ε'/ε in the Standard Model

Héctor Gisbert Mullor

In collaboration with V. Cirigliano,
A. Pich and A. Rodríguez-Sánchez

IFIC-Universitat de València-CSIC

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What is ε'/ε and why is it interesting?

- Explanation for $\mathrm{M}/\overline{\mathrm{M}}$ asymmetry in Universe requires CPV.
- Amount of CPV in SM is too low to describe the observed $\mathrm{M}/\overline{\mathrm{M}}$ asymmetry.
- ε'/ε constitutes a fundamental test for our understanding of CPV phenomena. In the SM,

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) \propto \mathrm{Im}(V_{td}V_{ts}^*), \qquad V_{\mathrm{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- ε and ε' parametrize different sources of CP violation in K decays:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon', \qquad \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\,\varepsilon'.$$

- Dominant effect from CP violation in K mixing is contained in ε:

$$|\varepsilon| = \frac{1}{3}|\eta_{00} + 2\eta_{+-}| = (2.228 \pm 0.011) \cdot 10^{-3}.$$

- ε' is a tinier effect and accounts for direct CP violation in K decays:

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4}$$

  demonstrates the existence of direct CP violation in K decays.
- Small size of ε' makes it particularly sensitive to new sources of CPV.

Dynamical features of $K \to \pi\pi$ decay

- **Isospin decomposition:**

$$A[K^0 \to \pi^+\pi^-] = A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}} (\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^0 \to \pi^0\pi^0] = A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \sqrt{2} (\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^+ \to \pi^+\pi^0] = \frac{3}{2} A_2^+\, e^{i\chi_2^+} = \frac{3}{2} \left(\mathcal{A}_{3/2} - \frac{2}{3} \mathcal{A}_{5/2}\right).$$

- **Measured $K \to \pi\pi$ branching ratios:**

$A_0 = (2.704 \pm 0.001) \cdot 10^{-7}$ GeV, $A_2 = (1.210 \pm 0.002) \cdot 10^{-8}$ GeV, $\chi_0 - \chi_2 = (47.5 \pm 0.9)^\circ$.

- **What do they tell us?**

$$\boxed{\varepsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)}\, \omega\, \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \left[1 - \frac{1}{\omega} \frac{\mathrm{Im} A_2}{\mathrm{Im} A_0}\right]}$$

1. **$\Delta I = 1/2$ rule: $\omega = \frac{\mathrm{Re} A_2}{\mathrm{Re} A_0} \approx 1/22$**
   - **ε' is suppresed by ω.**
   - **Small IB corrections to the ratio $\frac{\mathrm{Im} A_2}{\mathrm{Im} A_0}$ get amplified by $\omega^{-1} \approx 22$.**
2. **Strong FSI: $\frac{\mathrm{Abs}(\mathcal{A}_{1/2}/\mathcal{A}_{3/2})}{\mathrm{Dis}(\mathcal{A}_{1/2}/\mathcal{A}_{3/2})} \approx 1 \to$ Absorptive $\sim$ Dispersive.**

Lattice QCD predictions

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.4 \pm 7.0)\cdot 10^{-4} \quad \mathbf{2.1\,\sigma} \quad (\text{RBC-UKQCD '15})$$

- This discrepancy has triggered several analysis of possible contributions from new physics.
- Before claiming any evidence for NP, one should realize the technical limitations of the current lattice result:

$$\delta_2 = -(11.6 \pm 2.8)^\circ \quad \mathbf{1.0\,\sigma} \quad (\text{RBC-UKQCD '15}) \quad ✓$$

$$\delta_2|_{\text{exp}} = -(8.5 \pm 1.5)^\circ$$

$$\delta_0 = +(23.8 \pm 5.0)^\circ \quad \mathbf{2.9\,\sigma} \quad (\text{RBC-UKQCD '15}) \quad ✗$$

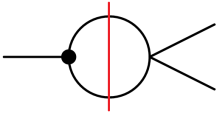
$$\delta_0|_{\text{exp}} = +(39.2 \pm 1.5)^\circ$$

Large uncertainty in δ_0!

