Evolution equations for medium-induced QCD cascades and their solutions

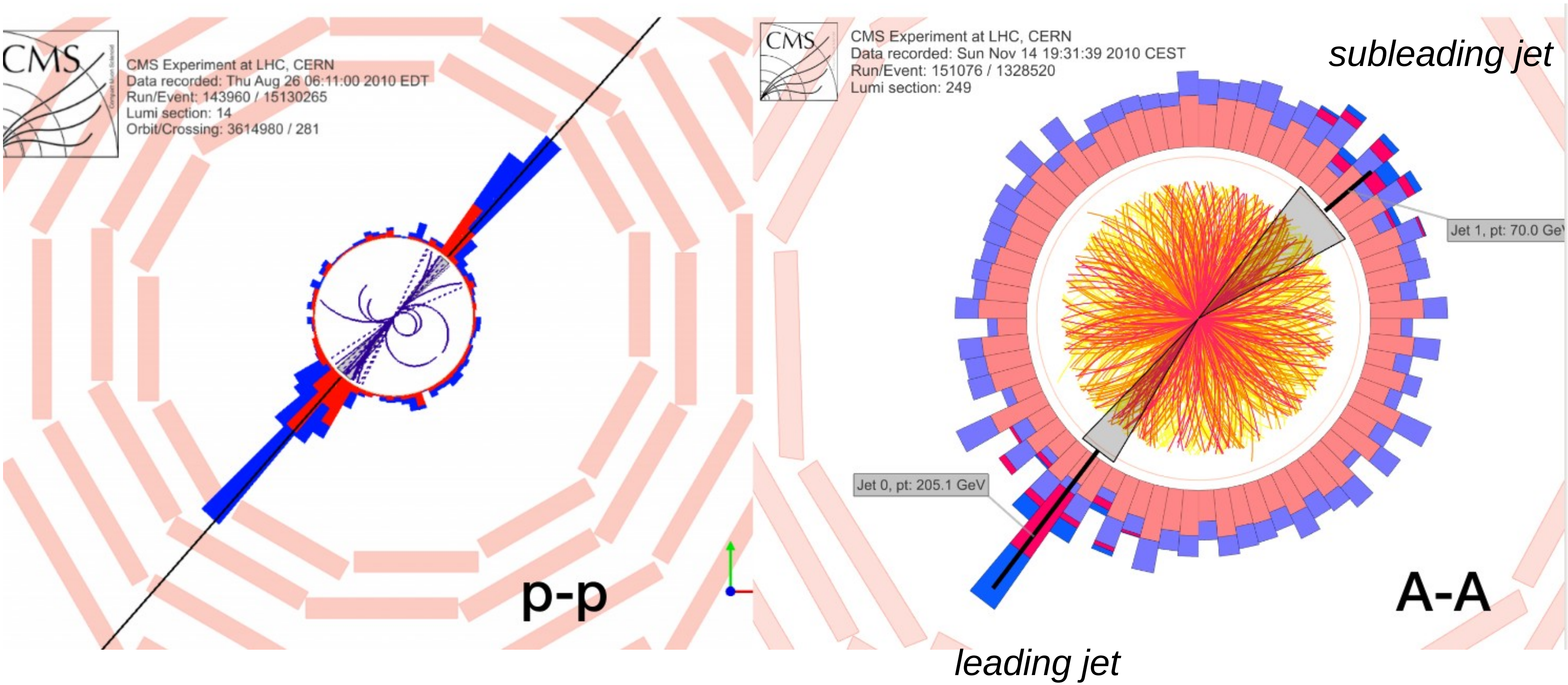
Krzysztof Kutak
IFJ PAN

Based on:

1811.06390 by K. Kutak, W. Płaczek, R. Straka

To appear in EPJ C

Jet quenching



Why jets are useful?

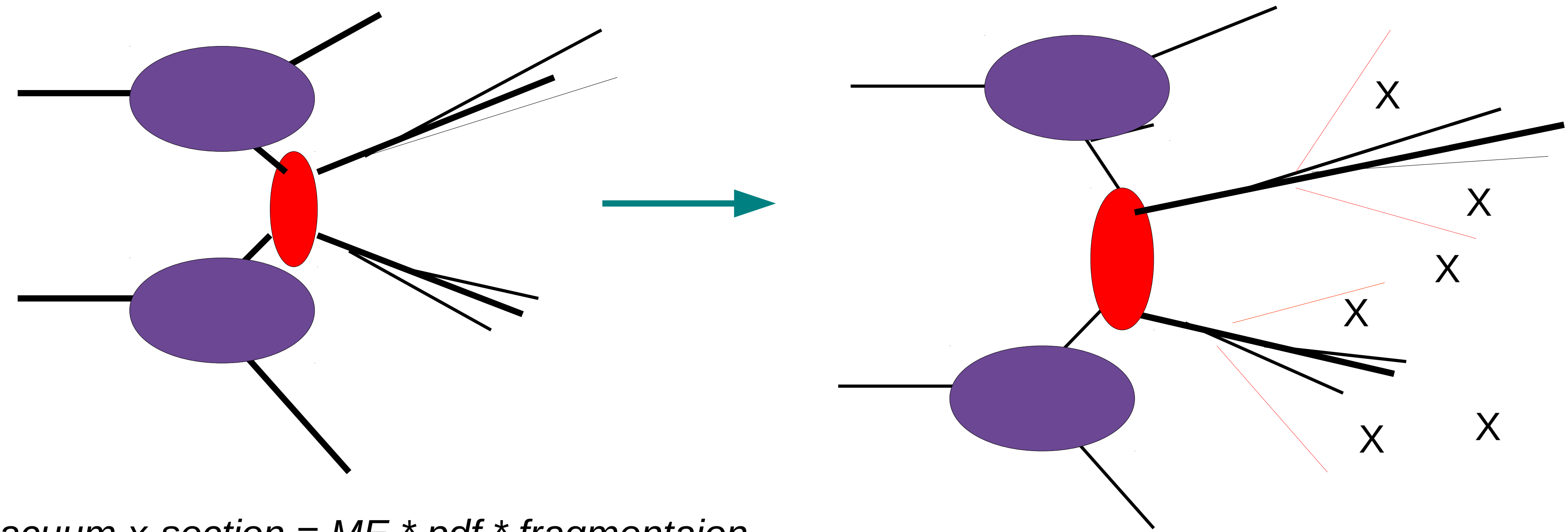
Plasma lasts for too short times (10–20 fm/c) and one can not at present use external probe to study plasma’s properties . Interactions of the outgoing partons with the hot and dense QCD medium are expected to modify the angular and momentum distributions of final-state jet fragments relative to those in proton-proton collisions.

This process is known as jet quenching → can be used to as probe QGP

Dominant process for jet quenching is radiative energy loss associated with medium induced gluon radiation

From vacuum to medium

vacuum *medium*



*vacuum x-section = ME * pdf * fragmentaion*

*complete x-section = ME * pdf * fragmentaion+*
*ME * pdf * distribution of minijets*

van Hameren, Kotko, Kutak, Sapeta 1903.01361

LPM and BDMPS-Z

Multiple soft scattering approximation (MSSA): high density effects are resummed to all orders. It is expected to be important for short mean free-path

Because medium-induced radiation can occur anywhere along the medium with equal probability, the radiation spectrum is expected to scale linearly with L.

