Exploring the QCD phase diagram

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375

A. Bzdak, VK, V. Skokov: arXiv:1612.05128

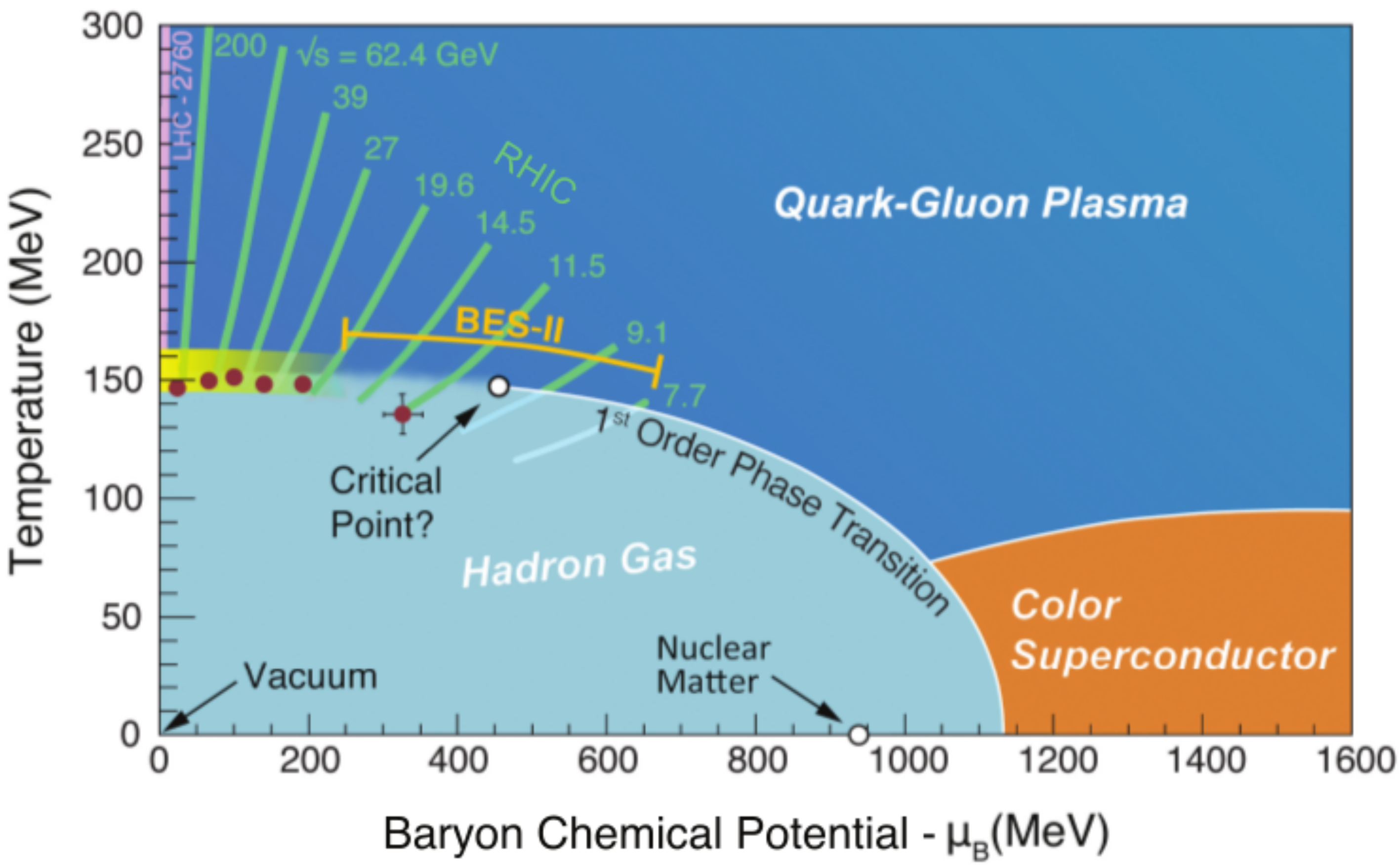
A. Bzdak, D. Oliinychenko, J. Steinheimer, VK: arXiv:1804.04463

A. Bzdak, VK: arXiv:1810.01913, arXiv:1811.04456

A. Bialas, A. Bzdak and VK: arXiv:1608.07041, arXiv:1711.09440

BEST COLLABORATION

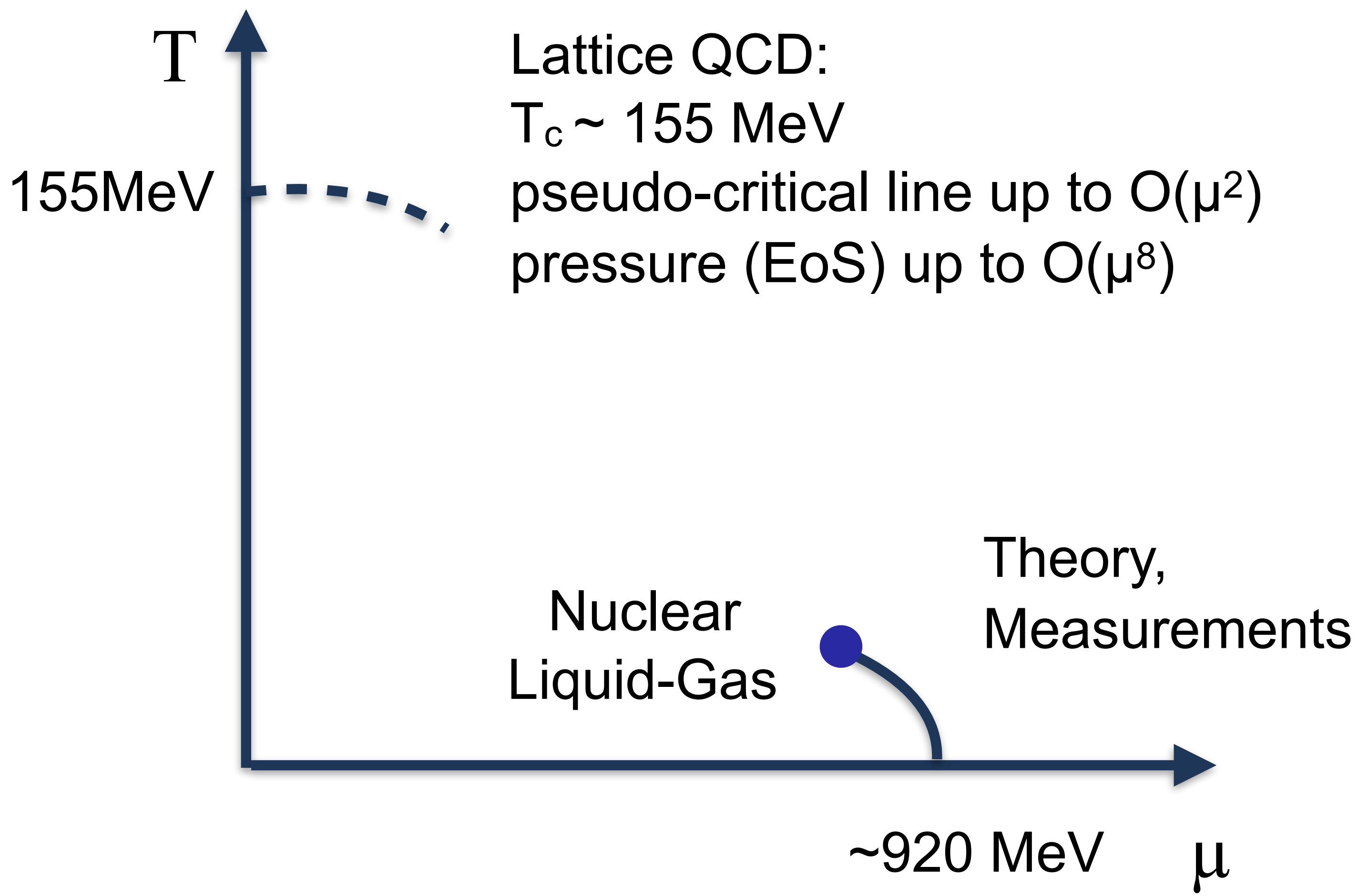
The phase diagram



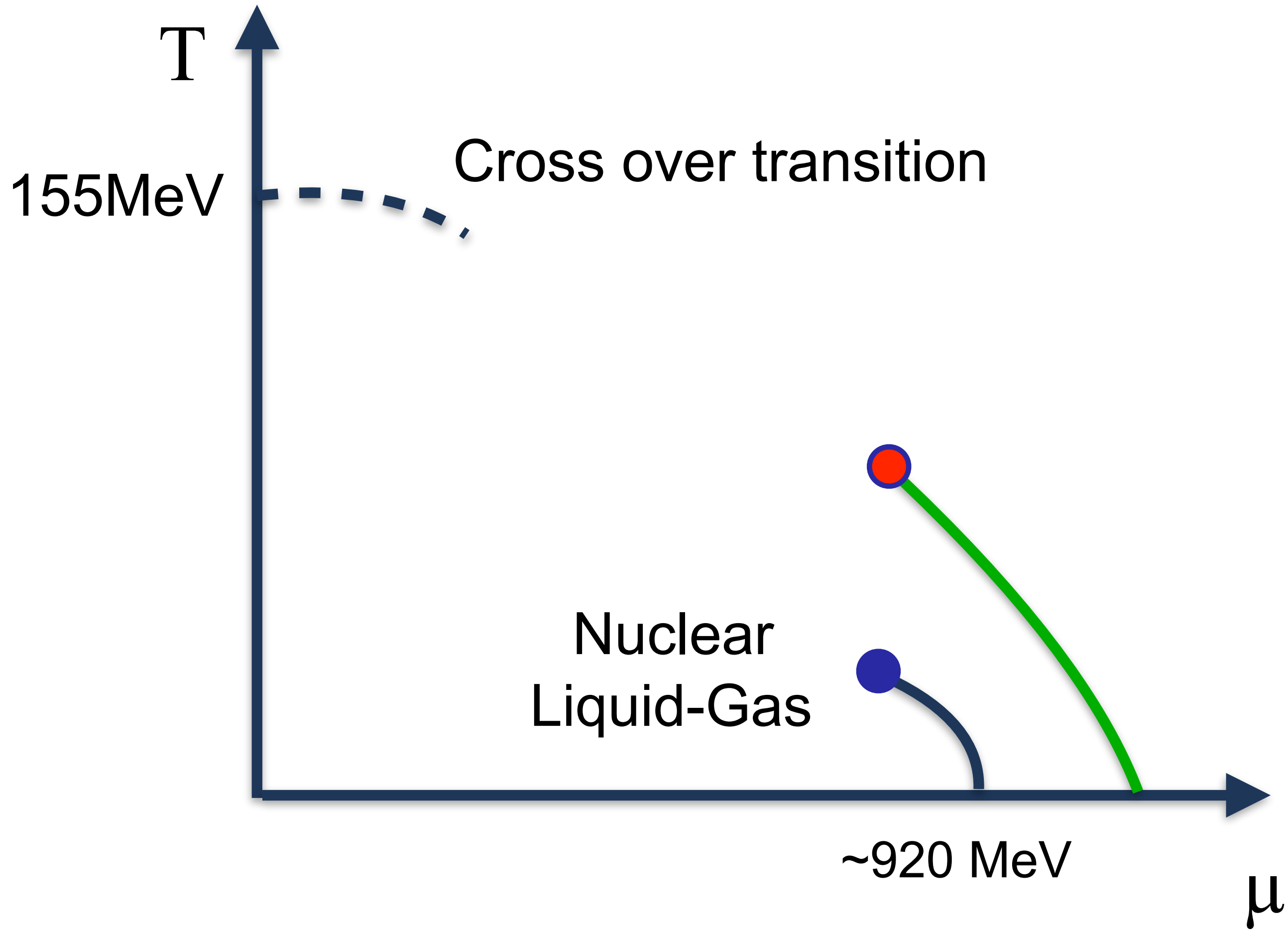
Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

