Spin polarization dynamics for the Bjorken hydrodynamic background

Rajeev Singh

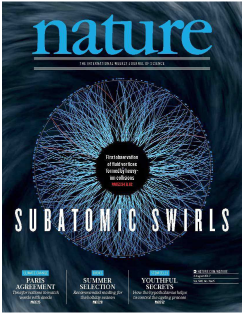
THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

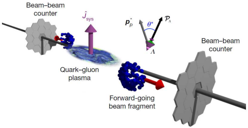
rajeev.singh@ifj.edu.pl

in collaboration with:
Radoslaw Ryblewski, Wojciech Florkowski and Avdhesh Kumar

XIV Polish Workshop on Relativistic Heavy-Ion Collisions
April 6-7, 2019
Krakow, Poland.

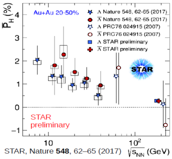
The most vortical fluid!





Liang-Wang 2005

Averaged vorticity from 7.7 GeV-200 GeV: $\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$ "Most vortical fluid!"



STAR, Nature **548**, 62–65 (2017)

LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*

Motivation:

Figure: Jupiter great red spot and Nanodroplets of superfluid helium.

Motivation:

The first positive measurement of $\Lambda(\bar{\Lambda})$ global spin polarization by STAR.

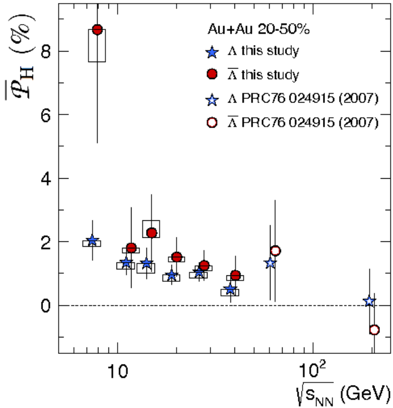


Figure: Average polarization $\bar{\mathcal{P}}_H$ (where $H = \Lambda$ or $\bar{\Lambda}$) versus collision energy in 20-50% central Au+Au collisions.

Source: L. Adamczyk et al.(STAR), Nature 548 (2017) 62-65

Motivation:

- Non-central relativistic heavy ion collisions creates global rotation of matter. This may induce spin polarization reminding us of Barnett effect and Einstein and de-Haas effect.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

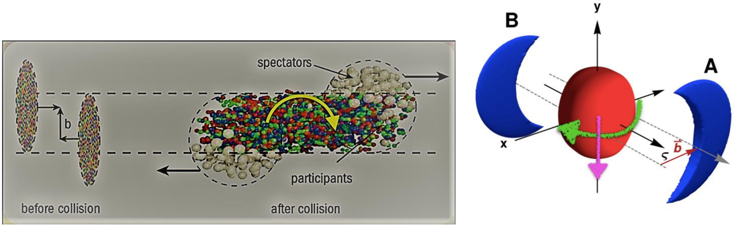
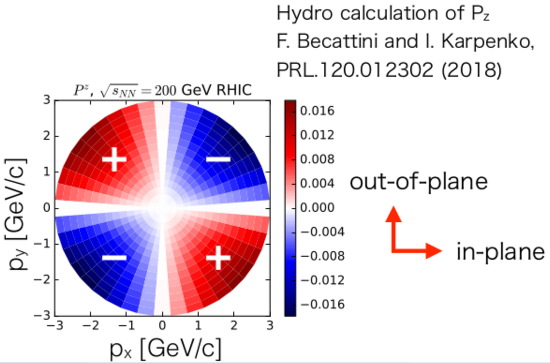


Figure: Schematic view of non-central heavy-ion collisions.

Other works:

- Other theoretical models used for the heavy-ions data interpretation dealt mainly with the spin polarization of particles at freeze-out, where the basic hydrodynamic quantity giving rise to spin polarization is the 'thermal vorticity' expressed as $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$.