Owing to the quantum nature of the radiation, many scattering centers act coherently during the radiation over time t_{coh} ($\ll t_{mfp}$).

With increasing frequency the coherence time increases. As a result, the effective number of scattering centers decreases.

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{t_{\mathrm{coh}}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

$$t_{\mathrm{coh}} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$

energy of observed gluon

maximal energy that can be taken by single gluon

$$\omega \frac{\mathrm{d}I_{\mathrm{BH}}}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\ell_{\mathrm{mfp}}} = \alpha_s N_{\mathrm{scatt}}$$

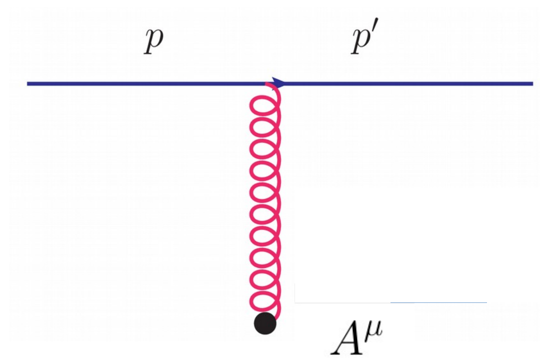
if $t_{coh} \sim t_{mfp}$ only one scattering is involved in radiation

Single interaction with medium

Introductory lectures on jet quenching in heavy ion collisions, Casalderrey-Solana, SalgadoActa Phys.Polon.B38:3731-3794,2007

Original papers by:

Mueller, Baier, Dokzhitzer, Schiff, Zakcharov '97, '98



$$x_{\pm} = \frac{1}{\sqrt{2}}(x_0 \pm x_3) \qquad p_{\pm} = \frac{1}{\sqrt{2}}(p_0 \pm p_3)$$

$$p \cdot x = p_+ x_- + p_- x_+ - \mathbf{p}_\perp \cdot \mathbf{x}_\perp$$

$$S_1(p', p) = \int d^4x \; \mathrm{e}^{i(p'-p)\cdot x} \; \bar{u}(p') \; igA^a_\mu(x) T^a \gamma^\mu \; u(p)$$

$$p^\mu A^a_\mu \simeq 2p_+ A^a_-$$

p+ is big

$$\tfrac{1}{2} \sum_\lambda \bar{u}^\lambda(p)\gamma^\mu u^\lambda(p) = 2p^\mu$$

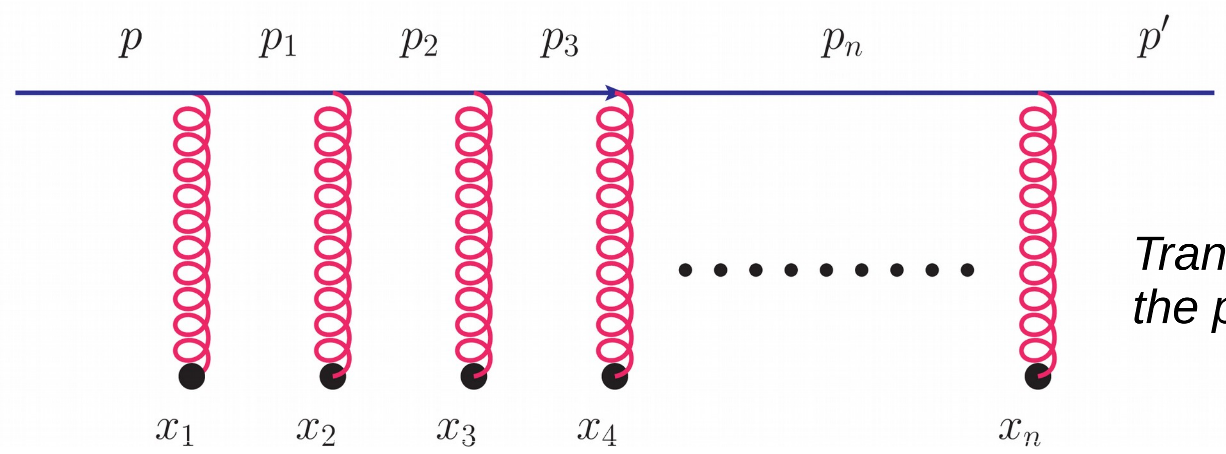
$$S_1(p', p) \simeq 2\pi\delta(p'_+ - p_+) 2p_+ \int d\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} \left[ig \int dx_+ A_-(x_+, \mathbf{x}_\perp) \right]$$

$$\boldsymbol{v} \equiv \boldsymbol{v}_\perp$$

$$\langle A^-_a(\boldsymbol{p}, t') A^{*-}_b(\boldsymbol{p}', t') \rangle = \delta_{ab} n(t) \delta(t - t') (2\pi)^2 \delta^{(2)}(\boldsymbol{p} - \boldsymbol{p}') w(\boldsymbol{p})$$

Landau Pomeranchuk effect: during the radiation process the system interacts with many scattering centers that act coherently as a single one. Medium is modeled as static centers → Gullsy-Wang model. Scale characterizing fluctuations of QGP is m_D which is much smaller than the energy of jet. Mean free path is longer than screening length → collisions are independent

Multiple interactions with medium



Transverse components in the propagators are neglected

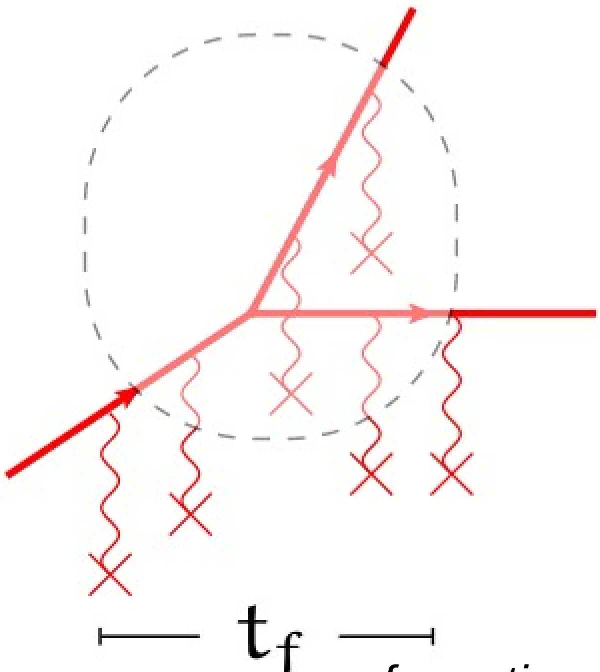
$$S_n(p',p) \simeq 2\pi\delta(p'_+ - p_+)2p_+ \int d\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} \frac{1}{n!}\mathcal{P}\left[ig\int dx_+ A_-(x_+,\mathbf{x}_\perp)\right]^n$$