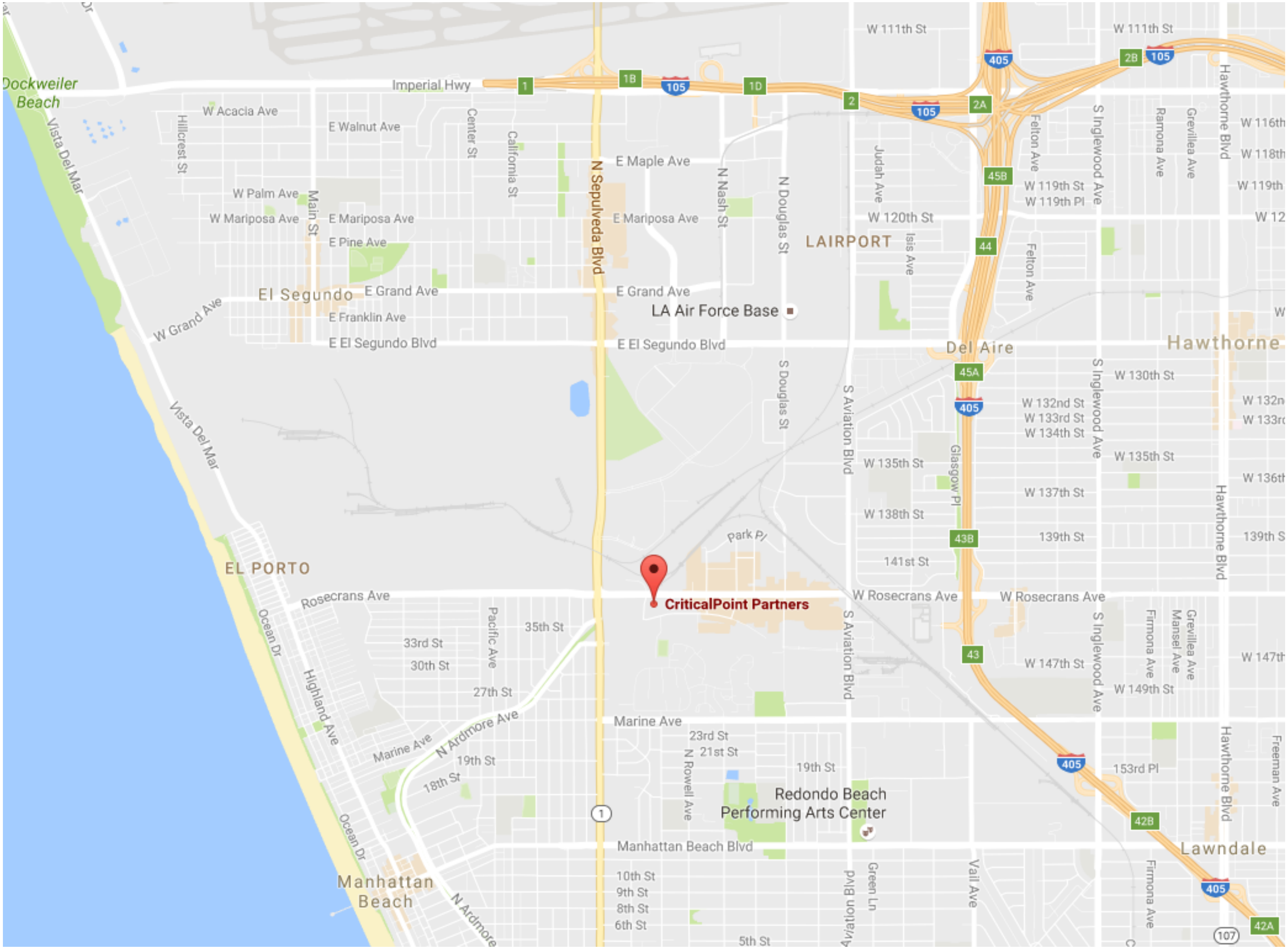
What we know about the Phase Diagram



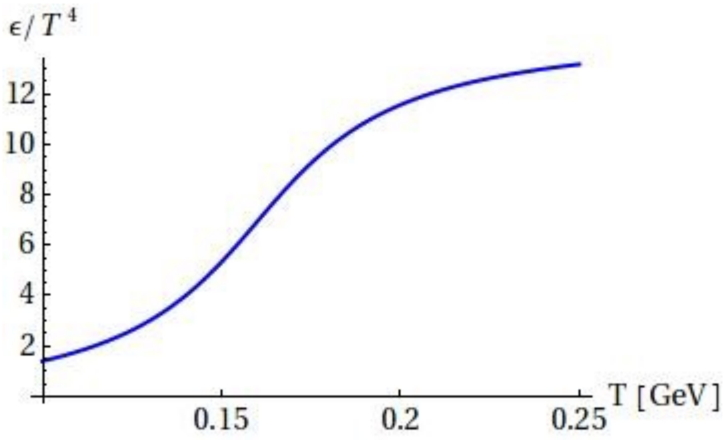
What we "hope" for



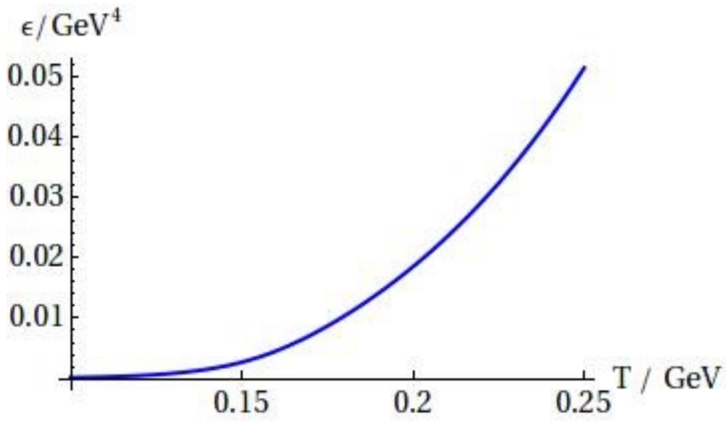
Is there a critical point?



Cumulants and phase structure



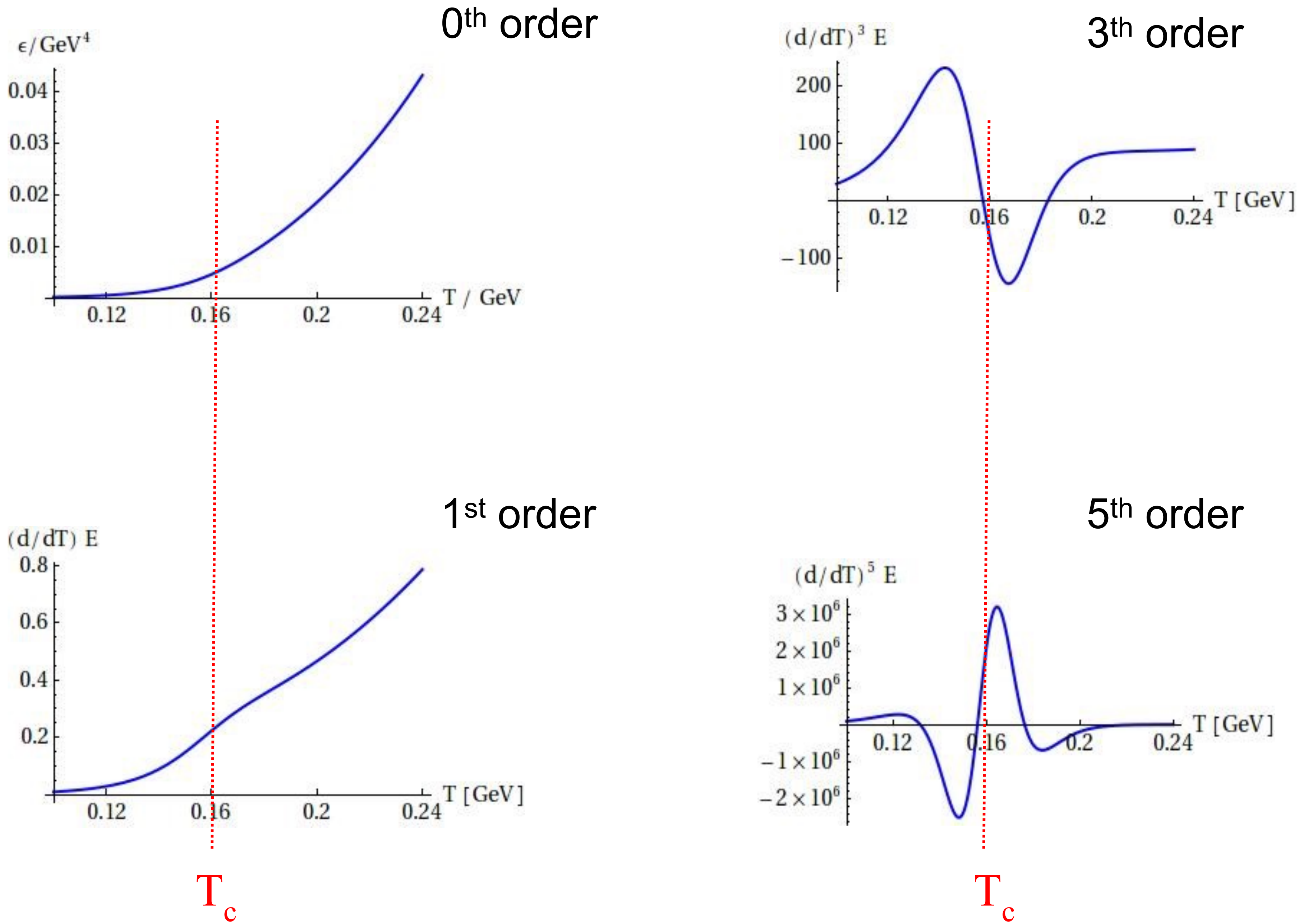
What we always see....



What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T}\right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T}\right) \langle E \rangle$$

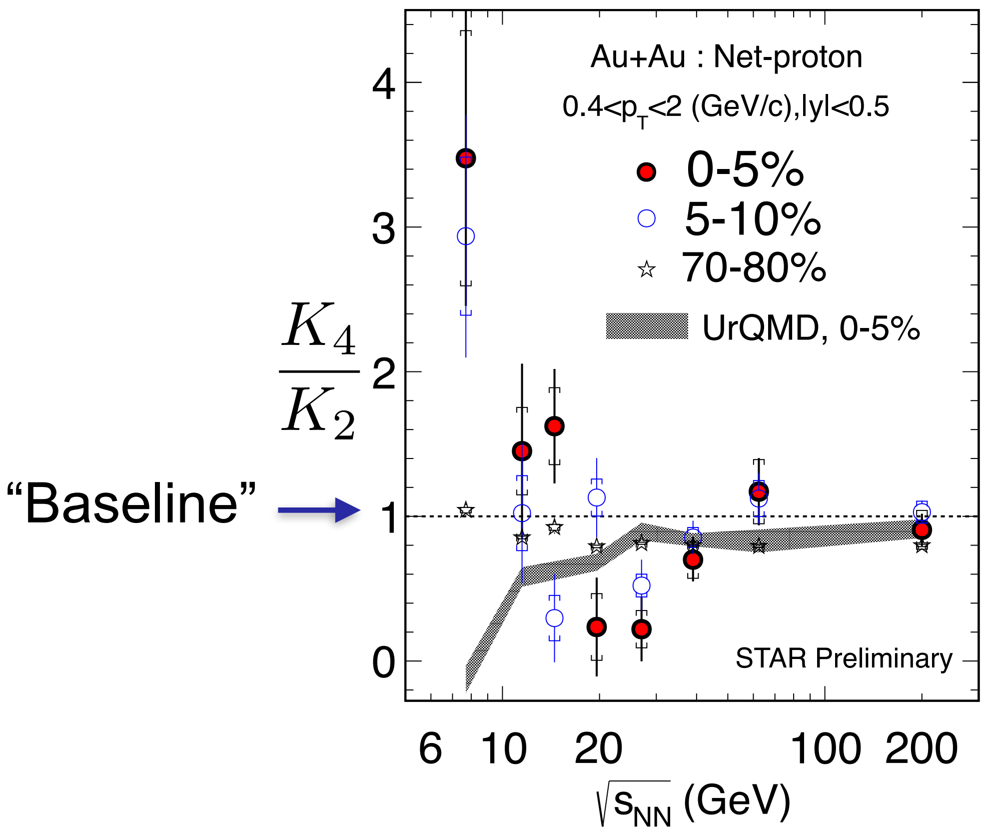
$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T}\right)^{n-1} \langle E \rangle$$

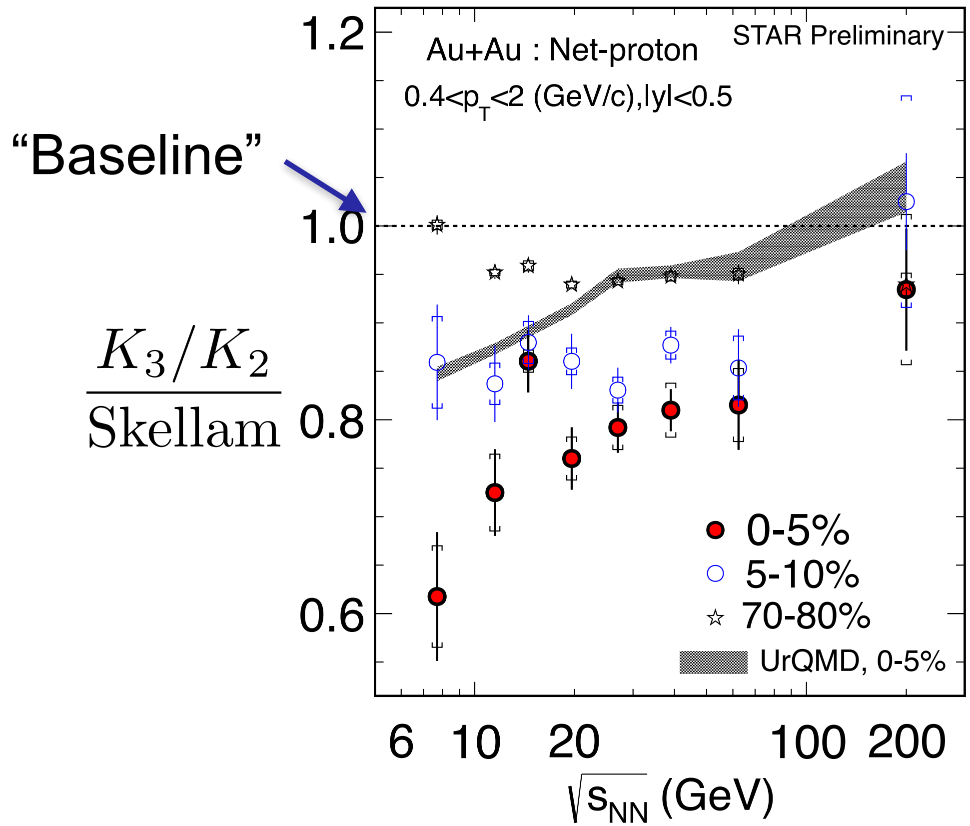
Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75





K_4/K_2 follows expectation for CP , K_3/K_2 no so much…..
URQMD totally fails to get trend for K_4/K_2 !

