The Physics Project and the Phase I & II Trigger activities in ATLAS of the AUTH/HOU group



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HEP 2019 - Conference on Recent Developments in High Energy Physics and Cosmology 17-20 April 2019, NCSR "DEMOKRITOS", Athens, Greece

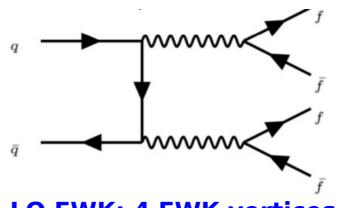
Outline

- A) The Physics Project: diboson production
 - Three recent measurements of WZ and ZZ production at 13 TeV
 - Limits on anomalous Triple Gauge Couplings
 - Vector Boson Scattering and probing of Quartic Gauge Couplings.
 - All results available at: https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults
- B) The Trigger projects: online tracking with hardware
 - The Fast TracKer (FTK) phase I upgrade
 - The Hardware Track Trigger (HTT) phase II upgrade

A) Diboson production - introduction

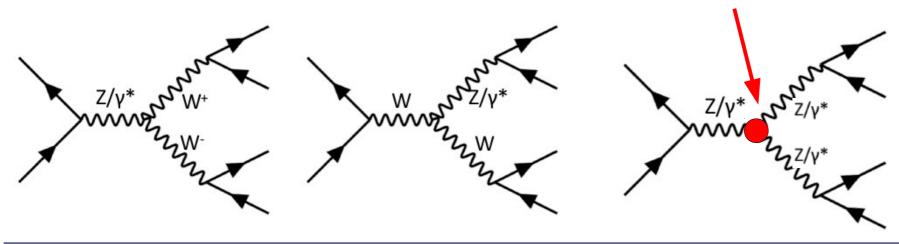
LO EWK WW/WZ/ZZ production diagrams

At Leading Order with all ElectroWeak vertices:

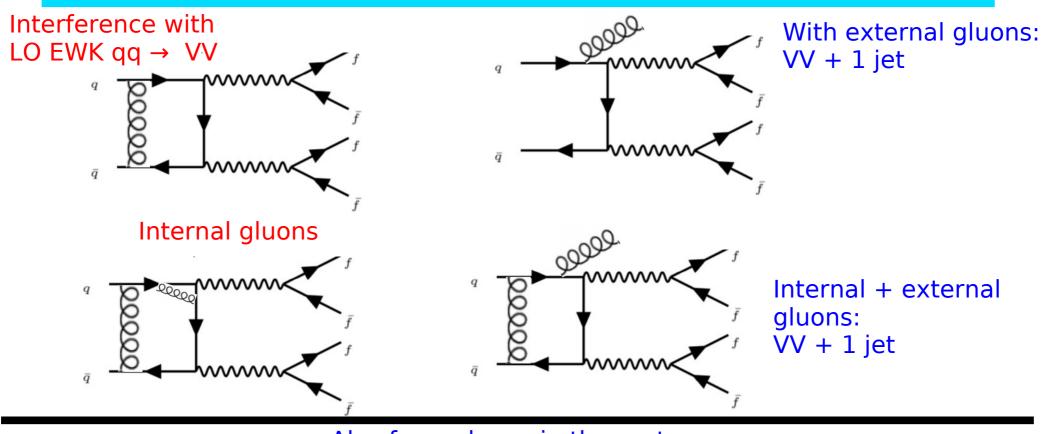


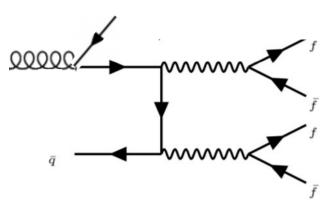
LO EWK: 4 EWK vertices

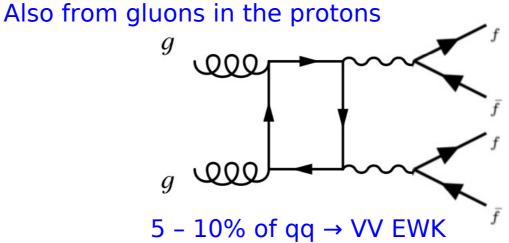
Also, **Triple Gauge Couplings** (not there in SM when all three neutral):



With extra QCD vertices

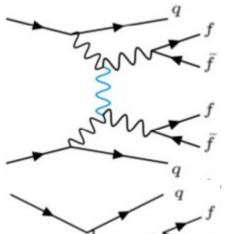






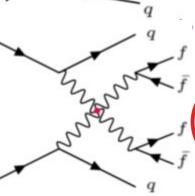
With >=5 EWK vertices, get VV+2jets: "Vector Boson Scattering" signature

Vector Boson Scattering: incoming quarks act as sources of colliding boson beams **Signature: VV + 2 forward jets**



6 EWK vertices: not only vector bosons in the t-channel, but also the Higgs:

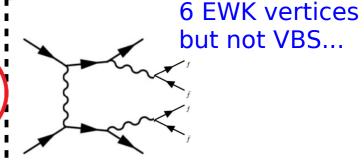
important for not letting the cross section explode at high energies (like the ZWW vertex was needed to limit the WW production cross section at e+ e- collisions at LEP)



5 EWK vertices

Quartic Gauge Couplings:

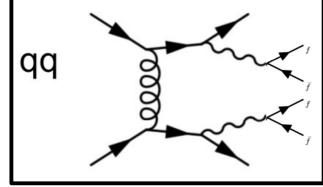
again, SM does not allow all neutral in the quartic vertex