Unitarity and Analyticity

1. **Unitarity:** $\delta_0(M_K) = (39.2 \pm 1.5)^\circ \longrightarrow \boxed{A_0 \approx 1.3 \times \mathrm{Dis}(A_0)}$

   (Colangelo-Gasser-Leutwyler '01)



$$A_I\, e^{i\,\delta_I} = \mathrm{Dis}(A_I) + i\,\mathrm{Abs}(A_I)$$

$$\tan\delta_I = \frac{\mathrm{Abs}(A_I)}{\mathrm{Dis}(A_I)}$$

$$A_I = \mathrm{Dis}(A_I)\,\sqrt{1+\tan^2\delta_I}$$

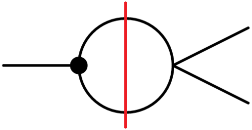
2. **Analyticity:** $\boxed{\mathrm{Dis}(A_I) \propto \mathrm{Abs}(A_I)}$

3. **Message:** $\boxed{\text{Large } \delta_0 \longrightarrow \text{Large } \mathrm{Abs}(A_0) \longrightarrow \text{Large } \mathrm{Dis}(A_0)}$

   - Lattice still doesn't have a good control of the $I = 0$ amplitudes.
   - Still premature to derive strong implications and RBC-UKQCD collaboration is making efforts to solve these problems.

Pallante-Pich-Scimemi approach

- Pallante-Pich-Scimemi '01 within a well-defined theoretical framework which takes into account the important role of the pion dynamics obtained a theoretical value compatible with the experimental result.



$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right)\big|_{\mathrm{SM}} = (17 \pm 9) \cdot 10^{-4}$$

- A lot of improvements since 2001:
  - Better knowledge of Low energy constants (LECs),
  - Much better precision of quark masses,
  - Strong coupling constant,
  - ...
- It is convenient to perform a complete updated of ε'/ε.

Multi-scale framework

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Big\downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)},\ \mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Big\downarrow$ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

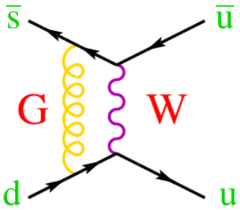
Short-distance description

$\Delta S = 1$ transitions for $K \to \pi\pi$

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu)\, Q_i(\mu)$$

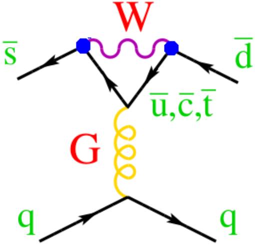
$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$



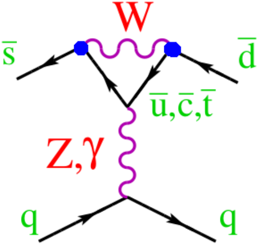
Current - Current operators

$Q_{1,2}$



QCD - Penguins operators

$Q_{3,4,5,6}$



Electroweak Penguins operators

$Q_{7,8,9,10}$

Long-distance description

- χPT is the QFT of QCD at low energies.
- χPT formulation is a consistent theoretical framework to describe pseudoscalar-octet dynamics.
- Perturbative expansion in powers of p^2/Λ_χ^2 where $\Lambda_\chi \sim 1\text{GeV}$.
- Chiral symmetry fixes all allowed χPT operators, at given order in p.
- $\boldsymbol{\mathcal{O}(G_F\, p^2)}$ **:** **Goldstone Interactions** (π, K, η)

$$\mathcal{L}_2^{\Delta S=1} \;=\; G_8\, F^4\, \mathrm{Tr}(\lambda\, L_\mu\, L^\mu) \,+\, G_{27}\, F^4 \left(L_{\mu\,23}\, L^\mu_{11} \,+\, \tfrac{2}{3}\, L_{\mu\,21}\, L^\mu_{13} \right)$$

$$G_{8,27} \;=\; -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V^*_{us}\, g_{8,27} \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda_{ij} \equiv \delta_{i3}\delta_{j2}; \quad U \equiv \exp\{i\sqrt{2}\phi/F\}$$

- Short-distance dynamics encoded in Low-Energy Couplings.

Estimation of the LECs

- From phenomenological data or with additional input from theory.
- Principle of calculation LECs: perform a matching between two EFTs.
- In the large N_C limit, the T-product of two colour singlet currents factorizes:

$$\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle \left\{ 1 + \mathcal{O}\left(\frac{1}{N_C}\right) \right\}$$

Since quark currents have a well-known representation in χPT, the matching between the EFTs can be done at leading order in $1/N_C$.