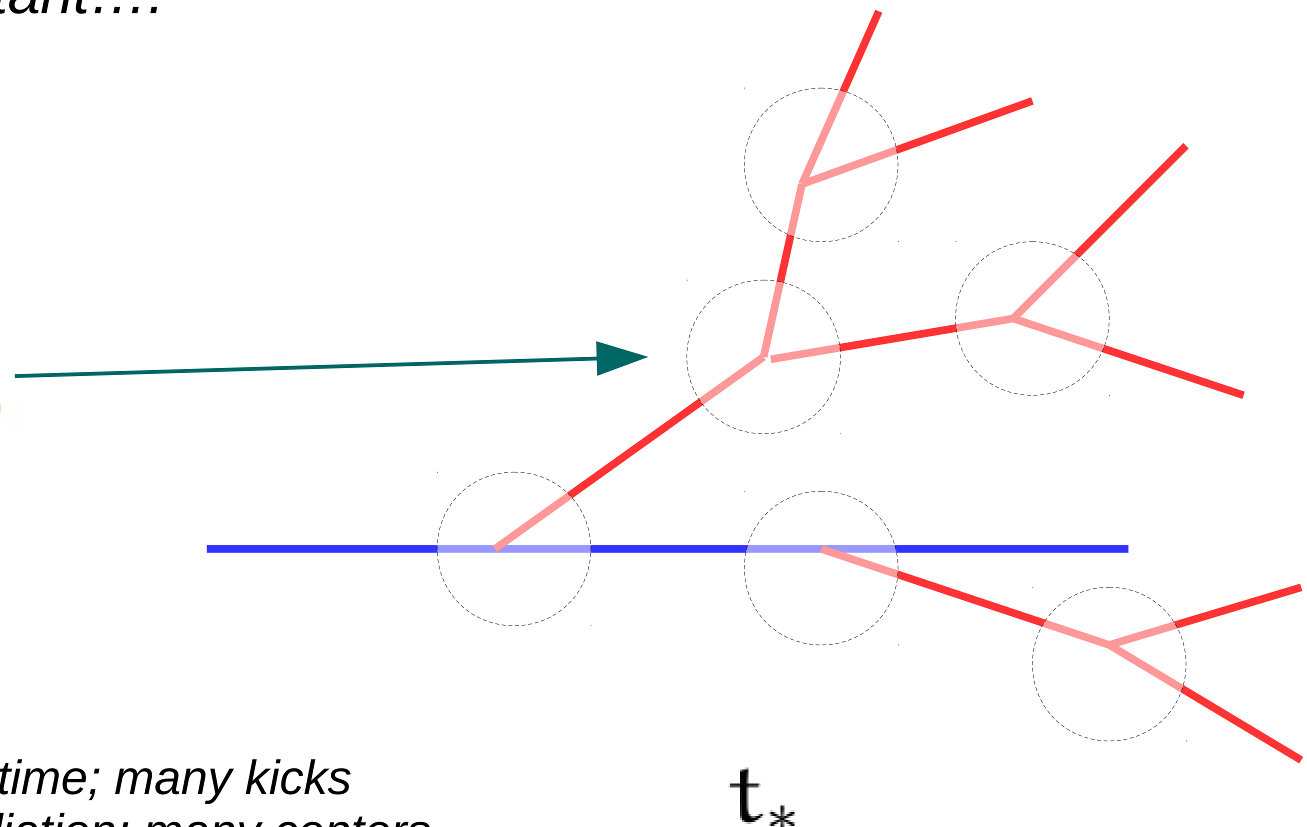
$$S(p',p) = \sum_{n=0}^{\infty} S_n(p',p) \simeq 2\pi\delta(p'_+ - p_+)2p_+ \int d\mathbf{x}_\perp \mathrm{e}^{-i\mathbf{x}_\perp(\mathbf{p}'_\perp - \mathbf{p}_\perp)} W(\mathbf{x}_\perp)$$

Wilson line in fundamental representation

Multiple branching

Beyond energy lost by the leading particle, the LHC data call for a more thorough analysis of the jet shape for which the effects of multiple branching at large angles are important….





formation time; many kicks before radiation; many centers act as a single source

from Y. Mehtar-Tani

stopping time → time at which energy has been emitted in form of soft gluons

Kinetic equation

Maxwell-Boltzmann equation

Not easy to derive in QCD. Impossible? - Kovchegov, Wu 17
Assumes thermalization of minijets.
Nonuniform plasma

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}\right) f(t, \boldsymbol{x}, \boldsymbol{p}) \;=\; C_{\rm el}[f] + C_{\rm br}[f]$$

$$x = \omega / E$$

energy of observed gluon

energy of leading particle

Example:

p *– transverse momentum of observed gluon*

$$C_{\rm el}[f] \;\simeq\; \frac{1}{4}\,\hat{q}\,\nabla_{\boldsymbol{p}} \cdot \left[\left(\nabla_{\boldsymbol{p}} + \frac{\boldsymbol{v}}{T}\right) f\right]$$

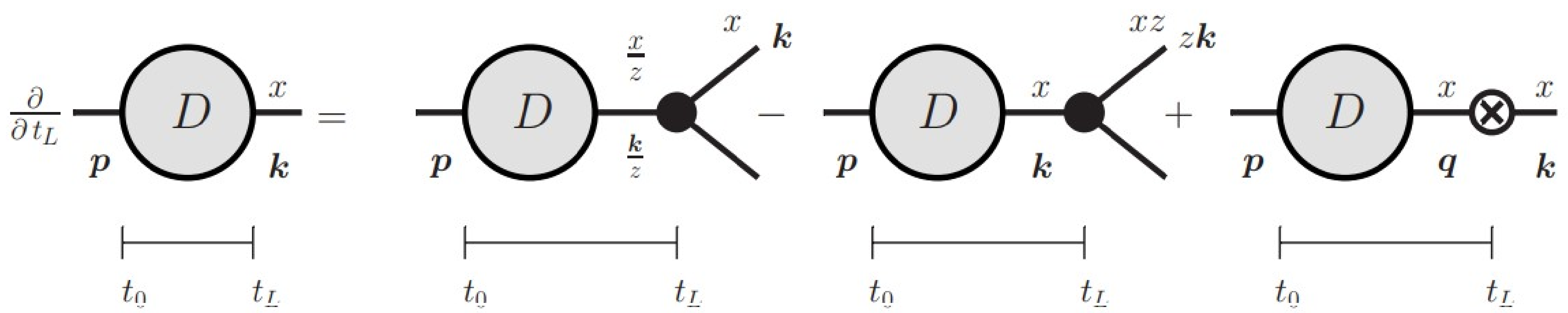
$$\frac{d\langle k_\perp^2\rangle}{dt} \simeq \frac{m_D^2}{l} \equiv \hat{q}$$

$$C_{\rm br}[f] \;\simeq\; \frac{1}{t_{\rm br}(p)} \int_0^1 {\rm d}x\,\mathcal{K}(x) \left[\frac{1}{x^{\frac{5}{2}}} f\left(t, \boldsymbol{x}, \frac{\boldsymbol{p}}{x}\right) - \frac{1}{2} f(t, \boldsymbol{x}, \boldsymbol{p})\right]$$

*where **V = p/**p*

Baier, Mueller, Schiff, Son '00
Iancu, Wu '15

The BDIM equation



Blaizot, Dominguez, Iancu, Mehtar-Tani '12

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$

Inclusive gluon distribution as produced by hard jet

$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t)$$

$$\frac{1}{t^*} = \frac{\bar{\alpha}}{\tau_{\rm br}(E)} = \bar{\alpha}\sqrt{\frac{\hat{q}}{E}}$$

$$C(\mathbf{q}) = w(\mathbf{q}) - \delta(\mathbf{q})\int d^2\mathbf{q}'\,w(\mathbf{q}')$$

$$\mathcal{K}(z) = \frac{[f(z)]^{5/2}}{[z(1-z)]^{3/2}} \qquad f(z) = 1-z+z^2$$

Equation describes interplay of rescatterings and branching. This particular equation has k_t independent kernel. This is an approximation. The whole broadening comes from rescattering. Energy of emitted gluon is much larger than its transverse momentum

Equation for energy

Integral of the former equation over kt

$$\frac{\partial}{\partial t}D(x,t)=\frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right)\theta(z-x)-\frac{z}{\sqrt{x}}D(x,t)\right]$$

In simplified version of the kernel - f(z)=1 - case analytical solution is possible.

$$D(x,\tau)=\frac{\tau}{\sqrt{x}(1-x)^{3/2}}\exp\left(-\pi\frac{\tau^2}{1-x}\right)$$

Blaizot, Iancu, Mehtar-Tani'13

The solution features so called turbulent behavior. Here that means that at low x the solution factorizes into x and t dependent distributions.

