It has been a long way since 2012:

We have recently established ttH and bbH couplings!
Our understanding has gotten pretty sophisticated -- 10-parameter fit has been performed:

Table 15: Results of the ten-parameter fit of $\mu_F^f = \mu_{ggF+ttH}^f$ and $\mu_V^f = \mu_{VBF+VH}^f$ for each of the five decay channels, and of the six-parameter fit of the global ratio $\mu_V^f/\mu_F^f = \mu_{VBF+VH}/\mu_{ggF+ttH}$ together with $\mu_F^f$ for each of the five decay channels. The results are shown for the combination of ATLAS and CMS, together with their measured and expected uncertainties. The measured results are also shown separately for each experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ATLAS+CMS</th>
<th>ATLAS+CMS</th>
<th>ATLAS</th>
<th>CMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Expected uncertainty</td>
<td>Measured</td>
<td>Measured</td>
</tr>
<tr>
<td>$\mu_V^{\gamma\gamma}$</td>
<td>1.05$^{+0.44}_{-0.41}$</td>
<td>+0.41</td>
<td>0.69$^{+0.63}_{-0.58}$</td>
<td>1.37$^{+0.62}_{-0.56}$</td>
</tr>
<tr>
<td>$\mu_V^{ZZ}$</td>
<td>0.47$^{+1.37}_{-0.92}$</td>
<td>+1.16</td>
<td>0.24$^{+1.60}_{-0.93}$</td>
<td>1.45$^{+2.32}_{-2.29}$</td>
</tr>
<tr>
<td>$\mu_V^{WW}$</td>
<td>1.38$^{+0.41}_{-0.37}$</td>
<td>+0.38</td>
<td>1.56$^{+0.52}_{-0.46}$</td>
<td>1.08$^{+0.65}_{-0.58}$</td>
</tr>
<tr>
<td>$\mu_V^{TT}$</td>
<td>1.12$^{+0.37}_{-0.35}$</td>
<td>+0.38</td>
<td>1.29$^{+0.58}_{-0.53}$</td>
<td>0.88$^{+0.49}_{-0.45}$</td>
</tr>
<tr>
<td>$\mu_V^{bb}$</td>
<td>0.65$^{+0.31}_{-0.29}$</td>
<td>+0.32</td>
<td>0.50$^{+0.39}_{-0.37}$</td>
<td>0.85$^{+0.47}_{-0.44}$</td>
</tr>
<tr>
<td>$\mu_V^{\gamma\gamma}$</td>
<td>1.16$^{+0.27}_{-0.24}$</td>
<td>+0.25</td>
<td>1.30$^{+0.37}_{-0.33}$</td>
<td>1.00$^{+0.33}_{-0.30}$</td>
</tr>
<tr>
<td>$\mu_F^{ZZ}$</td>
<td>1.42$^{+0.37}_{-0.33}$</td>
<td>+0.29</td>
<td>1.74$^{+0.51}_{-0.44}$</td>
<td>0.96$^{+0.53}_{-0.41}$</td>
</tr>
<tr>
<td>$\mu_F^{WW}$</td>
<td>0.98$^{+0.22}_{-0.20}$</td>
<td>+0.21</td>
<td>1.10$^{+0.29}_{-0.26}$</td>
<td>0.84$^{+0.27}_{-0.24}$</td>
</tr>
<tr>
<td>$\mu_F^{TT}$</td>
<td>1.06$^{+0.60}_{-0.56}$</td>
<td>+0.56</td>
<td>1.72$^{+1.24}_{-1.12}$</td>
<td>0.89$^{+0.67}_{-0.63}$</td>
</tr>
<tr>
<td>$\mu_F^{bb}$</td>
<td>1.15$^{+0.99}_{-0.94}$</td>
<td>+0.90</td>
<td>1.52$^{+1.16}_{-1.09}$</td>
<td>0.11$^{+1.85}_{-1.90}$</td>
</tr>
</tbody>
</table>
We have measured many of the following couplings with uncertainties of 10 – 30 % or larger:

**Couplings to massive gauge bosons**
\[
\left( \frac{2m_W^2}{v} h W_\mu^+ W^- \mu + \frac{m_Z^2}{v} h Z_\mu Z^\mu \right)
\]

**Couplings to massless gauge bosons**
\[
+ c_g \frac{\alpha_s}{12\pi v} h G^{a\mu} G^{a\nu} + c_\gamma \frac{\alpha}{8\pi v} h F_{\mu\nu} F^{\mu\nu} + c_{Z\gamma} \frac{\alpha}{8\pi v s_w} h F_{\mu\nu} Z^{\mu\nu}
\]

\[c_g^{(SM)}(125 \text{ GeV}) = 1 \, , \quad c_\gamma^{(SM)}(125 \text{ GeV}) = -6.48 \, , \quad c_{Z\gamma}^{(SM)}(125 \text{ GeV}) = 5.48 \, .\]

