What's the Matter with V_{ud}?

Misha Gorshteyn

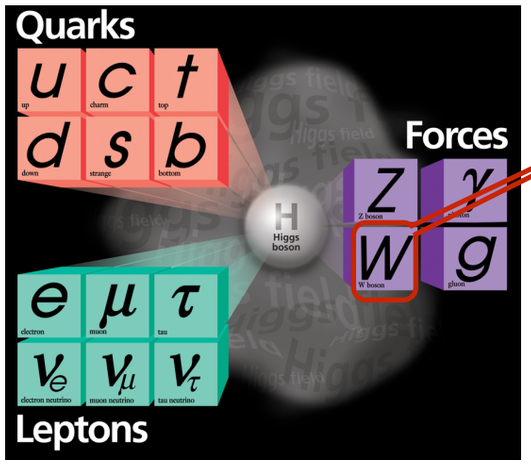
Johannes Gutenberg-Universität Mainz

Collaborators:
Chien-Yeah Seng
Hiren Patel
Michael Ramsey-Musolf

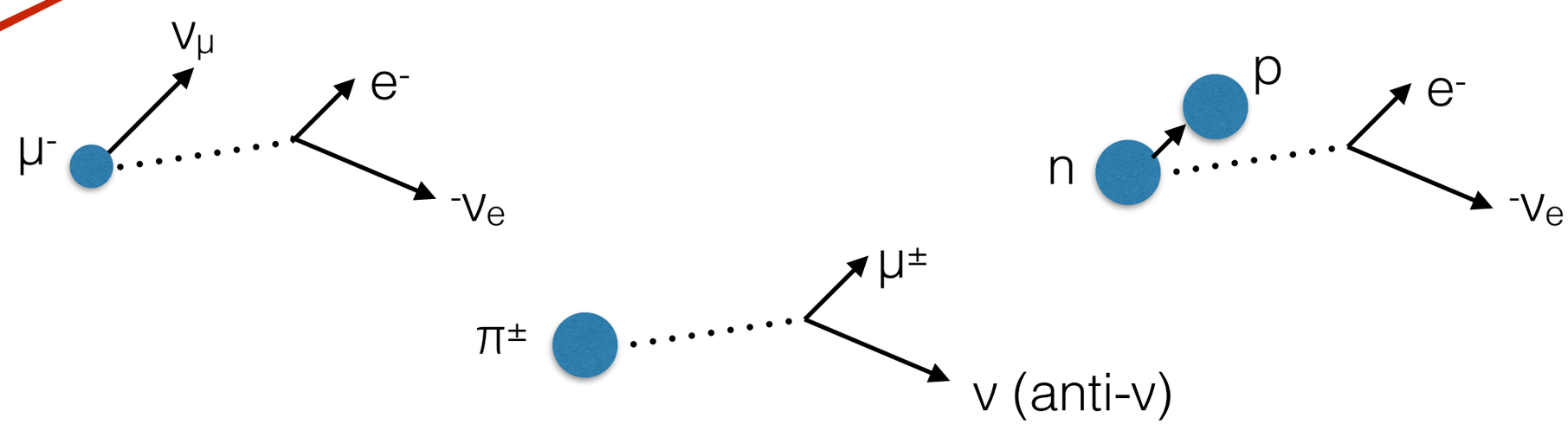
Based on 3 papers:

arXiv: 1807.10197
arXiv: 1812.03352
arXiv: 1812.04229

Precision measurements of V_{ud}



Charged current interaction - β-decay (μ, π±, n)



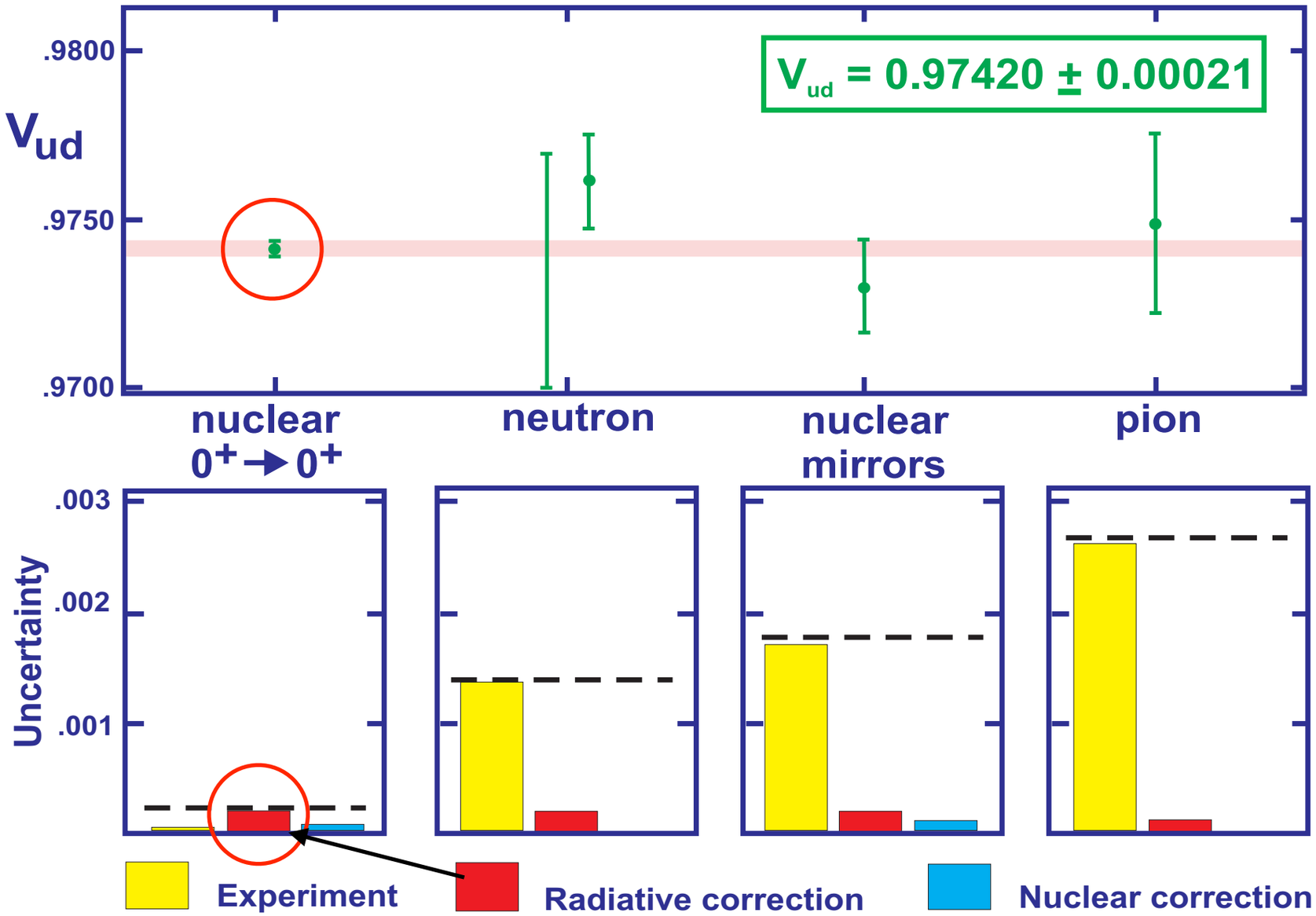
W coupling to leptons and hadrons very close but not exactly the same:
quark mixing - Cabbibo-Kabayashi-Maskawa matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM - Determines the relative strength of the weak CC interaction of quarks vs. that of leptons

CKM unitarity - measure of completeness of the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