Further insights: Correlations

Cumulants $$K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

C_2: Correlation Function

$$K_3 = \langle (\delta N)^3 \rangle$$

$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}$$

From Cumulants to Correlations (no anti-protons)

Defining integrated correlations function a.k.a factorial cumulants

$$C_n = \int dp_1 \ldots dp_n C_n(p_1, \ldots, p_n)$$

Simple Algebra leads to relation between correlations C_n and K_n

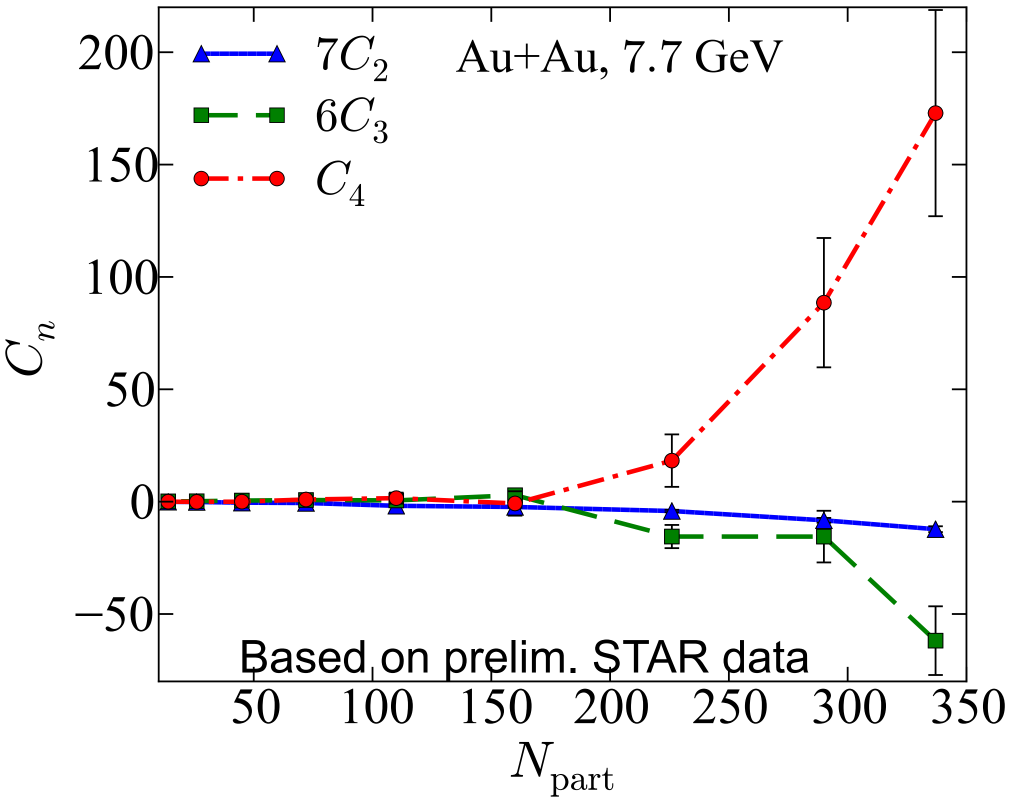
$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

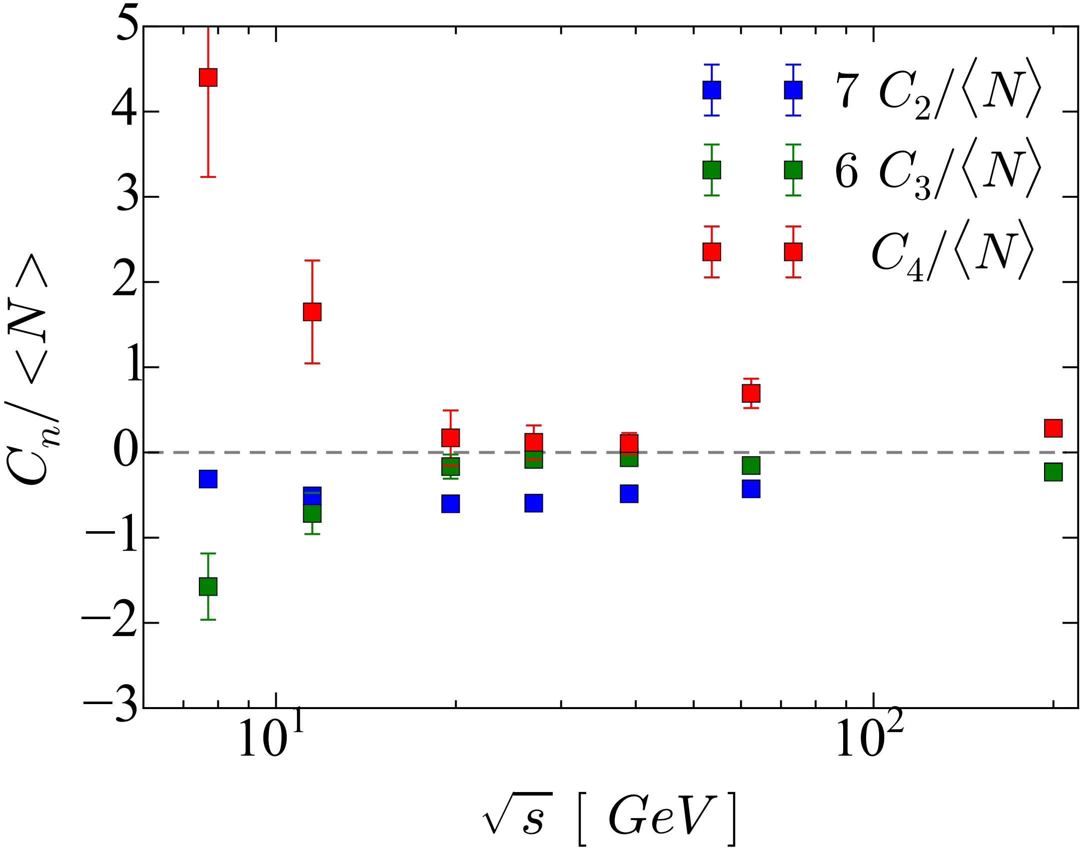
or vice versa

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))





Significant four particle correlations!

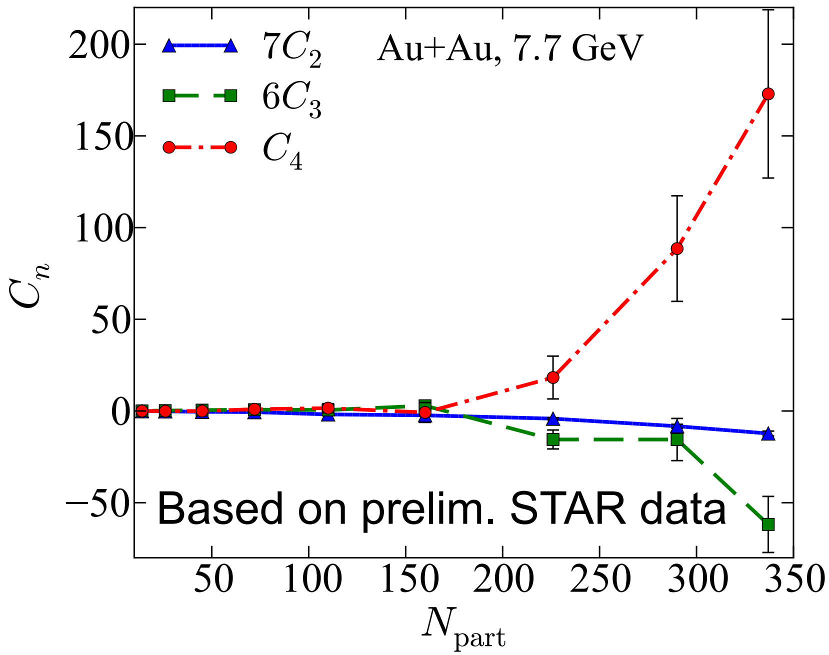
Four particle correlation dominate K_4 for central collisions at 7.7 GeV

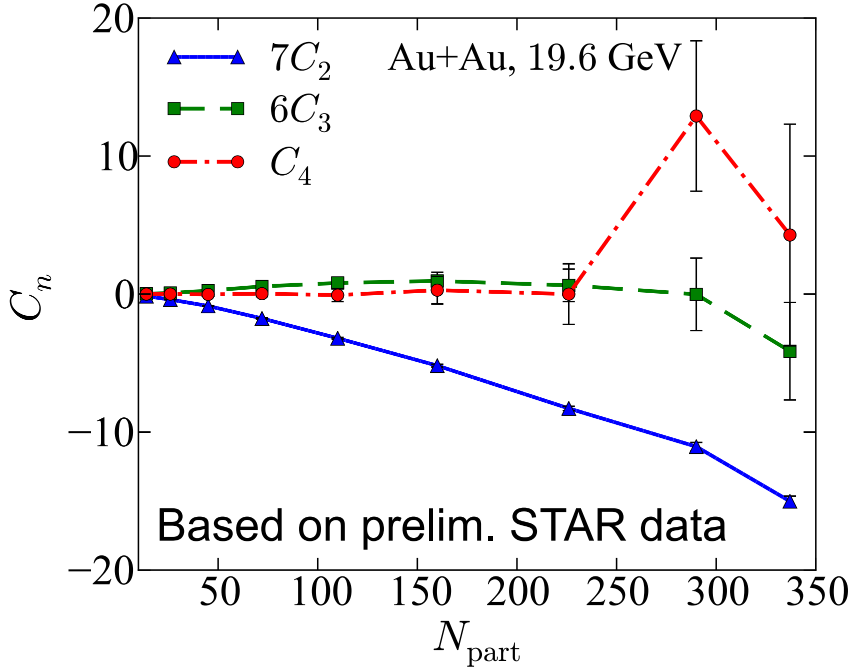
$$K_2 = \langle N\rangle + C_2$$

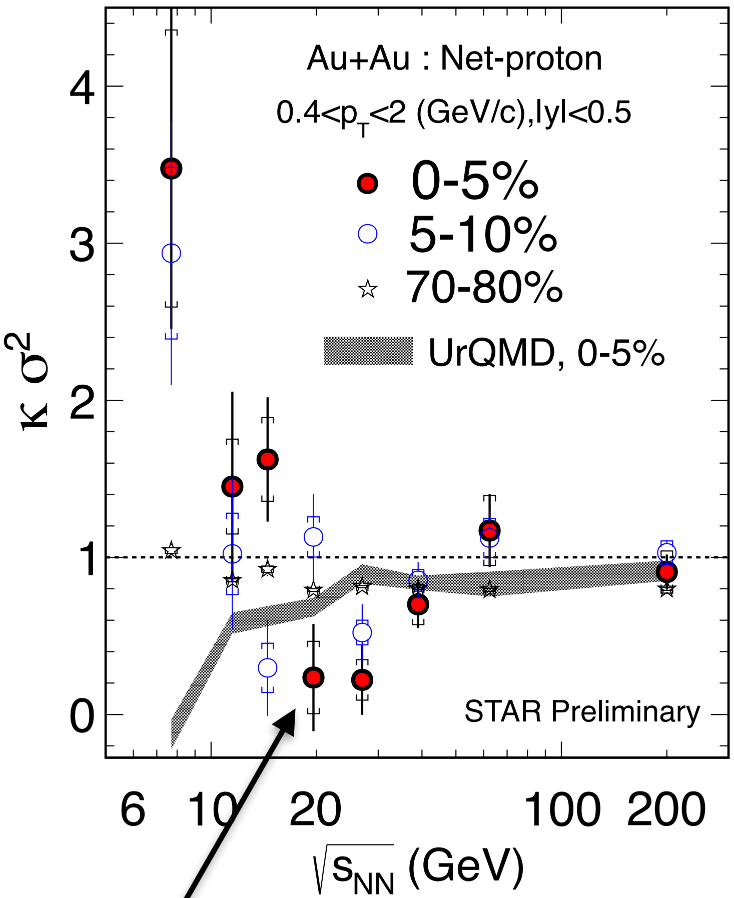
$$K_3 = \langle N\rangle + 3C_2 + C_3$$

$$K_4 = \langle N\rangle + 7C_2 + 6C_3 + C_4$$

Correlations







Dip at 19.6 GeV from
NEGATIVE TWO particle correlations !

Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \dots dy_k \rho_1(y_1) \dots \rho_1(y_k) c_k(y_1, \dots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \dots, y_k) \sim \delta(y_1 - y_2) \dots \delta(y_{k-1} - y_k)$$

$$C_k(\Delta Y) \sim \Delta Y \to K_k \sim \Delta Y$$

Long range correlations:

$$c_k(y_1, \dots, y_k) = const.$$

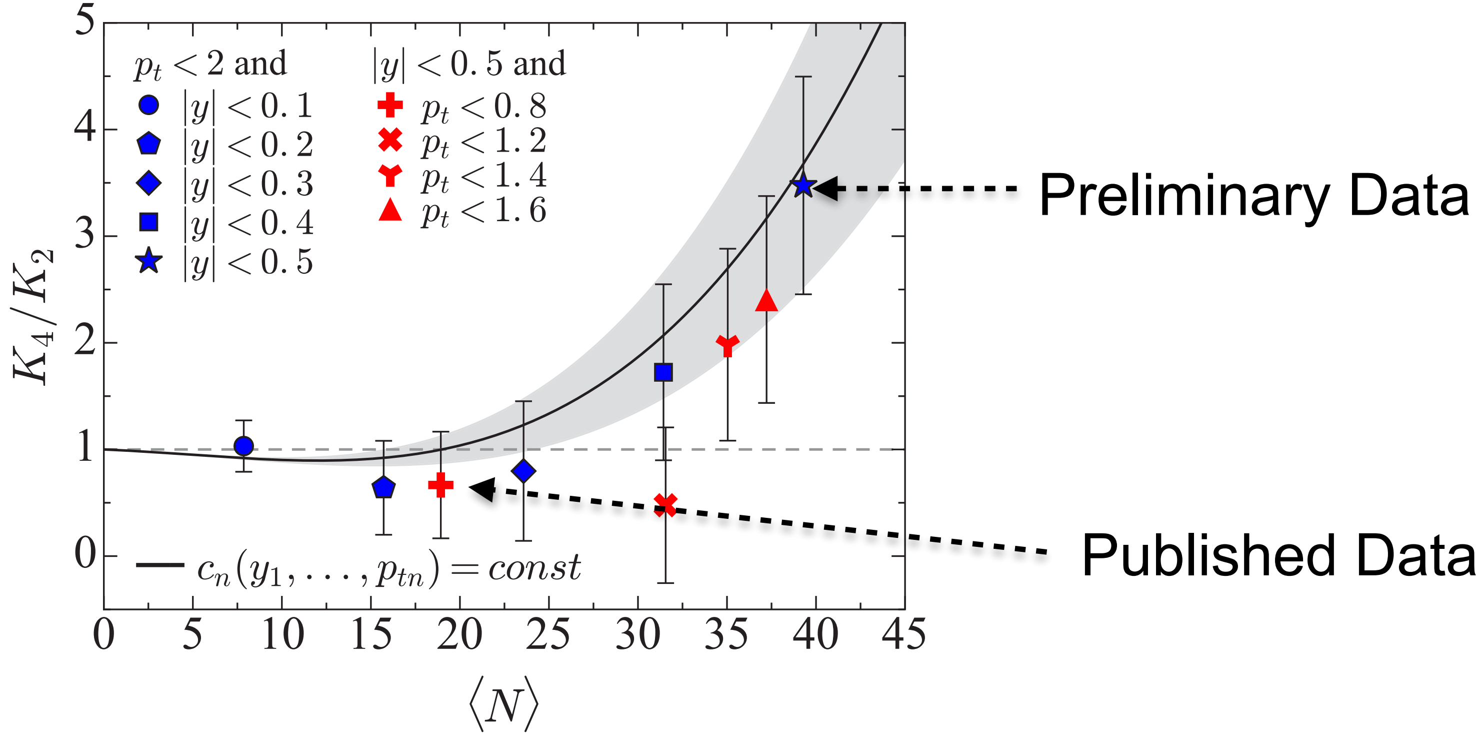
$$C_k(\Delta Y) \sim (\Delta Y)^k \sim \langle N \rangle^k$$