- Weak couplings of $\mathcal{O}(G_F\, p^2)$ and $\mathcal{O}(e^2\, G_8\, p^0)$:

$$\begin{aligned}
g_8^\infty &= -\frac{2}{5}\, C_1(\mu_{SD}) + \frac{3}{5}\, C_2(\mu_{SD}) + C_4(\mu_{SD}) - 16\, L_5\, B(\mu_{SD})\, C_6(\mu_{SD}), \\
g_{27}^\infty &= \frac{3}{5}\left[C_1(\mu_{SD}) + C_2(\mu_{SD})\right], \\
(e^2\, g_8\, g_{ewk})^\infty &= -3\, B(\mu_{SD})\, C_8(\mu_{SD}) - \frac{16}{3}\, B(\mu_{SD})\, C_6(\mu_{SD})\, e^2\, (K_9 - 2\, K_{10}).
\end{aligned}$$

where $B(\mu_{SD}) \equiv \frac{\langle \bar{q} q \rangle}{F_\pi^3} = \left[\frac{M_K^2}{(m_s+m_d)(\mu_{SD}) F_\pi}\right]^2 \left[1 - \frac{16 M_K^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{8 M_\pi^2}{F_\pi^2}\, L_5\right]$

Strong cancellation in simplified analysis

- Good numerical approximation to consider only $Q_{6,8}$ operators:

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2 \cdot 10^{-3} \left\{ B_6^{(1/2)} \left(1 - \underbrace{\Omega_{\mathrm{eff}}}_{\text{IB}}\right) - 0.48 \, B_8^{(3/2)} \right\}$$

$$\text{IB} \equiv \mathbf{O}[(m_u - m_d)p^2, e^2 p^2]$$

  - Large N_C:

    $B_6^{(1/2)} = B_8^{(3/2)} = 1$, $\Omega_{\mathrm{eff}} = 0.12 \longrightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 9.0 \cdot 10^{-4} \sim \mathcal{O}(10^{-3})$

  - Buras-Gorbahn-Jäger-Jamin '15:

    $B_6^{(1/2)} = 0.57$, $B_8^{(3/2)} = 0.76$, $\widehat{\Omega}_{\mathrm{eff}} = 0.18 \rightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2 \cdot 10^{-4} \sim \mathcal{O}(10^{-4})$

Strong cancellation between Q_6 and Q_8!

Why such small values? Missing FSI contributions

- Experimental value of phase shift: $\chi_0 - \chi_2 = (47.5 \pm 0.9)^\circ$.
- Could we include these LARGE corrections in naive way?
  - χPT prediction: $\chi_0 - \chi_2 = 34^\circ \neq 0^\circ$ to one-loop.
  - Including FSI in $B_{6,8}^{(1/2,3/2)}$:

$$B_6^{(1/2)} \to |1 + \Delta_L \mathcal{A}_{1/2}^{(8)}|\, B_6^{(1/2)} \approx 1.35\, B_6^{(1/2)}$$

$$B_8^{(3/2)} \to |1 + \Delta_L \mathcal{A}_{3/2}^{(g)}|\, B_8^{(3/2)} \approx 0.54\, B_8^{(3/2)}$$

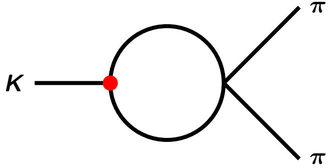
New simplified formula for $\mathrm{Re}(\varepsilon'/\varepsilon)$

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2 \cdot 10^{-3} \left\{ 1.35\, B_6^{(1/2)} \left(1 - \Omega_{\mathrm{eff}}\right) - 0.26\, B_8^{(3/2)} \right\}$$

Set-up	$B_6^{(1/2)}$	$B_8^{(3/2)}$	$\mathrm{Re}(\varepsilon'/\varepsilon)$
Large N_C + $\mathrm{FSI}_{I=0,2}$	1	1	$2.0 \cdot 10^{-3}$
Large $N_C\|_{I=0}$ + $\mathrm{LQCD}_{I=2}$ + $\mathrm{FSI}_{I=0}$	1	0.76	$1.6 \cdot 10^{-3}$
LQCD + $\mathrm{FSI}_{I=0,2}$	0.57(?)	0.76	$1.0 \cdot 10^{-3}$
LQCD + $\mathrm{FSI}_{I=0}$	0.57(?)	0.76	$0.6 \cdot 10^{-3}$

(?) Not reliable, unable to reproduce $\delta_{I=0}$.