V_{ud} and CKM unitarity in early 2018



From neutron decay

$$|V_{ud}|^2 = \frac{5099.34s}{\tau_n(1+3g_A^2)(1+\Delta_R)}$$

From superallowed decays

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005$$

CKM unitarity: V_{ud} the main contributor to the sum and to the uncertainty

0^+-0^+ nuclear decays $|V_{ud}|^2 = 0.94906 \pm 0.00041$

Kl3 and Kl2 average $|V_{us}|^2 = 0.05031 \pm 0.00022$

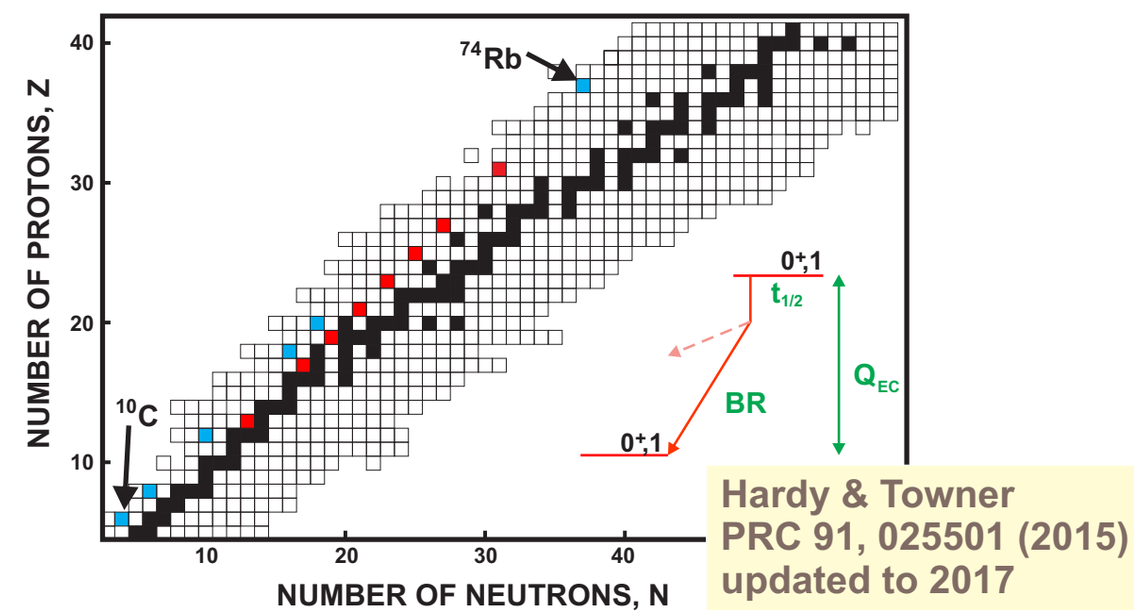
B decays $|V_{ub}|^2 = 0.00002$

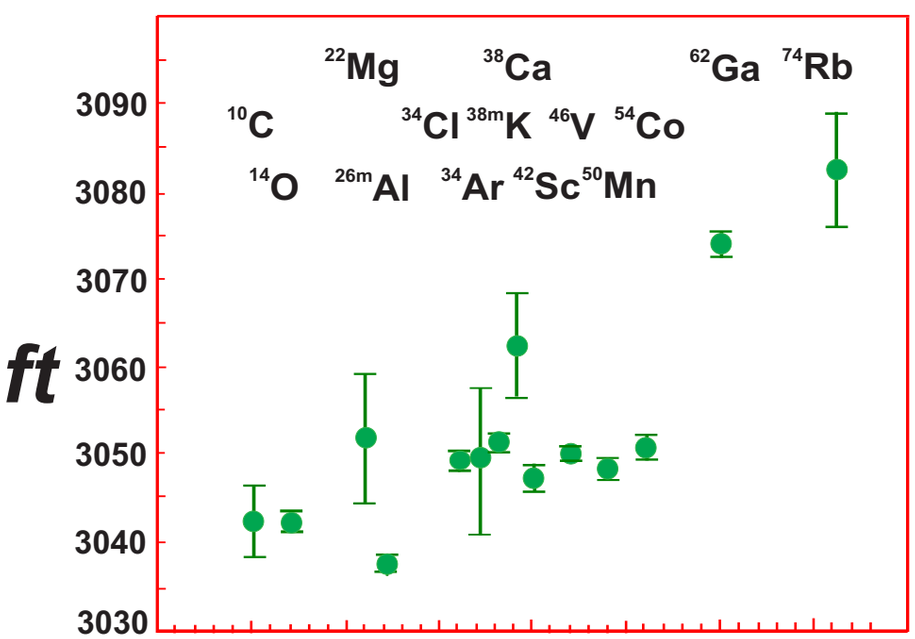
Why are superallowed decays special?

Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**





ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

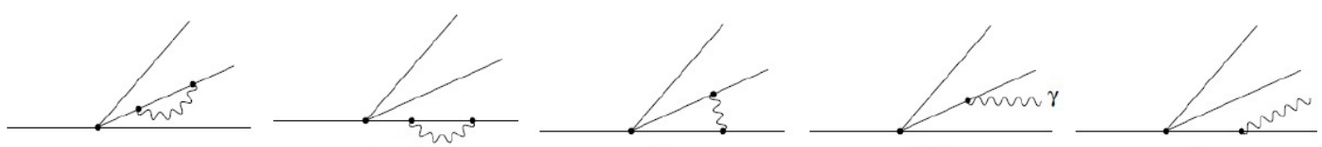
Why are superallowed decays special?

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1+\Delta_R^V)}$$

Modified ft-values to include these effects

$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C-\delta_{NS})]$$

δ'_R - “outer” correction (depends on e-energy) - QED



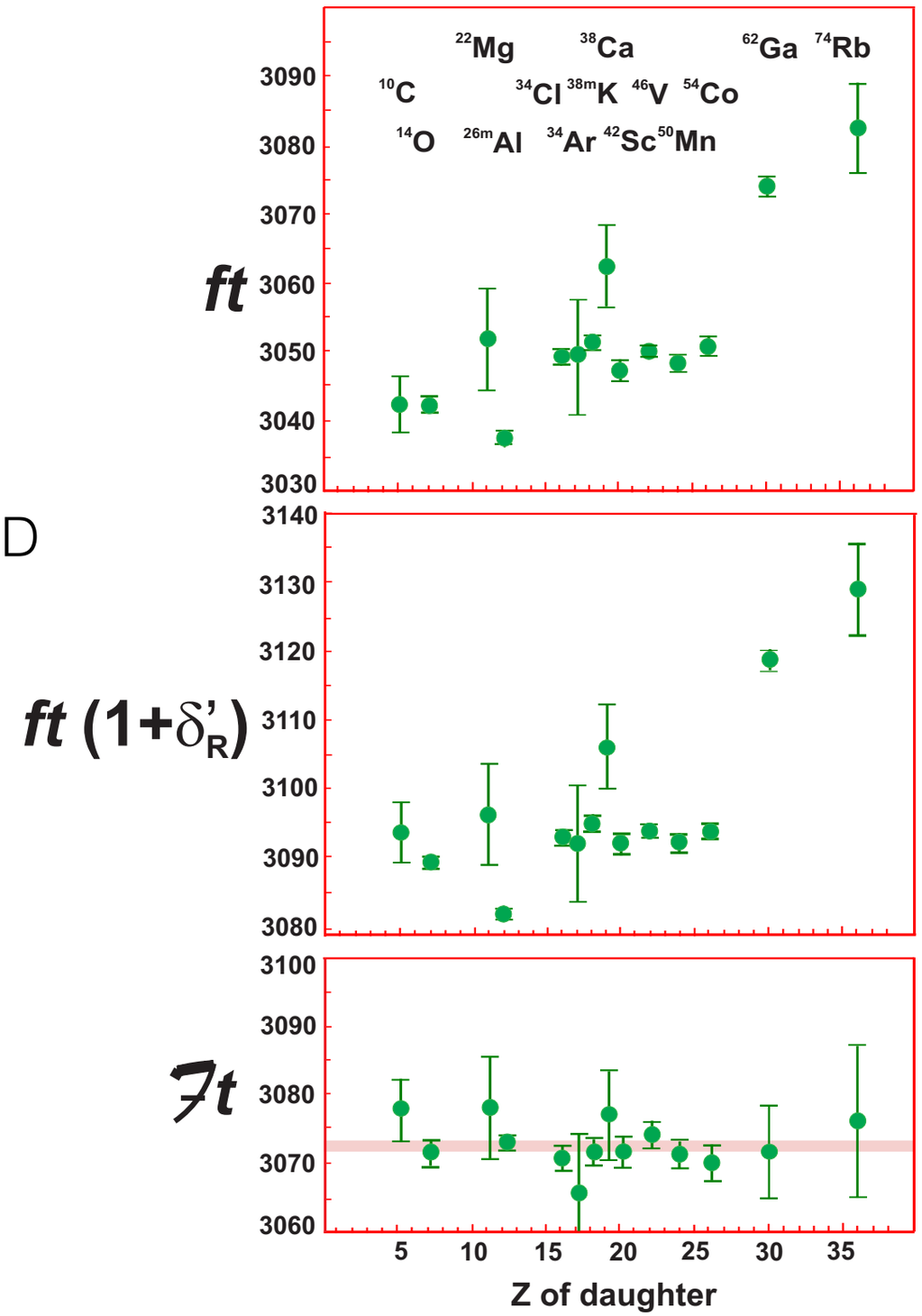
δ_C - SU(2) breaking in the nuclear matrix elements
- mismatch of radial WF in parent-daughter
- mixing of different isospin states