F. Becattini *et.al.*(arXiv:1303.3431), F. Becattini, L. Csernai, D. J. Wang (arXiv:1304.4427), F. Becattini *et.al.*(arXiv:1610.02506), Iu. Karpenko, F. Becattini (arXiv:1610.04717), F. Becattini, Iu. Karpenko(arXiv:1707.07984).



Our hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin
- Determination of the spin evolution in the hydrodynamic background
- Determination of the Pauli-Lubanski (PL) vector on the freeze-out hypersurface
- Calculation of the spin polarization of particles in their rest frame
- The spin polarization obtained is a function of the three-momenta of particles and can be directly compared with the experiment

Our hydrodynamic framework:

- In this work, we use relativistic hydrodynamic equations for polarized spin $1/2$ particles to determine the space-time evolution of the spin polarization in the system using forms of the energy-momentum and spin tensors proposed by de Groot, van Leeuwen, and van Weert (GLW).

  S. R. De Groot,Relativistic Kinetic Theory. Principles and Applications (1980).

- The calculations are done in a boost-invariant and transversely homogeneous setup. We show how the formalism of hydrodynamics with spin can be used to determine physical observables related to the spin polarization required for the modelling of the experimental data.

  Wojciech Florkowski *et.al.*(arXiv:1901.09655), Wojciech Florkowski *et.al.*(arXiv:1705.00587), Wojciech Florkowski *et.al.*(arXiv:1712.07676).

- Our hydrodynamic formulation does not allow for arbitrary large values of the spin polarization tensor, hence we have restricted ourselves to the leading order terms in the $\omega_{\mu\nu}$.

Spin polarization tensor:

The spin polarization tensor $\omega_{\mu\nu}$ is anti-symmetric and can be defined by the four-vectors κ^μ and ω^μ,

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta,$$

Using these constraints,

$$\kappa \cdot U = 0, \quad \omega \cdot U = 0$$

We can express κ_μ and ω_μ in terms of $\omega_{\mu\nu}$, namely

$$\kappa_\mu = \omega_{\mu\alpha} U^\alpha, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} \omega^{\alpha\beta} U^\gamma$$

Conservation of charge:

$$\partial_\alpha N^\alpha(x) = 0,$$
$$\text{where,} \quad N^\alpha = nU^\alpha, \quad n = 4\sinh(\xi)\; n_{(0)}(T).$$

The quantity $n_{(0)}(T)$ defines the number density of spinless and neutral massive Boltzmann particles,

$$n_{(0)}(T) = \langle p \cdot U \rangle_0 = \frac{1}{2\pi^2}\, T^3\, \hat{m}^2 K_2\left(\hat{m}\right)$$

where, $\langle \cdots \rangle_0 \equiv \int dP \left(\cdots\right) e^{-\beta \cdot p}$ denotes the thermal average, $\hat{m} \equiv m/T$ denotes the ratio of the particle mass (m) and the temperature (T), and $K_2\left(\hat{m}\right)$ denotes the modified Bessel function.

The factor, $4\, \sinh(\xi) = 2\left(e^{\xi} - e^{-\xi}\right)$ accounts for spin degeneracy and presence of both particles and antiparticles in the system and the variable ξ denotes the ratio of the chemical potential μ and the temperature T, $\xi = \mu/T$.

Conservation of energy and linear momentum:

$$\partial_\alpha T^{\alpha\beta}_{GLW}(x) = 0$$

where the energy-momentum tensor $T^{\alpha\beta}_{GLW}$ has the perfect-fluid form:

$$T^{\alpha\beta}_{GLW}(x) = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}$$

with energy density $\varepsilon = 4\cosh(\xi)\varepsilon_{(0)}(T)$ and pressure $P = 4\cosh(\xi)P_{(0)}(T)$

The auxiliary quantities are:

$\varepsilon_{(0)}(T) = \langle (p\cdot U)^2\rangle_0$ and $P_{(0)}(T) = -(1/3)\langle p\cdot p - (p\cdot U)^2\rangle_0$ are the energy density and pressure of the spin-less ideal gas respectively.