Same order of magnitude as experimental value!



$$\Longrightarrow \quad \mathrm{Re}(\varepsilon'/\varepsilon) \approx 10^{-3}$$

Anatomy of ε'/ε calculation in χPT

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right]$$

~~Strong cancelation:~~ $Q_6 \uparrow \quad - \quad Q_8 \downarrow \quad \neq 0$

1. **$\mathrm{O(p^4)}$ χPT Loops: Large correction** $\Delta_L \mathcal{A}_n^{(X)} \to \frac{1}{N_C}\log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3}\times 2$

$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\mathcal{A}_n^{(X)} + \Delta_C\mathcal{A}_n^{(X)}\right] \quad ; \quad X = 27,\ 8,\ \varepsilon,\ \gamma,\ Z,\ g.$$

$$\Delta_L\mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + i\,0.47\,, \quad \Delta_L\mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - i\,0.21\,, \ldots$$

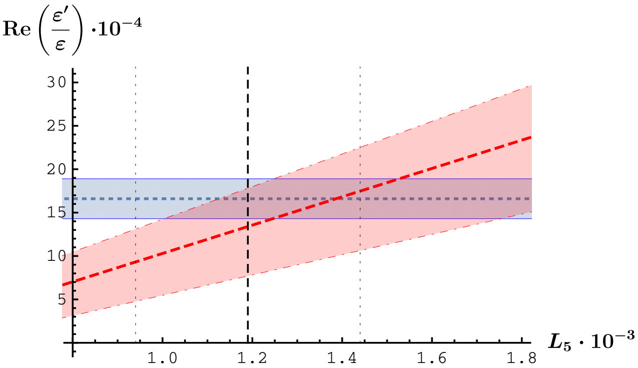
2. **$\mathrm{O(p^4)}$ LECs fixed at $\mathrm{N_C} \to \infty$: Small correction** $\Delta_C \mathcal{A}_n^{(X)}$

3. **Isospin Breaking $\mathrm{O[(m_u - m_d)p^2, e^2p^2]}$: Sizeable correction**

   $\Omega_{\mathrm{eff}} = (12 \pm 9)\cdot 10^{-2}$ (V.Cirigliano-HG-A.Pich-A.Rodríguez-Sánchez '19)

4. **Re g_8, Re g_{27} and $\chi_0 - \chi_2$ fixed from phenomenological fit.**

SM prediction of $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ V.Cirigliano-HG-A.Pich-A.Rodríguez-Sánchez '19



$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (15\,\pm\,7)\cdot 10^{-4} \longrightarrow \mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} = \left(13\,{}^{+6}_{-7}\right)\cdot 10^{-4}$$

in good agreement with the experimental value!

$$\mathrm{Re}\,(\varepsilon'/\varepsilon)\big|_{\mathrm{exp}} = (16.6\,\pm\,2.3)\cdot 10^{-4}$$

Future directions and improvements

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{SM}} = \left(13.1 \pm 0.4_{\,m_s}\, {}^{+2.2}_{-4.0}\,{}_{\mu_{\mathrm{SD}}}\, {}^{+3.0}_{-3.2}\,{}_{\nu_\chi} \pm 1.2_{\,\gamma_5} \pm 4.3_{\,L_{5,8}} \pm 1.1_{\,L_7} \pm 0.2_{\,K_i} \pm 0.3_{\,X_i}\right) \cdot 10^{-4}$$