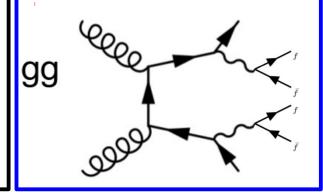


4 EWK + 2 QCD vertices:

same final state

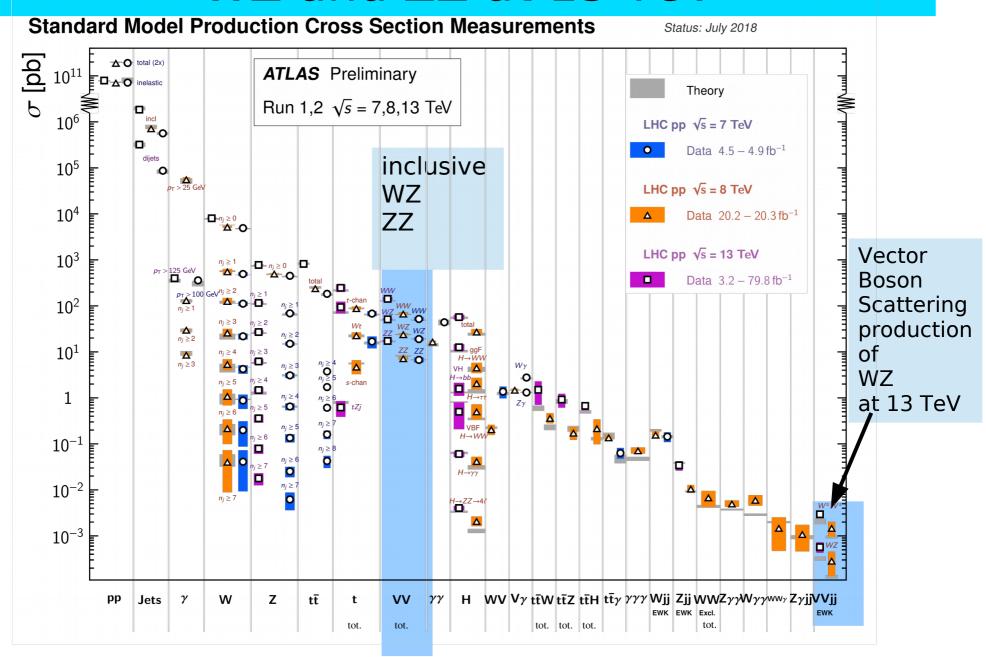
→ important background





+ ..

WZ and ZZ at 13 TeV



https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/SM/

A.2) ZZ and WZ production @ 13 TeV

All 13 TeV measurements so far are with leptonic states:

Increasing cross section

	ZZ	WZ	WW
Signature:	4 leptons	3 leptons + MET(1ν)	3 leptons + MET(2v)
Backgrounds:	WZ + 1 fake lepton; WW / tt / Drell-Yan + 2 fake leptons	Drell-Yan + 1 fake lepton; ZZ (missing lepton); WW / tt +1 fake lepton	tt ; Drell-Yan
Signal purity:	~98%	~79%	~70%

Increasing purity

Thessaloniki (with Hellenic Open University in ZZ) in all ZZ and WZ analyses

A.2.1) ZZ production

"ZZ → 4l cross-section measurements and search for anomalous triple gauge couplings in 13 TeV pp collisions with the ATLAS detector"

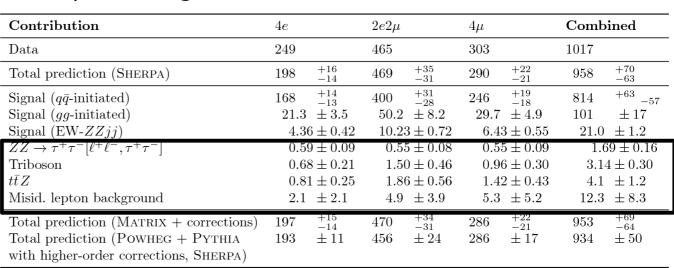
Phys. Rev. D 97 (2018) 032005

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/STDM-2016-15/

$ZZ \rightarrow 4I$

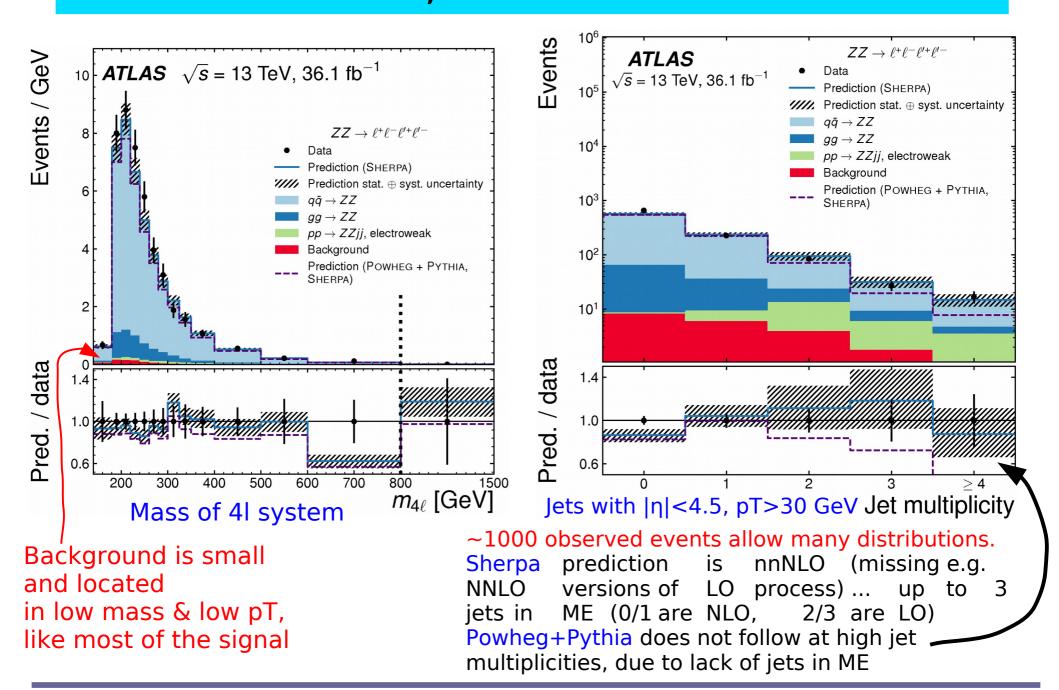
- 36.1 fb⁻¹ (2015+2016) \sqrt{s} = 13 TeV data
- Select events with at least 4 leptons with $|\eta| < 2.7$, pT>20,>15,>10 GeV; the rest >5 GeV
- Only on-shell: 66 < m_| < 116 GeV
- Fully leptonic final state is very clean:
 - 21.2 bkg events (12.3 from fake leptons) out of 958 total event yield predicted
 - Main background from fake leptons (e.g. in
 - Z + jet events)
 - SM processes
 with >4 leptons treated
 as background
 (e.g. ZZZ → 6l)

A very pure channel: bkg ~2% of total



GeV]	140	ATLAS $\sqrt{s} = 13 \text{ TeV}, 36.1 \text{ fb}^{-1}$	Jala everilis
ass [120	- 116 GeV Signal region	משוק
Leading- p_{T} Z cand date mass [GeV]	100		בֿ
ppu	80	101	
Z ca	60	66 GeV	
<i>d-</i> b⊤	40		
adin	20	100	
Le	0(Subleading- p_T Z candidate mass [GeV]	

