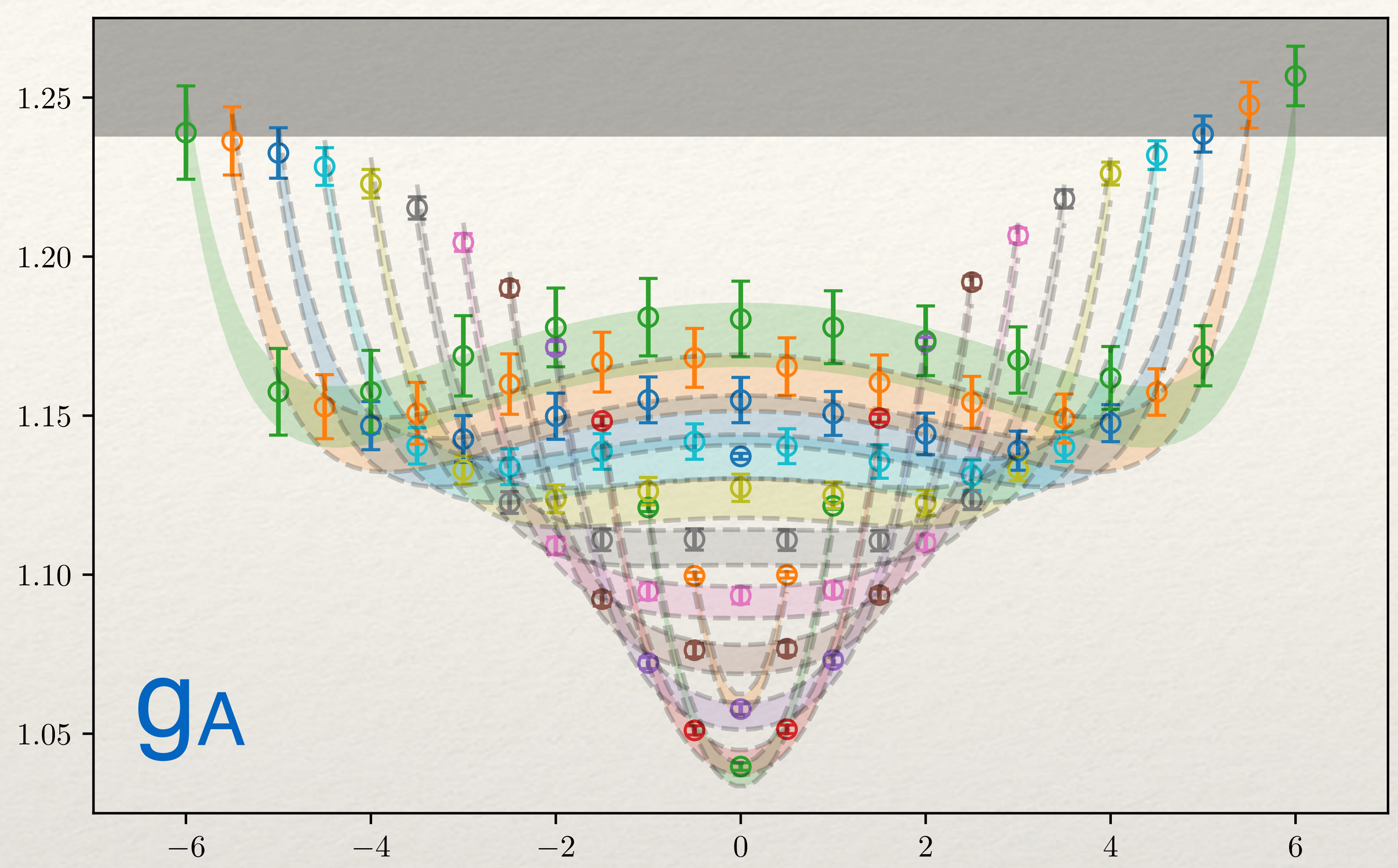
$$C_\Gamma^{\text{“}FH\text{”}}(t_{sep}) = \frac{C_\Gamma^{sum}(t_{sep}+1)}{C_2(t_{sep}+1)} - \frac{C_\Gamma^{sum}(t_{sep})}{C_2(t_{sep})}$$