**Couplings to fermions**
\[
\sum_f \frac{m_f}{v} h \bar{f}f \quad \text{for } bb, \, tt, \, \text{and } \tau\tau \text{ only!}
\]

**Self-couplings** is being probed in the double Higgs production channel:
\[
\frac{1}{2} m_h^2 h^2 + \frac{m_h^2}{v} h^3 + \frac{2m_h^2}{v^2} h^4
\]

- **Limits at 95% CL on self-coupling scale factor \(\kappa_h\):**
  - ATLAS: \(-5.0 < \kappa_h < 12.1\)
  - CMS: \(-11.8 < \kappa_h < 18.8\)
What are we missing still?
In my view, one class of couplings that has not received enough attention is the HHVV coupling:

\[ D_\mu H^\dagger D^\mu H \supset g^2 h^2 V_\mu V^\mu \]

HHVV and the self-couplings are two predictions of SM that have NOT been tested experimentally.
In addition, many important questions remain unanswered...

A few years back I was reminded by my (then) 7-year-old of one such question:

**What is it made of?**

A physics Ph.D. could rephrase slightly:

**What is the microscopic theory that gives rise to the Higgs boson and its potential?**

\[ V(H) = -\mu^2 |H|^2 + \lambda |H|^4 \]

Our colleagues in condensed matter physics are very used to asking, and studying, this kind of questions.
One of the most beautiful examples is the superconductivity discovered in 1911:

\[
V(\Psi) = \alpha(T)|\Psi|^2 + \beta(T)|\Psi|^4 \quad \alpha(T) \approx a^2(T - T_c) \quad \text{and} \quad \beta(T) \approx b^2
\]

What is the microscopic origin of the Ginzburg-Landau potential for superconductivity?
In 1957 Bardeen, Cooper and Schrieffer provided the **microscopic** (fundamental) theory that allows one to

1) interpret $|\Psi|^2$ as the number density of Cooper pairs

2) calculate coefficients of $|\Psi|^2$ and $|\Psi|^4$ in the potential.

We do not have the corresponding **microscopic** theory for the Higgs boson.

In fact, we have NOT even measured the Ginzburg-Landau potential of the Higgs!
The question can be reformulated in terms of **Quantum Criticality**:

$$V(\phi) = m^2|\phi|^2 + \lambda |\phi|^4$$

**Quantum Phase Diagram of EWSB**

- $m^2 > 0, \langle \phi \rangle = 0$
- $m^2 = 0$
- $m^2 < 0$, $\langle \phi \rangle \sim M_{\text{Planck}}$

$M_h = 125$ GeV. We are sitting extremely close to the criticality. **WHY??**
One appealing possibility – the critical line is selected dynamically.

This is the analogy of BCS theory for electroweak symmetry breaking. It goes by the name of “technicolor,” which is strongly disfavored experimentally.

Two popular “explanations:”

1. Postulate new global symmetries above the weak scale, and the Higgs boson arises as a (pseudo) Nambu-Goldstone boson.
   ➔ This class goes by the name of “composite Higgs models.”

2. The critical line is a locus of enhanced symmetry.
   ➔ This is the (broken) supersymmetry.
Supersymmetry v.s. Composite Higgs:

Neither of them is doing great --
Although that may be a difference of opinion...
The fact that we have not seen signs of SUSY or CHM only deepens the mystery, of why we are sitting close to the critical line of EWSB!

Some people argued that the SM by itself is UV-complete and, therefore, there’s no need for new physics.

This is a reasoning that has failed many times throughout the course of the history:

• QED (photons+electrons) is a UV-complete theory. But physics didn’t stop there.
• QCD (gluons+quarks) is also a UV-complete theory. Again physics didn’t stop there.
• SM with one generation of fermion is UV-complete. “WHO ORDERED THAT?”

Not to mention there is also all these empirical evidence for physics beyond the SM: Dark matter, Baryon asymmetry and etc.
It is a somewhat embarrassing realization that, after 40 years, our understanding of the electroweak symmetry breaking is still at the level of Ginzburg-Landau level!