In case of ideal relativistic gas of classical massive particles,

$$\varepsilon_{(0)}(T) = \frac{1}{2\pi^2}\, T^4\, \hat{m}^2\Big[3K_2\left(\hat{m}\right) + \hat{m}K_1\left(\hat{m}\right)\Big], \quad P_{(0)}(T) = Tn_{(0)}(T)$$

Above conservation laws provide closed system of five equations for five unknown functions: ξ, T, and three independent components of U^μ.

Conservation of total angular momentum:

$$\partial_\mu J^{\mu,\alpha\beta}(x) = 0, \quad J^{\mu,\alpha\beta}(x) = -J^{\mu,\beta\alpha}(x)$$

Total angular momentum consists of orbital and spin parts:

$$J^{\mu,\alpha\beta}(x) = L^{\mu,\alpha\beta}(x) + S^{\mu,\alpha\beta}(x),$$

$$L^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x)$$

Since the energy-momentum tensor is symmetric, the conservation of the angular momentum implies the conservation of its spin part.

$$\partial_\lambda J^{\lambda,\mu\nu}(x) = 0, \quad \partial_\mu T^{\mu\nu}(x) = 0 \quad \Longrightarrow \partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$$

Hence, the spin tensor $S^{\mu,\alpha\beta}(x)$ is separately conserved in GLW formulation.

Conservation of spin angular momentum:

$$\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW}\right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta GLW}$ is:

$$S^{\alpha,\beta\gamma}_{\Delta GLW} = \mathcal{A}_{(0)}U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_\delta$$
$$+\mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_\delta + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_\delta + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_\delta\right),$$

with,

$$\mathcal{B}_{(0)} = -\frac{2}{\hat{m}^2}s_{(0)}(T)$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$$

Basis for boost invariant and transversely homogeneous systems:

For our calculations, it is useful to introduce a local basis consisting of following 4-vectors,

$$
\begin{aligned}
U^\alpha &= \frac{1}{\tau}(t,0,0,z) = (\cosh(\eta),0,0,\sinh(\eta))\,,\\
X^\alpha &= (0,1,0,0)\,,\\
Y^\alpha &= (0,0,1,0)\,,\\
Z^\alpha &= \frac{1}{\tau}(z,0,0,t) = (\sinh(\eta),0,0,\cosh(\eta))\,.
\end{aligned}
$$

where, $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time and $\eta = \ln((t+z)/(t-z))/2$ is the space-time rapidity.

The basis vectors satisfy the following normalization and orthogonal conditions:

$$
\begin{aligned}
& U \cdot U = 1\\
X \cdot X &= Y \cdot Y = Z \cdot Z = -1,\\
X \cdot U &= Y \cdot U = Z \cdot U = 0,\\
X \cdot Y &= Y \cdot Z = Z \cdot X = 0.
\end{aligned}
$$

Boost-invariant form for the spin polarization tensor:

We use the following decomposition of the vectors κ^μ and ω^μ,

$$\begin{aligned}\kappa^\alpha &= C_{\kappa U} U^\alpha + C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\ \omega^\alpha &= C_{\omega U} U^\alpha + C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.\end{aligned}$$

Here, the scalar coefficients are functions of the proper time τ only due to boost invariance. Since $\kappa \cdot U = 0, \quad \omega \cdot U = 0$, therefore

$$\begin{aligned}\kappa^\alpha &= C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\ \omega^\alpha &= C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.\end{aligned}$$

$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$ can be written as,

$$\begin{aligned}\omega_{\mu\nu} &= C_{\kappa Z}(Z_\mu U_\nu - Z_\nu U_\mu) + C_{\kappa X}(X_\mu U_\nu - X_\nu U_\mu) + C_{\kappa Y}(Y_\mu U_\nu - Y_\nu U_\mu) \\ &\quad + \epsilon_{\mu\nu\alpha\beta} U^\alpha (C_{\omega Z} Z^\beta + C_{\omega X} X^\beta + C_{\omega Y} Y^\beta)\end{aligned}$$

In the plane $z = 0$ we find:

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & C_{\kappa X} & C_{\kappa Y} & C_{\kappa Z} \\ -C_{\kappa X} & 0 & -C_{\omega Z} & C_{\omega Y} \\ -C_{\kappa Y} & C_{\omega Z} & 0 & -C_{\omega X} \\ -C_{\kappa Z} & -C_{\omega Y} & C_{\omega X} & 0 \end{bmatrix}$$

Boost-Invariant form of fluid dynamics with spin:

- Conservation law of charge can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

Therefore, for Bjorken type of flow we can write,

$$\dot{n} + \frac{n}{\tau} = 0$$

- Conservation law of energy-momentum can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

Hence for the Bjorken flow,

$$\dot{\varepsilon} + \frac{(\varepsilon + P)}{\tau} = 0$$

Boost-Invariant form of fluid dynamics with spin:

Using the equations,

$$S^{\alpha,\beta\gamma}_{\Delta GLW} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}{}_\delta$$
$$+\mathcal{B}_{(0)} \left(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}{}_\delta + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}{}_\delta + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]}{}_\delta \right),$$

and

$$S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi) \left(n_{(0)}(T) U^\alpha \omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW} \right)$$

in

$$\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$$

Contracting the final equation with $U_\beta X_\gamma, U_\beta Y_\gamma, U_\beta Z_\gamma, Y_\beta Z_\gamma, X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$

$\mathcal{P}(\tau) = \mathcal{A}_1,$

$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$

$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$

$$\mathcal{A}_1 = \cosh(\xi)\left(n_{(0)} - \mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_2 = \cosh(\xi)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_3 = \cosh(\xi)\,\mathcal{B}_{(0)}$$

Background evolution:

Initial baryon chemical potential $\mu_0 = 800$ MeV
Initial temperature $T_0 = 155$ MeV
Particle mass $m = 1116$ MeV

Initial and final proper time is $\tau_0 = 1$ fm and $\tau_f = 10$ fm, respectively.

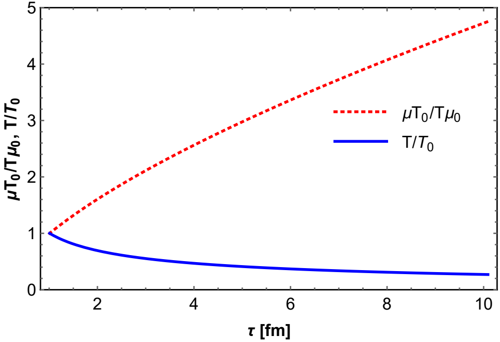


Figure: Proper-time dependence of T divided by its initial value T_0 (solid line) and the ratio of baryon chemical potential μ and temperature T re-scaled by the initial ratio μ_0/T_0 (dotted line) for a boost-invariant one-dimensional expansion.

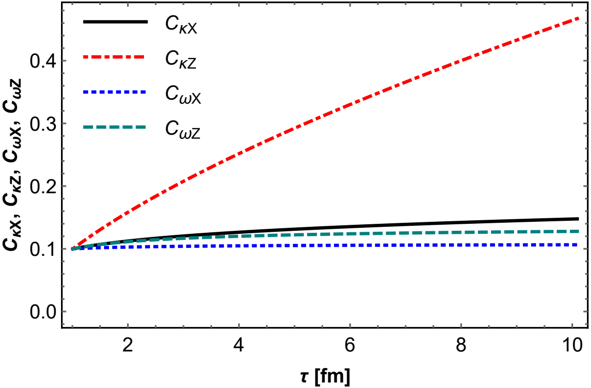


Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{\kappa Y}$ and $C_{\omega Y}$ satisfy the same differential equations as the coefficients $C_{\kappa X}$ and $C_{\omega X}$.