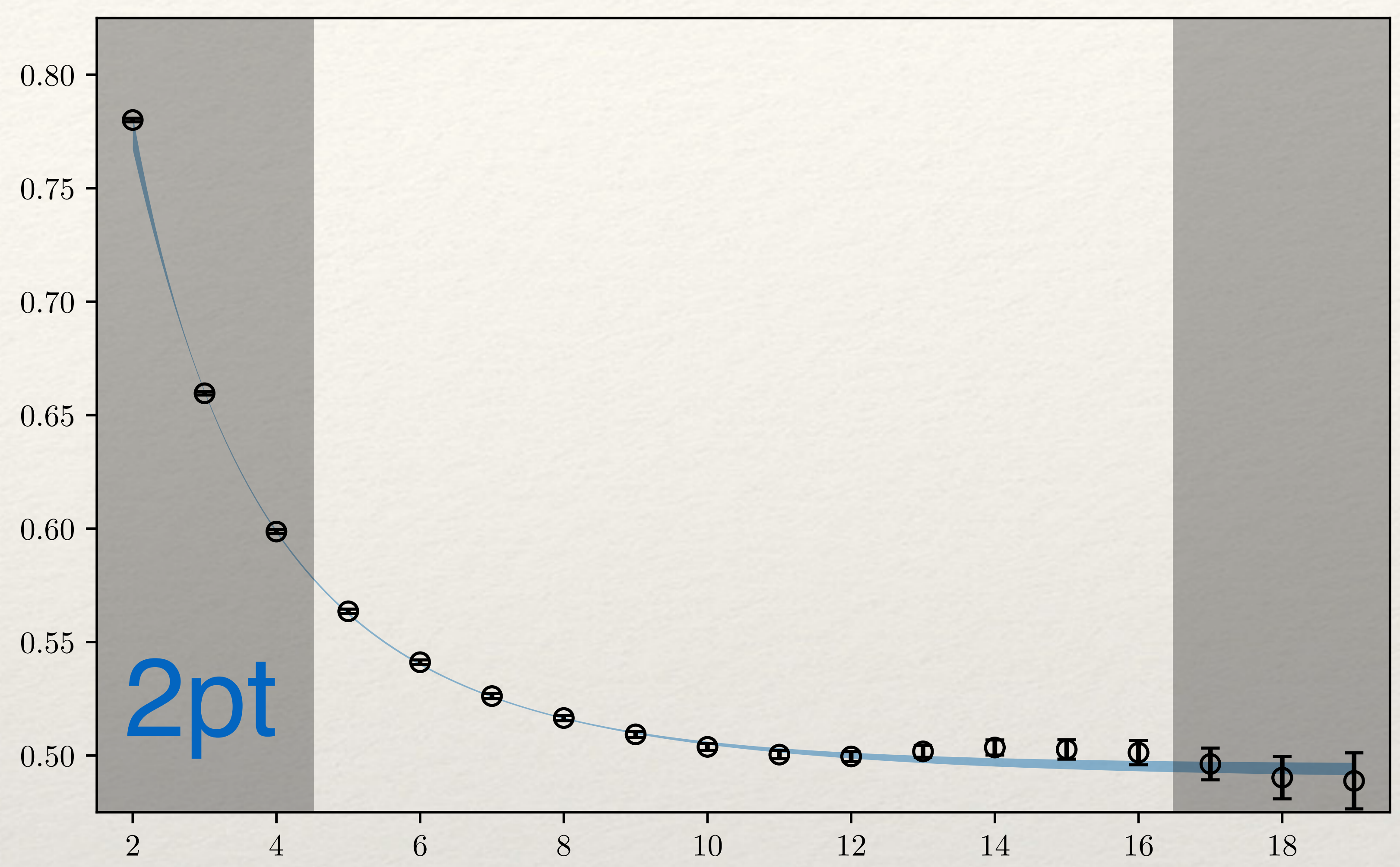
Nucleon Axial FormFactor

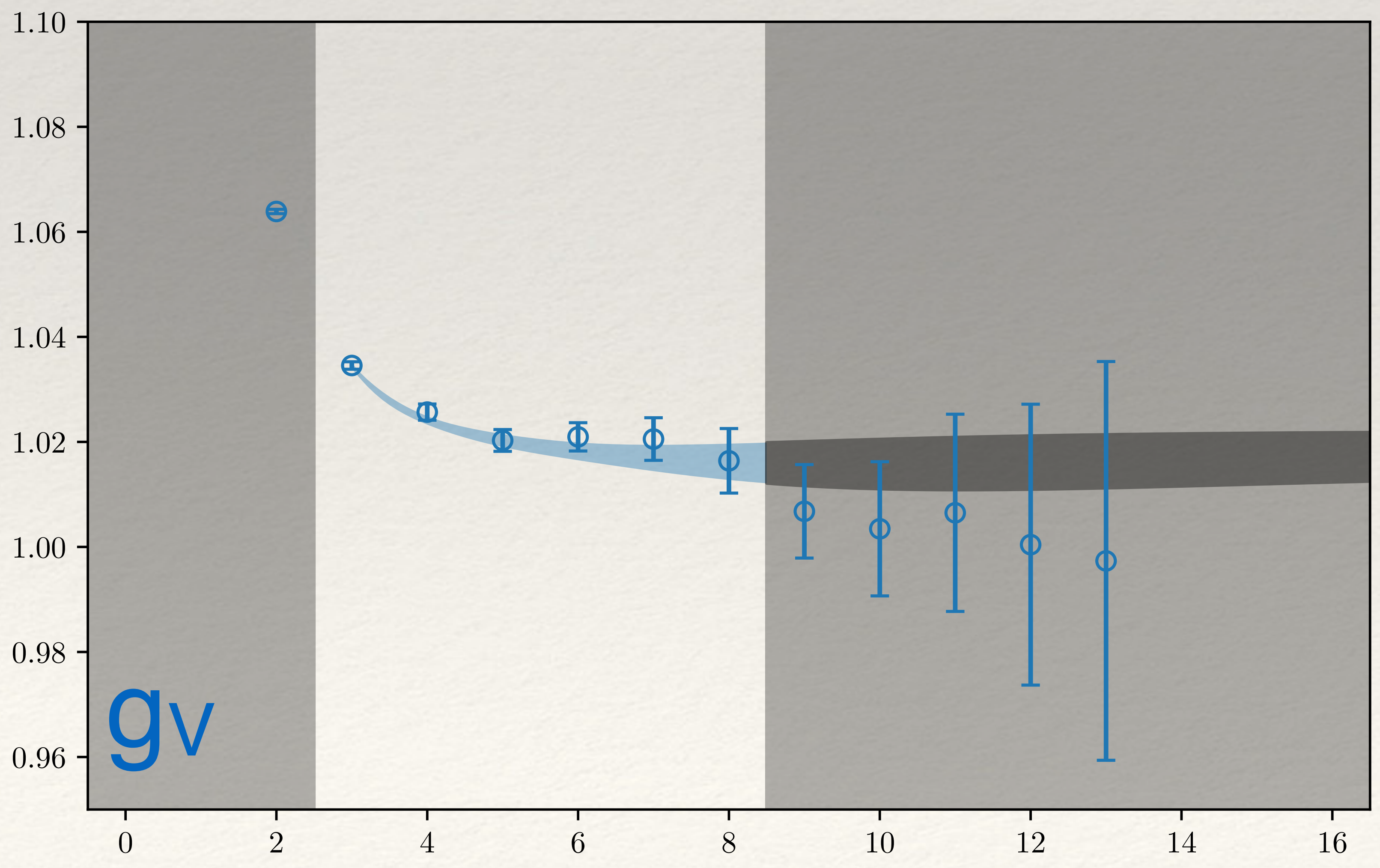
PRELIMINARY

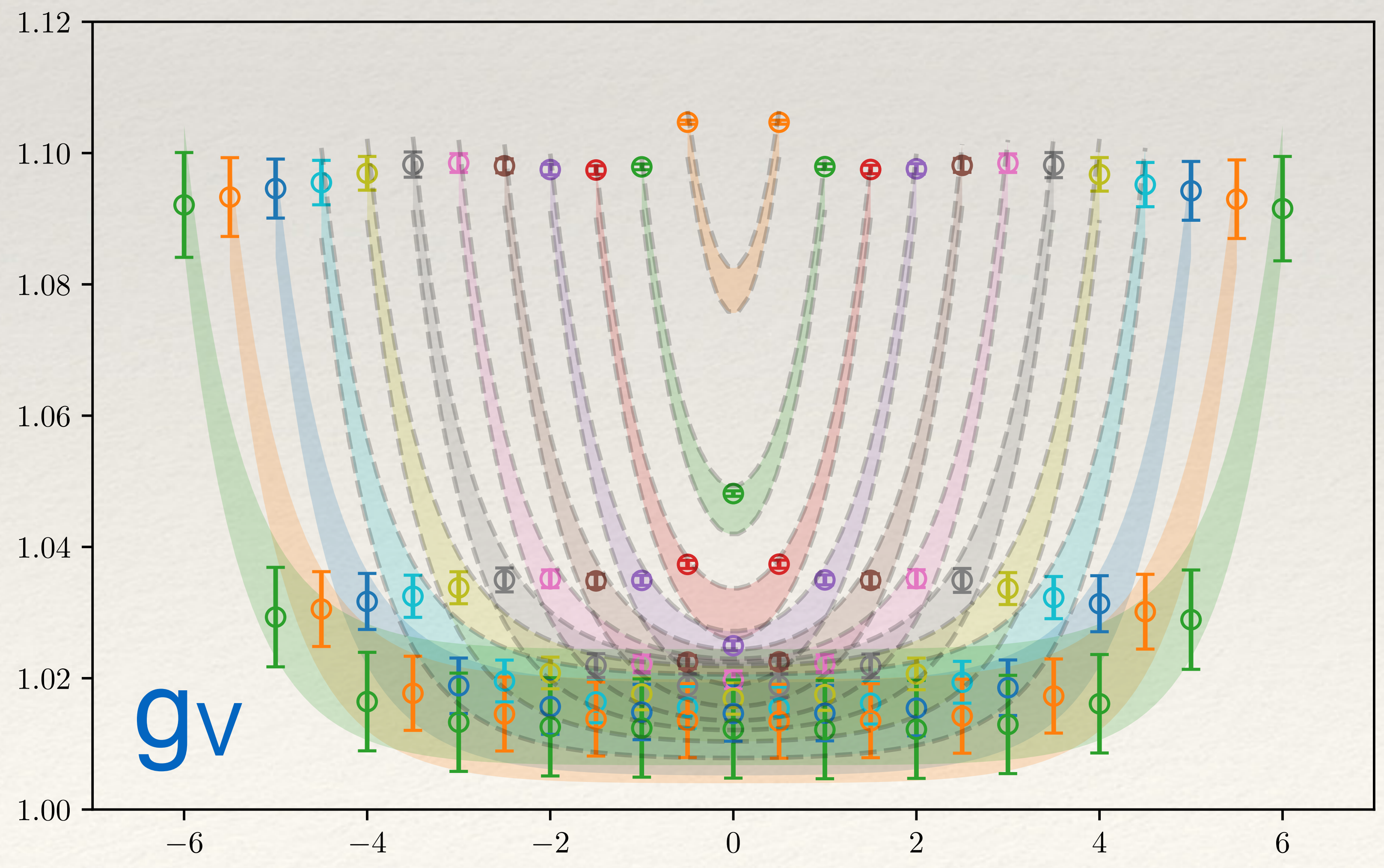
a09m310 - 8 sources - 1 coherent sink t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]











