Electroweak Vacuum Stability in the Early Universe

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Based on:

- Herranen, Markkanen, Nurmi & AR, PRL113 (2014) 211102
- AR & Stopyra, PRD95 (2017) 025008
- AR & Stopyra, PRD97 (2018) 025012
- Figueroa, AR & Torrenti, PRD98 (2018) 023532
Important in the Early Universe (EU)
Decouples from other fields and turns into a weakly interacting and slowly decaying spectator
Brexiton

Decays due to interactions
The Standard Model

- All renormalisable terms allowed by symmetries in Minkowski space
- 19 parameters – all have been measured
- Can be extrapolated all the way to Planck scale
- For central experimental values $M_H = 125.18$ GeV, $M_t = 173.1$ GeV
  - $\lambda$ becomes negative at $\mu_\Lambda \approx 9.9 \times 10^9$ GeV
  - Minimum value $\lambda_{\min} \approx -0.015$ at $\mu_{\min} \approx 2.8 \times 10^{17}$ GeV

(Buttazzo et al 2013)
Vacuum Instability

- Higgs effective potential
  \[ V(\phi) \approx \lambda(\phi)\phi^4 \]

- Becomes negative at \( \phi > \phi_c \approx 10^{10}\) GeV

- True vacuum at Planck scale?

- Current vacuum metastable against quantum tunnelling

- Barrier at
  \[ \phi_{\text{bar}} \approx 4.6 \times 10^{10} \text{ GeV}, \text{ height } V(\phi_{\text{bar}}) \approx (4.3 \times 10^9 \text{ GeV})^4 \]
Tunneling Rate

- Bubble nucleation rate:
  - $\Gamma \sim e^{-B}$, where
  - $B =$ “bounce” action (Coleman 1977)
  - Solution of Euclidean eoms
- Constant $\lambda < 0$:
  $$\phi(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}$$
- Action $B = \frac{8\pi^2}{3|\lambda|}$
- When $\lambda$ runs, $B \approx \frac{8\pi^2}{3|\lambda_{\text{min}}|} \approx 1800$ and $\Gamma \sim \mu_{\text{min}}^4 e^{-B}$
- Depends sensitively on Higgs and top masses
Number of bubbles in past lightcone: \( \langle \mathcal{N} \rangle \approx 0.125 \Gamma / H_0^4 \)

If \( \langle \mathcal{N} \rangle \ll 1 \), no contradiction – Metastable
Curved spacetime:
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \xi R \phi^\dagger \phi \]
(Chernikov&Tagirov 1968)

Symmetries allow one more renormalisable term:
Higgs-curvature coupling \( \xi \)

Required for renormalisability, runs with energy – Cannot be set to zero!

Last unknown parameter in the Standard Model
Running $\xi$

\[ \mu \frac{d\xi}{d\mu} = \left( \xi - \frac{1}{6} \right) \frac{12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2}{16\pi^2} \]

- Becomes negative if $\xi_{EW} = 0$
- Conformal value $\xi = 1/6$
- RG invariant at 1 loop but not beyond
Curved spacetime:
\[ \mathcal{L} = \mathcal{L}_{SM} + \xi R \phi^\dagger \phi \]

Ricci scalar $R$ very small today
\[ \implies \text{Difficult to measure } \xi \]

Colliders: Suppresses Higgs couplings (Atkins&Calmet 2012)
- LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$
- Future (?) ILC: $|\xi| \lesssim 4 \times 10^{14}$

In contrast, $R$ was high in the early Universe
Expected number of bubbles ($d\eta = dt/a$):

$$\langle N \rangle = \frac{4\pi}{3} \int_{\eta_0}^{\eta_0} d\eta \ a(\eta)^4 (\eta_0 - \eta)^3 \Gamma(\eta)$$
Assume:
- Light, subdominant Higgs
- Inflaton decoupled from the Higgs

Effective Higgs mass term $m_{\text{eff}}^2(t) = m_H^2 + \xi R(t)$

Ricci scalar in FRW spacetime:

$$R = 6 \left( \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right) = 3(1 - 3w)H^2$$

- Radiation dominated $w = 1/3$ \quad $R = 0$
- Matter dominated $w = 0$ \quad $R = 3H^2$
- Inflation / de Sitter $w = -1$ \quad $R = 12H^2$
Higgs Fluctuations from Inflation

- Inflation: $H \lesssim 9 \times 10^{13}$ GeV (Planck+BICEP2 2015)
- Equilibrium field distribution (Starobinsky&Yokoyama 1994)
  
  \[ P(\phi) \propto \exp \left[ -\frac{8\pi^2}{3H^4} V(\phi) \right] \]

- Tree-level potential
  \[ V(\phi) = \lambda(\phi^2 - v^2)^2 \]

- Nearly scale-invariant fluctuations with amplitude $\phi \sim \lambda^{-1/4}H$
Higgs Fluctuations from Inflation

- Equilibrium $P(\phi) \propto \exp \left[ -\frac{8\pi^2}{3H^4} V(\phi) \right]$

- Running $\lambda$: Fluctuations take the Higgs over the barrier if $H \gtrsim \phi_{\text{bar}} \approx 10^{10}\text{GeV}$ (Espinosa et al. 2008; Lebedev&Westphal 2013; Kobakhidze&Spencer-Smith 2013; Fairbairn&Hogan 2014; Hook et al. 2014)