The fact that the spectrum keeps the same x-dependence when t keeps increasing reflects the fact that the energy flows to x = 0 without accumulating at any finite value of x
→ *wave turbulence*

Rearrangement of the equation for energy

1811.06390 by K. Kutak, W. Płaczek, R. Straka

$$\frac{\partial}{\partial t} D(x,t) = \frac{1}{t^*}\int_0^1 dz\, \mathcal{K}(z)\left[\sqrt{\frac{z}{x}}\, D\left(\frac{x}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}\, D(x,t)\right]$$

mathematics: transformation of differential equation to integral equation

physics: resummation of virtual and unresolved real emissions

$$D(x,\tau) = e^{-\Phi(x)(\tau-\tau_0)} D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_0^{1-\epsilon} dz \int_0^1 dy\, \delta(x-zy)\sqrt{\frac{z}{x}}\, z\mathcal{K}(z)\, e^{-\Phi(x)(\tau-\tau')} D(y,\tau')$$

where

$$\Phi(x) \equiv \frac{1}{\sqrt{x}}\int_0^{1-\epsilon} dz\, z\mathcal{K}(z)$$

$$\tau = t/t^*$$

Sudakov form factor resumes virtual and unresolved real emissions

Rearrangement of the equation for gluon density

procedure almost the same as for energy distribution

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$
$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t),$$

1811.06390 by Kutak, Płaczek, Straka

$$\downarrow$$

$$D(x,\mathbf{k},\tau) = e^{-\Psi(x)(\tau-\tau_0)}D(x,\mathbf{k},\tau_0)$$
$$+\int_{\tau_0}^{\tau}d\tau'\int_0^1 dz\int_0^1 dy\int d^2\mathbf{k}'\int d^2\mathbf{q}\;\mathcal{G}(z,\mathbf{q})$$
$$\times\,\delta(x - zy)\,\delta(\mathbf{k}-\mathbf{q}-z\mathbf{k}')\,e^{-\Psi(x)(\tau-\tau')}D(y,\mathbf{k}',\tau')$$

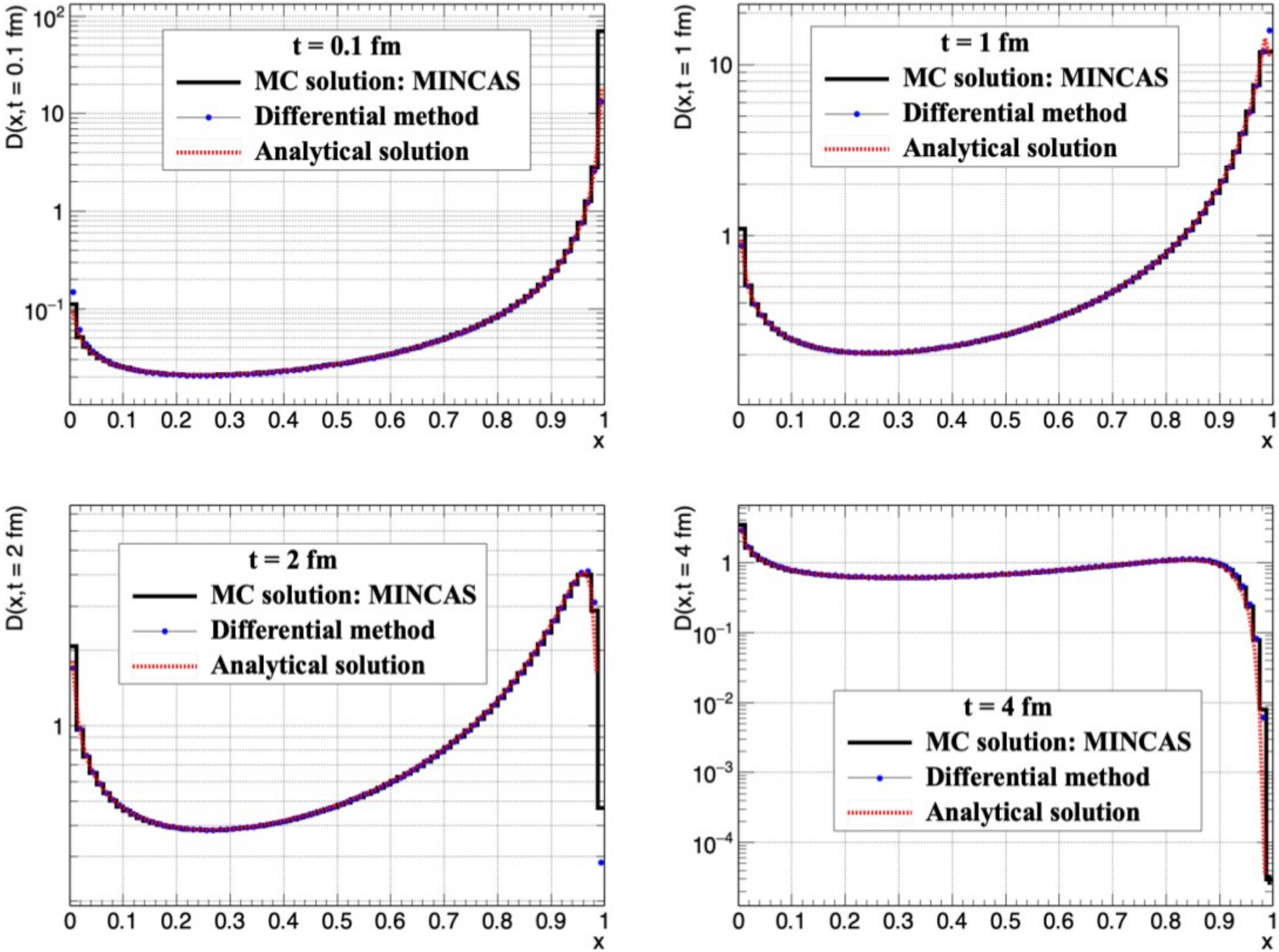
The solver - MINCAS

Newly developed Monte Carlo program by Wieslaw Płaczek (Jagiellonian University)