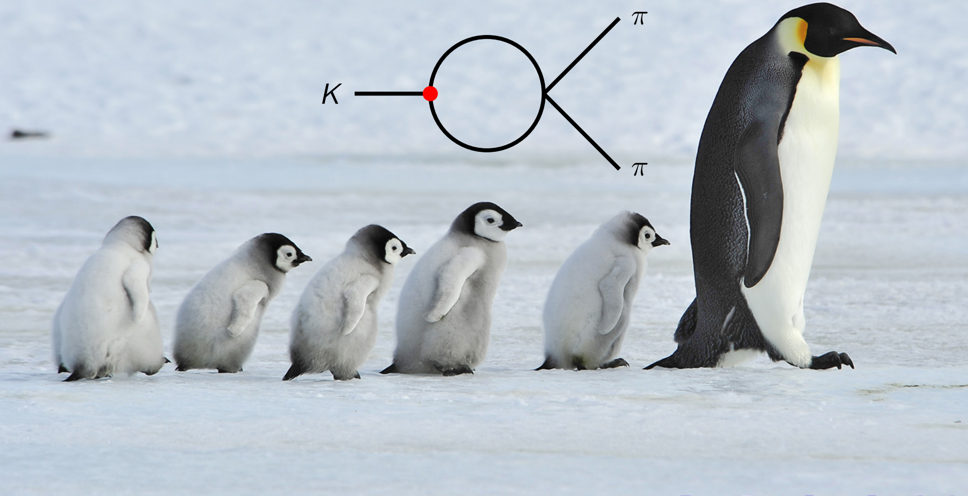
- **Close to be completed:**
  - **Wilson coefficients at NNLO** $(\pm 1.2_{\,\gamma_5})$ (Cerdà et al)
- **Future directions:**
  - **g_8 and higher-order LECs at NLO** $\left({}^{+2.2}_{-4.0}\,{}_{\mu_{\mathrm{SD}}}\, {}^{+3.0}_{-3.2}\,{}_{\nu_\chi}\right)$ New ideas
  - **Improved lattice input** $(\pm\, 4.5_{\mathrm{LECs}})$ Expected

Best strategy:

χPT (amplitudes) + Lattice (LECs) + New ideas

SM prediction of ε'/ε agrees well with the measured value!

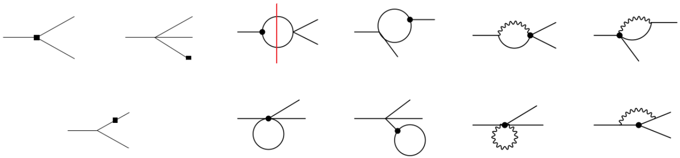
$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$ $\quad$ $\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} = \left(13\ {}^{+6}_{-7}\right) \cdot 10^{-4}$



BACKUP SLIDES

Amplitudes at NLO $\mathcal{O}(p^4,\ (m_u - m_d)p^2,\ e^2p^0,\ e^2p^2)$

Including strong isospin violation and electromagnetic corrections



$$\mathcal{A}_n = G_{27}\, F_\pi\, (M_K^2 - M_\pi^2)\, \mathcal{A}_n^{(27)}$$
$$+\ G_8\, F_\pi \left\{ (M_K^2 - M_\pi^2)\, \left[\mathcal{A}_n^{(8)} + \varepsilon^{(2)}\, \mathcal{A}_n^{(\varepsilon)} \right] - e^2\, F_\pi{}^2 \left[\mathcal{A}_n^{(\gamma)} + Z\, \mathcal{A}_n^{(Z)} + g_{\mathrm{ewk}}\, \mathcal{A}_n^{(g)} \right] \right\}$$

where $\mathcal{A}_n^{(X)} = a_n^{(X)} \left[1 + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right] \quad ; \quad X = 27,\ 8,\ \varepsilon,\ \gamma,\ Z,\ g.$

$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^*\, g_{8,27}$, $\quad \varepsilon^{(2)} = (\sqrt{3}/4)(m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $\quad Z \approx (M_{\pi^\pm}^2 - M_{\pi_0}^2)/(2e^2F_\pi^2) \approx 0.8$

Isospin-breaking corrections V.Cirigliano-HG-A.Pich-A.Rodríguez-Sánchez '19

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right]$$

$$\omega \equiv \omega_+(1+f_{5/2}),\ \omega_+ \equiv \frac{\mathrm{Re}A_2^+}{\mathrm{Re}A_0},\ \Omega_{\mathrm{IB}} = \frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_2^{(0)}}\,\frac{\mathrm{Im}A_2^{\mathrm{non-emp}}}{\mathrm{Im}A_0^{(0)}},\ \Omega_{\mathrm{eff}} \equiv \Omega_{\mathrm{IB}} - \Delta_0 - f_{5/2}$$