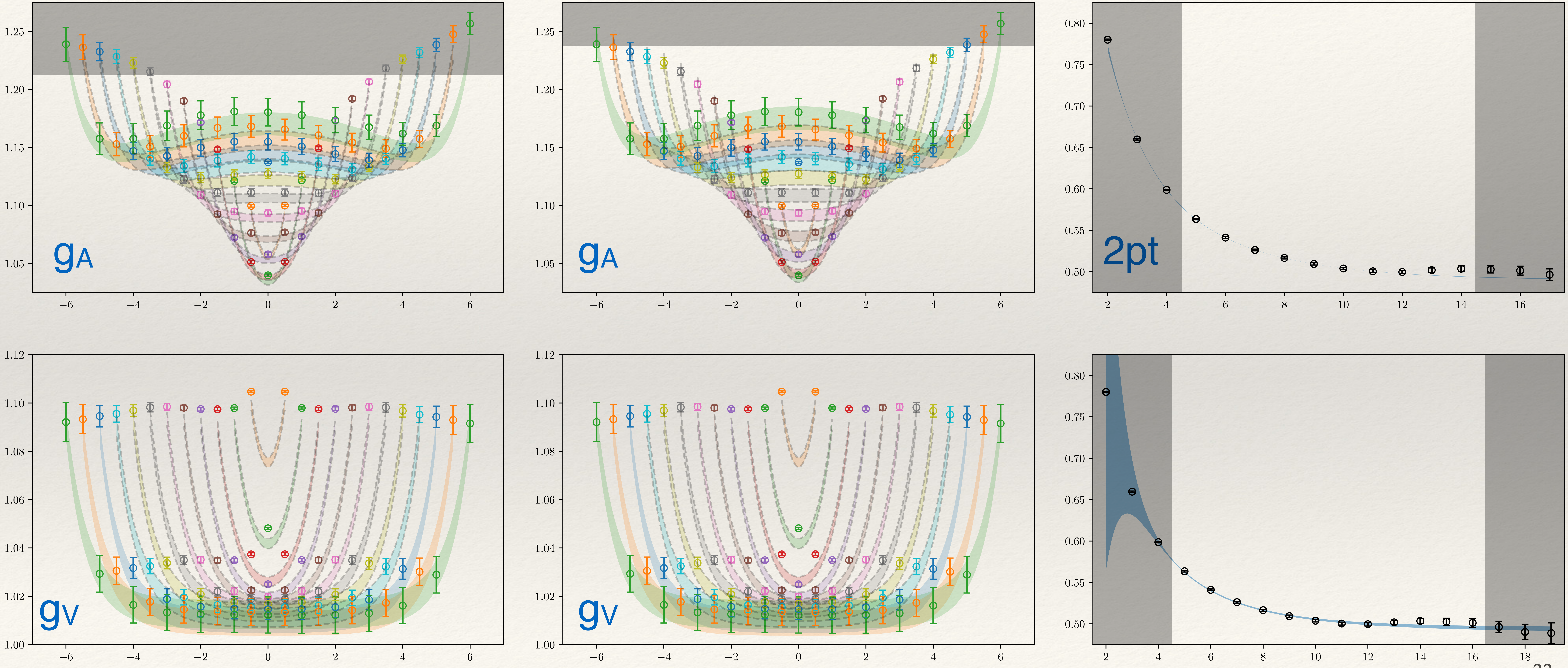
Add summed and then subtracted 3pt data to analysis

$$C_\Gamma^{sum}(t_{sep}) = \sum_{\tau=t_0+1}^{t_{sep}-1} C_3(t_{sep}, \tau_\Gamma)$$

$$C_\Gamma^{\text{“}FH\text{”}}(t_{sep}) = \frac{C_\Gamma^{sum}(t_{sep}+1)}{C_2(t_{sep}+1)} - \frac{C_\Gamma^{sum}(t_{sep})}{C_2(t_{sep})}$$

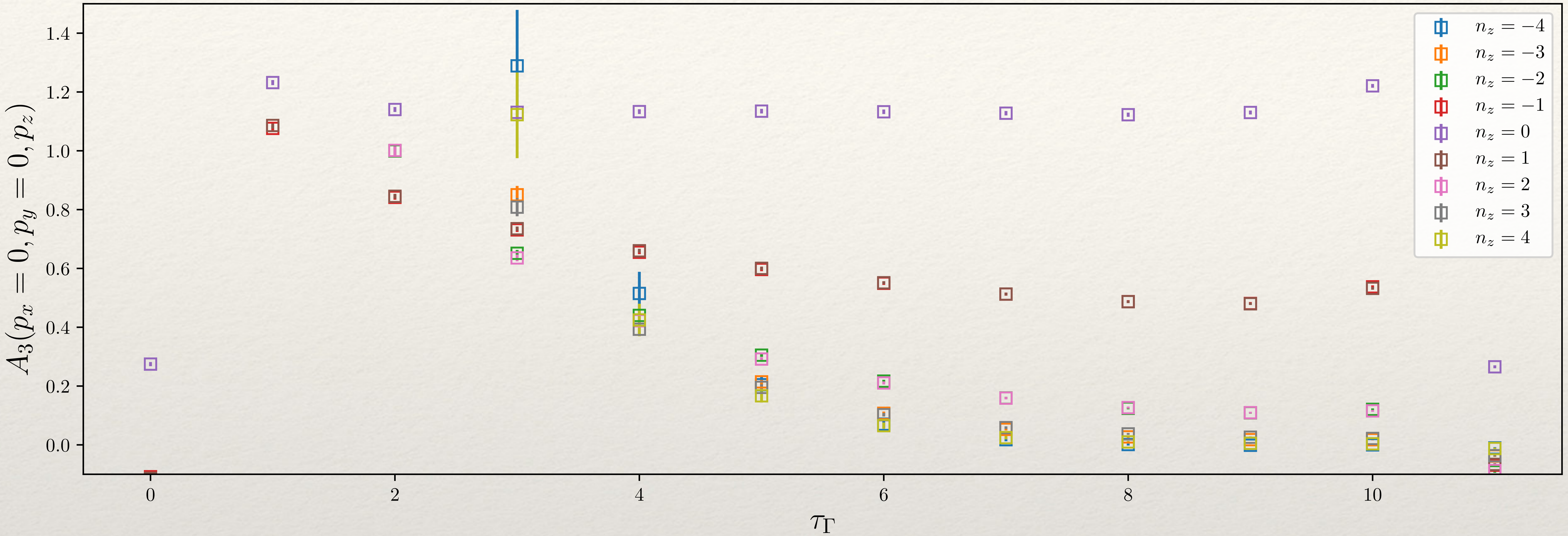
Nucleon Axial FormFactor PRELIMINARY

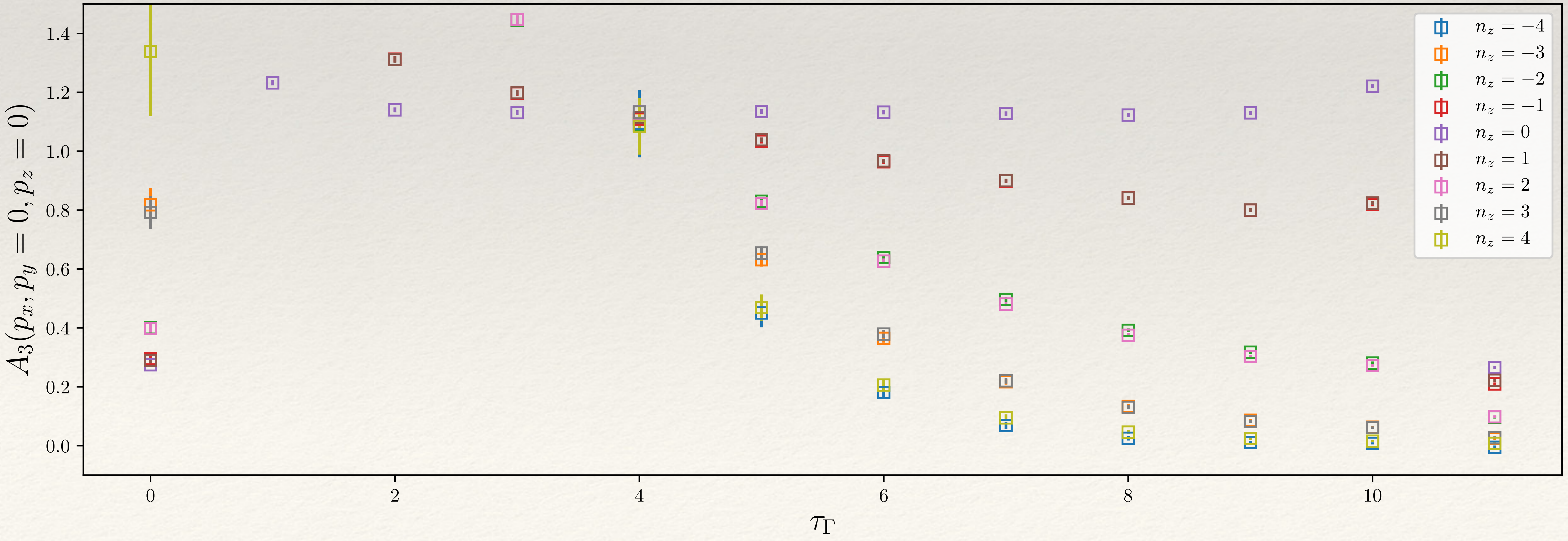
a09m310 t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]