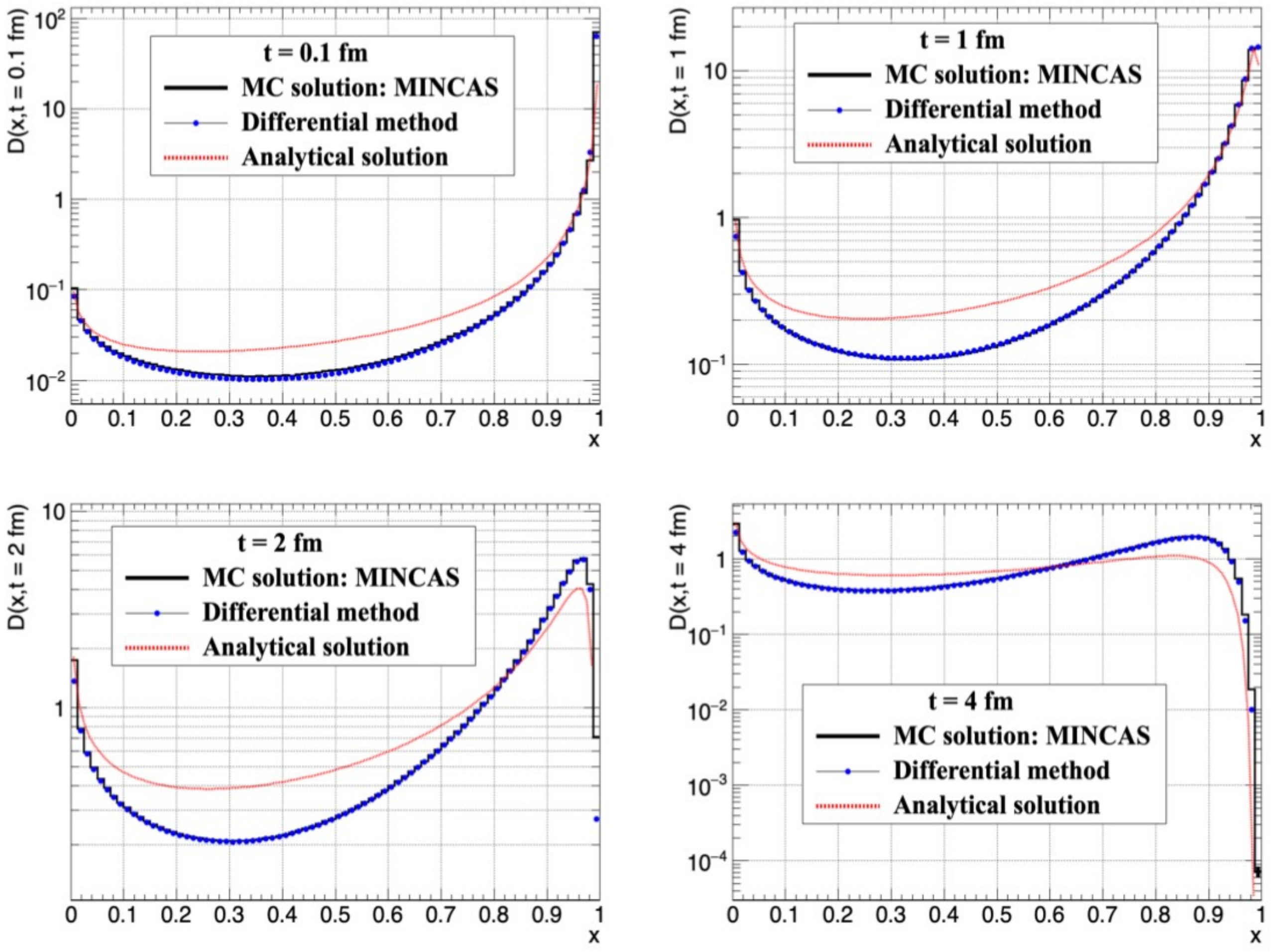
- *based on Markov chain algorithm*
- *written in C++*
- *with reasonable settings solves the equation in lets than 10 minutes*
- *can be promoted to parton shower for FSR*
- *easy to use user interface*
- *will be released for public use*

The results for the energy distribution were cross-checked by traditional method based on Runge-Kutta solver written by Robert Straka

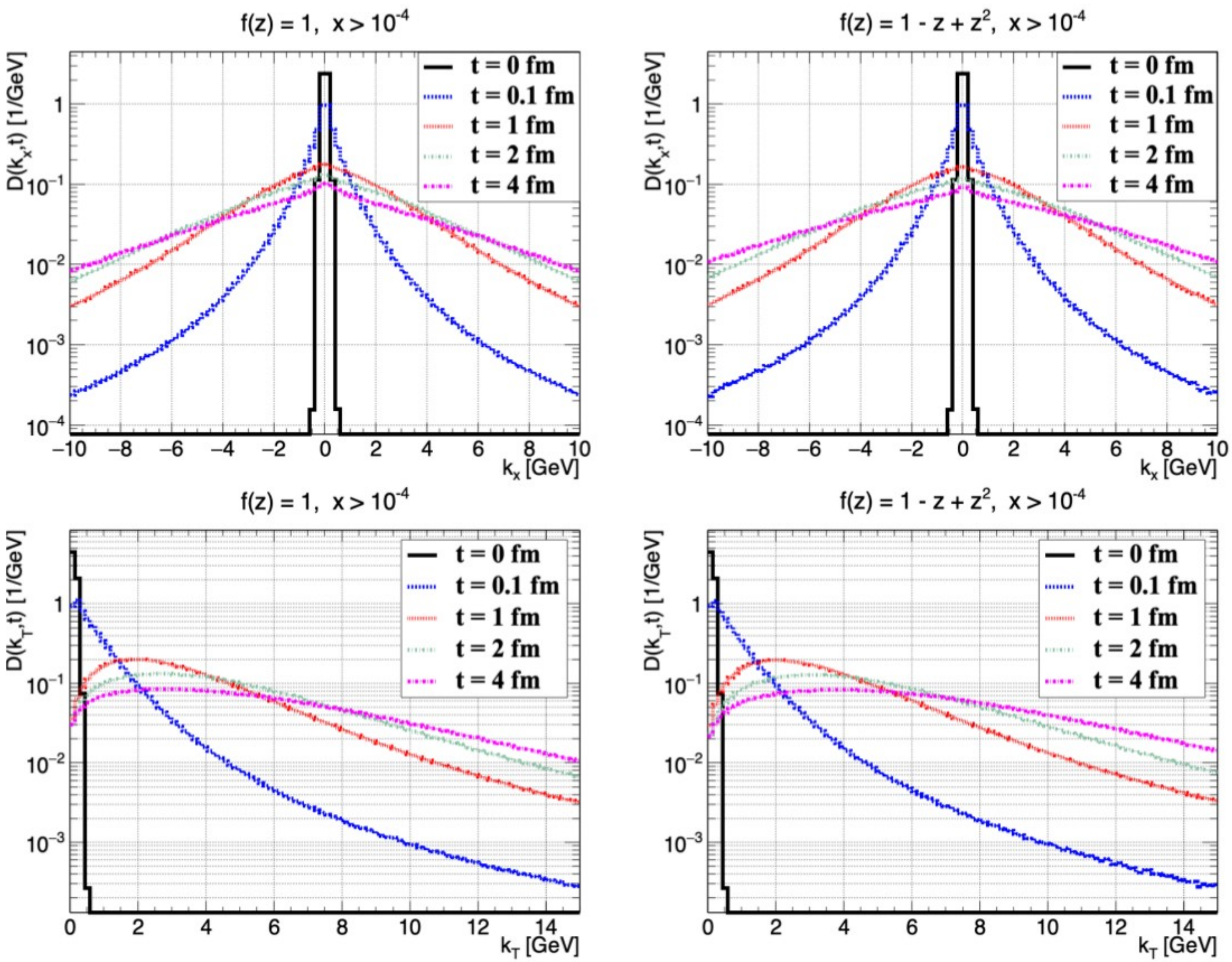
Solution for energy distribution – simplified kernel