δ_{NS} - RC depending on the nuclear structure

δ_C,δ_{NS} - energy independent

Average

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$

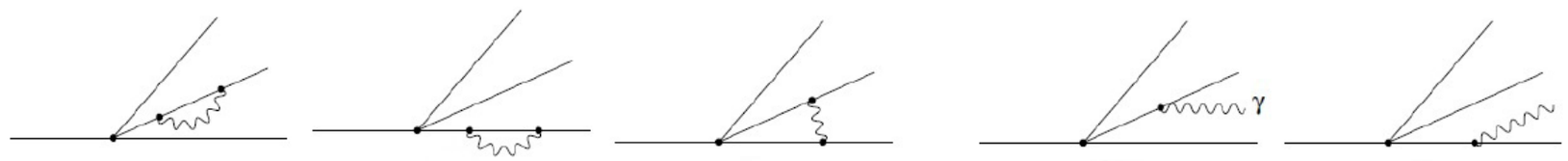


Hardy, Towner 1973 - 2018

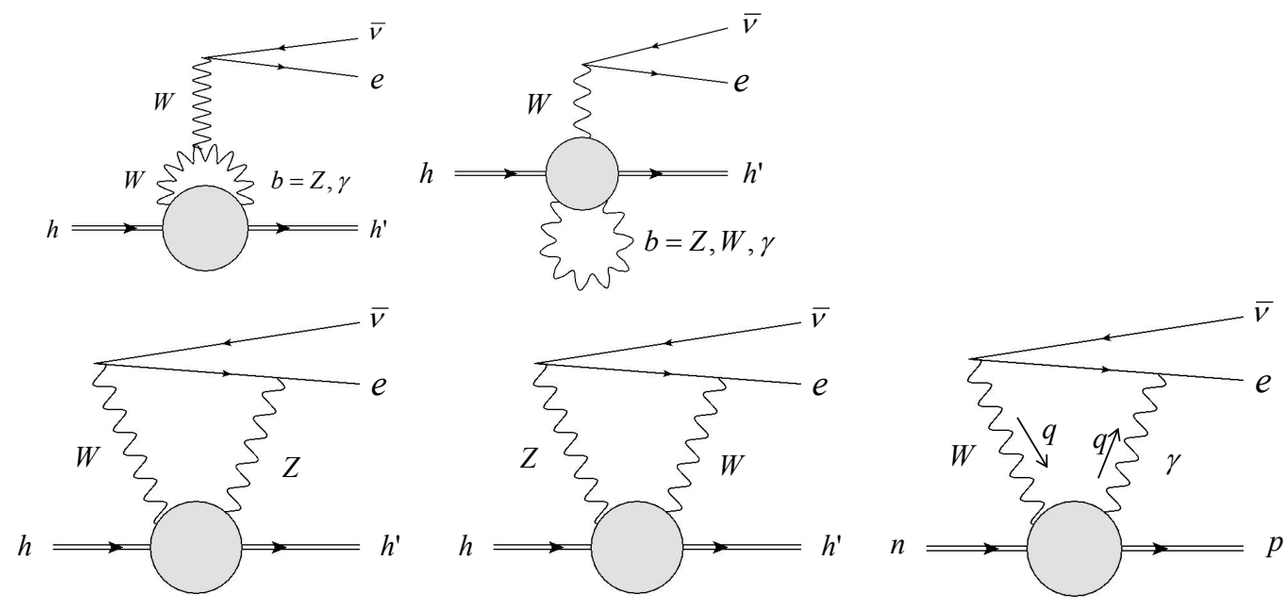
Radiative corrections - “inner” and “outer”

Splitting inner-outer possible due to scales hierarchy: $m_e \leq E_e \ll \Lambda \sim m_\pi \ll M_n$

Outer: soft photon exchange and emission (QED + magnetic moment + charge radius)



Inner: everything else

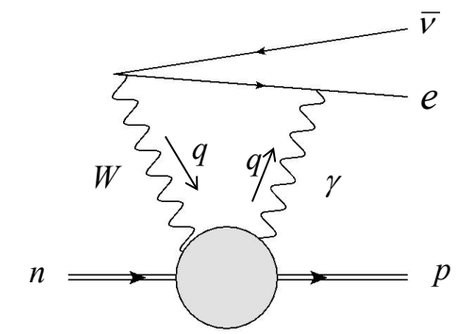


W,Z-exchange: UV-sensitive, pQCD; model-independent
When γ involved - sensitivity to long range physics —> model-dependent
V x V correlator protected by CVC - no hadronic uncertainty
Axial vector not conserved —> A x V correlator from γW box sensitive to hadron structure

γW box - the main source of the uncertainty $\Delta_R^V = 2\Box_{\gamma W}^{A\times V} + \text{model independent}$

γW-box by Marciano & Sirlin

$$\Box^{VA}_{\gamma W} = \frac{\alpha}{8\pi}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}F(Q^2)$$



Short distance: OPE

$$F^{DIS}(Q^2) = \frac{1}{Q^2}\left[1-\frac{\bar{\alpha}_s}{\pi}-C_2\left(\frac{\bar{\alpha}_s}{\pi}\right)^2-C_3\left(\frac{\bar{\alpha}_s}{\pi}\right)^3\right]$$

GLS and Bjorken SR to N3LO
Larin, Vermaseren 1997

$Q^2 > Q_2^2$

Interpolate: VDM Ansatz

$$F^{\rm INT}(Q^2) = -\frac{1.490}{Q^2+m_\rho^2}+\frac{6.855}{Q^2+m_A^2}-\frac{4.414}{Q^2+m_{\rho'}^2}$$

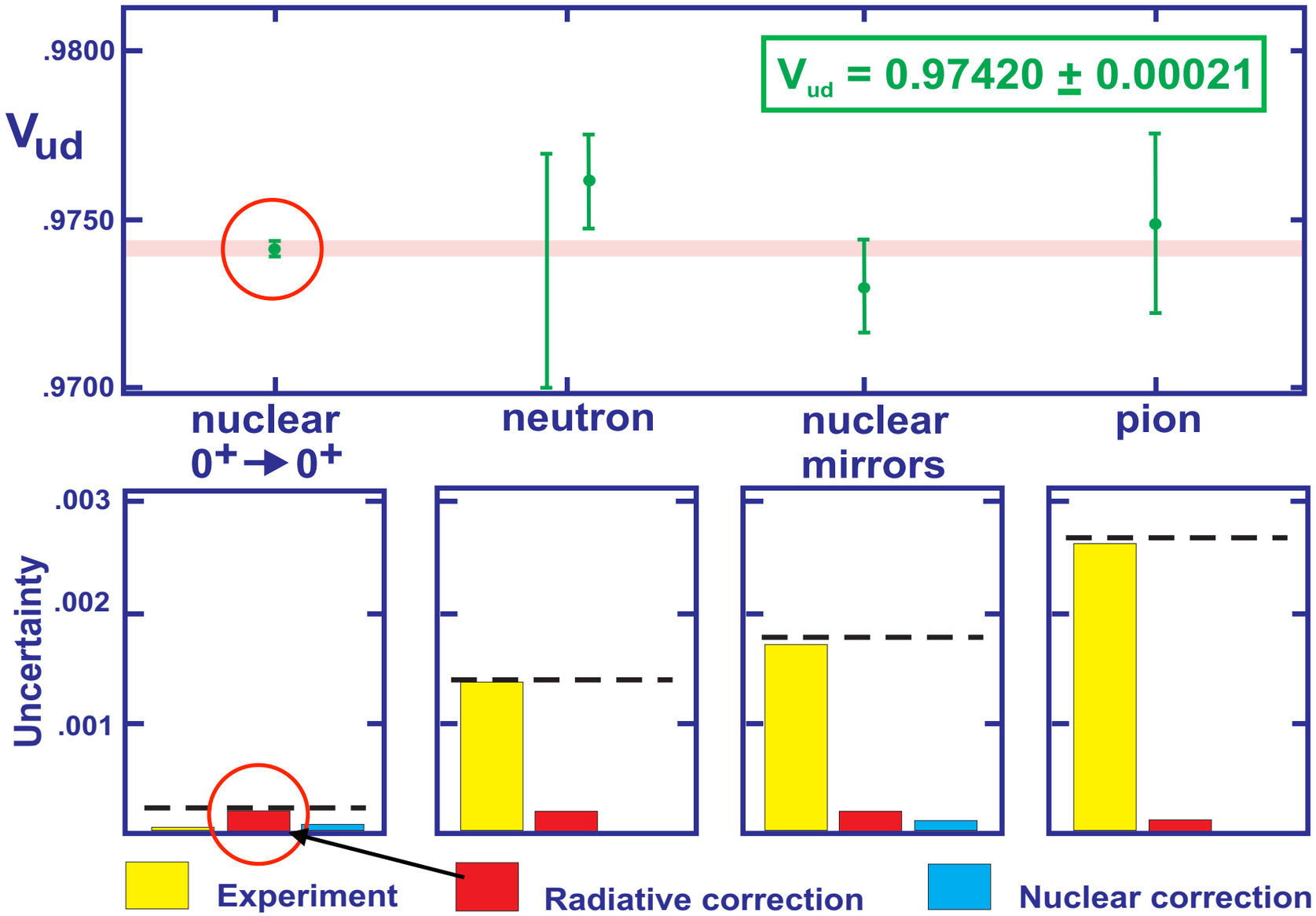
$Q^2 < Q_1^2$

Long distance: Born (axial and magnetic form factors)

Marciano & Sirlin 2006

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\ln\underbrace{\frac{M_W}{\Lambda}}_{\sim 3.77}+\underbrace{A_g}_{0.14(14)}+\underbrace{C^{Int}+2C_B}_{1.66}\right]$$