ZZ → 4I, kinematics etc



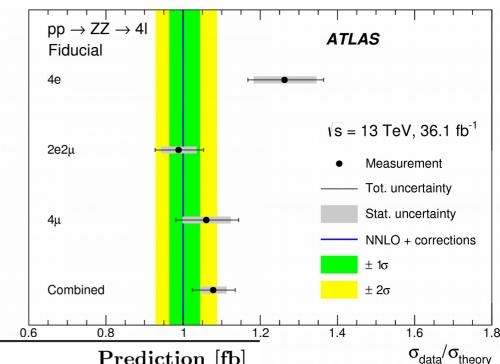
ZZ fiducial cross section

 "Fiducial cross sections" reported: a fraction A of the total, corresponding to the reduced phase-space and the decay channels of the actual measurements (so it includes the Branching Ratios, BR)

 $\sigma^{fid}(pp \rightarrow VV + X, V \rightarrow leptons) = \frac{N - B}{L * C}$

N-B: Observed events – bkg estimate C: detector efficiency , L: integrated luminosity

- "NNLO + corrections":
 - NNLO calculation from Matrix
 - gg-initiated contribution (Sherpa)
 multiplied by a global NLO
 correction factor of 1.67.
 - global NLO EW correction factor of 0.95, except to the gg-initiated loop-induced contribution, and the contribution of around 2.5% from EW-ZZjj generated with Sherpa is added.



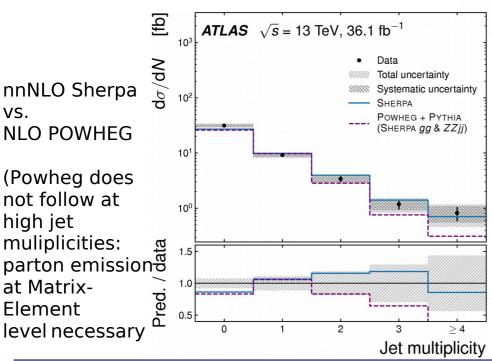
$\mathbf{Channel}$	Measurement [fb]	Prediction	- -
4e	$13.7_{-1.0}^{+1.1} \left[\pm 0.9 \text{ (stat.) } \pm 0.4 \text{ (syst.) } \right]_{-0.4}^{+0.5} \left[\pm 1.0 \text{ (stat.) } \pm 0.6 \text{ (syst.) } \right]_{-0.6}^{+0.7} \left[\pm 1.0 \text{ (stat.) } \pm 0.6 \text{ (syst.) } \right]_{-0.6}^{+0.7}$	$10.9^{+0.5}_{-0.4}$	•Statistics limited, dominant
$2e2\mu$	$20.0^{+1.4}$ [± 1.0 (gtat) ± 0.6 (gyat) ± 0.7 (lymin)	$91.9^{+0.9}$	systematics lepton
4μ	$11.5^{+0.9}_{-0.9}$ [± 0.7 (stat.) ± 0.4 (syst.) ± 0.4 (lumi.)]	$10.9^{+0.5}_{-0.4}$	officiencies (tension in 46
Combined	$46.2^{+2.5}_{-2.3}$ [± 1.5 (stat.) $^{+1.2}_{-1.1}$ (syst.) $^{+1.6}_{-1.4}$ (lumi.)]	$42.9_{-1.5}^{+1.9}$	channel)

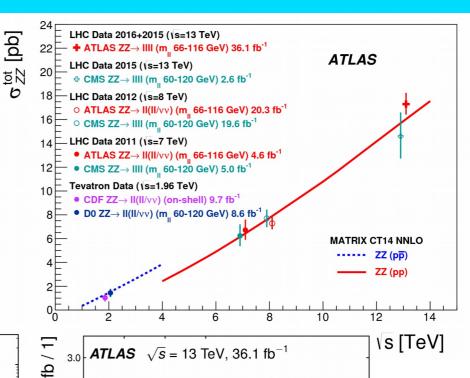
ZZ total cross section & differential fiducial

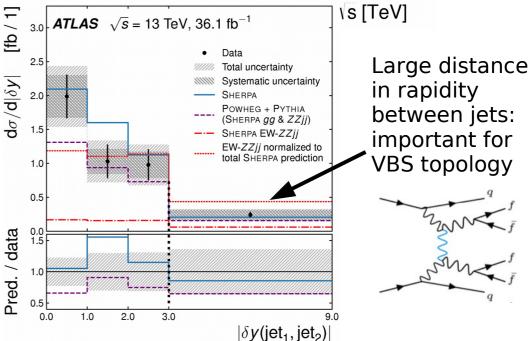
- Fiducial measurement extrapolated to "total cross sections" by correcting for the BRs and the Acceptance, A.
- Total pp → ZZ production:

 17.3 ± 0.9 [± 0.6 (stat.) ± 0.5 (syst.) ± 0.6 (lumi.)] pb

 (Fiducial) differential crosssections provided in 20 variables, a lot of them for first time:







A.2.2) Inclusive WZ production

"Measurement of W^{\pm} Z production cross sections and gauge boson polarisation in pp collisions at s= $\sqrt{13}$ TeV with the ATLAS detector"

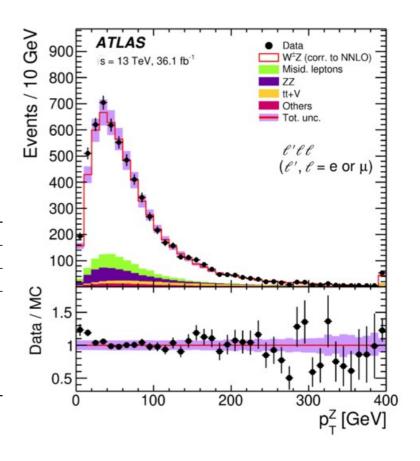
https://arxiv.org/abs/1902.05759, submitted to EPJC

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/STDM-2018-03/

$WZ \rightarrow 3l v$: fiducial cross section

- 36.1 fb⁻¹ $\sqrt{s} = 13$ TeV data
- Includes 3e, 3μ, μ2e, and e2μ final states
- Biggest uncertainty from fake-lepton estimate (3%, from a total of ~7%)

Channel	$e\epsilon$	ee	$\mu\epsilon$	ee	$e\mu$	μ	$\mu \mu$	$\iota \mu$	A	X ll
Data	12	79	128	81	16	71	19:	29	61	.60
Total Expected	1221	± 7	1281	± 6	1653	± 8	1830	± 7	5986	± 14
WZ	922	± 5	1077	± 6	1256	± 6	1523	± 7	4778	± 12
Misid. leptons	138	± 5	34	± 2	193	± 5	71	± 2	436	± 8
ZZ	86	± 1	89	± 1	117	± 1	135	± 1	426	\pm 3
$ZZ \ tar{t} + { m V}$	50.0	± 0.7	54.0	± 0.7	56.1	± 0.7	63.8	± 0.8	225	\pm 1
tZ	23.1	± 0.4	24.8	± 0.4	28.8	± 0.4	33.5	± 0.5	110	\pm 1
VVV	2.5	± 0.1	2.8	± 0.1	3.2	± 0.1	3.6	± 0.1	12.0	± 0.5