Solution for energy distribution
– simplified kernel vs. complete kernel

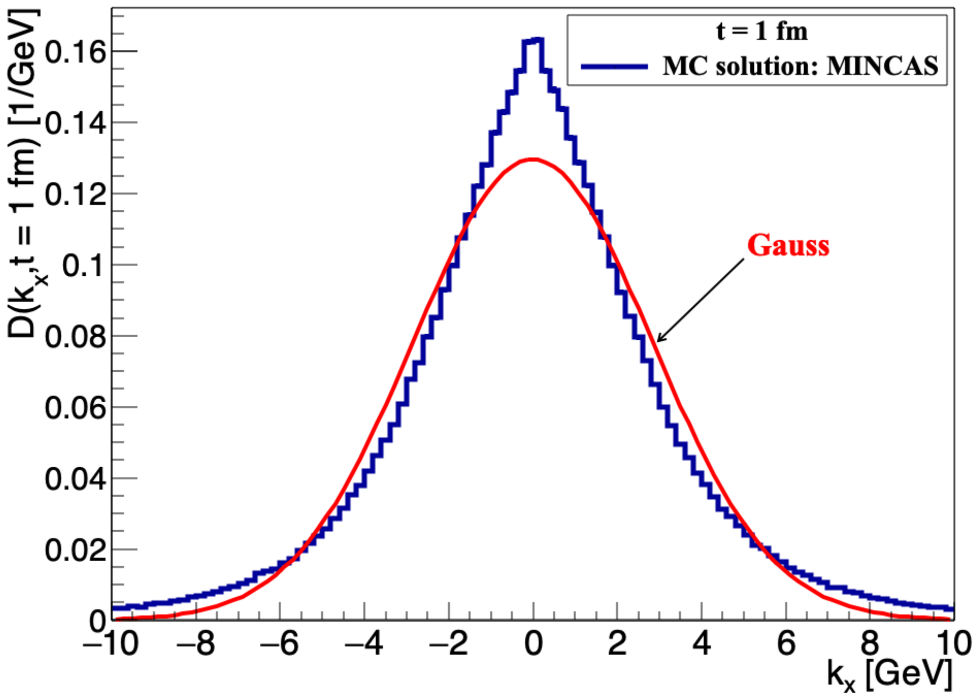


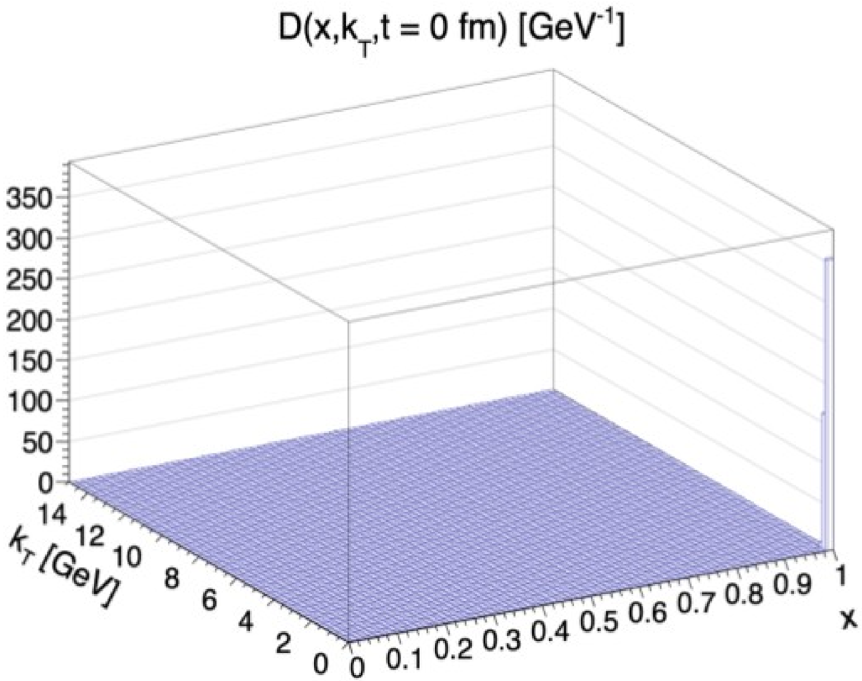
k_t – dependent distributions



The equation forgets quickly about the initial condition. The distribution is not thermal. The maximum moves to lower scales