Nucleon Axial FormFactor PRELIMINARY

a09m310 non-zero momentum, t_{sep} = 11





|n|=1, Q=0.196 GeV
|n|=2, Q=0.393 GeV
|n|=3, Q=0.589 GeV
|n|=4, Q=0.785 GeV

Nucleon Axial FormFactor

Spin averaging

PRELIMINARY

In the literature - we see both

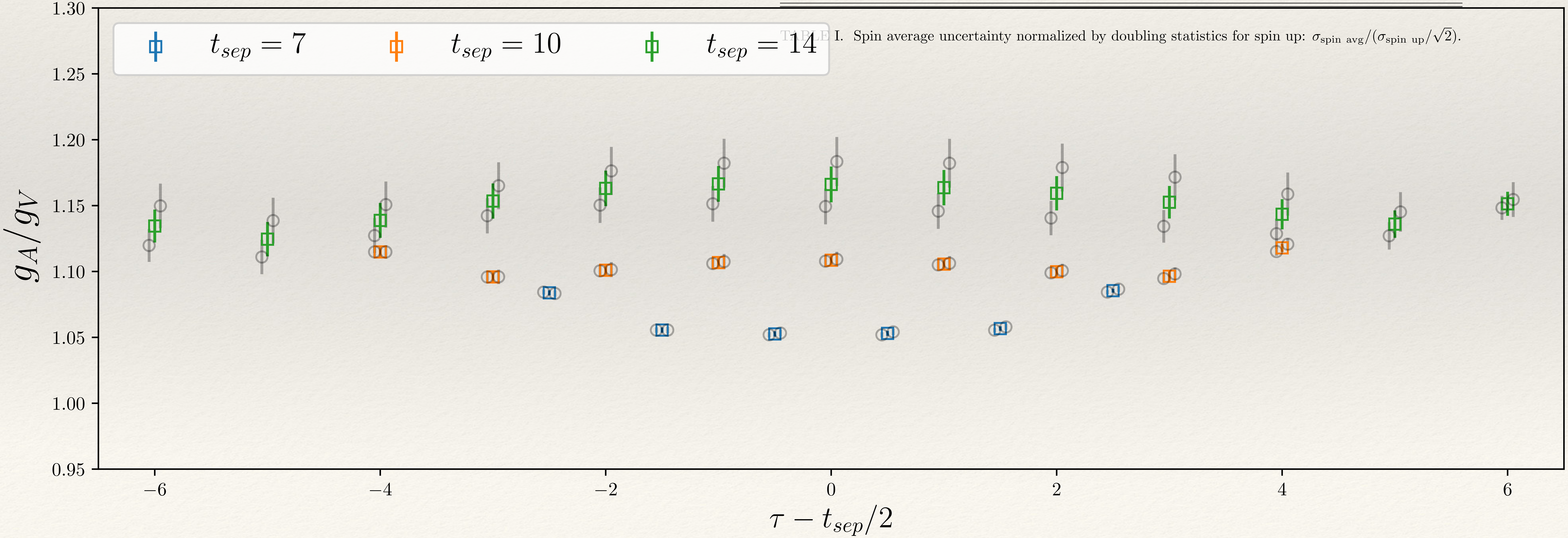
$\mathcal{P}_{3pt} \propto 1 + i\gamma_5\gamma_3$ spin up only

$\mathcal{P}_{3pt} \propto i\gamma_5\gamma_3$ spin up - spin dn

t_{sep}	$t_0+\tau_T$												
3	1.23	1.23											
4	1.19	1.20	1.18										
5	1.14	1.15	1.15	1.12									
6	1.09	1.11	1.12	1.10	1.06								
7	1.07	1.08	1.10	1.09	1.06	1.03							
8	1.06	1.07	1.09	1.09	1.06	1.03	1.01						
9	1.05	1.06	1.08	1.08	1.07	1.05	1.03	1.02					
10	1.04	1.05	1.06	1.07	1.07	1.06	1.05	1.05	1.04				
11	1.03	1.03	1.04	1.05	1.05	1.06	1.07	1.07	1.07	1.07			
12	1.02	1.02	1.02	1.03	1.04	1.06	1.08	1.10	1.10	1.10	1.09		
13	1.02	1.02	1.01	1.01	1.03	1.06	1.08	1.10	1.12	1.12	1.13	1.12	
14	1.01	1.01	1.01	1.01	1.03	1.05	1.08	1.10	1.12	1.13	1.14	1.14	1.13

TABLE I. Spin average uncertainty normalized by doubling statistics for spin up: $\sigma_{\text{spin avg}}/(\sigma_{\text{spin up}}/\sqrt{2})$.

$$\frac{\sigma_{\text{spin avg}}}{\sigma_{\text{spin up}}/\sqrt{2}}$$



arXiv.org > hep-lat > arXiv:1612.06963
Phys. Rev. D96 (2017)

High Energy Physics – Lattice

On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

Feynman-Hellmann theorem

$$\partial_\lambda E_n|_{\lambda=0} = \langle n|H_\lambda|n\rangle$$

"Follow your nose" in QFT

"Feynman-Hellmann" correlation function

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C_\lambda(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C_\lambda(t)}\right]_{\lambda=0}$$

$$= g_\lambda + z\left(e^{-(t+1)\Delta_{10}} - e^{-t\Delta_{10}}\right) + \cdots$$

$\Delta_{10} = E_1 - E_0$ more than exponentially suppressed

arXiv.org > hep-lat > arXiv:1612.06963
Phys. Rev. D96 (2017)

High Energy Physics – Lattice

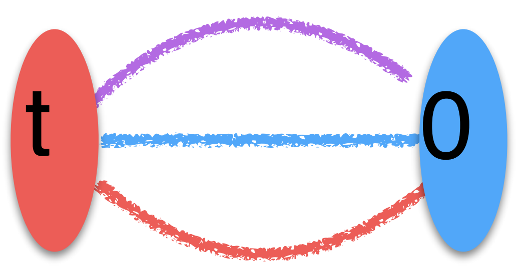
On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