In order to probe the microscopic nature of the Higgs, we need to pursue a program to simultaneously study HVV and HHVV couplings.
Let me elaborate –

Suppose the SM is just an effective description:

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^n} \mathcal{O}_i^{(n-4)}
\]

At the weak scale, the HVV and HHVV couplings deviate from their SM expectations, both in coupling strength and the tensor structure,

\[
\mathcal{L}^{(2)} = \frac{1}{2} \partial_\mu h \partial^\mu h + b_{nh} \left(\frac{h}{v}\right)^n \left( m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right)
\]

There are also operators carrying “four-derivative”:

\[
\begin{align*}
\frac{h}{v} V_{1\mu} D^{\mu\nu} V_{2\nu} , & \quad \frac{h}{v} V_{1\mu\nu} V_{2\mu} , & & \quad D^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2 \\
\frac{h^2}{v^2} V_{1\mu} D^{\mu\nu} V_{2\nu} , & \quad \frac{h^2}{v^2} V_{1\mu\nu} V_{2\mu} , & \quad \frac{\partial_\mu h \partial_\nu h}{v^2} V_{1\mu} V_{2\nu}
\end{align*}
\]
In a given BSM model, coefficients of these corrections can be calculated.

Generically, these coefficients are independent parameters depending on various masses and couplings in the UV model.

However, in composite Higgs models these anomalous HVV and HHVV couplings are controlled by only a small number of parameters because there is a symmetry relating the coefficients.

\[
\mathcal{L}^{(2)} = \frac{1}{2} \partial_\mu h \partial^\mu h + b_{nh} \left( \frac{h}{v} \right)^n \left( m_W^2 W_\mu^+ W^-\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right)
\]

\[
b_h = 1 - 2\xi
\]

\[
b_{3h} = -\frac{4}{3} \xi \sqrt{1 - \xi}
\]

\[
b_{5h} = \frac{4}{15} \xi^2 \sqrt{1 - \xi}
\]

\[
b_{2h} = 2 \sqrt{1 - \xi}
\]

\[
b_{4h} = \frac{1}{3} \xi (2\xi - 1)
\]

\[
b_{6h} = \frac{2}{45} \xi^2 (1 - 2\xi)
\]

\[
\ldots
\]
For this class of theories, the two-derivative Lagrangian can be written in a compact way:

\[
\mathcal{L}^{(2)} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4} \sin^2(\theta + h/f) \left( W^+_\mu W^{-\mu} + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right)
\]

\[
\sin^2 \theta = \xi = \frac{v^2}{f^2}
\]

In the unitary gauge, the “symmetry” that enforces this particular form is highly disguised and non-trivial.

**One way to “detect” the presence of such disguised symmetry is to measure HVV and HHVV couplings to see if they are controlled by the same parameter.**
More concretely, consider the following “anomalous” HVV and HHVV couplings:

\[
\mathcal{L}_{NL} = \sum_i \frac{m_W^2}{m_\rho^2} \left( C^h_i \mathcal{I}^h_i + C^{2h}_i \mathcal{I}^{2h}_i + C^{3V}_i \mathcal{I}^{3V}_i \right)
\]

<table>
<thead>
<tr>
<th>$\mathcal{I}^h_i$</th>
<th>$\mathcal{I}^{2h}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\frac{h}{v} Z_\mu D^{\mu\nu} Z_\nu$</td>
<td>(1) $\frac{h^2}{v^2} Z_\mu D^{\mu\nu} Z_\nu$</td>
</tr>
<tr>
<td>(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$</td>
<td>(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$</td>
</tr>
<tr>
<td>(3) $\frac{h}{v} Z_\mu D^{\mu\nu} A_\nu$</td>
<td>(3) $\frac{h^2}{v^2} Z_\mu D^{\mu\nu} A_\nu$</td>
</tr>
<tr>
<td>(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$</td>
<td>(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$</td>
</tr>
</tbody>
</table>

An example of “Universal Relations” is

\[
\frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2} \cos \theta = \frac{1}{2} \sqrt{1 - \xi}
\]

Z. Yin, D. Liu and IL: 1805.00489; 1809.09126
Simultaneous measurements on HVV and HHVV coupling tensor structures allows to detect the presence of new symmetry relating the multi-Higgs couplings to electroweak gauge bosons.

But what is this “symmetry”?

Observation:
Secretly this is a symmetry relating multi-Higgs self-interactions – Recall in the unitary gauge the longitudinal components of the W/Z gauge bosons are related to the 125 GeV Higgs by SU(2)xU(1).
In fact, everyone knows an example of such a symmetry.