From event count

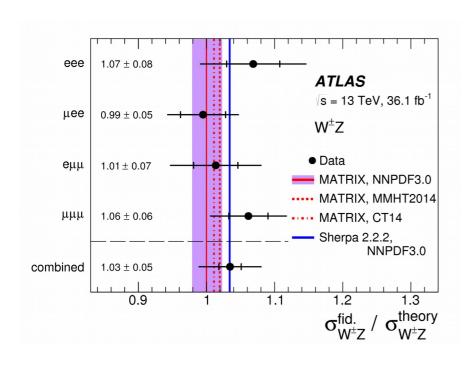
(Nobs = 6160, with NexpectedBkg = 1208)

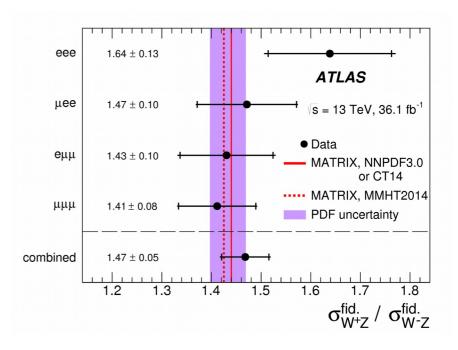
to fiducial cross section:

$$\sigma_{W^{\pm}Z \to \ell' \nu \ell \ell}^{\text{fid.}} = 63.7 \pm 1.0 \text{ (stat.)} \pm 2.3 \text{ (syst.)} \pm 1.4 \text{ (lumi.) fb.}$$

$WZ \rightarrow 3l v : ratios of W+ Z \& W- Z$

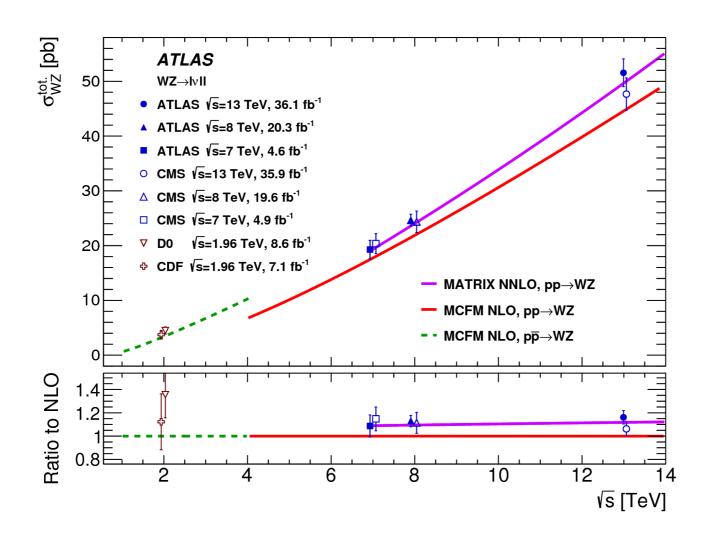
- 36.1 fb⁻¹ $\sqrt{s} = 13$ TeV data
- W+ Z and W- Z compared to theory and to each other (ratio sensitive to PDFs)
- Theoretical predictions: NNLO QCD from MATRIX & NLO QCD from Sherpa





$WZ \rightarrow 3l v$: total cross section

- 36.1 fb⁻¹ $\sqrt{s} = 13$ TeV data
- W Z cross section extrapolated to total phase-space and comparison to theory

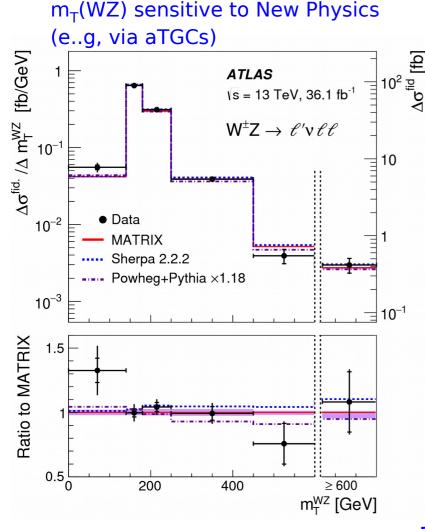


The need for the extra order in QCD calculations is evident.

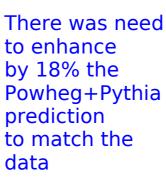
Measurements are in agreement with these NNLO QCD predictions

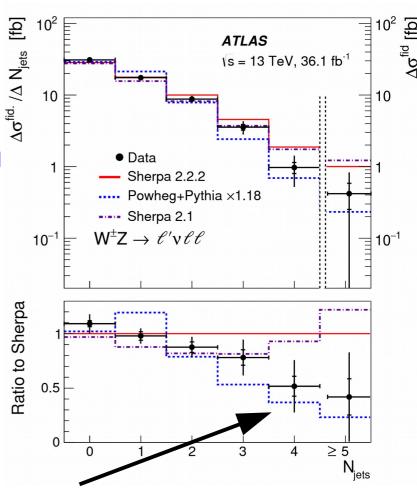
$WZ \rightarrow 3l \ v$: differential fiducial cross sections

• Differential fiducial cross section measured as well: in p_T^Z , m_T^{WZ} , N_{jets}



Njets sensitive to QCD modelling





Powheg needs extra jets in Matrix Element (Powheg+Pythia does not follow data, While Sherpa2.2 does, as we also saw in ZZ)

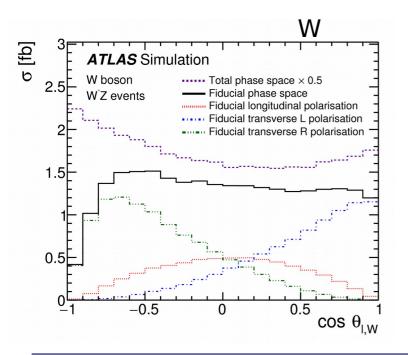
WZ → 3I v : boson polarization fractions