Feynman-Hellmann theorem $\partial_\lambda E_n|_{\lambda=0} = \langle n|H_\lambda|n\rangle$

derivative correlation function

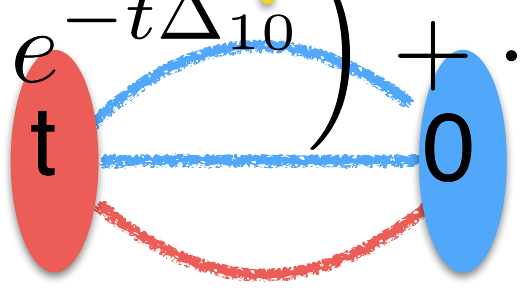


"Follow your nose" in QFT

"Feynman-Hellmann" correlation function

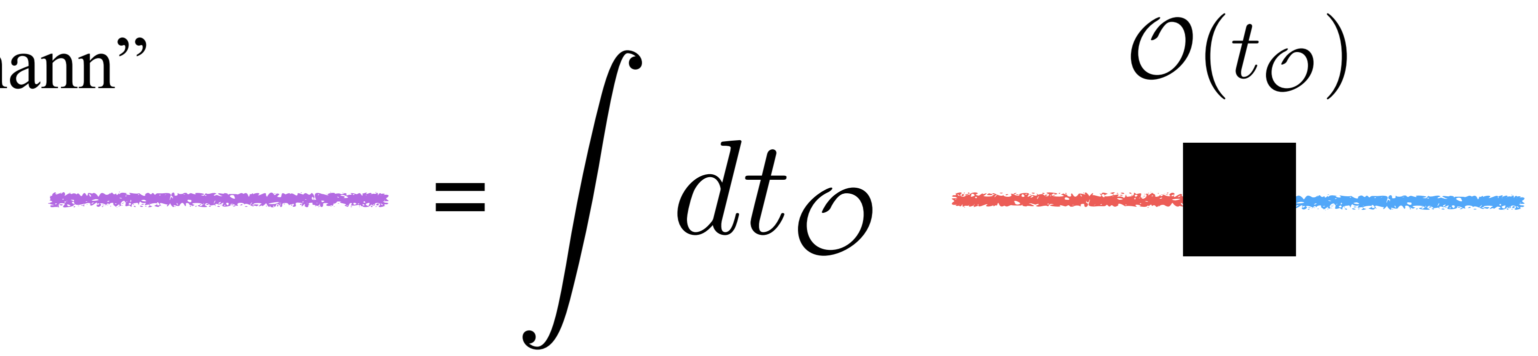
$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C_\lambda(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C_\lambda(t)}\right]_{\lambda=0}$$

$$= g_\lambda + z\left(e^{-(t+1)\Delta_{10}} - e^{-t\Delta_{10}}\right) + \cdots$$



standard 2-point function

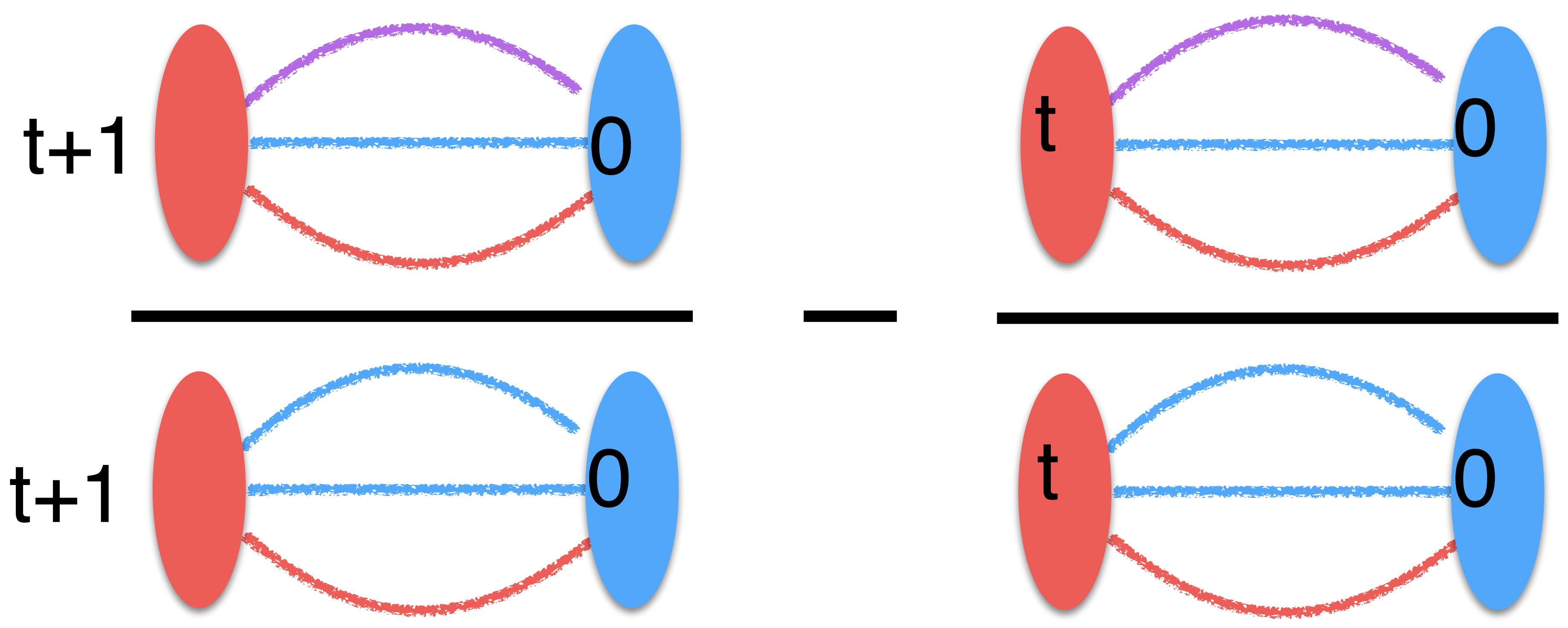
"Feynman-Hellmann" propagator



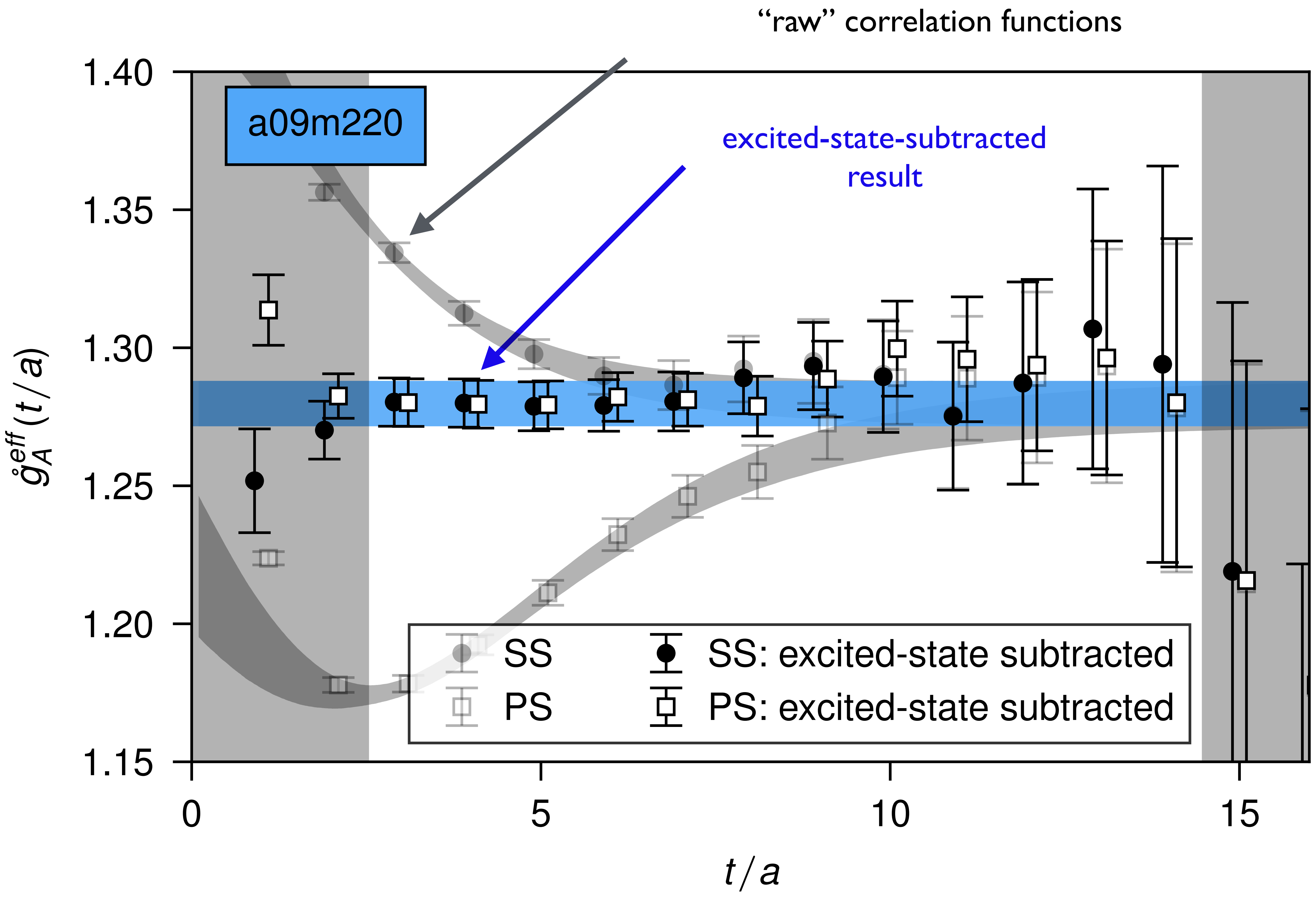
On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