$$\Rightarrow K_n = K_n(\langle N \rangle)$$

Long range correlations

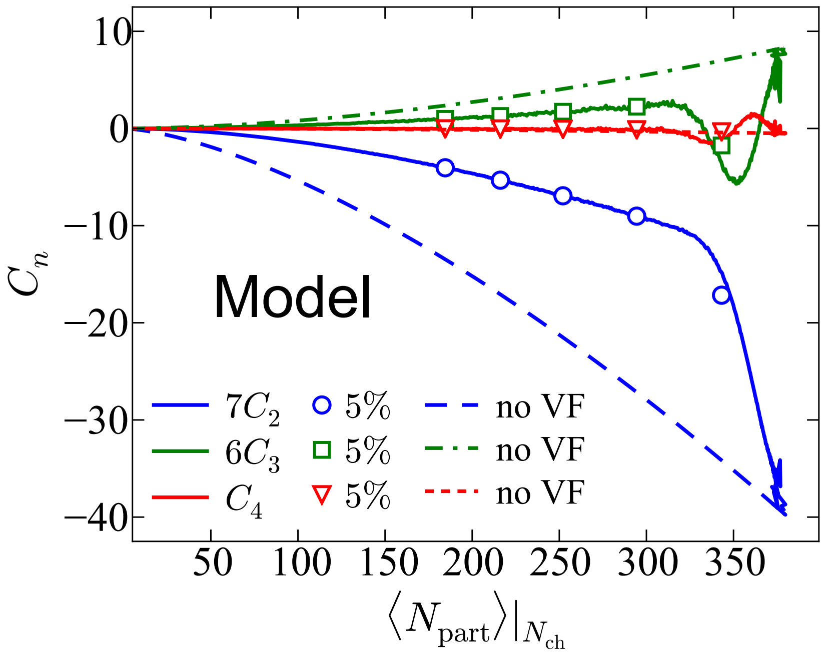
$$C_k = \langle N \rangle^k c_k$$

$$c_k = const. \Rightarrow K_n = K_n(\langle N \rangle)$$



Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation



STAR:
$7C_2$ ~ -20
$6C_3$ ~ -60
C_4 ~ 170

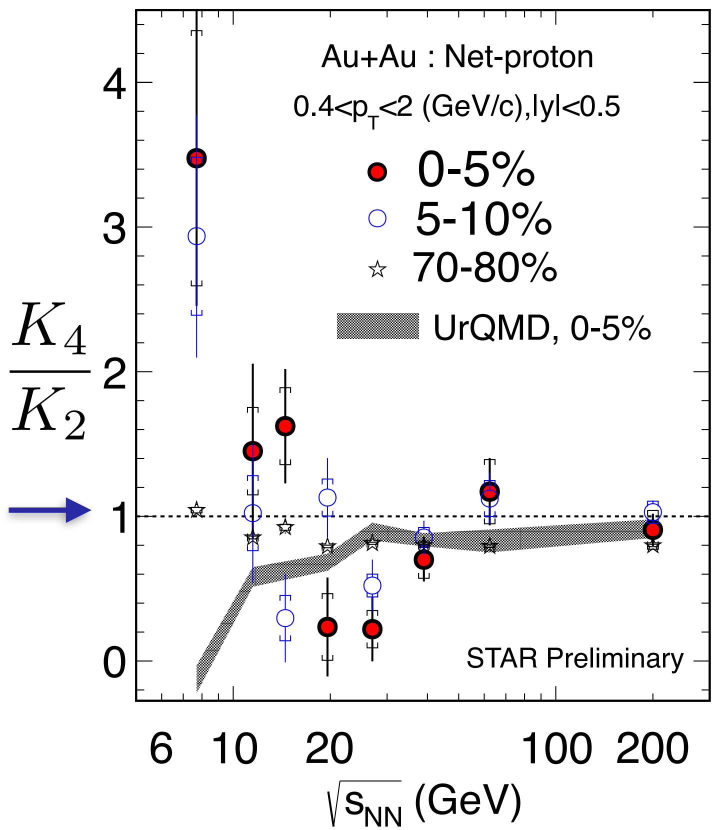
Four particle correlations are orders of magnitudes larger in the data

Also seen in URQMD calculations by He et al. PLB774 (2017) 623

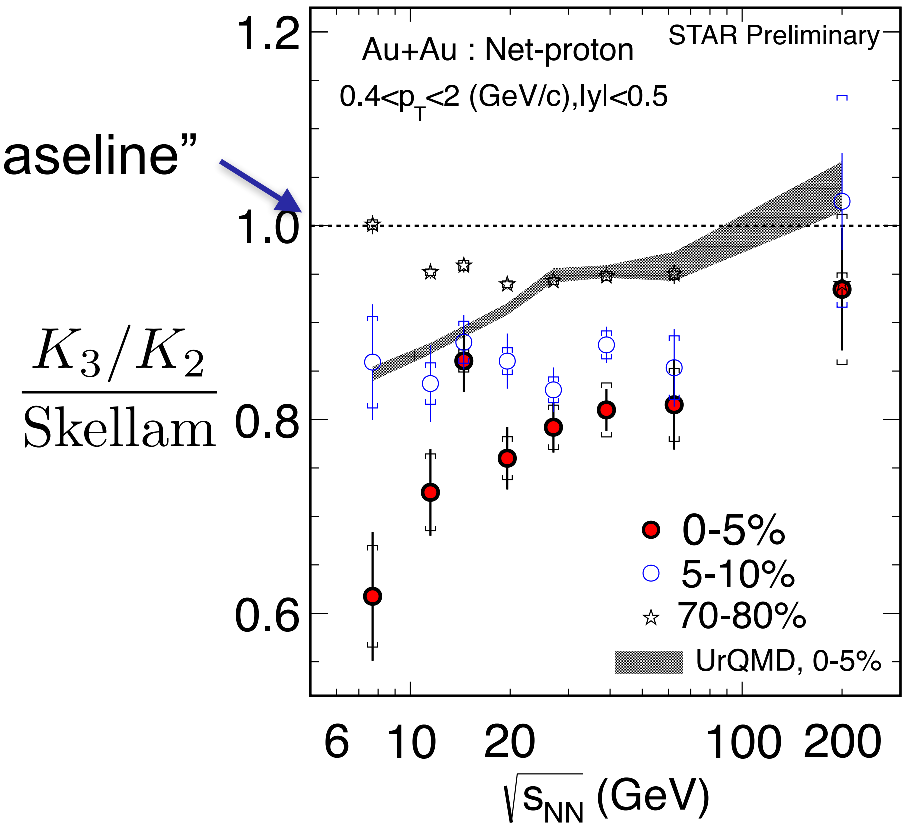
Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75

“Baseline”



“Baseline”