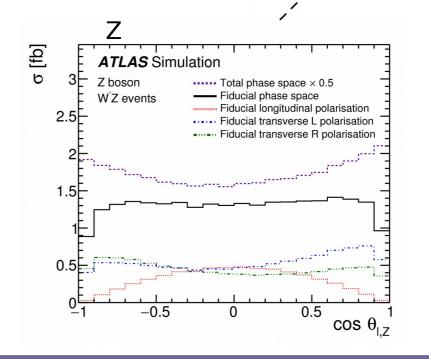
- Gauge-Boson polarization determined from angular distribution of decay leptons
- For the W (and similarly for the Z):

$$\frac{1}{\sigma_{W^{\pm}Z}}\frac{d\sigma_{W^{\pm}Z}}{d\cos\theta_{l,W}} = \frac{3}{8}f_L(1\mp\cos\theta_{l,W}) + \frac{3}{8}f_R(1\pm\cos\theta_{l,W}) + \frac{3}{4}f_0\sin^2\theta_{l,W}$$

 $\theta_{l,W}$ \Rightarrow decay angle in the W[±] rest frame of the charged lepton w.r.t the W[±] in the W[±]Z centre-of-mass frame

 $f_L, f_R, f_0 \Rightarrow helicity fractions$





Templates at particle level

q

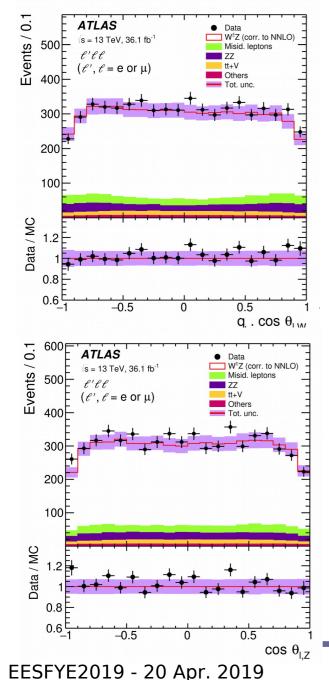
production

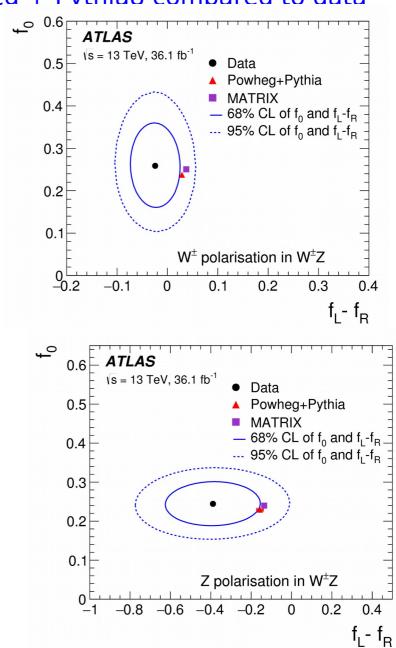
axis of the lv

Already sculpted by pT and η fiducial requirements

$WZ \rightarrow 3I v$: boson polarization fractions

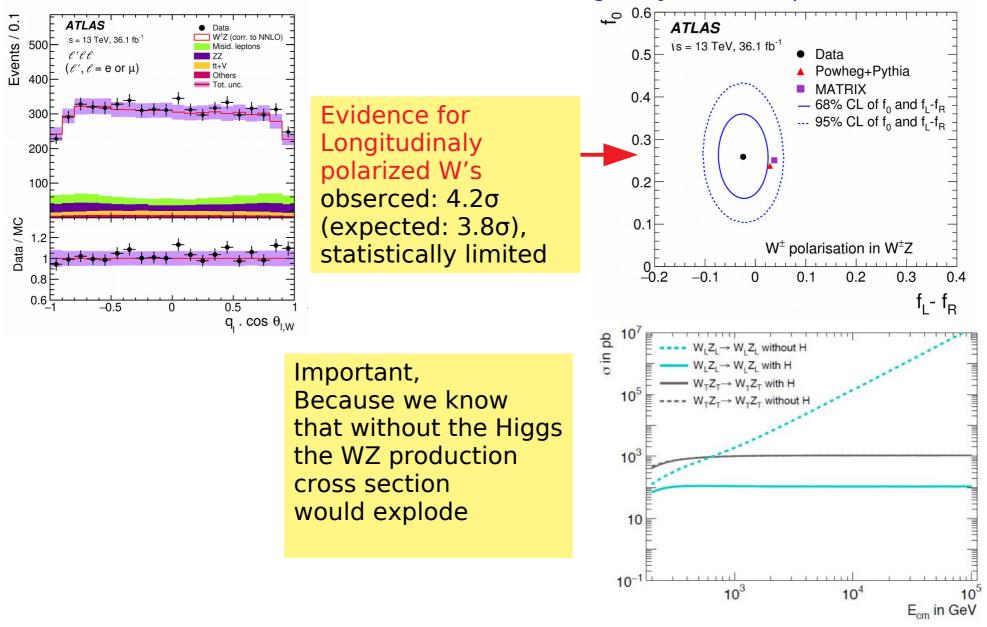
Templates at reconstruction level from Powhea + Pvthia8 compared to data





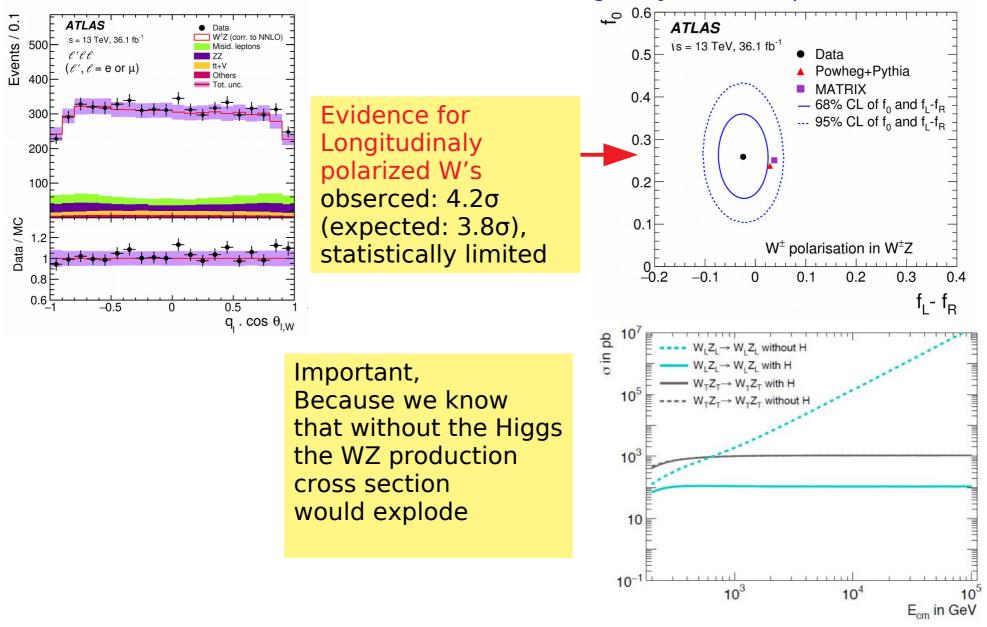
WZ → 3I v : boson polarization fractions