Non aussianity

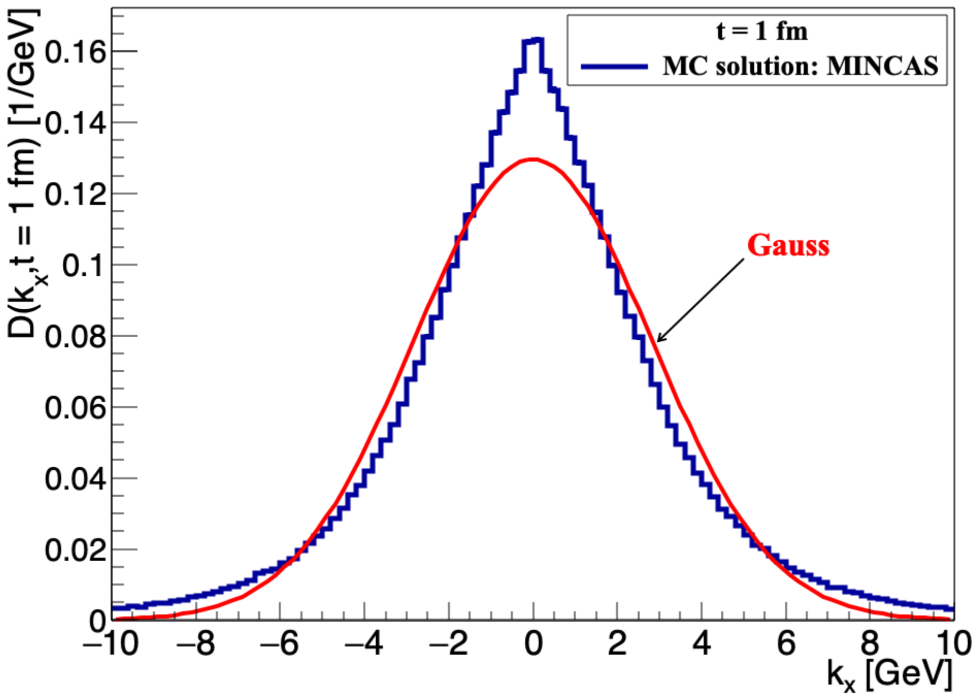


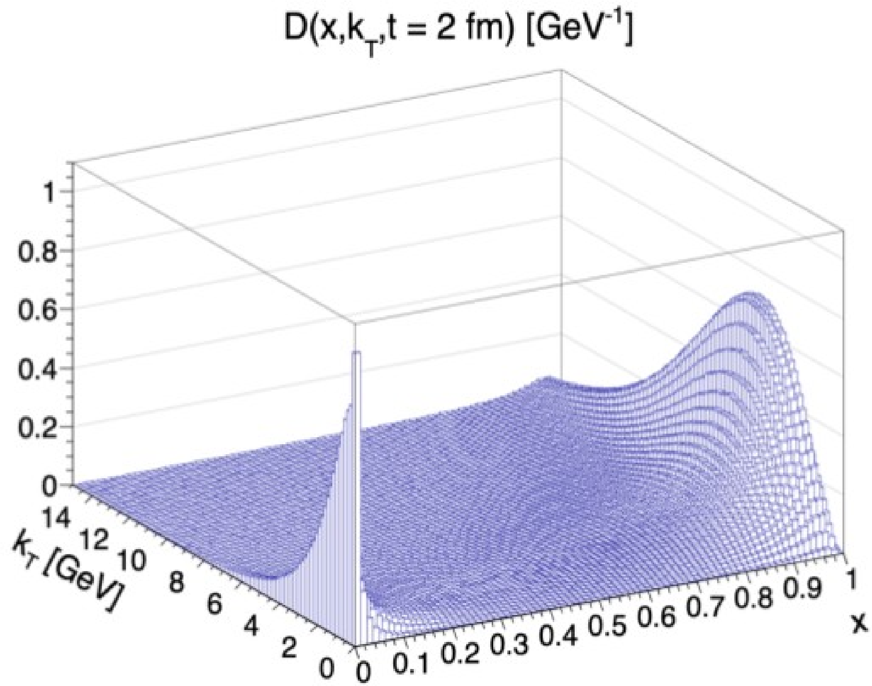


Sum of many gaussians with different width.

This is a result of the exact treatment of the gluon transverse-momentum broadening due to an arbitrary number of the collisions with the medium together with its shrinking due an arbitrary number of the emission branching

Non gaussianity



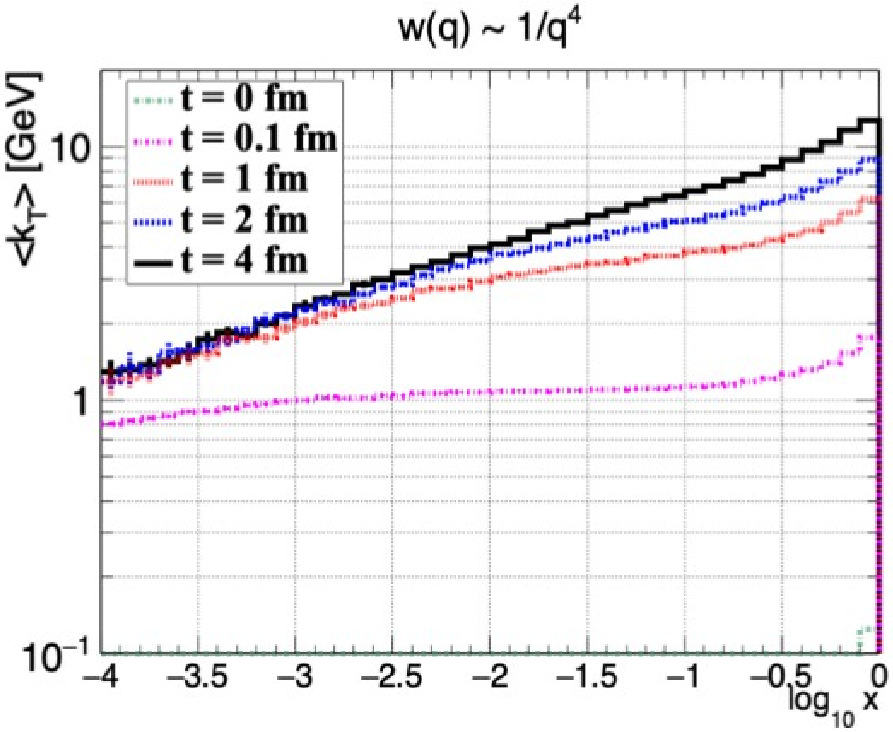


Sum of many gaussians with different width.

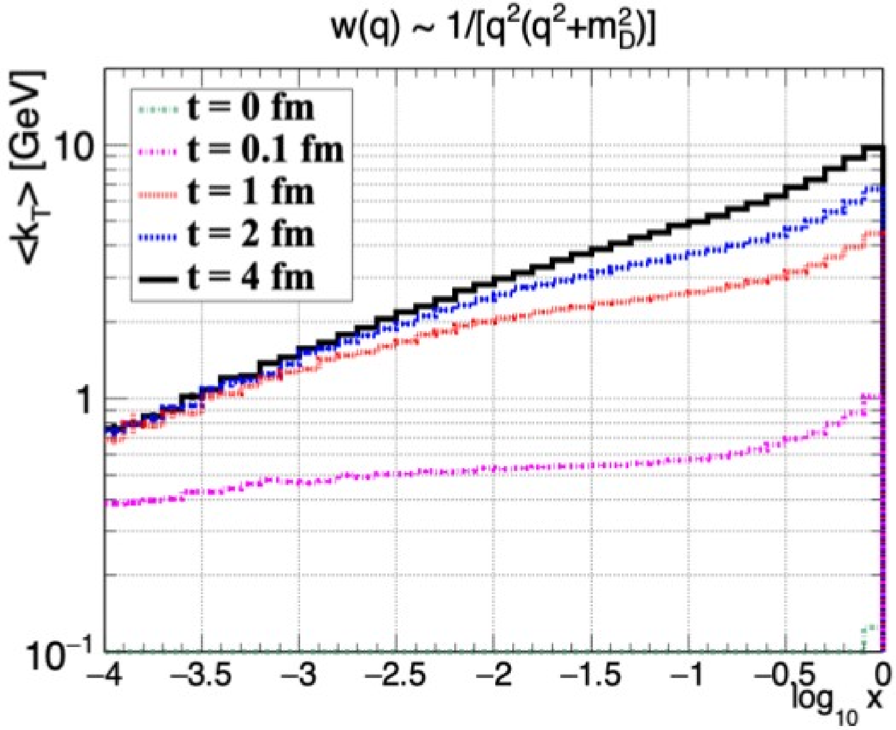
This is a result of the exact treatment of the gluon transverse-momentum broadening due to an arbitrary number of the collisions with the medium together with its shrinking due an arbitrary number of the emission branching.