Spin polarization of particles at the freeze-out:

Average spin polarization per particle $\langle \pi_\mu(p) \rangle$ is given as:

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

where, the total value of the Pauli-Lubański vector for particles with momentum p is:

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p} \, \tilde{\omega}_{\mu\beta} p^\beta$$

momentum density of all particles is given by:

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p}$$

and freeze-out hypersurface is defined as:

$$\Delta\Sigma_\lambda = U_\lambda dxdy \, \tau d\eta$$

Assuming that freeze-out takes place at a constant value of τ and parameterizing the particle four-momentum p^λ in terms of the transverse mass m_T and rapidity y_p, we get:

$$\Delta\Sigma_\lambda p^\lambda = m_T \cosh\left(y_p - \eta\right) dxdy \, \tau d\eta$$

Boost to the local rest frame (LRF) of the particle:

Polarization vector $\langle\pi^\star_\mu\rangle$ in the local rest frame of the particle can be obtained by using the canonical boost. Using the parametrizations $E_p = m_T \cosh(y_p)$ and $p_z = m_T \sinh(y_p)$ and applying the appropriate Lorentz transformation we get,

$$\langle\pi^\star_\mu\rangle = -\frac{1}{8m}\begin{bmatrix} 0 \\ \left(\frac{\sinh(y_p)p_x}{m_T\cosh(y_p)+m}\right)\left[\chi\left(C_{\kappa X}p_y - C_{\kappa Y}p_x\right) + 2C_{\omega Z}m_T\right] + \frac{\chi\, p_x \cosh(y_p)(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m} + 2C_{\kappa Z}p_y - \chi C_{\omega X}m_T \\ \left(\frac{\sinh(y_p)p_y}{m_T\cosh(y_p)+m}\right)\left[\chi\left(C_{\kappa X}p_y - C_{\kappa Y}p_x\right) + 2C_{\omega Z}m_T\right] + \frac{\chi\, p_y \cosh(y_p)(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m} - 2C_{\kappa Z}p_x - \chi C_{\omega Y}m_T \\ -\left(\frac{m\cosh(y_p)+m_T}{m_T\cosh(y_p)+m}\right)\left[\chi\left(C_{\kappa X}p_y - C_{\kappa Y}p_x\right) + 2C_{\omega Z}m_T\right] - \frac{\chi\, m \sinh(y_p)(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m} \end{bmatrix}$$

where,

$\chi\left(\hat{m}_T\right) = \left(K_0\left(\hat{m}_T\right) + K_2\left(\hat{m}_T\right)\right)/K_1\left(\hat{m}_T\right)$,

$\hat{m}_T = m_T/T$

Momentum dependence of polarization:

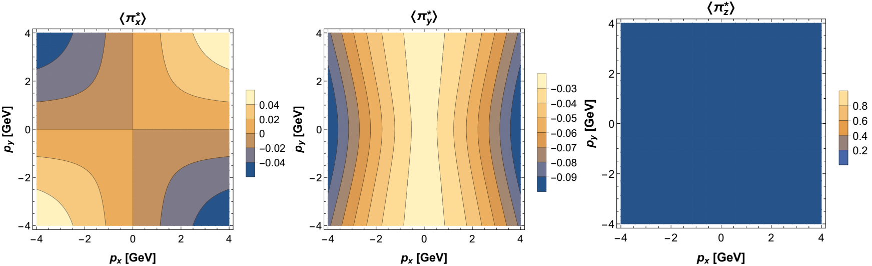
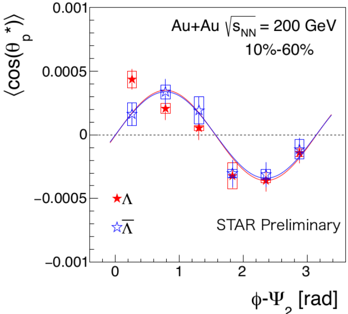


Figure: Components of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0, 0, 0)$, and $\boldsymbol{C}_{\omega,0} = (0, 0.1, 0)$ for $y_p = 0$.

Summary:

- We have discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- For boost invariant and transversely homogeneous set-up we show how our hydrodynamic framework with spin can be used to determine the spin polarization observables measured in heavy ion collisions.
- Since we worked with 0+1 dimensional expansion, our results cannot be compared with the experimental data.
- Our future work is to extend our hydrodynamic approach for 1+3 dimensions and interpret the experimental data correctly.