V_{ud} and CKM unitarity in early 2018



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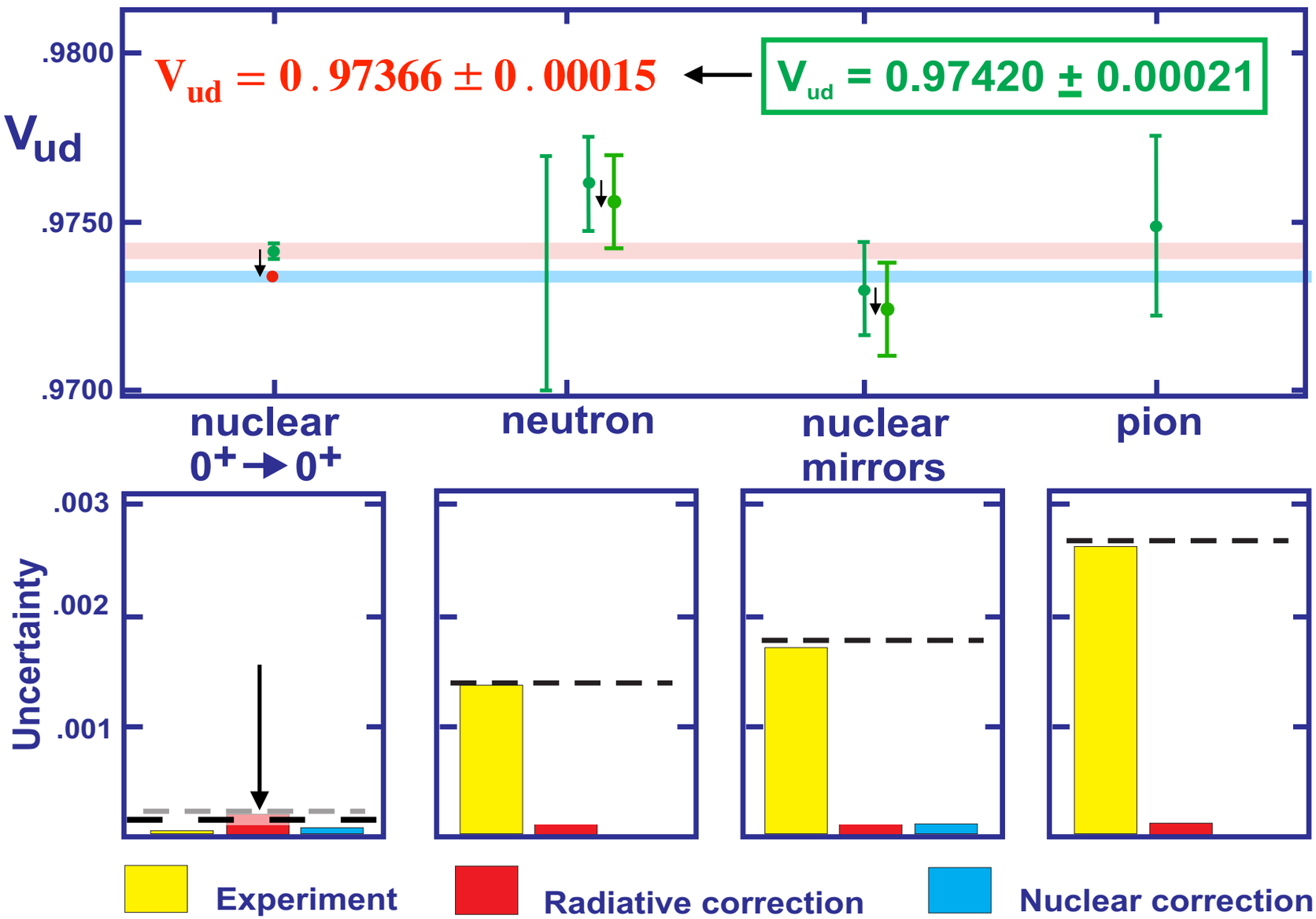
V_{ud} and CKM unitarity in Fall 2018

Seng et al., 1807.10197



$$|V_{ud}|^2 + |V_{us}|^2 + V_{ub}|^2 = 0.9979 \pm 0.0004$$

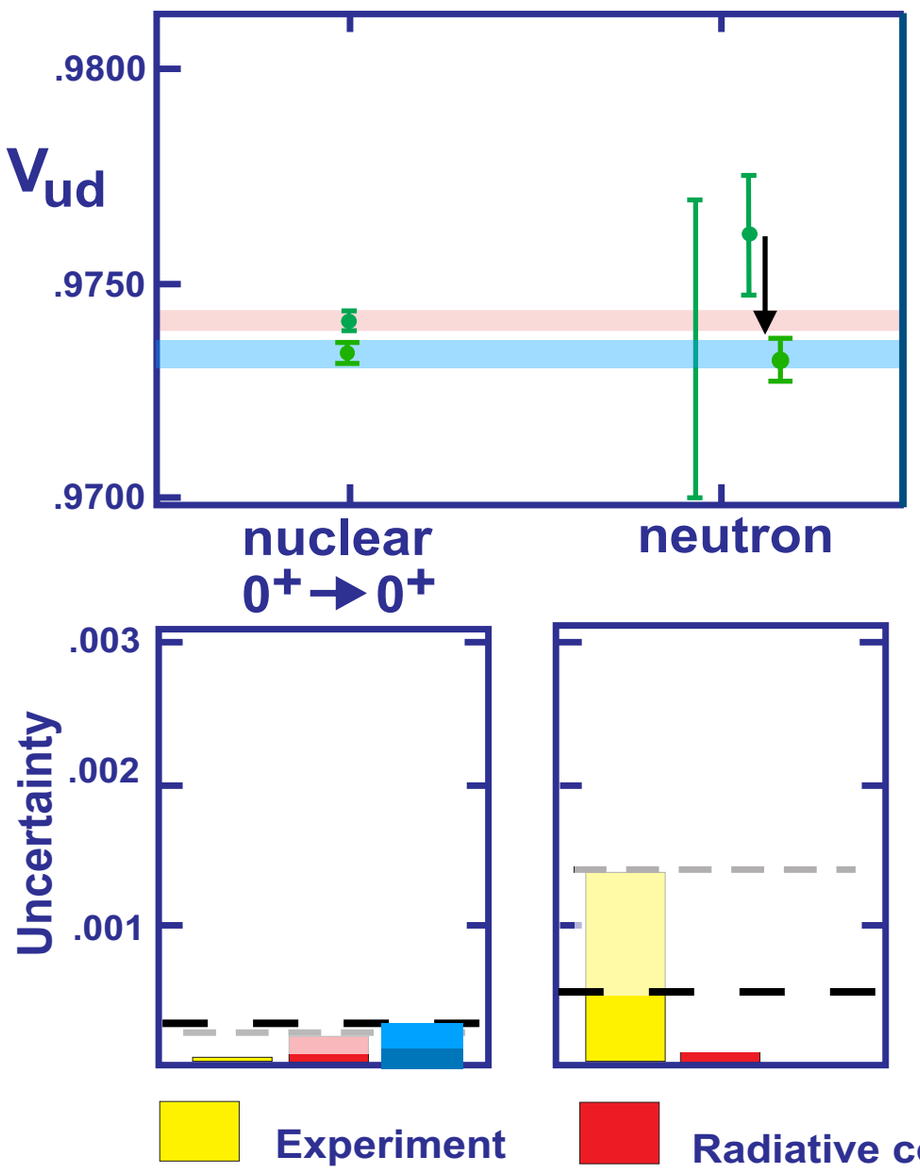
CKM unitarity: V_{ud} und V_{us} contribute equally to the uncertainty

0^+-0^+ nuclear decays $|V_{ud}|^2 = 0.94801 \pm 0.00029$

Kl3 decays $|V_{us}|^2 = 0.04987 \pm 0.00027$

Bazavov et al. (FNAL/MILC), 1809.02827

V_{ud} and CKM unitarity in December 2018



Major improvement in exp. determination of g_A

PDG2018 — PERKEO-III

Märkisch et al., 1812.04666

$$g_A = -1.2723(23) \longrightarrow g_A = -1.2764(6)$$

V_{ud} from free neutron decay

$$V_{ud} = 0.9763(5)_{\tau_n}(15)_{g_A}(2)_{\mathrm{RC}} \longrightarrow V_{ud} = 0.9735(5)_{\tau_n}(3)_{g_A}(1)_{\mathrm{RC}}$$

Revision of nuclear corrections to 0^+- 0^+-beta decay

Seng et al., 1812.03352; MG 1812.04229

$$V_{ud} = 0.97366(10)_{\mathrm{Ft}}(10)_{\mathrm{RC}} \longrightarrow V_{ud} = 0.97366(32)_{\mathrm{Ft}}(10)_{\mathrm{RC}}$$

Free neutron decay becomes competitive;
Limitation: the lifetime (also, beam vs. bottle)

Scrutiny of nuclear corrections with new methods

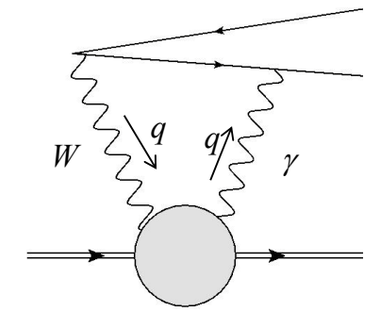
Top-row unitarity: 2,5-3,5σ deficit

$$\Delta_u = -(0.0016 - 0.0021) \pm 0.0006$$

(depending on V_{us})

γW-box from Dispersion Relations

Box at zero momentum transfer* (but with energy dependence)



$$T_{\gamma W} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e)\gamma^\nu (1-\gamma_5) v_\nu}{q^2[(k-q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\mu\nu}^{\gamma W}$$

***Precision goal: 10^{-4}; RC ~ $\alpha/2\pi$ ~ 10^{-3}; recoil on top - negligible**

Hadronic tensor: two-current correlator $\quad T_{\gamma W}^{\mu\nu} = \int dx e^{iqx} \langle f | T[J_{em}^\mu(x) J_W^{\nu,\pm}(0)] | i \rangle$

General gauge-invariant decomposition of a spin-independent tensor

$$T_{\gamma W}^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{(p\cdot q)} \left(p - \frac{(p\cdot q)}{q^2} q\right)^\mu \left(p - \frac{(p\cdot q)}{q^2} q\right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} T_3$$

Loop integral with generally unknown forward amplitudes

$$T_{\gamma W} = -\frac{\alpha}{2\pi} G_F V_{ud} \int \frac{d^4q M_W^2}{q^2(M_W^2 - q^2)} \bar{u}_e \gamma_\beta (1-\gamma_5) u_\nu \sum_i C_i^\beta(E,\nu,q^2) T_i^{\gamma W}(\nu, q^2)$$