K_4/K_2 above baseline K_3/K_2 below baseline

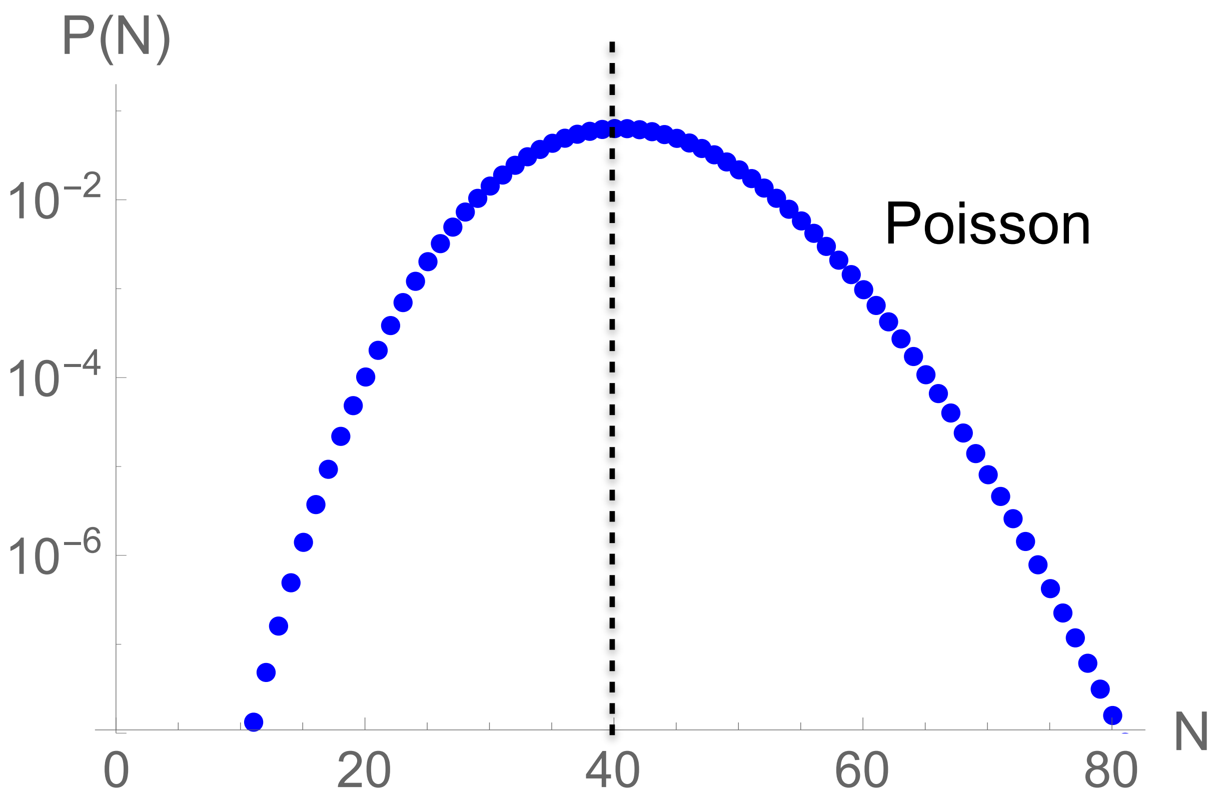
Shape of probability distribution

$K_3 < \langle N \rangle$

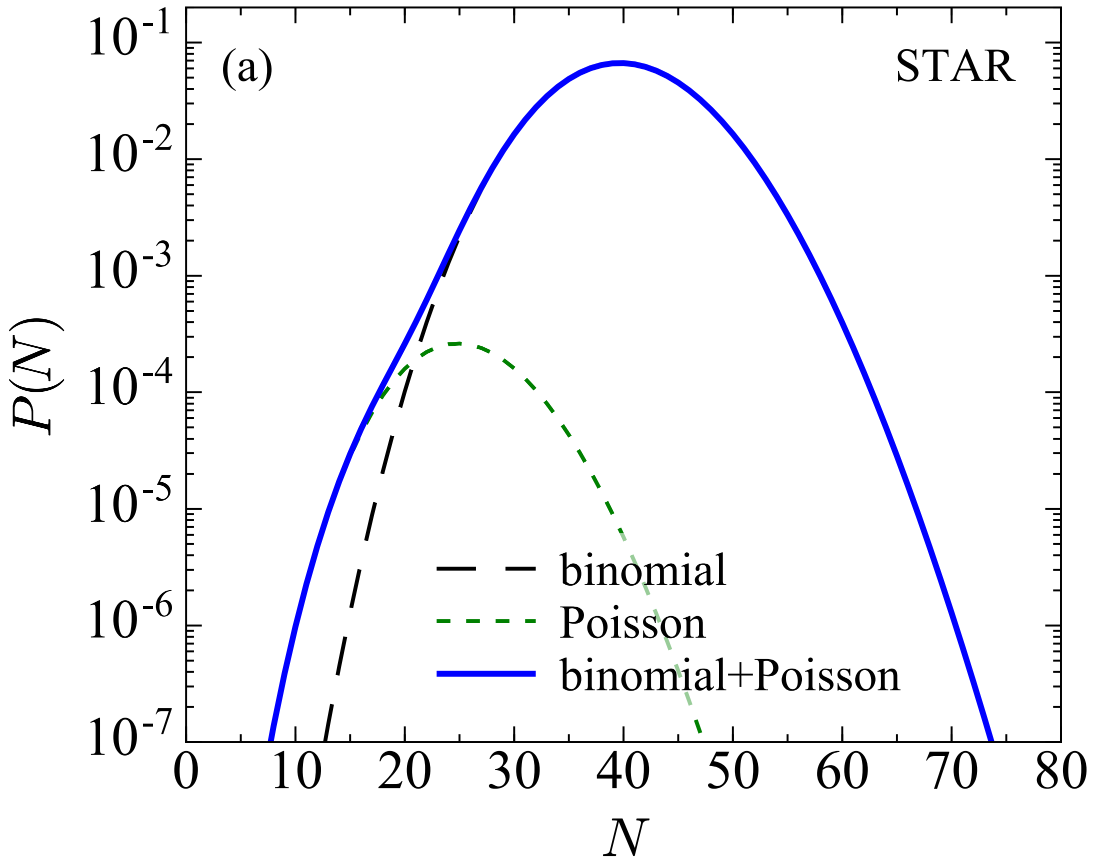
$K_4 > \langle N \rangle$

$$K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$

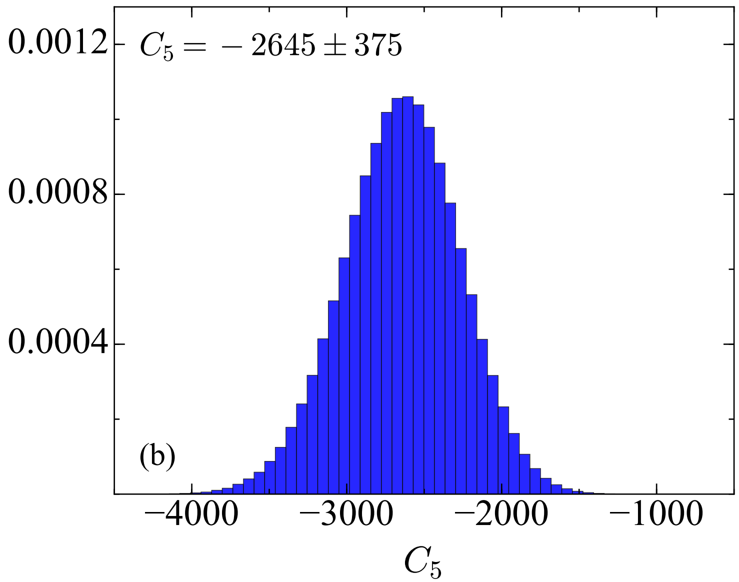


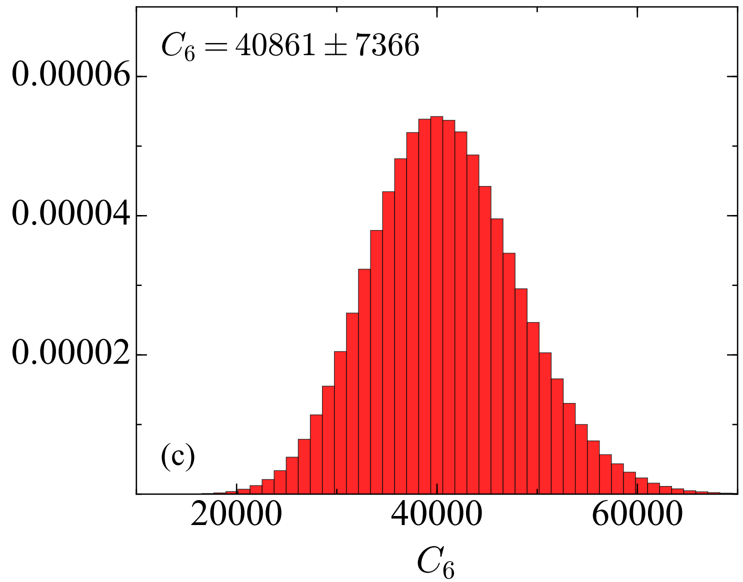
Simple two component model



Weight of small component: ~0.3%

Two component model is Statistics "friendly"





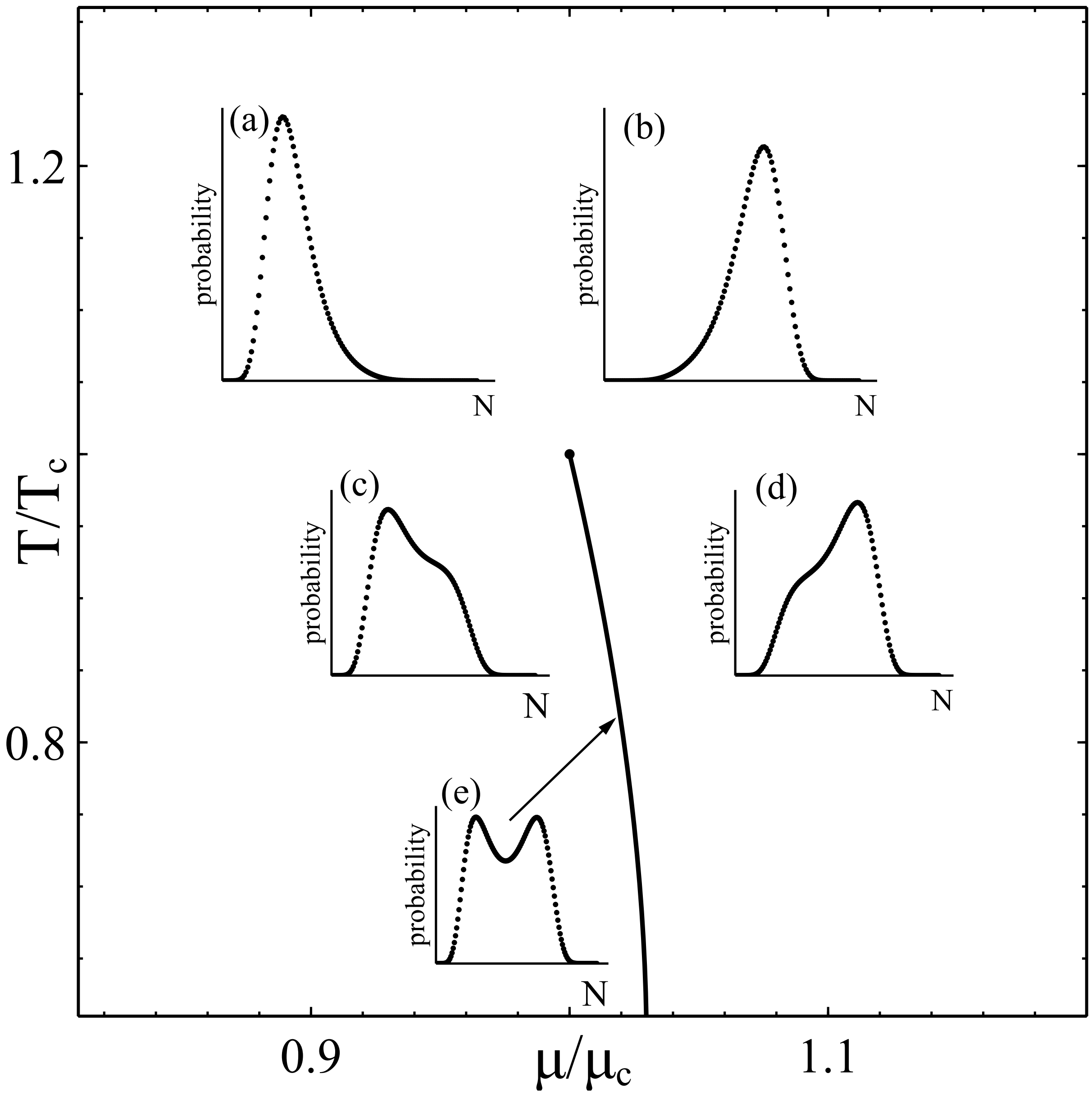
Model prediction:

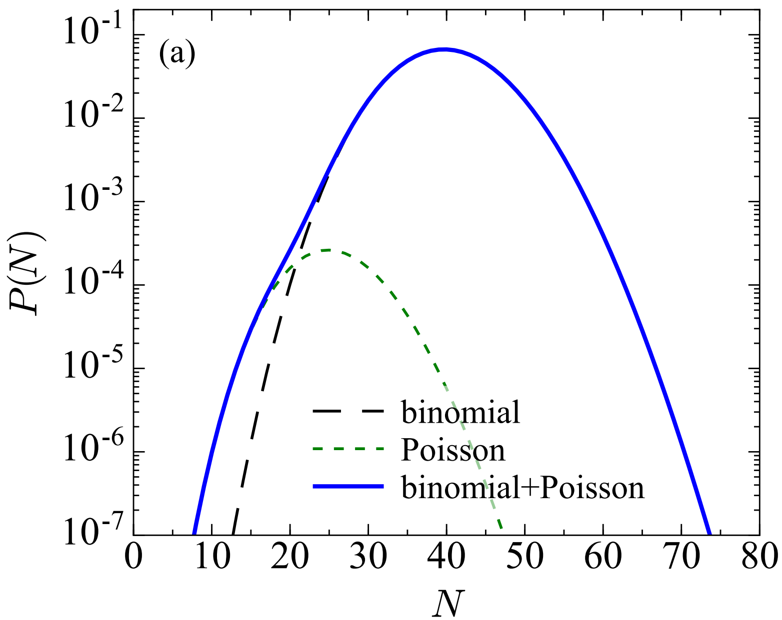
$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
$$C_7 = -615135\,(1 \pm 0.26), \quad C_8 = 8520220\,(1 \pm 0.42)$$

Efficiency corrected

Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

Speculation





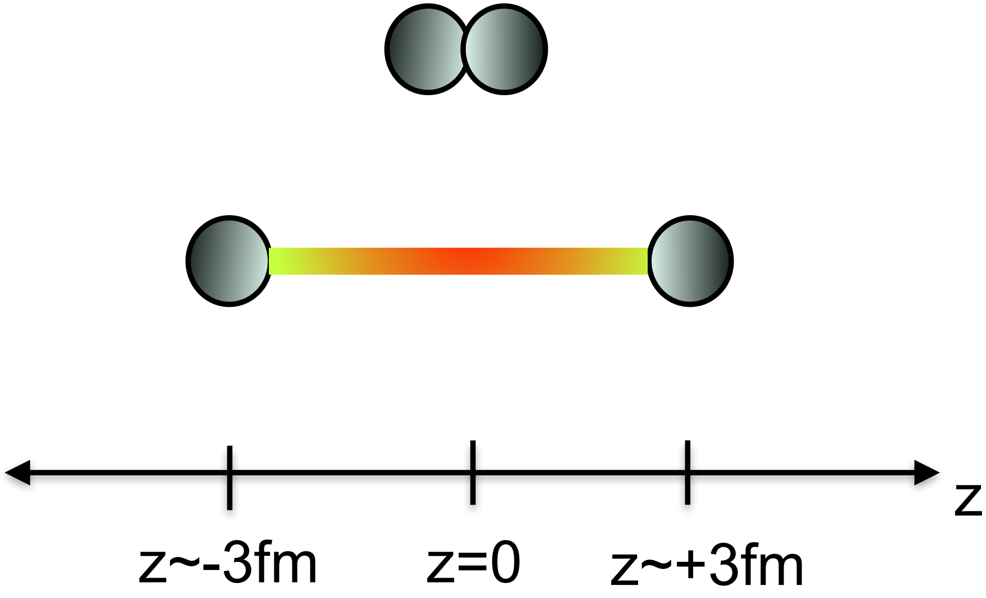
To be checked by STAR: experiment picks up "wrong" events

Baryon Stopping

Question: Where in CONFIGURATION space are the stopped protons

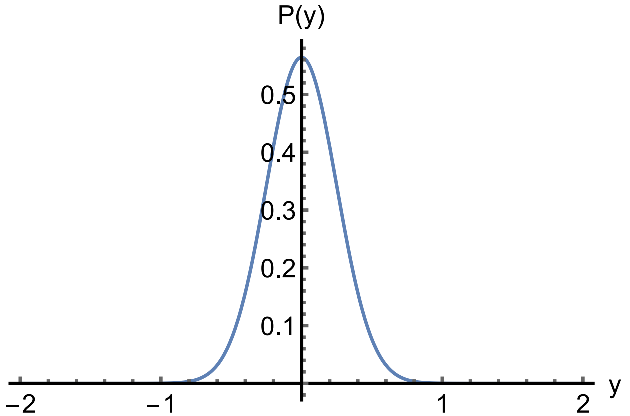
Basic observation:
It takes some distance in space to come to a full stop!