The example is pions in low-energy QCD. There are many ways to write down the effective Lagrangian of pions. One possibility is

\[ U(x) = \frac{1}{f} \left[ \sigma(x) + i \vec{\pi} \cdot \vec{\pi}(x) \right], \quad \sigma(x) = \sqrt{f^2 - \vec{\pi}^2(x)}, \]

\[ \mathcal{L}^{(2)} = \frac{1}{4} f^2 \text{Tr}[D_\mu U(D^{\mu}U)^\dagger] \]

When expanding the two-derivative in \( "1/f" \), all “multi-pion” vertices are controlled by one single parameter \( "f" \).

This is similar to the case we discussed in Higgs:

\[ \mathcal{L}^{(2)} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4} \sin^2(\theta + h/f) \left( W^+ W^- + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right) \]
Pions are (pseudo)-Nambu-Goldstone bosons arising from the chiral symmetry breaking:

\[ \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2)_V \]

The “symmetry” enforcing relations among multi-pion vertices is the result of degenerate vacua and the unbroken \( \text{SU}(2)_V \) isospin symmetry.

Similarly, if we detect relations among HVV and HHVV couplings, it’s the clearest signal that the 125 GeV Higgs is a (pseudo-)Nambu-Goldstone boson!
The theory space of composite Higgs models is large:

<table>
<thead>
<tr>
<th>$\mathcal{G}$</th>
<th>$\mathcal{H}$</th>
<th>$C$</th>
<th>$N_G$</th>
<th>$r_{\mathcal{H}} = r_{SU(2) \times SU(2)} (r_{SU(2) \times U(1)})$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(5) × U(1)</td>
<td>SU(2) × U(1)</td>
<td>5</td>
<td>$2_{\pm 1/2} + 1_0$</td>
<td>10, 35</td>
<td></td>
</tr>
<tr>
<td>SU(4)</td>
<td>Sp(4)</td>
<td>✓</td>
<td>5</td>
<td>$(2, 2)_{\pm 2} = 2 \cdot (2, 2)$</td>
<td>29, 47, 64</td>
</tr>
<tr>
<td>SU(4)</td>
<td>$[SU(2)]^2 \times U(1)$</td>
<td>✓*</td>
<td>8</td>
<td>$6 = 2 \cdot (1, 1) + (2, 2)$</td>
<td>63</td>
</tr>
<tr>
<td>SO(7)</td>
<td>SO(6)</td>
<td>✓</td>
<td>6</td>
<td>$7 = (1, 3) + (2, 2)$</td>
<td>66</td>
</tr>
<tr>
<td>SO(7)</td>
<td>G₂</td>
<td>✓*</td>
<td>7</td>
<td>$10_0 = (3, 1) + (1, 3) + (2, 2)$</td>
<td>−</td>
</tr>
<tr>
<td>SO(7)</td>
<td>$[SU(2)]^3$</td>
<td>✓*</td>
<td>12</td>
<td>$(2, 2, 3) = 3 \cdot (2, 2)$</td>
<td>−</td>
</tr>
<tr>
<td>Sp(6)</td>
<td>Sp(4) × SU(2)</td>
<td>✓</td>
<td>8</td>
<td>$(4, 2) = 2 \cdot (2, 2)$</td>
<td>63</td>
</tr>
<tr>
<td>SU(5)</td>
<td>SO(5)</td>
<td>✓*</td>
<td>14</td>
<td>$14 = (3, 3) + (2, 2) + (1, 1)$</td>
<td>9, 47, 49</td>
</tr>
<tr>
<td>SO(8)</td>
<td>SO(7)</td>
<td>✓</td>
<td>7</td>
<td>$7 = 3 \cdot (1, 1) + (2, 2)$</td>
<td>−</td>
</tr>
<tr>
<td>SO(9)</td>
<td>SO(8)</td>
<td>✓</td>
<td>8</td>
<td>$8 = 2 \cdot (2, 2)$</td>
<td>67</td>
</tr>
<tr>
<td>SO(9)</td>
<td>SO(5) × SO(4)</td>
<td>✓*</td>
<td>20</td>
<td>$(5, 4) = (2, 2) + (1 + 3, 1 + 3)$</td>
<td>34</td>
</tr>
<tr>
<td>[SU(3)]^2</td>
<td>SU(3)</td>
<td>8</td>
<td>$8 = 1_0 + 2_{\pm 1/2} + 3_0$</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>[SO(5)]^2</td>
<td>SO(5)</td>
<td>✓*</td>
<td>10</td>
<td>$10 = (1, 3) + (3, 1) + (2, 2)$</td>
<td>32</td>
</tr>
<tr>
<td>SU(4) × U(1)</td>
<td>SU(3) × U(1)</td>
<td>7</td>
<td>$3_{-1/3} + 3_{1/3} + 1_0 = 3 \cdot 1_0 + 2_{\pm 1/2}$</td>
<td>35, 41</td>
<td></td>
</tr>
<tr>
<td>SU(6)</td>
<td>Sp(6)</td>
<td>✓*</td>
<td>14</td>
<td>$14 = 2 \cdot (2, 2) + (1, 3) + 3 \cdot (1, 1)$</td>
<td>30, 47</td>
</tr>
<tr>
<td>[SO(6)]^2</td>
<td>SO(6)</td>
<td>✓*</td>
<td>15</td>
<td>$15 = (1, 1) + 2 \cdot (2, 2) + (3, 1) + (1, 3)$</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 1: Symmetry breaking patterns $\mathcal{G} \rightarrow \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension $N_G$ of the coset, while the fifth contains the representations of the GB’s under $\mathcal{H}$ and $SO(4) \cong SU(2)_L \times SU(2)_R$ (or simply $SU(2)_L \times U(1)_Y$ if there is no custodial symmetry). In case of more than two $SU(2)$’s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.