Set-up	Δ_0	$f_{5/2}$	Ω_{IB}	Ω_{eff}
Central 2004	0.083	0.084	0.227	0.060
Central 2019	0.057	0.082	0.260	0.121
σ_μ	$^{+0.007}_{-0.002}$	$^{+0.000}_{-0.001}$	$^{+0.008}_{-0.002}$	$^{+0.001}_{-0.000}$
σ_{ν_χ}	0.002	$^{+0.023}_{-0.024}$	0.034	$^{+0.057}_{-0.055}$
σ_{γ_5}	0.007	0.001	0.001	0.004
$\sigma_{L_{5,8}}$	0.014	0.002	0.040	0.029
σ_{L_7}	0.001	0.000	0.060	0.061
σ_{K_i}	0.002	0.003	0.018	0.013
σ_{X_i}	0.002	0.000	0.003	0.005

Simplified Estimate of ε'/ε

1. CP violation comes mainly from the QCD and EW penguin operators.

   QCD: $Q_{3,4}$ $_{(V-A)\otimes(V-A)}$ and $Q_{5,6}$ $_{(V-A)\otimes(V+A)}$.

   EW: $Q_{7,8}$ $_{(V-A)\otimes(V+A)}$ and $Q_{9,10}$ $_{(V-A)\otimes(V-A)}$.

2. Chiral enhancement of the operators $(V-A)\otimes(V+A)$.

   - Fierz rearrangement of numerically relevant operators:

   $$Q_4 = \sum_q (\bar{s}q)_{\mathrm{V-A}} (\bar{q}d)_{\mathrm{V-A}}, \qquad Q_{10} = \frac{3}{2}\sum_q e_q\, (\bar{s}q)_{\mathrm{V-A}} (\bar{q}d)_{\mathrm{V-A}},$$

   $$Q_6 = \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L), \qquad Q_8 = -12 \sum_q e_q\, (\bar{s}_L q_R)(\bar{q}_R d_L).$$

   - Large N_C: $\langle J\cdot J\rangle = \langle J\rangle\langle J\rangle \{1+\mathcal{O}(1/N_C)\}$

   $$\mathcal{A}_{LL} \equiv \langle\pi^+\pi^-|(\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)|K^0\rangle = \langle\pi^+|\bar{u}_L\gamma_\mu d_L|0\rangle\langle\pi^-|\bar{s}_L\gamma^\mu u_L|K^0\rangle = \frac{i\sqrt{2}}{4} F_\pi\,(M_K^2 - M_\pi^2),$$

   $$\mathcal{A}_{LR}(\mu) \equiv \langle\pi^+\pi^-|(\bar{s}_L u_R)(\bar{u}_R d_L)|K^0\rangle = \langle\pi^+|\bar{u}_R d_L|0\rangle\langle\pi^-|\bar{s}_L u_R|K^0\rangle = \frac{i\sqrt{2}}{4} F_\pi \left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2.$$

   At $\mu = 1$ GeV: $\mathcal{A}_{LR}(\mu)/\mathcal{A}_{LL} \sim M_K^2/[m_s(\mu)+m_d(\mu)]^2 \sim 14$.

3. Good numerical approximation to consider only $Q_{6,8}$ operators.

4. Ignoring all other contributions to the CP-violating decay amplitudes:

$$\mathrm{Im}A_0|_{Q_6} = \frac{G_F}{\sqrt{2}} A^2\lambda^5\eta\, V_{td}V_{ts}^*\, y_6(\mu)\, 4\sqrt{2}\,(F_K - F_\pi)\left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2 B_6^{(1/2)},$$

$$\mathrm{Im}A_2|_{Q_8} = -\frac{G_F}{\sqrt{2}} A^2\lambda^5\eta\, V_{td}V_{ts}^*\, y_8(\mu)\, 2\,F_\pi \left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2 B_8^{(3/2)}.$$

5. Including the isospin breaking effects:

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right]$$

6. Simplified estimate: **Strong cancellation between Q_6 and Q_8!**

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2\cdot 10^{-3}\left\{B_6^{(1/2)}(1-\Omega_{\mathrm{eff}}) - 0.48\, B_8^{(3/2)}\right\}$$