Quenching line

Non-termalized medium

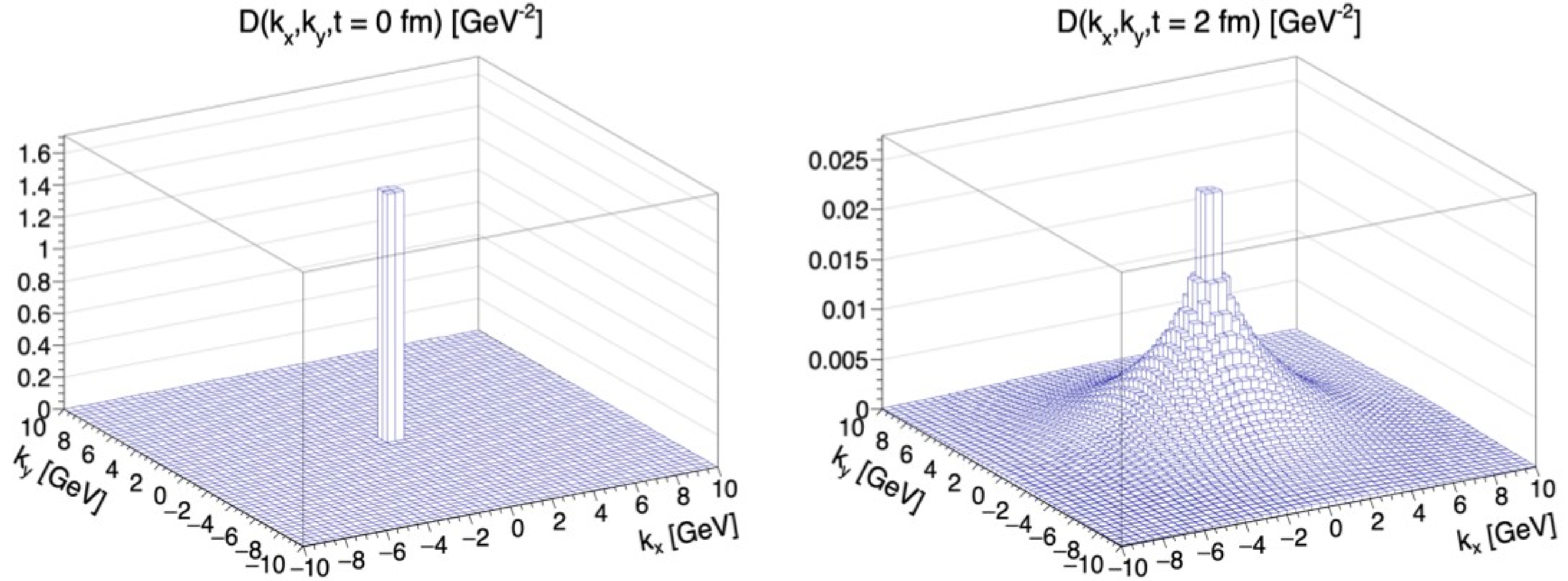


Termalized medium



Thermalized medium suppresses jets stronger
Universal behavior at larger times
The jet gets delocalized in transverse in transverse plane and lower and lower "x"

Broadening of jet



Summary and plans for the future

We obtained new formulation of the equation for jet quenching

The first complete - numerical- solution has been obtained - MINCASC program

The solution has shown that the equation has features not studied in the literature so far

More detailed analysis of the solution of the equation

Relax approximation of small momentum transfer in the branching term

Calculation of some final state observables combined with Initial state effects

Compare to solution of more general kinetic equation

Generalize to momentum dependent quenching parameter

Use another function for the medium field

Back up

Rearrangement of the equation for energy

1811.06390 by K. Kutak, W. Płaczek, R. Straka

$$\frac{\partial}{\partial\tau}D(x,\tau) + D(x,\tau)\int_0^1 dz\,\frac{z}{\sqrt{x}}\,\mathcal{K}(z) = \int_0^1 dy\int_0^1 dz\,\delta(x-yz)\,z\sqrt{\frac{z}{x}}\,\mathcal{K}(z)D(y,\tau)$$

$$D(x,\tau) \equiv \bar{D}(x,\tau)\,e^{-\Phi(x)\tau} \qquad\qquad \Phi(x) \equiv \frac{1}{\sqrt{x}}\int_0^{1-\epsilon} dz\,z\mathcal{K}(z)$$

$$e^{-\Phi(x)\tau}\,\frac{\partial}{\partial\tau}\bar{D}(x,\tau) - \bar{D}(x,\tau)\,\Phi(x)\,e^{-\Phi(x)\tau} + \bar{D}(x,\tau)\,\Phi(x)\,e^{-\Phi(x)\tau}$$
$$= \int_0^1 dy\int_0^{1-\epsilon} dz\,\delta(x-yz)\,z\sqrt{\frac{z}{x}}\,\mathcal{K}(z)D(y,\tau)$$

$$\frac{\partial}{\partial\tau}\bar{D}(x,\tau) = e^{\Phi(x)\tau}\int_0^1 dy\int_0^{1-\epsilon} dz\,\delta(x-yz)\,z\mathcal{K}(z)\sqrt{\frac{z}{x}}\,D(y,\tau)$$

$$\bar{D}(x,\tau) = \bar{D}(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' e^{\Phi(x)\tau'}\int_0^1 dy\int_0^{1-\epsilon} dz\,\delta(x-yz)\,z\mathcal{K}(z)\sqrt{\frac{z}{x}}\,D(y,\tau')$$

$$D(x,\tau) = e^{-\Phi(x)(\tau-\tau_0)}D(x,\tau_0)$$
$$+\int_{\tau_0}^{\tau} d\tau'\int_0^{1-\epsilon} dz\int_0^1 dy\,\delta(x-zy)\sqrt{\frac{z}{x}}\,z\mathcal{K}(z)e^{-\Phi(x)(\tau-\tau')}D(y,\tau')$$

Resummation

Procedure almost the same as for energy distribution

$$\Phi(x) = \frac{1}{\sqrt{x}} \int_0^{1-\varepsilon} dz\, z\, \mathcal{K}(z) \qquad W = t^* \int_{|\mathbf{q}|>q_{\min}} d^2\mathbf{q}\, \frac{w(\mathbf{q})}{(2\pi)^2}$$

$$\Psi(x) = \Phi(x) + W$$

$$\mathcal{G}(z,\mathbf{q}) = \sqrt{\frac{z}{x}}\, z\mathcal{K}(z)\, \theta(1-\varepsilon-z)\, \delta(\mathbf{q}) + t^* \frac{w(\mathbf{q})}{(2\pi)^2}\, \theta(|\mathbf{q}|-q_{\min})\delta(1-z)$$

$$\begin{aligned} D(x,\mathbf{k},\tau) = {} & e^{-\Psi(x)(\tau-\tau_0)} D(x,\mathbf{k},\tau_0) \\ & + \int_{\tau_0}^{\tau} d\tau' \int_0^1 dz \int_0^1 dy \int d^2\mathbf{k}' \int d^2\mathbf{q}\; \mathcal{G}(z,\mathbf{q}) \\ & \times \delta(x - zy)\, \delta(\mathbf{k}-\mathbf{q}-z\mathbf{k}')\, e^{-\Psi(x)(\tau-\tau')} D(y,\mathbf{k}',\tau') \end{aligned}$$

Generating functional

$$D(x,\boldsymbol{k},t) = k^+ \frac{\mathrm{d}N}{\mathrm{d}k^+ \mathrm{d}^2\boldsymbol{k}} \equiv k^+ \left\langle \sum_{n=1}^{\infty} \sum_{j=1}^{n} \delta^{(3)}(\vec{k}_j - \vec{k}) \right\rangle$$

Inclusive gluon distribution as produced by hard jet

$$= k^+ \frac{\delta \mathcal{Z}_{p_0}[t,t_0|u]}{\delta u(\vec{k})}\bigg|_{u=1}$$

$$\mathcal{Z}_{p_0}[t,t_0|u] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \left(\prod_{i=1}^{n} \mathrm{d}\Omega_i \right) P_n(\vec{k}_1,\cdots,\vec{k}_n;t,t_0)\, u(\vec{k}_1)\cdots u(\vec{k}_n)$$

Blaizot, Dominguez, Iancu, Mehtar-Tani'13

t is lightcone time x+

P_n is a probability density to find exactly n gluons

u(k) generic function

Calculable in QCD