Bellazzni, Csaki and Serra:1401.2457
A common ingredient for viable composite Higgs models:

The unbroken group $H$ always contains a $SO(4)$ subgroup, under which the 125 GeV Higgs is the fundamental representation.

The nonlinear Higgs interaction is a universal feature!

Experimental confirmation of the nonlinear symmetry would be a striking indication on the NGB nature of the 125 GeV Higgs boson.

→ Opens up a new experimental frontier.
Four different ways to test HHVV couplings at future colliders:

(a) Double Higgs production through vector boson fusion at a hadron collider.

(b) Double Higgs production through vector boson fusion at a lepton collider.

(c) Double Higgs production in association with a vector boson.

(d) Off-shell Single Higgs decay.

One could also measure the triple gauge boson couplings (TGC) to test the nonlinear symmetry.

Liu, IL and Yin: 1809.09126
The required precision is high and it’s important to employ advanced analysis technique!

One example is the $H \rightarrow 4L$ decays, where the full kinematic distributions can be constructed. Both “rate information” (in signal strength) and “shape information” (in differential spectra) are available.

Using the “Golden 4L channel”, we can

- obtain a more proper experimental limit on the nonlinear parameter $\xi$.
- constrain the Wilson coefficients of $O(p^4)$ operators in the nonlinear Higgs Lagrangian!

$$\mathcal{L}^{(1h)} = \frac{m_W^2}{M^2} \left[ C_1^h \frac{\hbar}{\nu} Z_\mu D^{\mu \nu} Z_\nu + C_2^h \frac{\hbar}{\nu} Z_{\mu \nu} Z^{\mu \nu} + C_3^h \frac{\hbar}{\nu} Z_\mu D^{\mu \nu} A_\nu ight. $$

$$+ C_4^h \frac{\hbar}{\nu} Z_{\mu \nu} A^{\mu \nu} + C_5^h \frac{\hbar}{\nu} (W_\mu^+ D^{\mu \nu} W^-_\nu + \text{h.c.}) + C_6^h \frac{\hbar}{\nu} W^+_{\mu \nu} W^\nu_{-\mu \nu} \right]$$
Measurements on the nonlinear parameter $\xi$ using “rate information:”

- Signal strength in 4L channel prefers a negative $\xi$, which corresponds to a non-compact coset structure in the UV.

Liu, IL, Vega-Morales: 1904.00026
Measurements on the nonlinear parameter $\xi$ using fully differential spectra:

- Depends on which anomalous HVV coupling is “turned on,” $\xi < 0.5$ or $> -0.5$ is still allowed.

Liu, IL, Vega-Morales: 1904.00026
First limits on the Wilson coefficients in nonlinear Lagrangian:

Projections at HL-LHC:

Liu, IL, Vega-Morales: 1904.00026
Concluding Remarks:

• The Higgs boson is the most exotic state of matter in Nature.

• The electroweak criticality is the most bizarre type of quantum criticality.

• Our understanding is still preliminary, at the level of Ginzburg-Landau picture for the superconductivity. 
  
  Need to pin down a microscopic picture.

• Nonlinear dynamics of a PNGB Higgs is the most salient feature, and is universal among viable composite Higgs models.
• Testing the nonlinear Higgs interaction opens up a new experimental frontier:

  – Simultaneous measurements of HVV, HHVV and TGCs could test the underlying symmetry in nonlinear Higgs interactions.

  – HHVV coupling is the least studied coupling in Higgs physics.

  – Need to verify the tensor structure of the coupling, in the same fashion as in the studies of HVV coupling.

  – The required precision to test the universal relations is high. Need to introduce advanced analysis techniques.