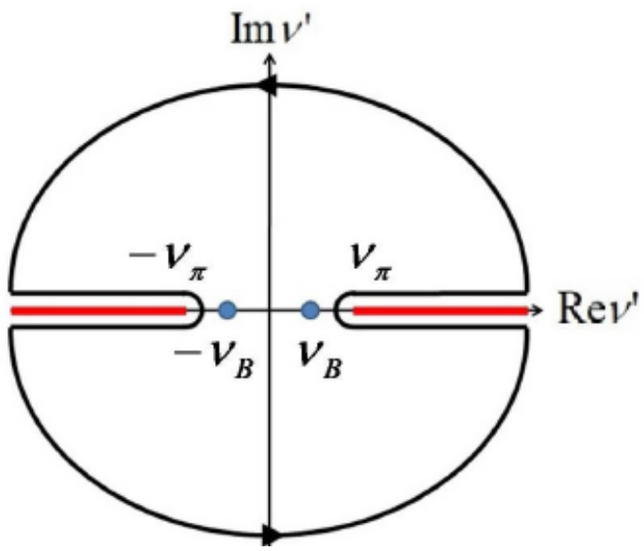
$p^\mu = (M, \vec{0})$

$E = (pk)/M$

$\nu = (pq)/M$

Known algebraic functions of external energy E and loop variables ν, q^2

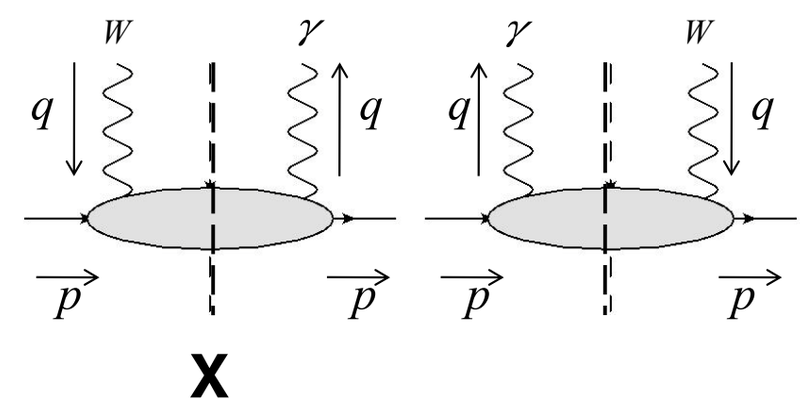
γW-box from Dispersion Relations



$T_{1,2,3}$ - analytic functions inside the contour C in the complex v-plane determined by their singularities on the real axis - poles + cuts

$$T_i^{\gamma W}(\nu, Q^2) = \frac{1}{2\pi i}\oint dz \frac{T_i^{\gamma W}(z, Q^2)}{z-\nu}, \quad \nu \in C$$

Forward amplitudes T_i - unknown;
Their absorptive parts can be related to production of on-shell intermediate states
—> a γW-analog of structure functions $F_{1,2,3}$

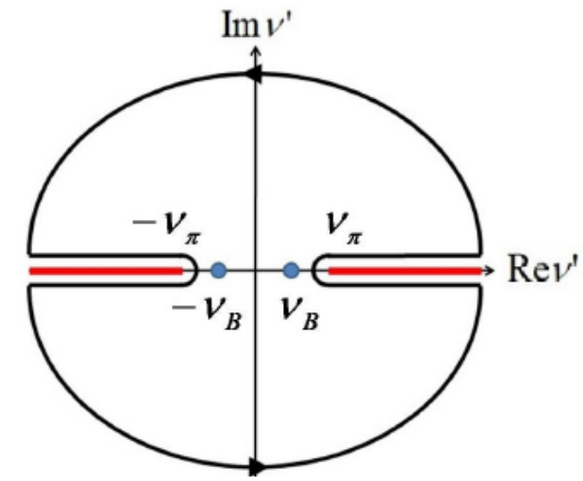


X = inclusive strongly-interacting on-shell physical states

Structure functions $F_i^{\gamma W}$ are NOT data
But they can be related to data

$$\mathrm{Im}\, T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)$$

γW-box from Dispersion Relations



Crossing behavior: relate the left and right hand cut
Mismatch between the initial and final states - asymmetric;
Symmetrize - γ is a mix of I=0 and I=1

$$T_i^{\gamma W,a} = T_i^{(0)}\tau^a + T_i^{(-)}\frac{1}{2}[\tau^3,\tau^a]$$

$$T_i^{(I)}(-\nu,Q^2) = \xi_i^{(I)}T_i^{(I)}(\nu,Q^2)$$

$$\xi_1^{(0)} = +1,\ \ \xi_{2,3}^{(0)} = -1;\quad \xi_i^{(-)} = -\xi_i^{(0)}$$

Two types of dispersion relations for scalar amplitudes

$$T_i^{(I)}(\nu,Q^2) = 2\int_0^\infty d\nu'\left[\frac{1}{\nu'-\nu-i\epsilon} + \frac{\xi_i^{(I)}}{\nu'-\nu-i\epsilon}\right]F_i^{(I)}(\nu',Q^2)$$

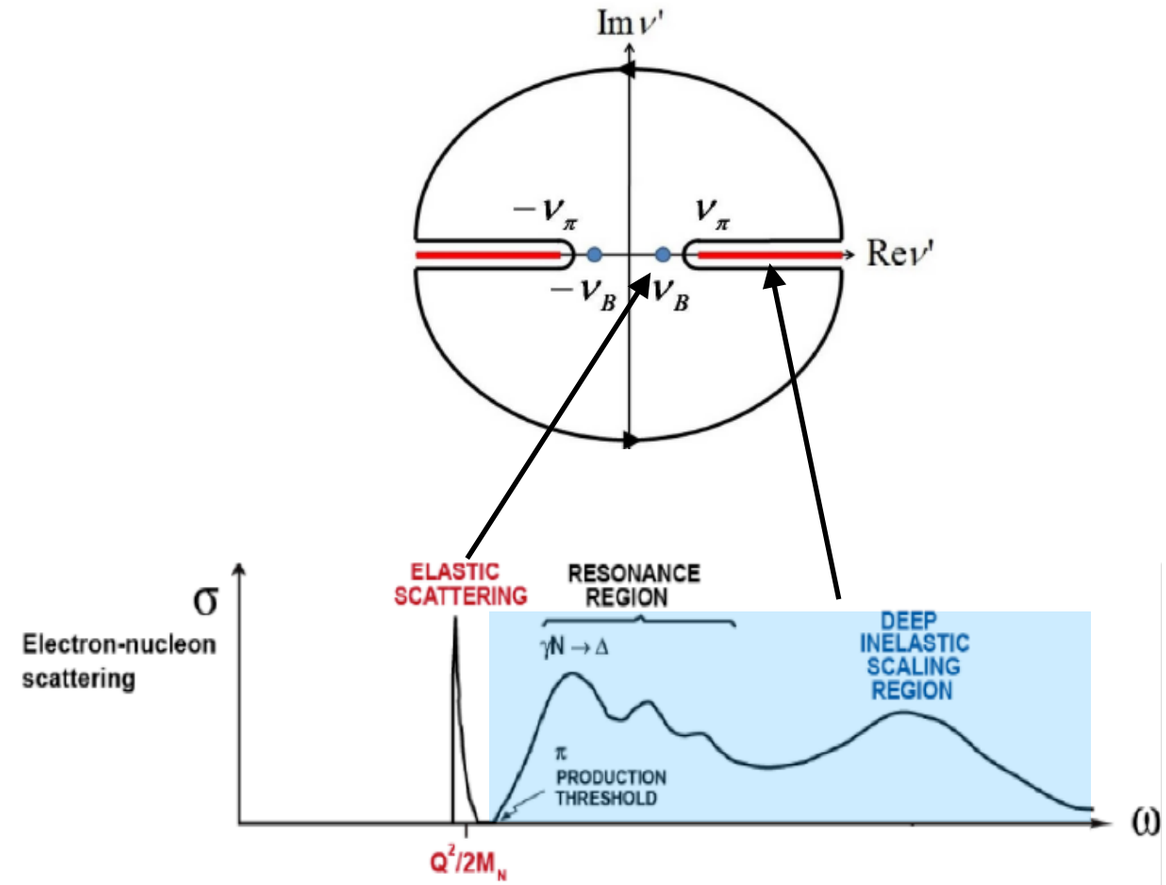
Substitute into the loop and calculate leading energy dependence

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi N}\int_0^\infty dQ^2\int_{\nu_{thr}}^\infty d\nu\frac{F_3^{(0)}}{M\nu}\frac{\nu+2q}{(\nu+q)^2} + O(E^2)$$

