Some Clever Stuff we do for Fermi Likelihood Fitting

1. Loop over only the filled pixels for this term

$$-\ln \mathcal{L} = \sum_i^{\text{pix}} \sum_k^{\text{energy}} N_{ik} \ln M_{ik} - M_{ik},$$

2. Pre-compute sum over pixels for this term

3. Do integration with log-log quadrature

$$M_{ijk} = (m_{ijk} S_{jk} E_k + m_{ijk+1} S_{jk+1} E_{k+1}) \cdot \frac{1}{2} \ln \frac{E_{k+1}}{E_k},$$

m_{ijk} = Template for source j (flux convolved with IRFs)
S_{jk} = Spectrum for source j

4. Pre-compute "spectral weights"

$$\tilde{S}^-_{jk} = S_{jk} E_k \frac{r_k}{2},$$

$$\tilde{S}^+_{jk} = S_{jk+1} E_{k+1} \frac{r_k}{2}.$$

$$M_{ijk} = m_{ijk} \tilde{S}^-_{jk} + m_{ijk+1} \tilde{S}^+_{jk}.$$

More Clever Stuff we do for Fermi Likelihood Fitting

4. Use the chain rule and factor out the spectral derivatives

$$\frac{-\partial \ln \tilde{\mathcal{L}}}{\partial x_\alpha} = \sum_i^{\text{pix}} \sum_k^{\text{energy}} \frac{w_{ik} N_{ik}}{M_{ik}} \frac{\partial M_{ik}}{\partial x_\alpha} - w_{ik} \frac{\partial M_{ik}}{\partial x_\alpha}. \qquad \delta_{jk\alpha} = \frac{\partial S_{jk}}{\partial x_\alpha}$$

$$\frac{\partial M_{ijk}}{\partial x_\alpha} = \xi_{jk} \cdot (m_{ijk} \delta_{jk\alpha} E_k + m_{ijk+1} \delta_{jk+1\alpha} E_{k+1}) \frac{r_k}{2}.$$

5. For making TS maps, use the same PSF image at each pixel

6. In many cases, we can compute the Hessian analytically:

$$\frac{\partial^2(-\log \mathcal{L})}{\partial \alpha_j \partial \alpha_k} = \sum_i \frac{\partial^2 P(\vec{x}|\vec{\alpha})}{\partial \alpha_j \partial \alpha_k} (1 - \frac{n_i}{P(\vec{x}|\vec{\alpha})}) + \frac{n_i}{P^2(\vec{x}|\vec{\alpha})} \frac{\partial P(\vec{x}|\vec{\alpha})}{\partial \alpha_j} \frac{\partial P(\vec{x}|\vec{\alpha})}{\partial \alpha_k}$$