Phys. Rev. D96 (2017)
arXiv:1612.06963

our unconventional method



$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$



$$\left.\partial_\lambda m_\lambda^{eff}(t)\right|_{\lambda=0} = g_{00} + z(e^{-(t+1)\Delta_{10}} - e^{-t\Delta_{10}}) + \cdots$$

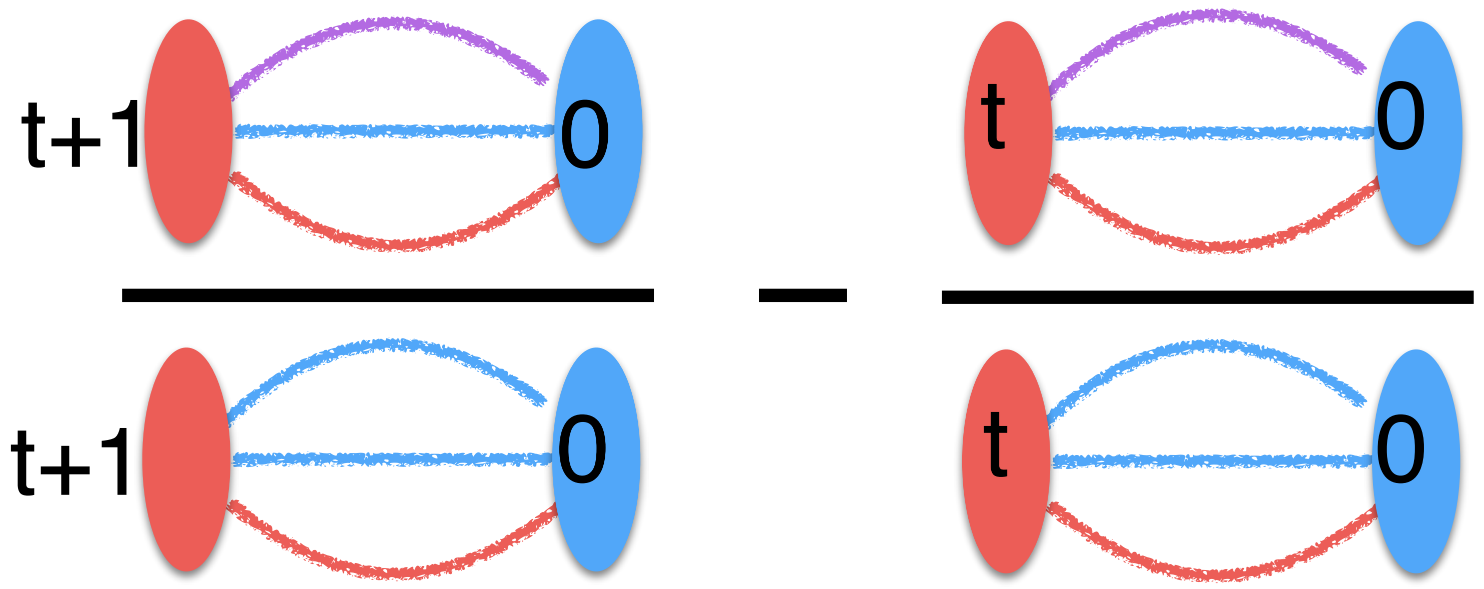
Key features of this method

- The correlation function is given by
- excited state contamination is demonstrably controlled
- we can access very early Euclidean time, allowing the use of exponentially more precise numerical points
- **No background field is used** - the FH-theorem is used to "derive" our "Feynman-Hellmann Correlation function" analytically

On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

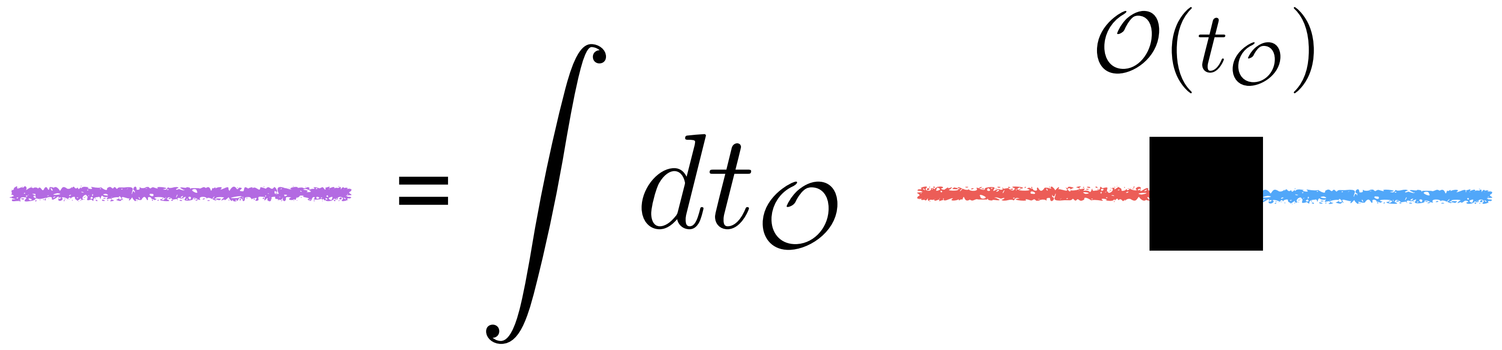
Phys. Rev. D96 (2017)
arXiv:1612.06963

our unconventional method



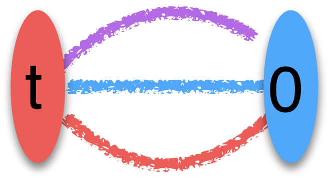
$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