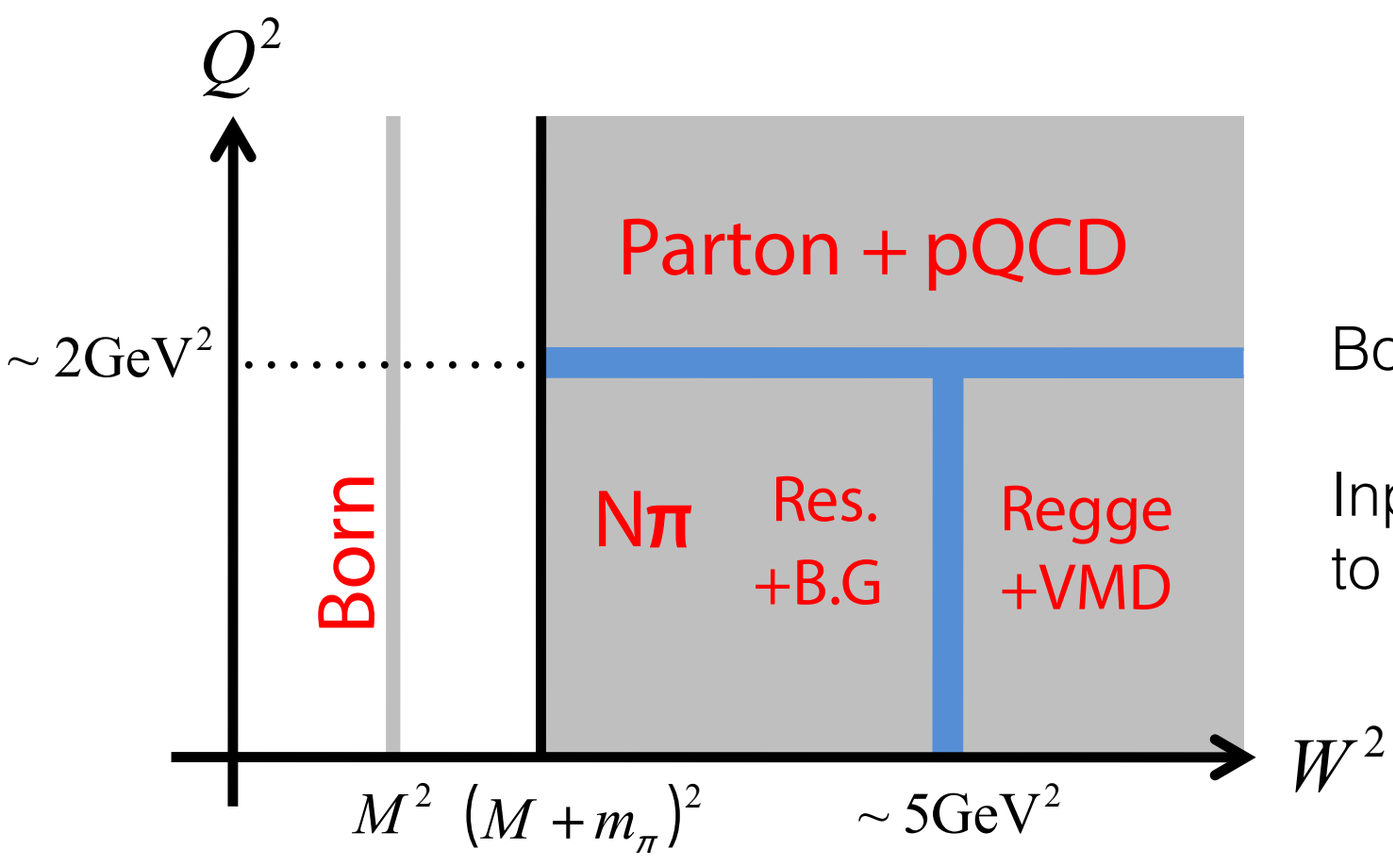
$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E) = \frac{8\alpha E}{3\pi NM}\int_0^\infty dQ^2\int_{\nu_{thr}}^\infty \frac{d\nu}{(\nu+q)^3}\left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu+q)}{2Q^2}+1\right)\frac{M}{\nu}F_2^{(0)} + \frac{\nu+3q}{4\nu}F_3^{(-)}\right] + O(E^3)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q^2:
scanning dominant physics pictures



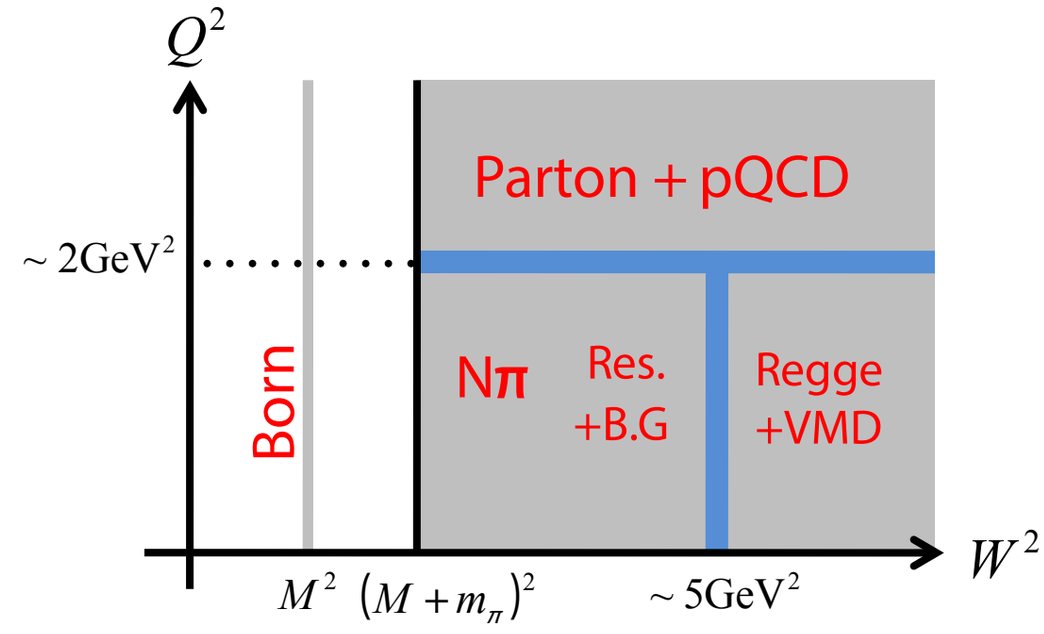
Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral

Our parametrization of the needed SF follows from this diagram

$$F_3^{(0)} = F_{\rm Born} + \begin{cases} F_{\rm pQCD}, & Q^2 \gtrsim 2\ {\rm GeV}^2 \\ F_{\pi N} + F_{\rm res} + F_{\mathbb{R}}, & Q^2 \lesssim 2\ {\rm GeV}^2 \end{cases}$$



Born: elastic FF from e^-, ν scattering data

πN: relativistic ChPT calculation plus nucleon FF

Resonances: axial excitation from PCAC (**Lalakulich et al, 2006**) - neutrino scattering;
electron and γ inelastic scattering

Above resonance region: multiparticle continuum described by Regge exchanges

Inelastic contributions quantify how strongly the decay electron polarizes the proton

Universal RC from DR

γW-box at zero energy

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty \frac{d\nu}{\nu}\frac{\nu+2q}{(\nu+q)^2}F_3^{(0)}(\nu,Q^2)$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd}(E=0)=0$$

Connection to MS: rewrite in terms of the first Nachtmann moment of F_3

$$M_3^{(0)}(1,Q^2) = \frac{4}{3}\int_0^1 dx \frac{1+2\sqrt{1+4M^2x^2/Q^2}}{(1+\sqrt{1+4M^2x^2/Q^2})^2}F_3^{(0)}(x,Q^2) \qquad x=\frac{Q^2}{2M\nu}$$

$$\mathrm{Re}\,\Box_{\gamma W}^{even} = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2M_W^2}{Q^2(M_W^2+Q^2)}M_3^{(0)}(1,Q^2) = \frac{\alpha}{8\pi}\int_0^\infty \frac{dQ^2M_W^2}{M_W^2+Q^2}F_{MS}(Q^2)$$

MS loop fn. $F(Q^2)$ directly related to $M_3^{(0)}$

$$F_{MS}(Q^2) = \frac{12}{Q^2}M_3^{(0)}(1,Q^2)$$

SF F_3 - commutator of em and weak currents - insert complete set of on-shell hadronic states

$$F_3^{(0)} \propto \int dx e^{iqx}\langle p|[J_{em}^{\mu,(0)}(x),J_W^{\nu,+}(0)]|n\rangle \sim \int dx e^{iqx}\sum_X \langle p|J_{em}^{\mu,(0)}(x)|X\rangle\langle X|J_W^{\nu,+}(0)|n\rangle$$

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W\,(0)}$

Data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2 F_1 + \left(1 - y - \frac{Mxy}{2E}\right)F_2 \pm x\left(y - \frac{y^2}{2}\right)F_3\right]$$

$$\sigma^{\nu p} - \sigma^{\bar{\nu} p} \sim F_3^{\nu p} + F_3^{\bar{\nu} p} = u_v^p(x) + d_v^p(x)$$

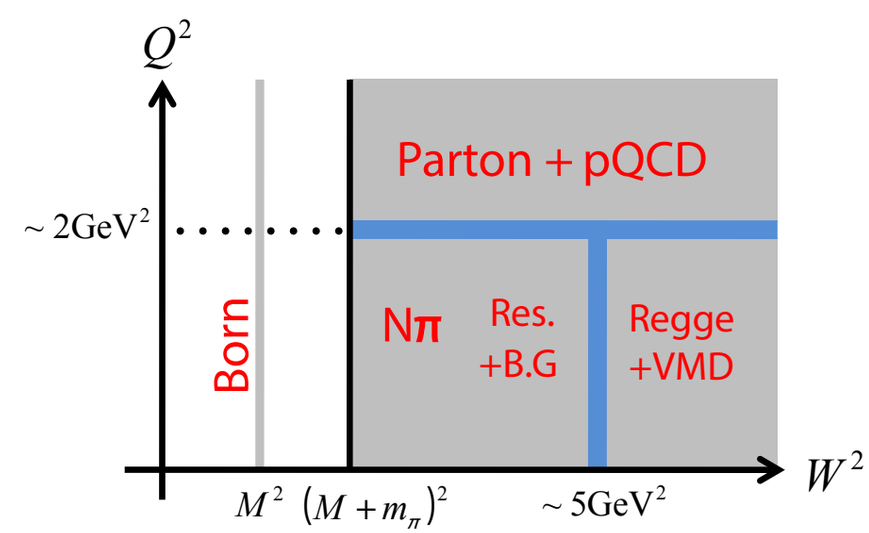
Gross-Llewellyn-Smith sum rule $\int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$