Templates at reconstruction level from Powheg + Pythia8 compared to data



WZ → 3I v : boson polarization fractions

Templates at reconstruction level from Powheg + Pythia8 compared to data



A.2.3) Electroweak WZ production

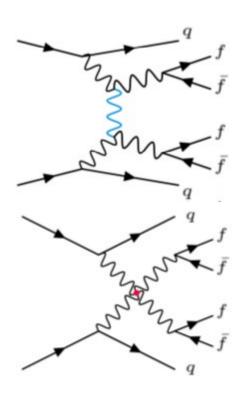
"Observation of electroweak W[±]Z boson pair production in association with two jets in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS Detector"

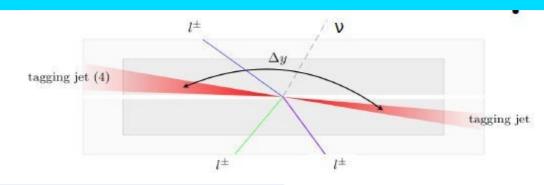
https://arxiv.org/abs/1812.09740 submitted to PLB

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/STDM-2015-20/

WZ → 3I v : EWK-enhanced production in VBS topology

• 36.1 fb⁻¹ $\sqrt{s} = 13$ TeV data





	SR
Data	161
Total predicted	167 ± 11
WZi i-EW (signal)	44 ±11
<i>WZjj</i> −QCD	91 ± 10
Misid. leptons	7.8 ± 3.2
ZZjj-QCD	11.1 ± 2.8
tZj	6.2 ± 1.1
$t\bar{t} + V$	4.7 ± 1.0
ZZjj–EW	1.80 ± 0.45
VVV	0.59 ± 0.15

$$\sigma_{W^{\pm}Zjj}^{\text{fid.}} = 68 \pm 0.25 \text{ fb}$$

$$\sigma_{WZjj-\text{EW}}^{\text{fid.}} = 0.57^{+0.14}_{-0.13} (\text{stat.}) {}^{+0.05}_{-0.04} (\text{exp. syst.}) {}^{+0.05}_{-0.04} (\text{mod. syst.}) {}^{+0.01}_{-0.01} (\text{lumi.}) \text{ fb}$$

For details, see Iro Koletsou's talk, later today

A 5.3σ Observation (3.2 expected)

A lesson from comparing the measurements to theory perdictions

- Overall good agreement with the Standard Model
 - NNLO QCD improves agreement substantially
 - NNLO reduces uncertainty to 10~20% from NLO at 60% (arXiv: 1604.08576)
- Almost all recent measurements are limited by systematic uncertainties (only ZZ is almost equal to statistics)

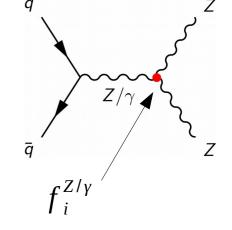
These and many more results in:

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/SM/

A.3. Anomalous Gauge Couplings (Triple or Quartic : aTGCs or aQGCs)

anomalous Triple Gauge Couplings (aTGCs), 1

- "Traditional", effective Lagrangian, approach: add terms to the SM Lagrangian to describe the Triple Gauge vertices; the deviation of the triple vector boson couplings from the SM predicted values are introduced as dimensionless anomalous couplings:
 - For the WW+Z/ γ vertices, 5 parameters: Δg_1^Z , $\Delta \kappa_Z$, λ_Z , $\Delta \kappa_\gamma$, λ_γ * Just 3 in LEP scenario: $\lambda_\gamma = \lambda_Z \\ \Delta g_1^Z = \Delta \kappa_Z + \tan^2 \theta_W \Delta \kappa_\gamma$
 - For the ZZ+Z/ γ vertices: f_4^{γ} , f_4^{Z} , f_5^{γ} , f_5^{Z}
- The contribution of anomalous couplings to the diboson production cross section grows with the partonic centre-of-mass energy s-hat, and quadratically with the TGC value.
- Anomalous TGCs will lead to excesses in high-end tails of sensitive observables, related to the s-hat of the partonic system



With one aTGC active: $d\sigma_{SM+TGC} = F_0 + fF_1 + f^2F_2$

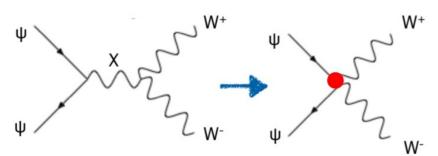
anomalous Triple Gauge Couplings (aTGCs), 2

• Effective Field Theory (EFT) approach: Standard Model is the low energy limit of a more fundamental theory at scale $\Lambda \gg \sqrt{s}$

Beyond SM Theory

EESFYE2019 - 20 Apr. 2019

Low Energy limit



At low energies (E $<<\Lambda$) interactions between SM fields only look like Fermi's contact interaction (which was indeed valid when much below W mass scale)

Add to the SM Lagrangian a linear combination of operators of mass dimension higher than four. Independent operators can lead to anomalous triple vector boson couplings.

$$L_{EFT} = L_{SM} + \sum_{d \ge 5} L_{EFT}^d$$
 with $L_{EFT}^d = \sum_i \frac{C_i^d}{\Lambda^{d-4}} O_i^d$

The dimensionless coefficients C_i represent the strength of the new couplings.

Charged TGC: first contributing operators have dimension $6 \Rightarrow$ coupling parameters c/ Λ^2

Neutral TGC: first contributing operators have dimension $8 \Rightarrow$ coupling parameters c/ Λ^4

There is a one-to-one mapping between EFT coupling parameters and traditional aTGCs e.g:

Traditional ⇔ EFT

$$g_1^Z = 1 + c_W \frac{m_Z^2}{2\Lambda^2}$$

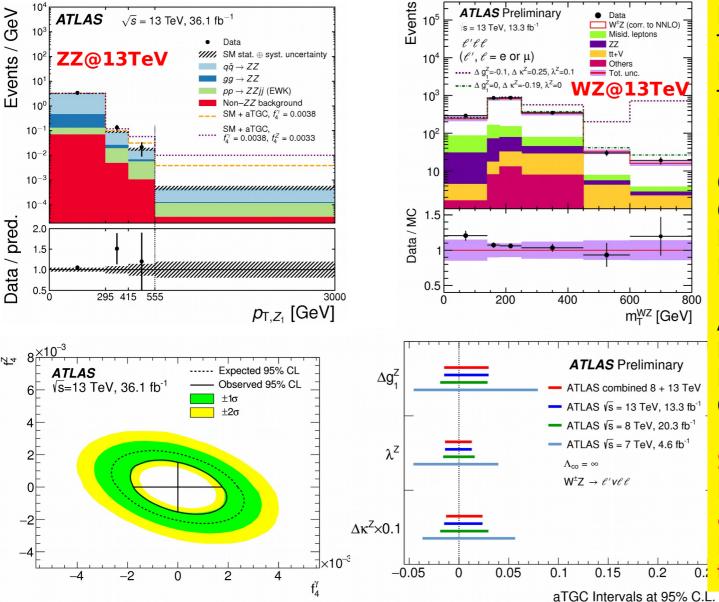
$$\kappa_{\gamma} = 1 + (c_W + c_B) \frac{m_W^2}{2\Lambda^2}$$

$$\kappa_Z = 1 + (c_W - c_B \tan^2 \theta_W) \frac{m_W^2}{2\Lambda^2}$$

$$\lambda_{\gamma} = \lambda_Z = c_{WWW} \frac{3g^2 m_W^2}{2\Lambda^2}$$

aTGCs from ZZ, WZ

 Anomalous TGCs will lead to enhanced event yields in high-end tails of sensitive observables, related to the s-hat of the partonic system system.



In the EFT approach,
There are many operators
Potentially contributing
To the Beyond the SM
Part of the Lagrangian:

Check which Operators Can single-handendly Produce an observable Enhancement

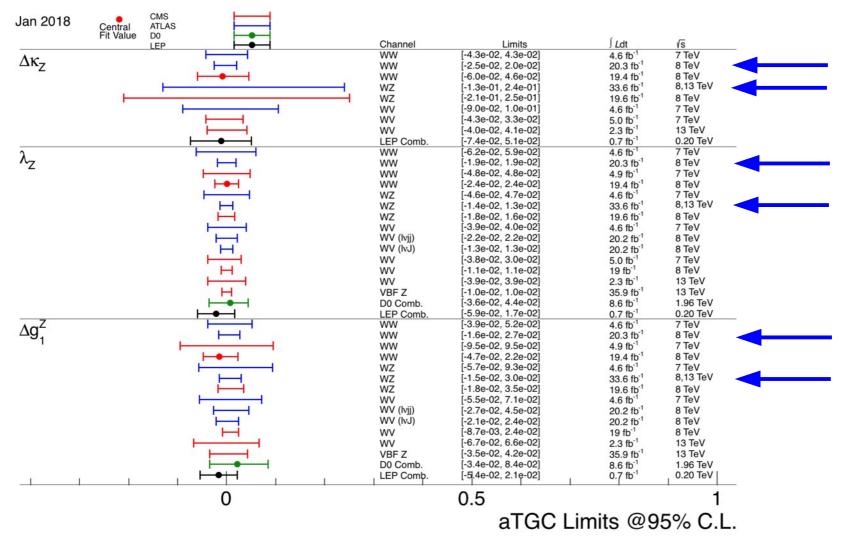
Absence of such an Enhancement set Limits
On their contribution

See Despina Sampsonidou and Eirini Kasimi's talks yesterday

Charged aTGCs status

Limits comparable between ATLAS and CMS, for similar datasets

These aTGC limits are better than LEP results by now



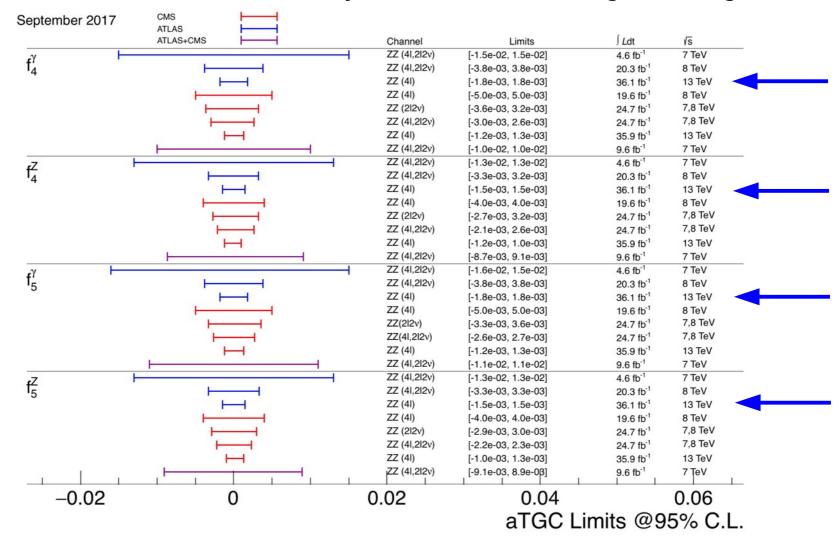
Compilation of ATLAS and CMS results on Triple and Quartic Gauge Couplings at:

https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC

Neutral aTGCs status

Limits comparable between ATLAS and CMS, for similar datasets

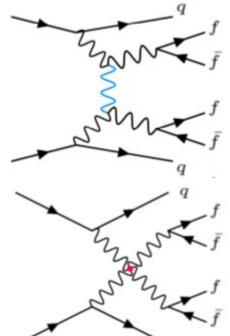
These aTGC limits constrain a variety of BSM models at higher energies



Compilation of ATLAS and CMS results on Triple and Quartic Gauge Couplings at: https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC

VBS and anomalous Quartic Couplings

Vector Boson Scattering: incoming quarks act as sources of colliding boson beams **Signature: VV + 2 forward jets**



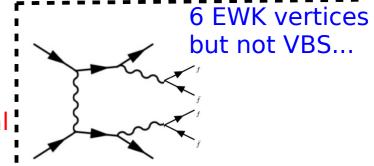
6 EWK vertices: not only vector bosons in the t-channel, but also the Higgs:

important for not letting the cross section explode at high energies (like the ZWW vertex was needed to limit the WW production cross section at e+ e- collisions at LEP)



Quartic Gauge Couplings:

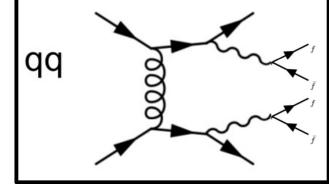
again, SM does not allow all neutral in the quartic vertex

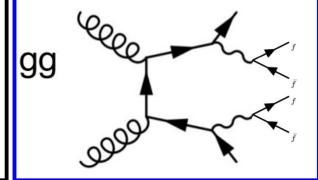


4 EWK + 2 QCD vertices:

same final state

→ important background





+ ..