"Feynman-Hellmann" propagator



$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

Our unconventional method is similar too

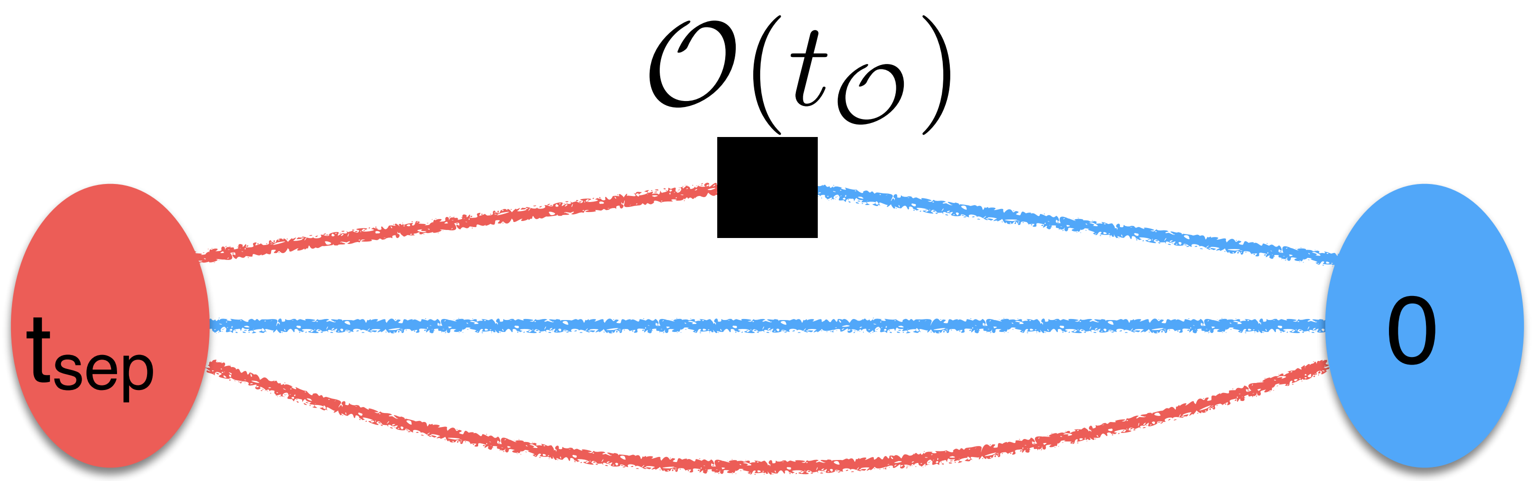
- traced back to Maiani, Martinelli, Paciello and Taglienti **Nucl. Phys. B293 (1987):** Güsken, Low, Mutter, Sommer, Patel, Schilling **PLB227 (1989)** first computed $-\partial_\lambda C_\lambda(t)$ 

- Bulava, Donnellan, Sommer, **JHEP 1201 (2012):** combined above with GEVP
- de Divitiis, Petronzio, Tantalo, **PLB718 (2012):** computed derivatives of form factors
- Chambers et al. **PRD90 (2014), PRD92 (2015)** Savage et al. **PRL199 (2017);** used unconventional method with background field (λ) varying strength of field to extract derivative
- **Our method:**
  - uses analytic representation of derivative correlator instead of background field (cheaper)
  - uses complete spectral decomposition of correlator, including contact operators
  - analysis was pushed to greater detail, showing stability of analysis (PRD96 [1612.06963], [1704.01114], Nature 558 [1805.12130])

On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

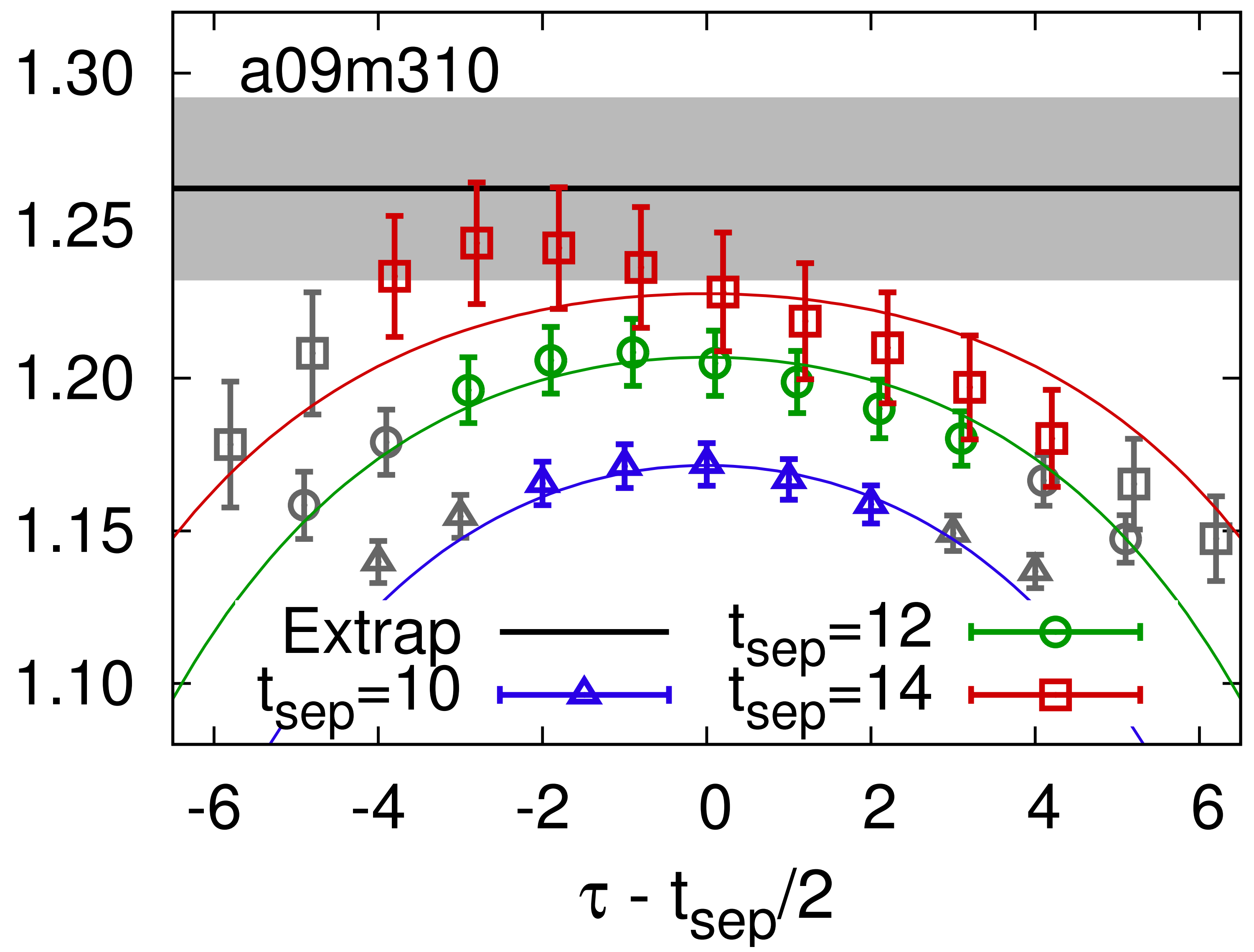
Phys. Rev. D96 (2017)
arXiv:1612.06963

standard method



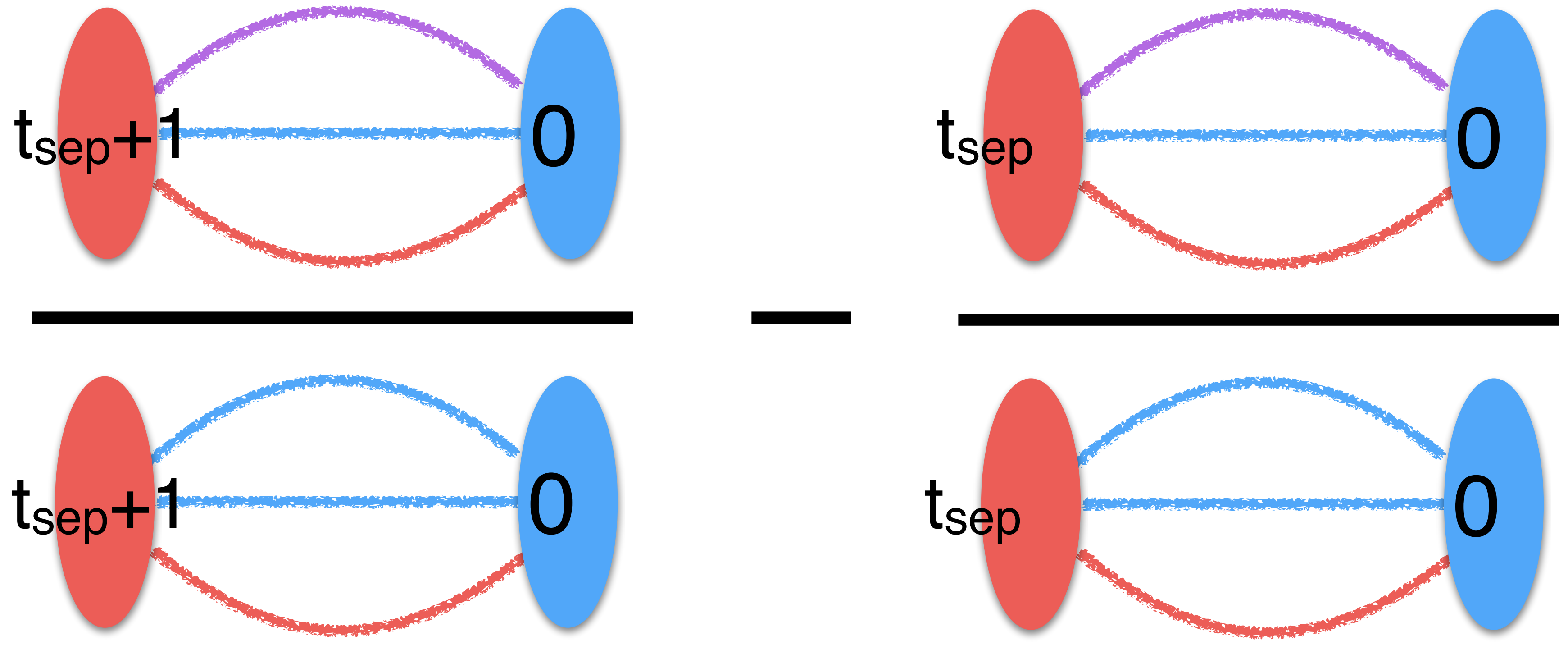
fixed source-sink separation time, **t_{sep}**
repeat for a few different t_{sep}