Example:
E_{cm} = 20 GeV
σ = 3 GeV/fm

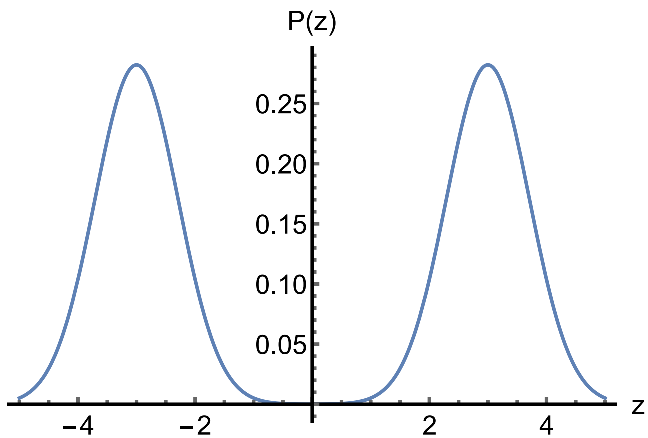


Baryon stopping

Stoppend Baryons

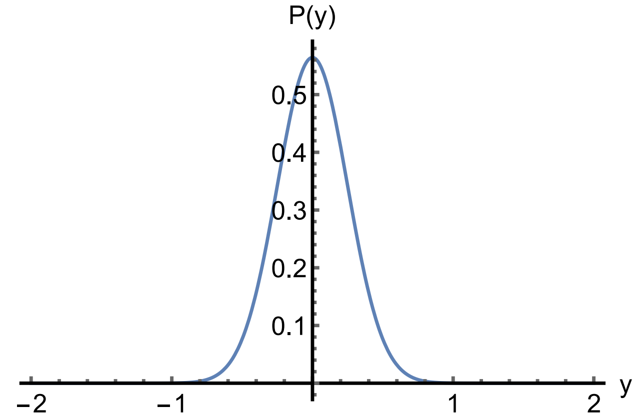


Momentum Space

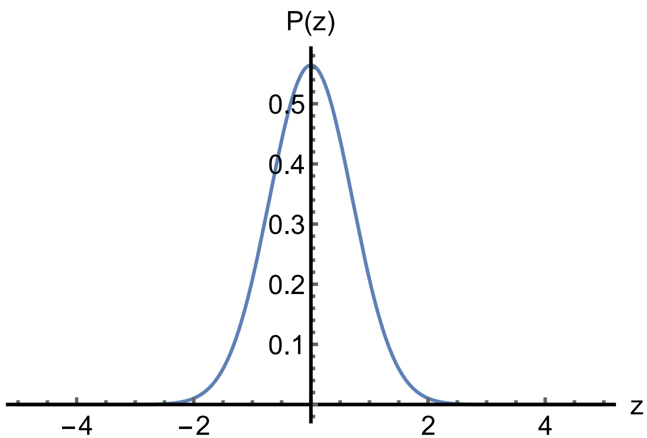


Configuration Space

Produced Baryons



Momentum Space



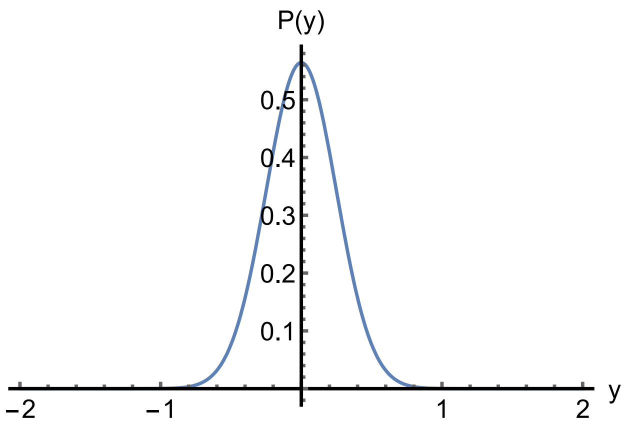
Configuration Space

Baryon stopping

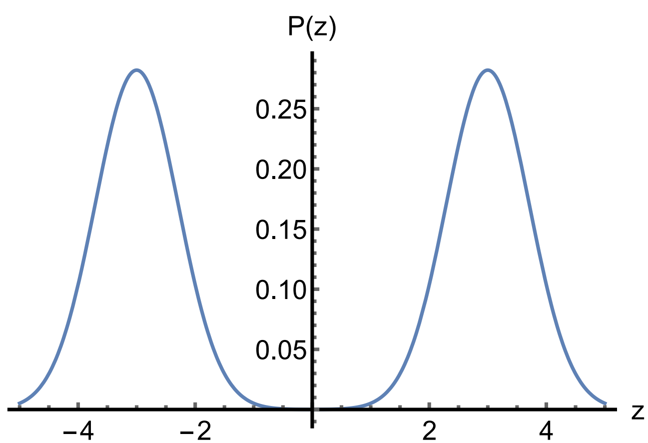
Stoppend Baryons

Produced Baryons

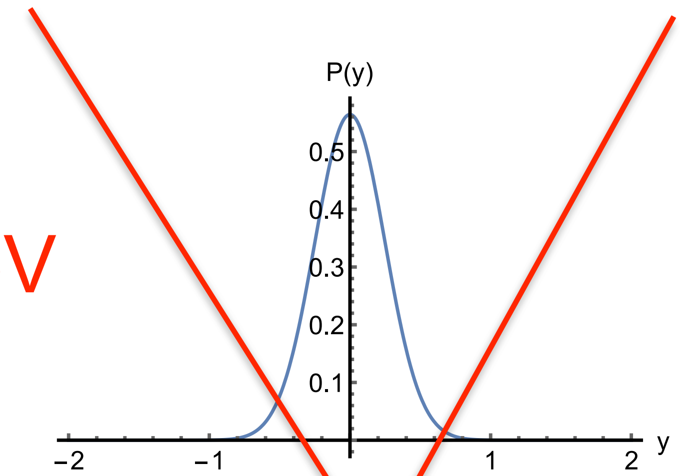
$E_{cm} < 20$ GeV



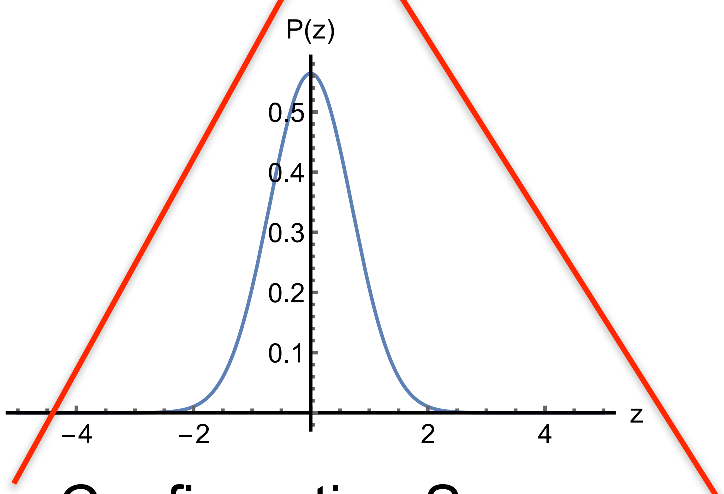
Momentum Space



Configuration Space

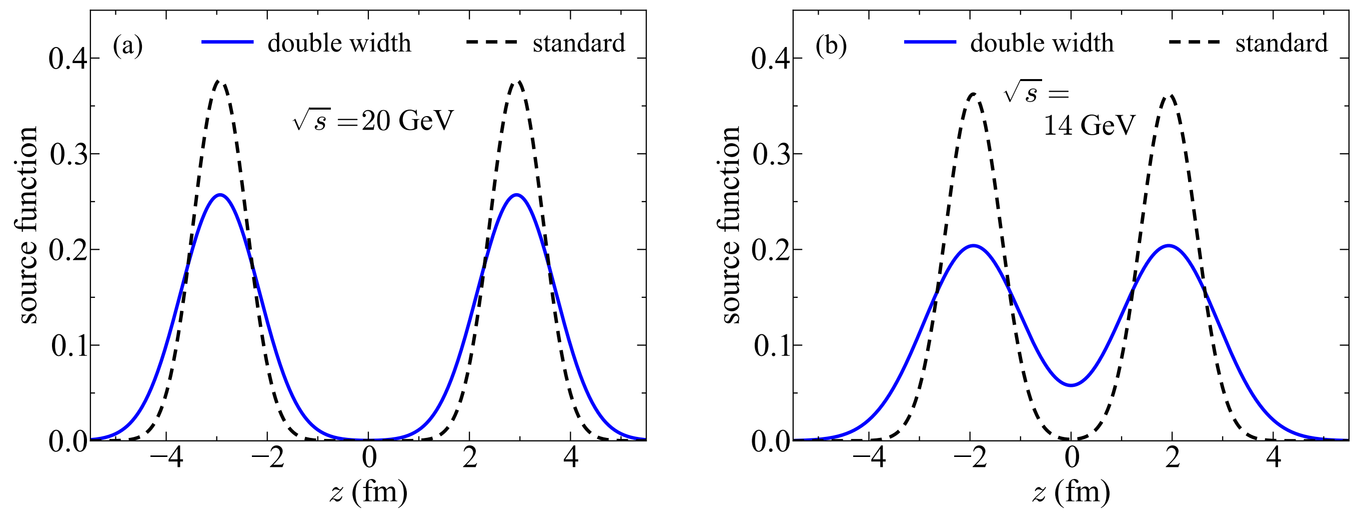


Momentum Space

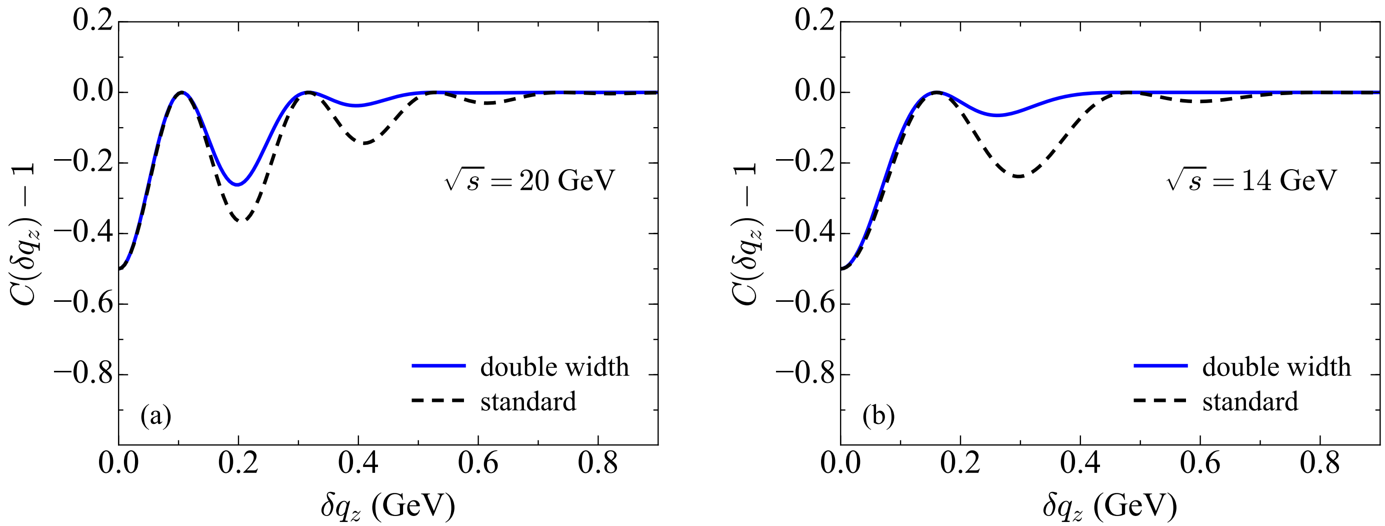


Configuration Space

Baryon Stopping

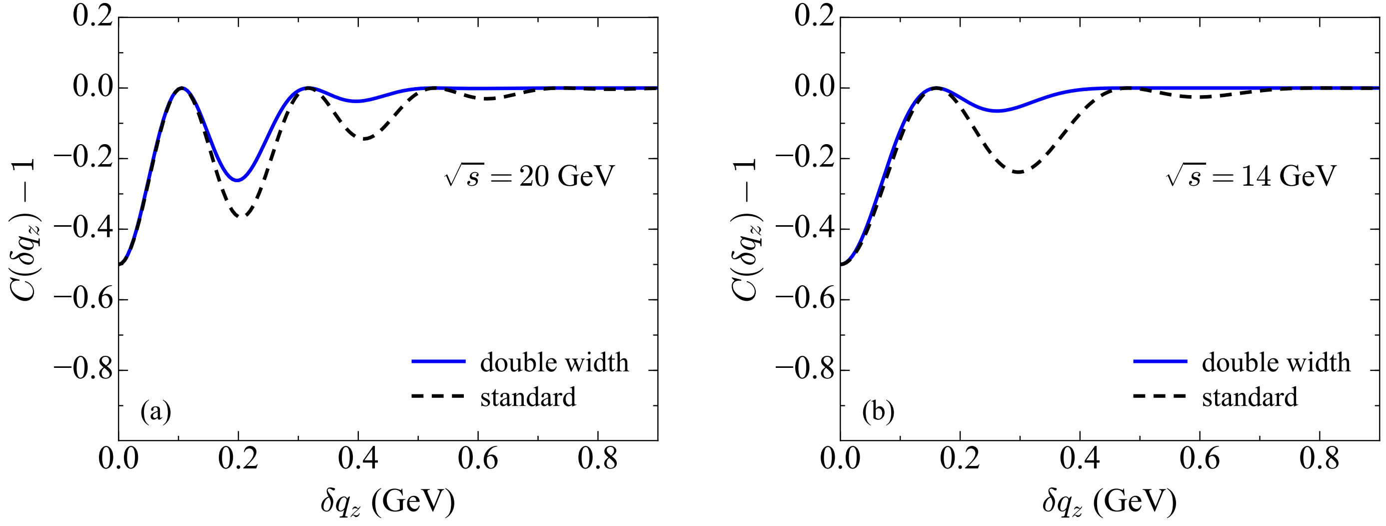


Femtoscopy correlation function

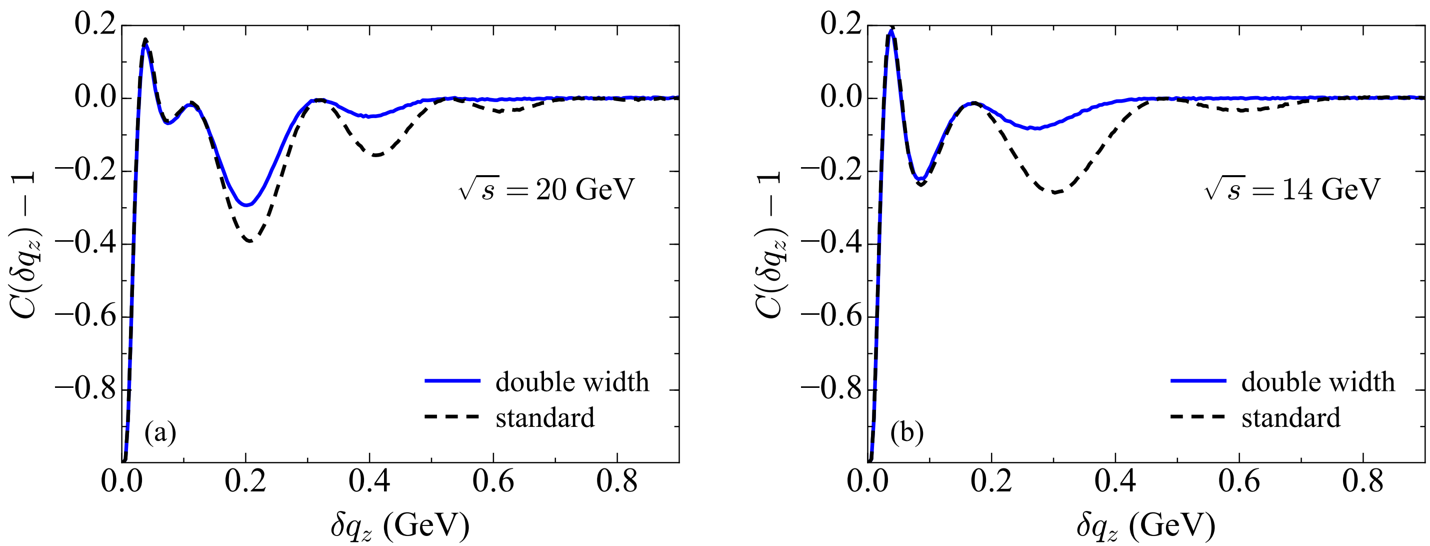


Details: (AB)[2]VK arXiv:1608.07041,1711.09440

Femtoscopy



Including strong interaction



Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
- Fluctuations of N_{part}, stopping, and baryon conservation:
  - May explain 2-particle correlations
  - Fail to reproduce the magnitude of 3- and 4- particle correlations
  - 3 and 4 particle correlations are HUGE!
- “Bi-Modal” distribution works
  - Can be tested RIGHT NOW by STAR.
- Stopped protons do NOT sit a z=0!
  - Can be checked with femtoscopy
  - If correct: back to the drawing board