Validate the model for CC process; apply an isospin rotation to obtain γW

$$F_{3,\,\mathrm{low}-Q^2}^{\nu p+\bar{\nu}p} = F_{3,\,el.}^{\nu p+\bar{\nu}p} + F_{3,\,\pi N}^{\nu p+\bar{\nu}p} + F_{3,\,R}^{\nu p+\bar{\nu}p} + F_{3,\,\mathrm{Regge}}^{\nu p+\bar{\nu}p}$$

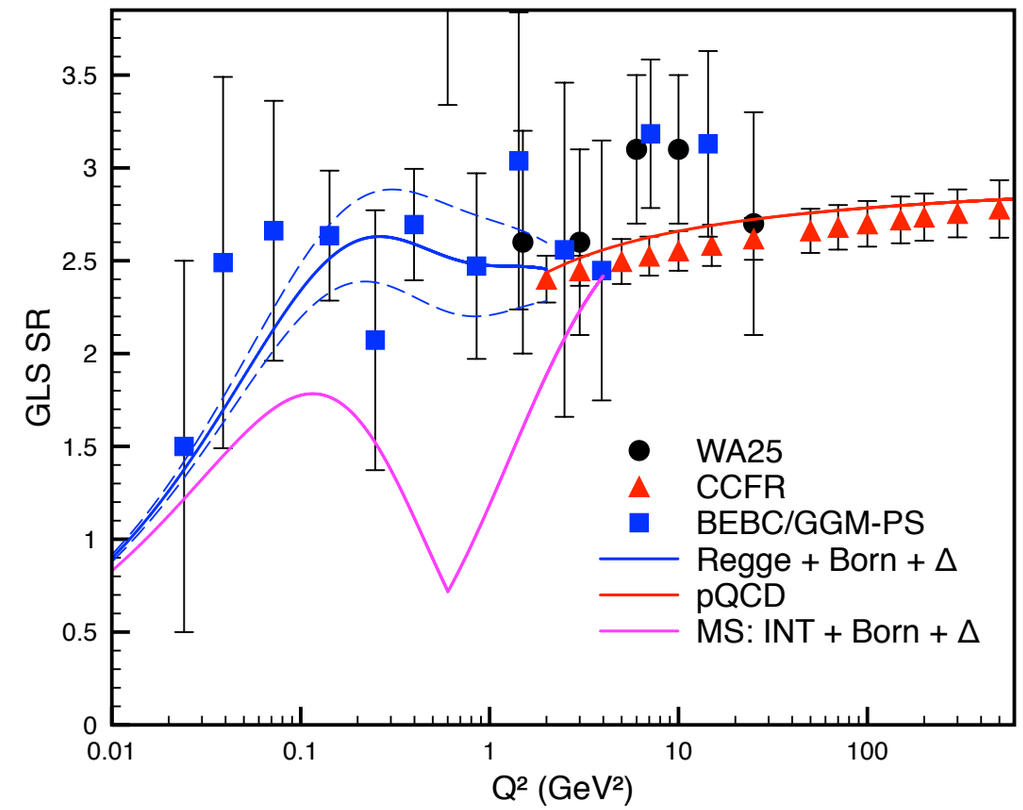
Low-W part of spectrum:
neutrino data from MiniBooNE, Minerva, …
- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^{\nu} \sim x^{-\alpha}$, $\alpha \sim 0.5$-0.7



Use of neutrino data via isospin rotation

Low $Q^2 < 0.1$ GeV^2: Born + Δ(1232) dominate
Not fitted: modern data more precise but
cover only limited energy range
Fit driven by 4 data points between 0.2 and 2 GeV^2



Model & Uncertainty fully specified
- compare M&S vs This work

$M_3^{WW}(1,Q^2)$

Isospin symmetry

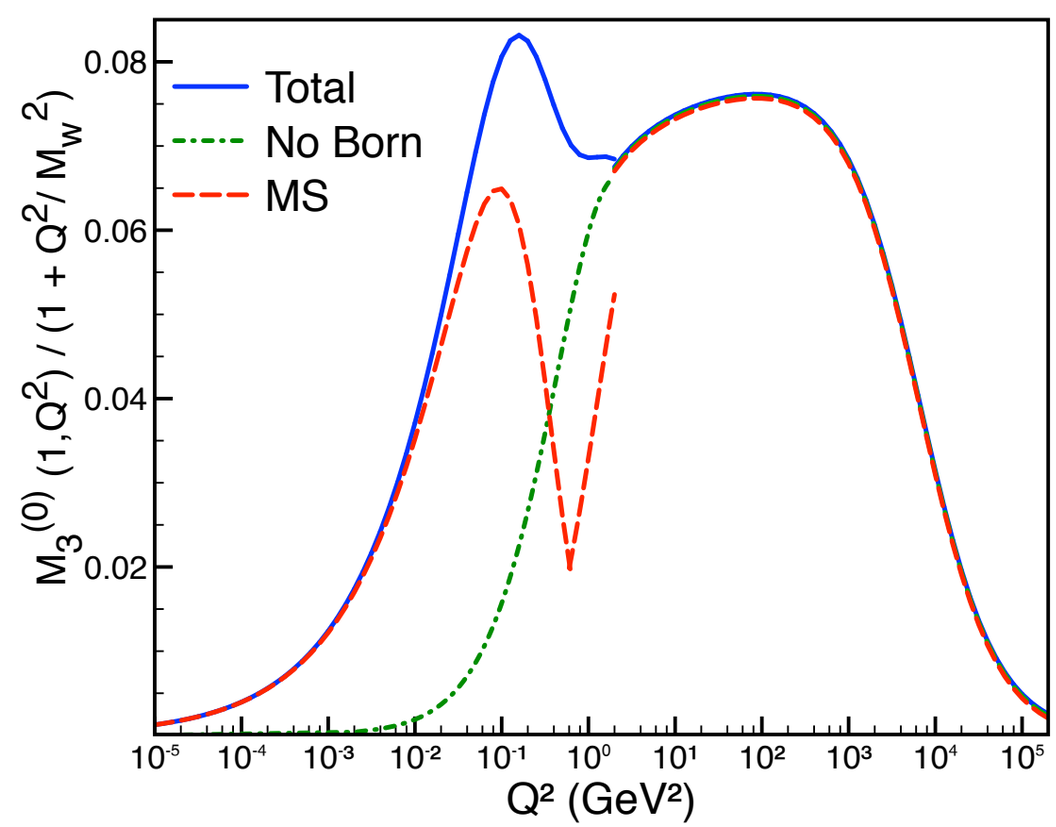
$M_3^{\gamma W}(1,Q^2)$

M&S: integrand discontinuous at $Q^2 = 2.25$ GeV^2

Log scale for x-axis: integral = surface under the curve

$$\text{MS Total}: \quad \square_{\gamma W}^{(0)} = 0.00324 \pm 0.00018$$
$$\text{New Total}: \quad \square_{\gamma W}^{(0)} = 0.00379 \pm 0.00010$$

Uncertainty reduced by almost factor 2;
~ 3 sigma shift from the old value



Consequences for the V_{ud} extraction

Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

In July 2019 Czarnecki, Marciano and Sirlin published an update in which they largely agreed that the RC was underestimated:

Czarnecki, Marciano, Sirlin, 1907.06737

C-M-S 2019: $\Delta_R^V = 0.02421(32)$ —> $|V_{ud}| = 0.97391(10)_{Ft}(15)_{RC}$

Seng, Meissner, 1903.07969:
γW-box can be calculated on the lattice via Feynman-Hellman Theorem

Nuclear Structure Modification

Long-range RC: the decay electron polarizes the proton
Nuclear polarizabilities are much different from nucleon - need to account for that

General structure of RC for nuclear decay

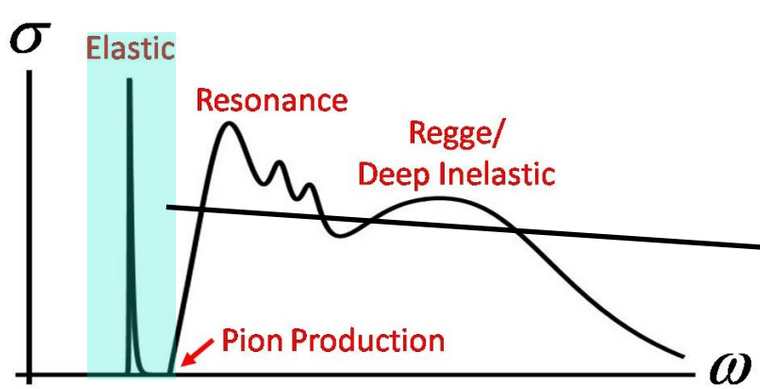
$$ft(1+RC) = Ft(1+\delta'_R)(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