Dziękuję bardzo !

Back-Up Slides

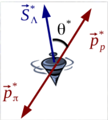
Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\rm H}\mathbf{P}_{\rm H} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

P_H: Λ polarization
p_p^*: proton momentum in the Λ rest frame
α_H: Λ decay parameter
($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



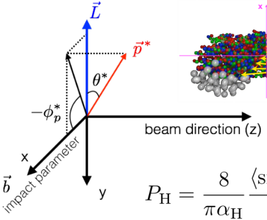
$$\Lambda \to p + \pi^-$$

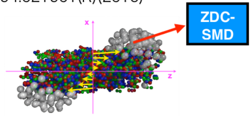
(BR: 63.9%, cτ ~7.9 cm)

C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)
- S. Voloshin and TN, PRC94.021901(R)(2016)





$$P_{\rm H} = \frac{8}{\pi\alpha_{\rm H}} \frac{\langle \sin(\Psi_1 - \phi_p^*)\rangle}{{\rm Res}(\Psi_1)}$$

Ψ_1: azimuthal angle of b
ϕ_p^*: ϕ of daughter proton in Λ rest frame
STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

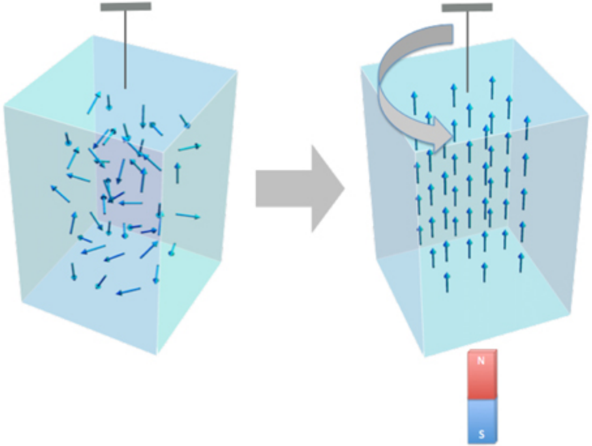


Figure: Einstein-De Haas Effect

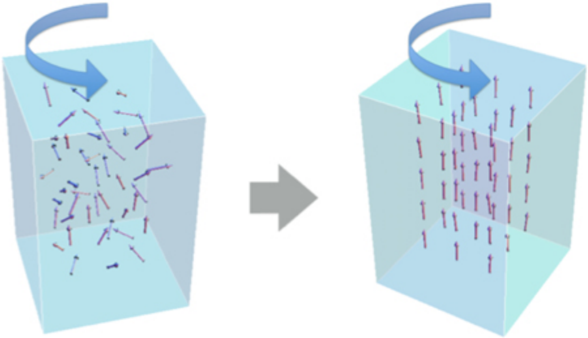


Figure: Barnett Effect

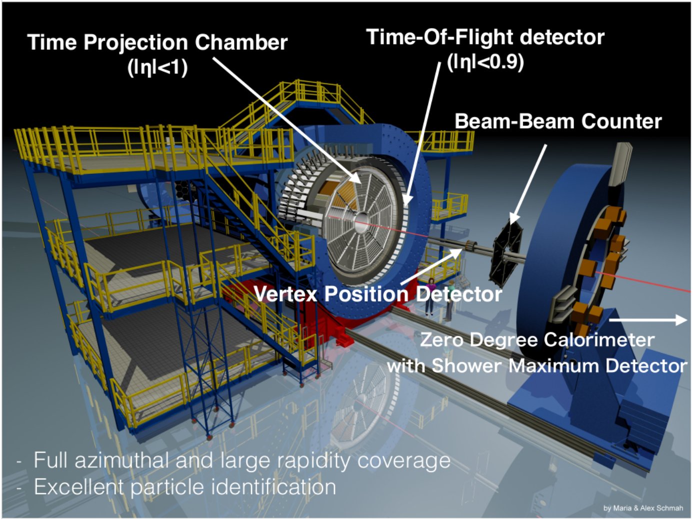


Figure: Schematic view of STAR Detector