Thank You

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\overline{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$\begin{aligned}
C_2 =& C_2^{(a)} - \alpha \left\{ \overline{C}_2 - (1-\alpha)\overline{N}^2 \right\} \\
C_3 =& C_3^{(a)} - \alpha \left\{ \overline{C}_3 + (1-\alpha) \left[(1-2\alpha)\overline{N}^3 - 3\overline{N}\,\overline{C}_2 \right] \right\} \\
C_4 =& C_4^{(a)} - \alpha \left\{ \overline{C}_4 - (1-\alpha) \left[\left(1 - 6\alpha + 6\alpha^2\right) \overline{N}^4 - 6(1-2\alpha)\overline{N}^2\overline{C}_2 + 4\overline{N}\,\overline{C}_3 + 3(\overline{C}_2)^2 \right] \right\}
\end{aligned}$$

$$\overline{C}_n = C_n^{(a)} - C_n^{(b)},$$

For Poisson, $C_{(a)}$, $C_{(b)}$=0

Fit to STAR data: $\langle N_{(a)} \rangle \simeq 40, \;\; \langle N_{(b)} \rangle \simeq 25, \;\; \alpha \simeq 0.003$

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$

$$K_n^B : \text{ Cumulant of Bernoulli distribution}$$

$$\alpha \ll 1, K_n^B = \alpha \;\Rightarrow\; C_n \simeq \alpha(-1)^n \bar{N}^n$$

$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR (i.e. “infinite” correlation length)

predict:

$$\frac{C_4}{C_3} = \frac{C_5}{C_4} = \frac{C_{n+1}}{C_n} = -\bar{N} \qquad \bar{N} \simeq 15$$

Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

This model can be tested RIGHT NOW!

Model prediction:

$$C_5 = -2645\,(1 \pm 0.14), \quad C_6 = 40900\,(1 \pm 0.18),$$
$$C_7 = -615135\,(1 \pm 0.26), \quad C_8 = 8520220\,(1 \pm 0.42)$$

Efficiency corrected

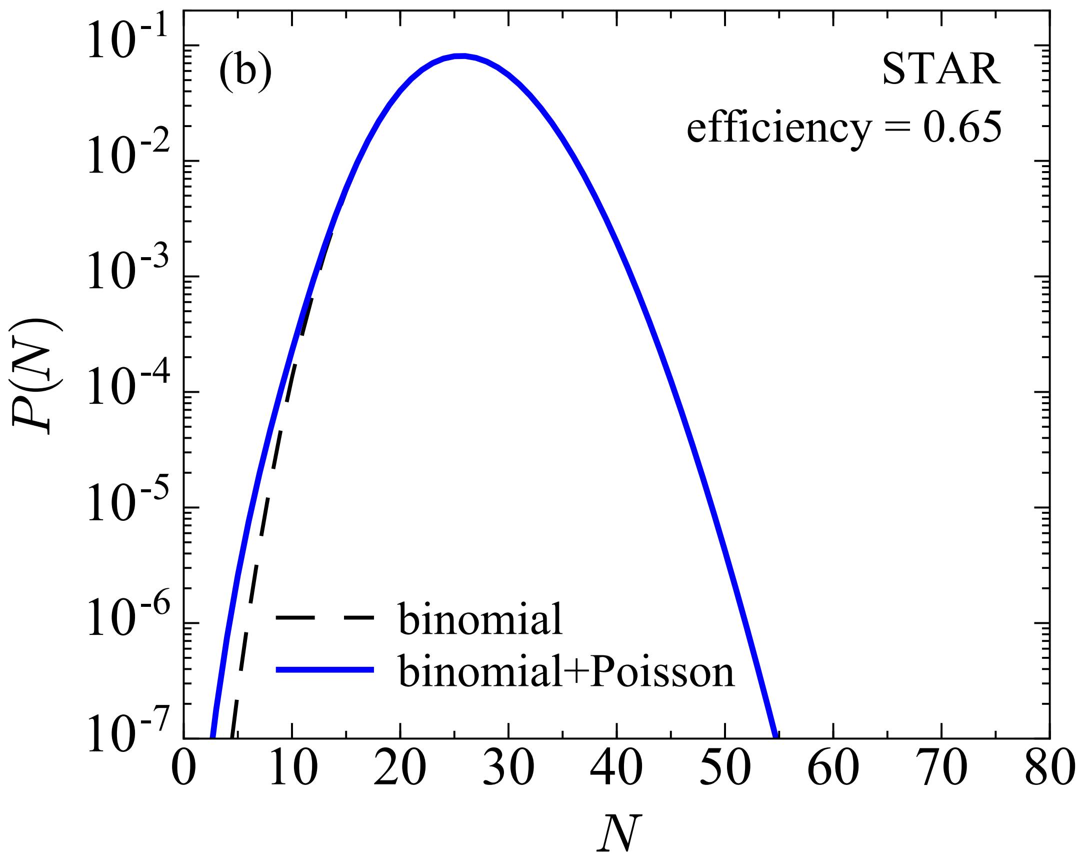
$$C_5 = -307\,(1 \pm 0.31), \quad C_6 = 3085\,(1 \pm 0.41),$$
$$C_7 = -30155\,(1 \pm 0.61), \quad C_8 = 271492\,(1 \pm 1.06),$$

Efficiency UN-corrected

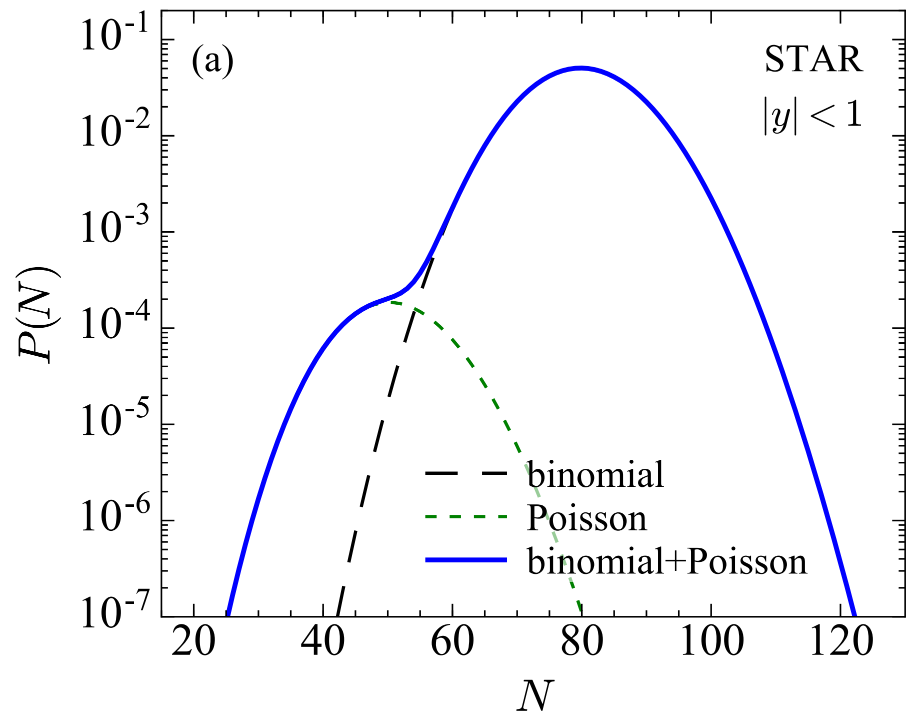
Based on 144393 events (same as STAR 0-5% at 7.7 GEV)