NS correction reflects this extraction of the free box

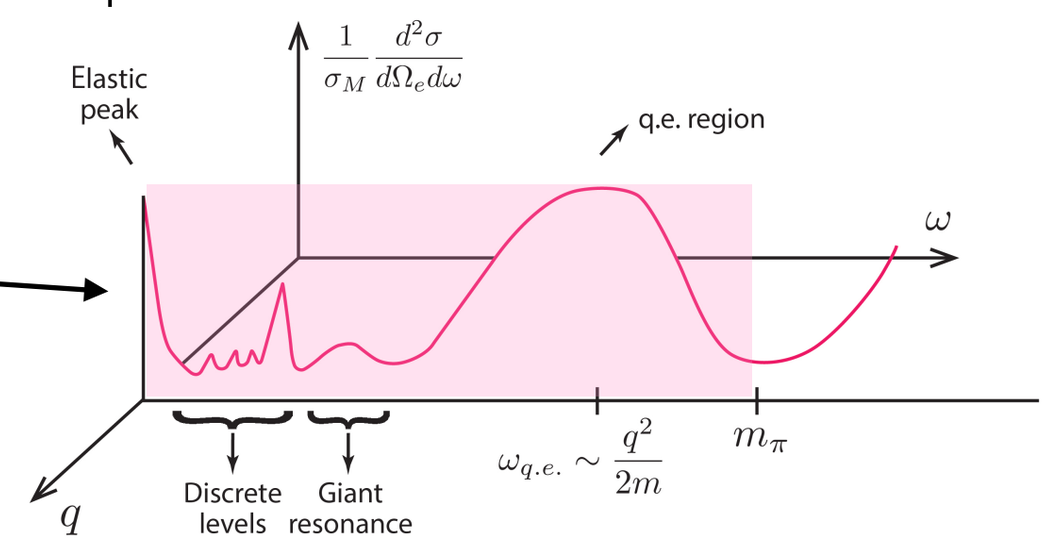
$$\Box_{\gamma W}^{\text{VA, Nucl.}} = \Box_{\gamma W}^{\text{VA, free n}} + \left[\Box_{\gamma W}^{\text{VA, Nucl.}} - \Box_{\gamma W}^{\text{VA, free n}}\right]$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC



Input in the DR for the RC on a nucleus



Nuclear Structure Modification

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352
MG, arXiv: 1812.04229

δ_{NS} from DR with energy dependence averaged over the spectrum

$$\delta_{NS} = \frac{2\alpha}{\pi NM}\int_0^{1\,\mathrm{GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,Nucl.} - F_3^{(0),B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,Nucl.}\right]$$

$$\langle E\rangle = \frac{\int_m^Q dEEp(Q-E)^2E}{\int_m^Q dEEp(Q-E)^2}$$

Hardy & Towner 1994 on: ad hoc description by nuclear quenching of spin operators
Compare the effect on the average Ft value:

HT value 2018: $\mathscr{F}t = 3072.1(7)s$

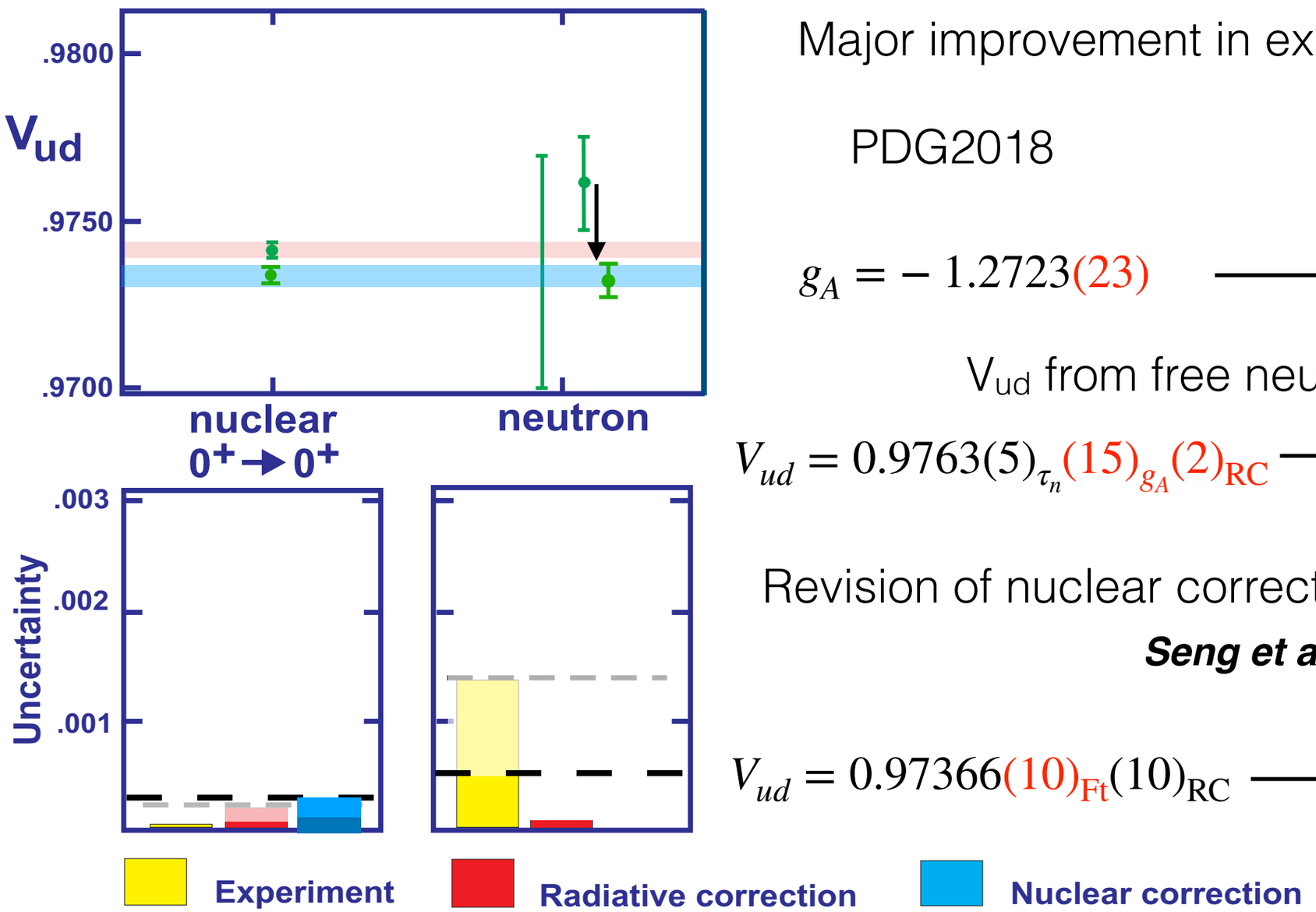
Old estimate: $\delta\mathscr{F}t = -(1.8\pm0.4)s \quad + \quad (0\pm0)s$

New estimate: $\delta\mathscr{F}t = -(3.5\pm1.0)s \quad + \quad (1.6\pm0.5)s$

Two 2σ corrections that cancel each other;
The cancellation is delicate: the two terms are highly correlated
Larger E-dep. term will correspond to a smaller negative E-indep. term and vv.

Conservative uncertainty estimate: 100% $\mathscr{F}t = (3072\pm2)s$

V_{ud} and CKM unitarity in December 2018



Major improvement in exp. determination of g_A

PDG2018 — PERKEO-III

Märkisch et al., 1812.04666

$$g_A = -1.2723(23) \longrightarrow g_A = -1.2764(6)$$

V_{ud} from free neutron decay

$$V_{ud} = 0.9763(5)_{\tau_n}(15)_{g_A}(2)_{\mathrm{RC}} \longrightarrow V_{ud} = 0.9735(5)_{\tau_n}(3)_{g_A}(1)_{\mathrm{RC}}$$

Revision of nuclear corrections to 0^+- 0^+-beta decay

Seng et al., 1812.03352; MG 1812.04229

$$V_{ud} = 0.97366(10)_{\mathrm{Ft}}(10)_{\mathrm{RC}} \longrightarrow V_{ud} = 0.97366(32)_{\mathrm{Ft}}(10)_{\mathrm{RC}}$$

Free neutron decay becomes competitive;
Limitation: the lifetime (also, beam vs. bottle)

Scrutiny of nuclear corrections with new methods

Top-row unitarity: 2,5-3,5σ deficit

$$\Delta_u = -(0.0016 - 0.0021) \pm 0.0006$$

(depending on V_{us})

Discrepancy - BSM?

BSM explanation: non-standard CC interactions —> new V,A,S(PS),T(PT) terms

$$H_{S+V} = (\overline{\psi}_p \psi_n)(C_S \overline{\phi}_e \phi_{\overline{\nu}_e} + C'_S \overline{\phi}_e \gamma_5 \phi_{\overline{\nu}_e}) + (\overline{\psi}_p \gamma_\mu \psi_n)\left[C_V \overline{\phi}_e \gamma_\mu (1+\gamma_5) \phi_{\overline{\nu}_e}\right]$$

Scalar and Tensor interactions: distort the beta decay spectra

Complementarity to LHC searches

Exp. high precision measurement of ^{6}He spectrum (O. Naviliat-Cuncic, A. Garcia, …)

$$N(E)dE = p_e E (E_m - E)^2 \left[1 + C_1 E + b \frac{m_e}{E}\right]$$

C_1 = 0.00650(**7**) MeV^{-1} - effect of weak magnetism - positive slope
b $\sim \pm 0.001$- negative slope

Energy-dep. polarizability correction —> C'_1 ~ 0.00020(**20**) MeV^{-1} — at the level 3σ of C_1

Conclusions & Outlook

- The γW-box in the forward dispersion relation framework
- Hadronic and nuclear corrections in a unified framework
- Nuclear structure leaks in the outer correction, distorts the beta decay spectrum
- Nuclear uncertainties shift the emphasis on free neutron decay
- Tensions with CKM unitarity: $\Sigma_{i=d,s,b} |V_{ui}|^2 - 1 = -0.0016(4\text{-}6)$

Nuclear correction δ_{NS}
DR allow to address hadronic and nuclear parts of the calculation on the same footing
The full nuclear correction should be calculated (not just QE) - further test of H&T δ_{NS}

Decay spectra and nuclear polarizabilities
Can contaminate the extraction of Fierz interference from precise spectra!

Further applications
γW box correction to GT rate (nuclear; nucleon - comparison of g_A w. lattice)
γW box correction to Kl3 decays and V_{us} - potential to reconcile Kl3 and Kl2?
γW box on the lattice - both for n and πl3 possible!