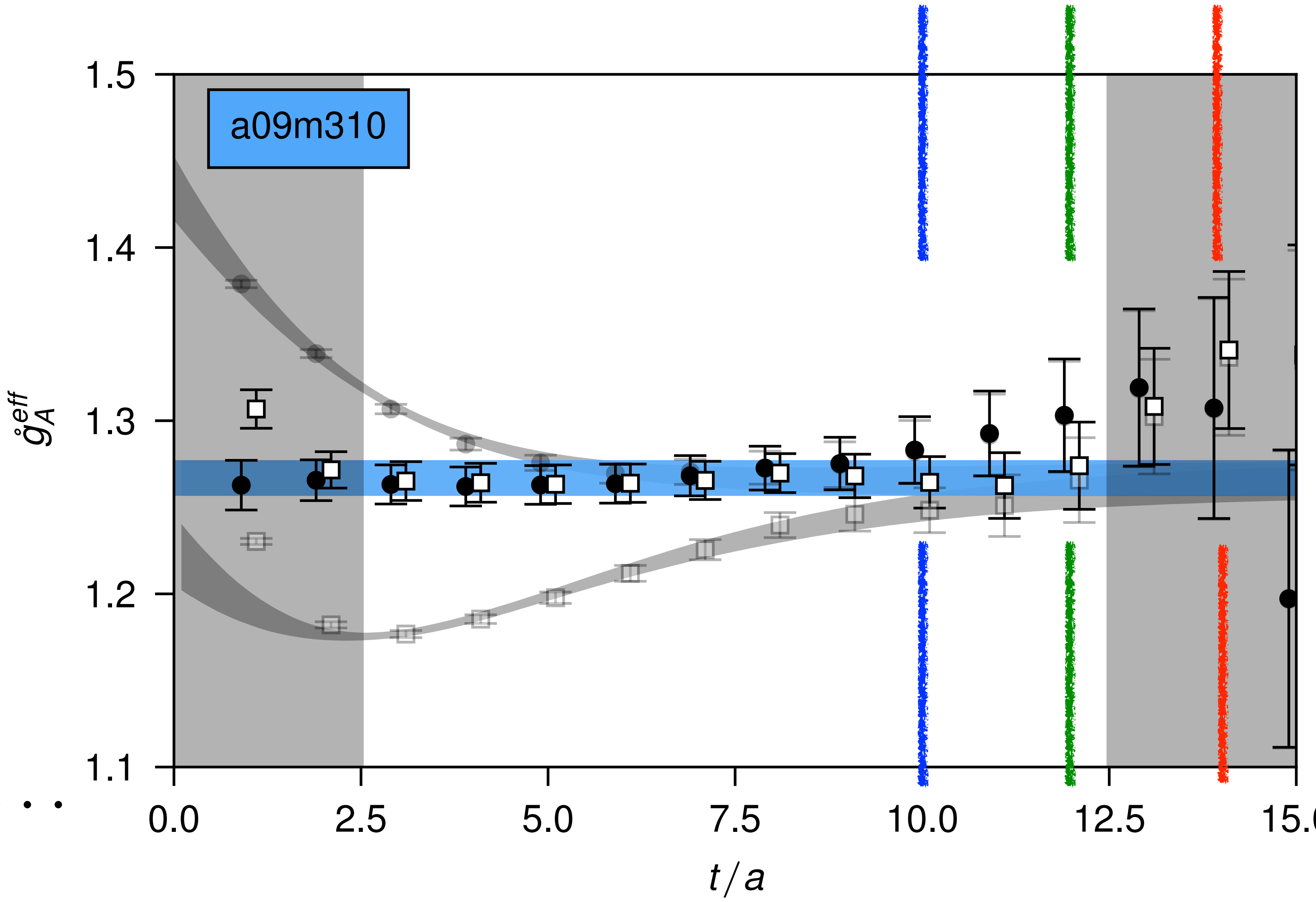
PNDME arXiv:1606.07049



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

our unconventional method





$$\partial_\lambda m_\lambda \Big|_{\lambda=0} = g_\lambda + z\left(e^{-(t_{sep}+1)\Delta_{10}} - e^{-t_{sep}\Delta_{10}}\right) + \cdots$$

Analysis Details

Extrapolations Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

- ChiPT: EFT expanding around m_π= 0
  - best hope for model-independent extrapolation
  - not guaranteed to converge around m_π 135 MeV
- Mild m_π a dependence
  - Taylor expansion works well for extrapolation/interpolation

Slide adapted from A. Nicholson

Analysis Details

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\begin{aligned}
\text{NNLO } \chi\text{PT}: &\quad \text{Eq. (S8)} + \delta_a + \delta_L \\
\text{NNLO+ct } \chi\text{PT}: &\quad \text{Eq. (S8)} + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi^2: &\quad c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L \\
\text{NLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + \delta_a + \delta_L \\
\text{NNLO Taylor } \epsilon_\pi: &\quad c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L
\end{aligned}$$

Slide adapted from A. Nicholson

Analysis Details

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3 \epsilon_\pi^3$$

NNLO XPT

NNLO χPT :	Eq. (S8) $+ \delta_a + \delta_L$
NNLO+ct χPT :	Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$
NNLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$
NNLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Slide adapted from A. Nicholson

Analysis Details

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3\epsilon_\pi^3$$

NNLO XPT

NNLO χPT :	Eq. (S8) $+ \delta_a + \delta_L$
NNLO+ct χPT :	Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$
NNLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$
NNLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Slide adapted from A. Nicholson

Analysis Details

Extrapolations Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$\delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

$$F_1(x) = \sum_{\mathbf{n}\neq 0}\left[K_0(x|\mathbf{n}|) - \frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}\right]$$

$$\delta_L = \underline{\frac{8}{3}\varepsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right]} + \underline{f_3\varepsilon_\pi^3 F_1(m_\pi L)}$$

$$F_3(x) = -\frac{3}{2}\sum_{\mathbf{n}\neq 0}\frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}$$

Beane and Savage
Phys.Rev.D70 [hep-ph/0404131]

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3\epsilon_\pi^3$$

NNLO XPT

parameterization of higher order volume corrections

NNLO χPT :	Eq. (S8) + $\delta_a + \delta_L$
NNLO+ct χPT :	Eq. (S8) + $c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$
NNLO Taylor ϵ_π^2 :	$c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$
NLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$
NNLO Taylor ϵ_π :	$c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

Slide adapted from A. Nicholson