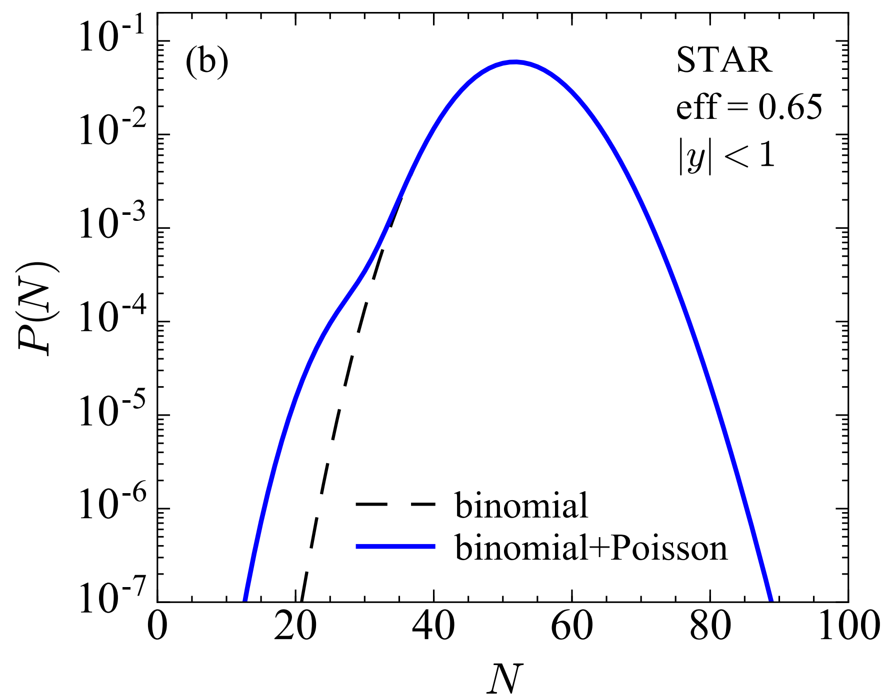
Simple two component model

Difficult to see in the real data with efficiency ε=0.65



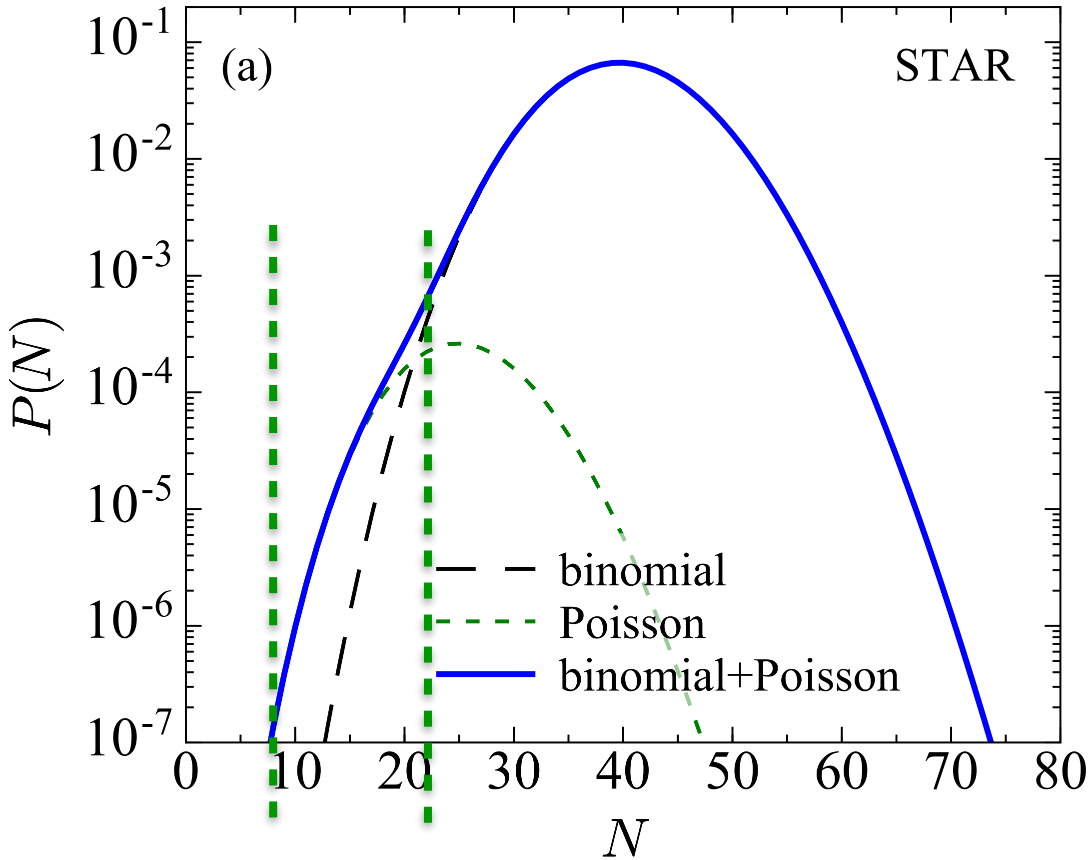
Double the acceptance





Should be visible in raw (unfolded) data

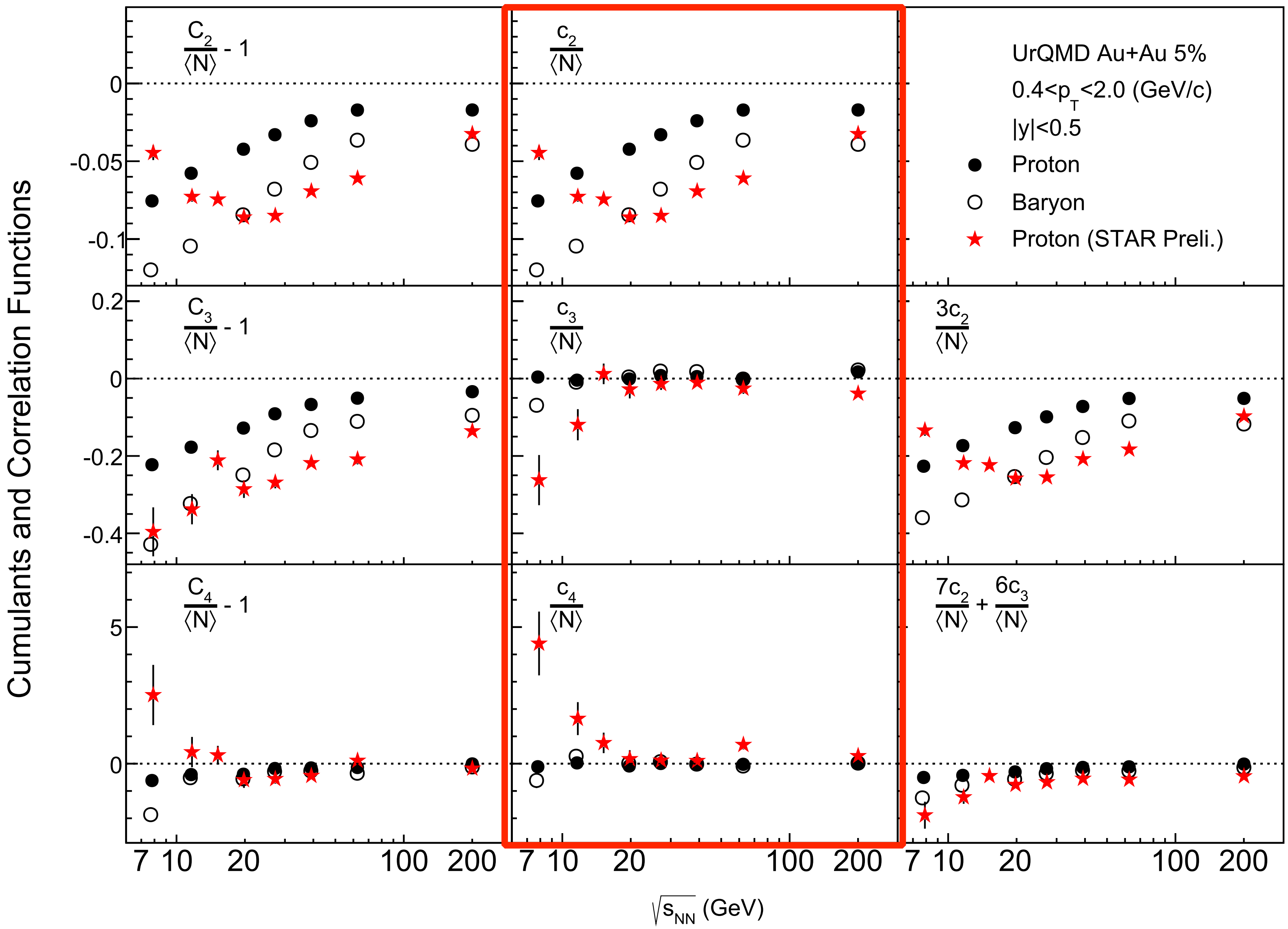
Simple two component model



Analyse data for N_p <20

- Is flow etc different?
- "Inspect by eye (<1% of all events)

URQMD



He, Luo PLB774 (2017) 623

Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle \,, \quad K_2 = \langle N - \langle N \rangle \rangle^2 \,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

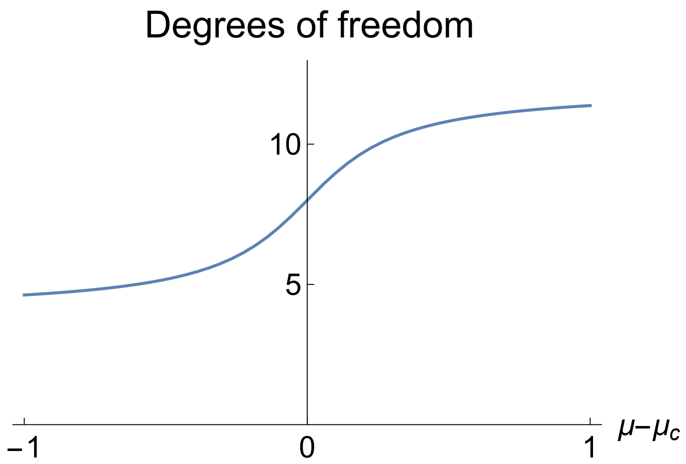
Volume not well controlled in heavy ion collisions

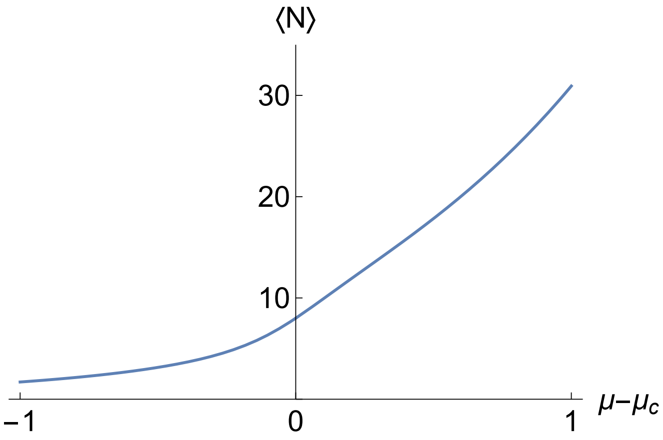
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \quad \frac{K_3}{K_2}, \quad \frac{K_4}{K_2}$

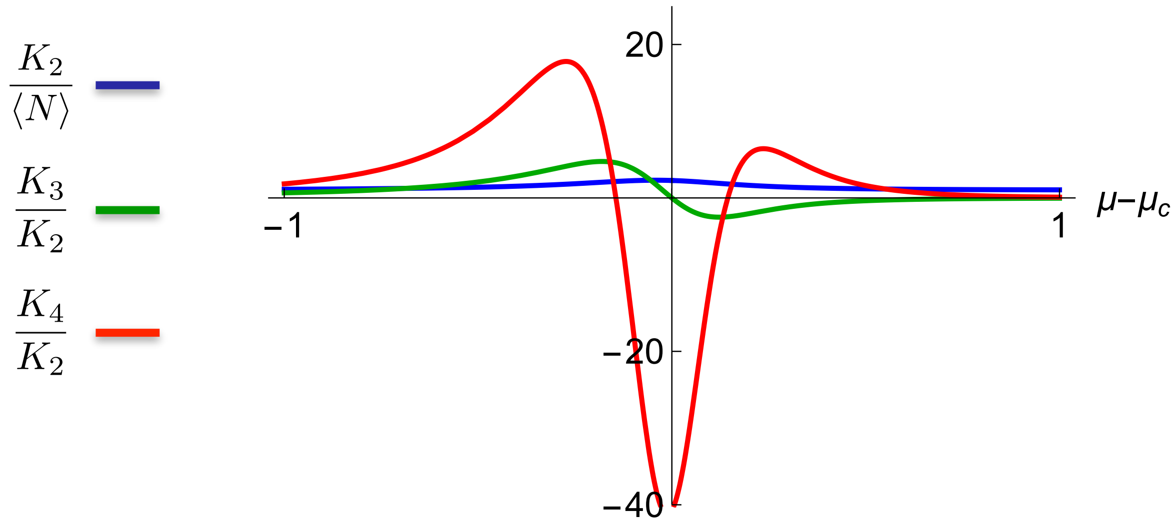
Simple model

Change degrees of freedom
at phase transition

$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$







Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants K_n with correlation length ξ

$$K_2 \sim \xi^2, \;\; K_3 \sim \xi^{4.5}, \;\; K_4 \sim \xi^7$$

Cumulants from Correlations

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Consequently:

$$C_2 \sim \xi^2, \;\; C_3 \sim \xi^{4.5}, \;\; C_4 \sim \xi^7$$

Correlations C_n pick up the most divergent pieces of cumulants K_n!

Reduced correlation function

Reduced correlation function

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)\, c_k(y_1,...,y_k)\, dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)\, dy_1\cdots dy_k} \qquad C_k = \langle N\rangle^k c_k$$

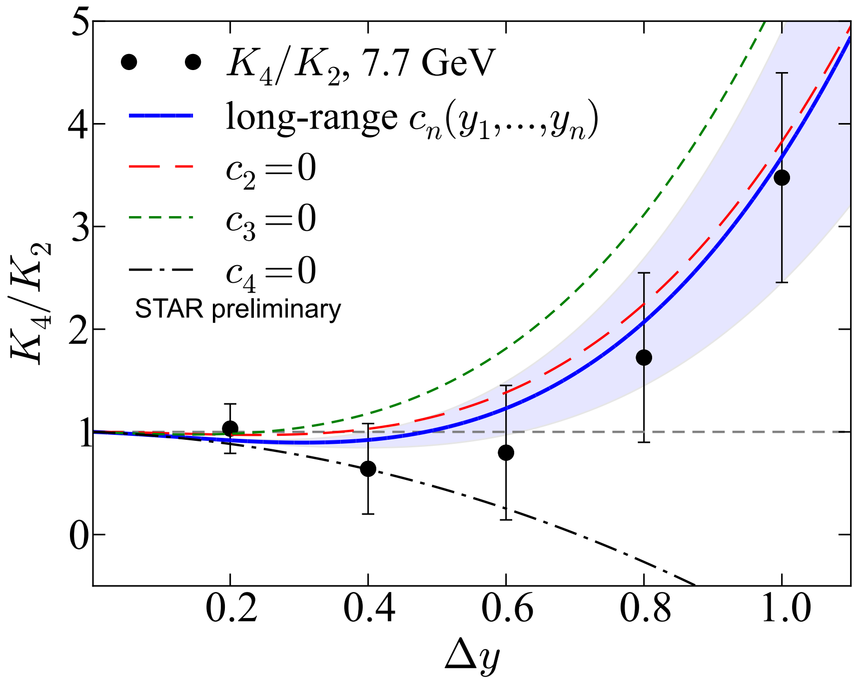
Independent sources such as resonances, cluster, p+p:

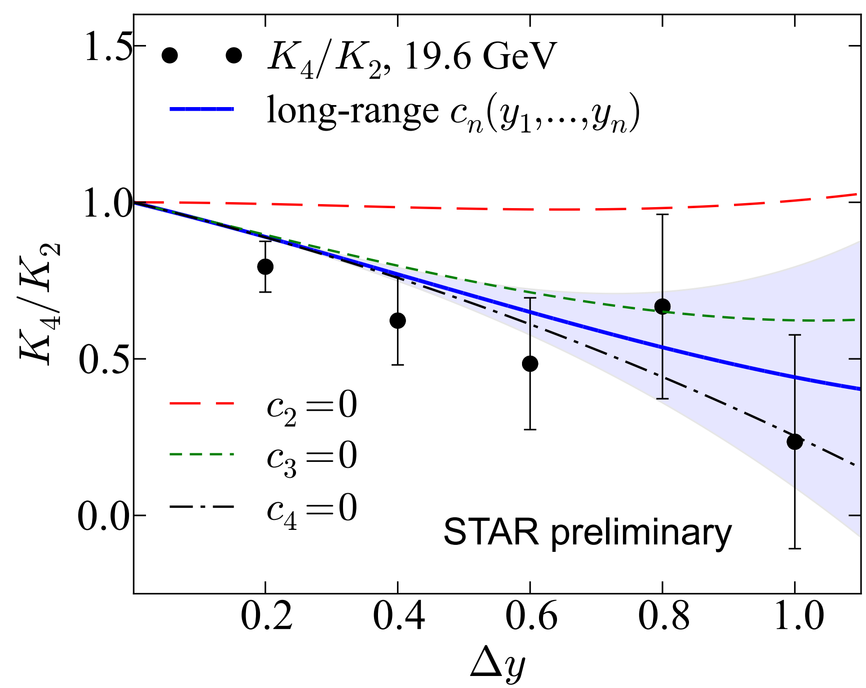
$$c_k \sim \frac{\langle N_s\rangle}{\langle N\rangle^k} \sim \frac{1}{\langle N\rangle^{k-1}}$$

For example two particle correlations:

$$c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N\rangle}$$

Preliminary Star data are consistent with "long range" correlations





7.7 GeV
central

19.6 GeV
central

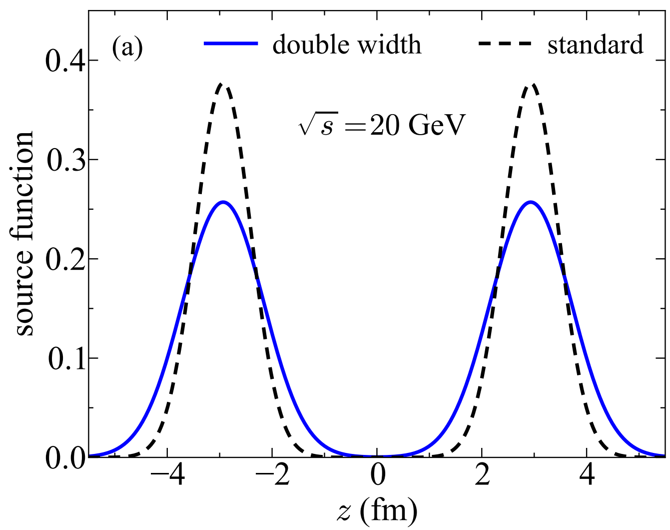
Also true for transverse momentum correlations

Femtoscopy

$$C_F(\delta q_z;\delta q_0) - 1 = -\frac{1}{2}\frac{|\Phi_F(\delta q_z;\delta q_0)|^2}{|\Phi_F(\delta q_z=0;\delta q_0=0)|^2}$$

$$\Phi_F(\delta q_z,\delta q_0;P_i,P_f) \sim e^{-\frac{\Gamma_c^2+\Gamma_F^2/\sigma^2}{4}(\delta q_z^2+\delta q_0^2)}\left[\cos(\delta q_z\Delta Z)\cosh\left(\delta q_0\delta q_z\frac{\Gamma_F^2}{2\sigma^2}\right)+i\sin(\delta q_z\Delta Z)\sinh\left(\delta q_0\delta q_z\frac{\Gamma_F^2}{2\sigma^2}\right)\right]$$

Source



Femtoscopy