- Does this imply $H \lesssim 10^{10}\text{GeV}$?
Higgs During Inflation

- Inflation: Constant $R = 12H^2$
- Effective mass term
  
  $$m_{\text{eff}}^2 = m_H^2 + \xi R = m_H^2 + 12\xi H^2$$

- Tree level: (Espinosa et al 2008)
  - $\xi > 0$: Increases barrier height
    Makes the low-energy vacuum more stable
  - $\xi < 0$: Decreases barrier height
    Makes the low energy vacuum less stable

- $H$ contributes to loop corrections:
  For $H \gg \phi$, $V(\phi) \approx \lambda(H)\phi^4 \Rightarrow \text{No barrier!}$ (HMNR 2014)
Effective Potential in Curved Spacetime

One-loop computation in de Sitter:

\[
V_{\text{SM}}^{\text{eff}}(\varphi(\mu)) = -\frac{1}{2} m^2(\mu)\varphi^2(\mu) + \frac{\xi(\mu)}{2} R\varphi^2(\mu) + \frac{\lambda(\mu)}{4} \varphi^4(\mu) + V(\mu) - 12\kappa(\mu)H^2 + \alpha(\mu)H^4
\]

\[
+ \frac{1}{64\pi^2} \sum_{i=1}^{31} \left\{ n_i M_i^4(\mu) \left[ \log \left( \frac{|M_i^2(\mu)|}{\mu^2} \right) - d_i \right] + n'_i H^4 \log \left( \frac{|M_i^2(\mu)|}{\mu^2} \right) \right\}. \quad (5.3)
\]

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<th>$n_i$</th>
<th>$d_i$</th>
<th>$n'_i$</th>
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(MNRS 2018)
One-loop computation for $\xi = 0$ (in units of $\mu_{\text{inst}} \approx 6.6 \times 10^9$ GeV)
(De-)Stabilising the Potential

\[ V_{\text{max}} \sim \frac{36 \xi^2 H^4}{|\lambda|} \]

Minkowski

\[ \xi \lesssim 0.02 \]

\[ \xi \gtrsim 0.06 \]
If $H \gtrsim \mu_{\text{inst}} = 6.6 \times 10^9 \text{GeV}$ and there is no new physics, vacuum stability during inflation requires $\xi \gtrsim 0$
In real inflationary models, $H$ depends on time:
Affects decay rate $\Gamma$ and volume of past light cone

Simplest case:
Quadratic chaotic inflation

$$V(\chi) = \frac{1}{2} m^2 \chi^2$$

Most bubbles produced near the end of inflation

Stability requires
$$\xi \gtrsim 0.044$$
(Mantziris, Markkanen & AR, in progress)
Quantum Tunneling

- Multiple coexisting solutions (AR&Stopyra, PRD 2018)
- Tunnelling rate $\Gamma \sim e^{-B}$ nearly constant until Hawking-Moss starts to dominate
Multiple Solutions

(AR&Stopyra, PRD 2018)
End of Inflation

- Reheating: Inflation \((R = 12H^2) \rightarrow\) radiation \((R = 0)\)

\[ R(t) = \frac{2m^2 \chi^2 - \dot{\chi}^2}{M_{Pl}^2} \]

- Effective Higgs mass \(m_{\text{eff}}^2 = m_H^2 + \xi R\) oscillates:
  - Parametric resonance ("Geometric preheating")
    (Bassett&Liberati 1998, Tsujikawa et al. 1999)

- \(R\) goes negative when \(\chi \sim 0\)
  - If \(\xi > 0\), Higgs becomes tachyonic (HMNR 2015)
  - Exponential amplification

\[
\langle \phi^2 \rangle_H \sim \frac{2}{3\sqrt{3} \xi} \left( \frac{H}{2\pi} \right)^2 e^{\frac{2\sqrt{\xi} \chi_{\text{ini}}}{M_{\text{Pl}}}} \sim \frac{H^2}{\xi} e^{2\sqrt{\xi}}
\]
Vacuum Decay at the End of Inflation

Not enough growth

Becomes nonlinear

Instability!

\( \Delta h \approx 10 \Lambda_f \)

\( \Delta h \approx 10^{2/3} \Lambda_f \)

Field backreaction

\( \frac{H}{\varphi_{\text{bar}}} \)

\( \xi \)

\( \text{(HMNR 2015)} \)
Lattice Simulations

\[ V(\chi) = \frac{1}{2} m^2 \chi^2, \ M_{\text{top}} = 172.12 \text{ GeV} \]

Figueroa, AR & Torrenti, 2018
Stability depends on top mass and speed of reheating

\[ M_{\text{top}} = 173.34 \text{ GeV}: \text{vacuum decay before } mt = 100 \text{ if } \xi \gtrsim 9 \]
Constraints on $\bar{\chi}$

- Minimal scenario:
  Standard Model + $m^2\chi^2$ chaotic inflation, no direct coupling to inflaton
  \[0.044 \lesssim \bar{\chi} \lesssim 9\]

- 15 orders of magnitude stronger than the LHC bound
  \[|\bar{\chi}| \lesssim 2.6 \times 10^{15}\]

- Caveats:
  - Assumes no direct coupling to inflaton
    - Would still need $|\bar{\chi}| \lesssim O(1)$
  - Assumes no new physics
    - Could stabilise potential altogether
  - Assumes high scale inflation $H \gtrsim 10^9$ GeV