VBS and anomalous Quartic Couplings

- Searches of anomalous QGC always assume aTGC=0
- •The first operators leading to aQGC but no aTGC have dimension 8
- \Rightarrow coupling parameters c/ Λ^4

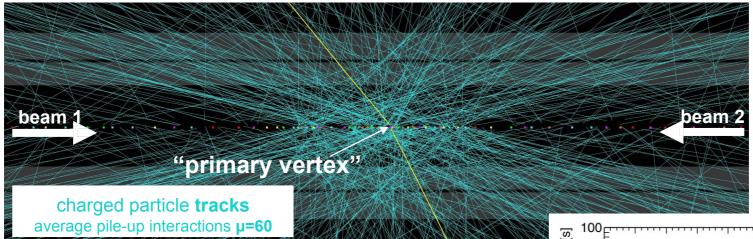
No time here to show results; You'll see dedicated discussion on VBS physics by Iro Koletsou

On ZZ EWK production, wihich is not yet observed firmely:

- * CMS has ~seen signs of EWK production of ZZ \rightarrow 4l (~2.7 σ , expected 1.6 σ) and set limits on aQGC parameters with 36/fb of data
- * In ATLAS, we are working with HOU on ZZ with full data set $\sim\!165$ /fb to establish firmely the EWK production of ZZ , and set limits on aQGCs.

B) Trigger activities in Phases I and II

Motivation for helping the online tracking



CPU time to reconstruct all tracks in an event → explodes for high pile-up

Trigger events in 2 steps: Level 1 (L1: hardware) and then a High Level Trigger (HLT)

Help the farm with Hardware-based tracking

L1 rate: 100 kHz accepts → the HLT has to cope with thesc.

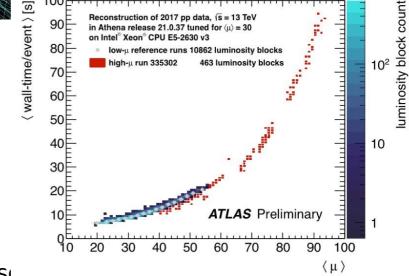
Help with the Fast TracKer (FTK): do full-tracking in all L1 accepted events (*all, can

be tuned with L1 trigger-type)

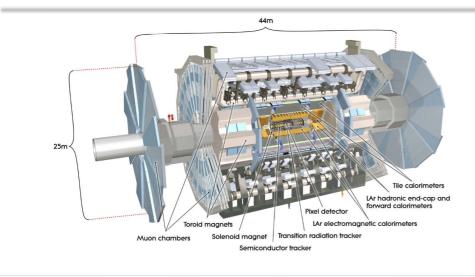
In phase-II , L1 accepts: >=1 MHz

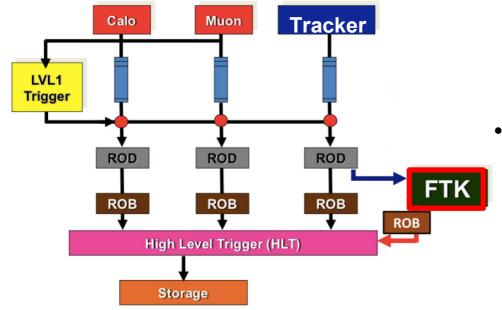
Hardware Track Trigger (HTT): do full-event or regional tracking on-demand

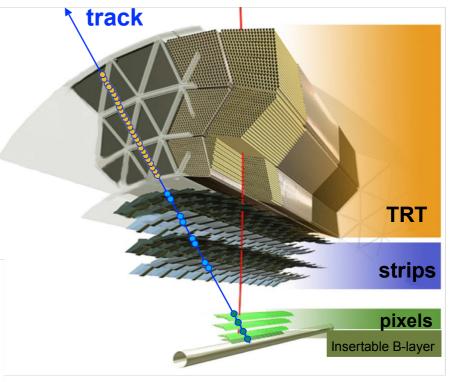
Option to do regional at Level1



FTK







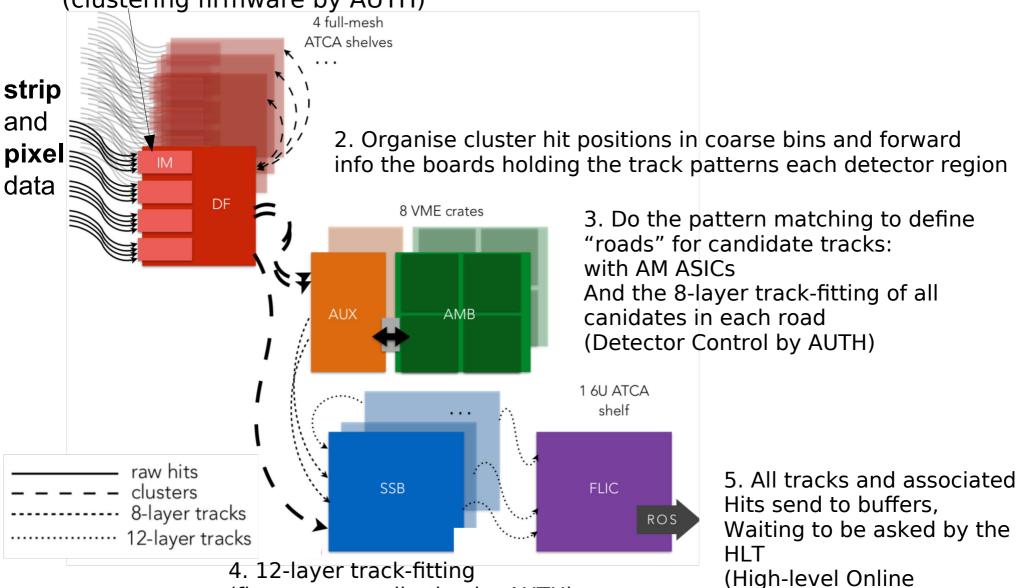
 FTK uses ASICS (AM chips) and FPGAs to reconstruct tracks with p_T > 1 GeV over full detector

Tracking is performed in **two** stages:

- 1st stage considers 8 tracker layers
- 2nd stage extends 8-layer tracks to all
 12 layers

FTK Hardware

1. Cluster pixels togrther to estimate points particles cross each detector layer (clustering firmware by AUTH)



(firmware contribution by AUTH)

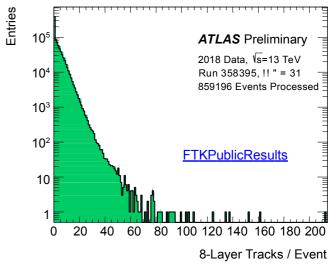
K. Kordas - AUTH/HOU in ATLAS physics and trigger

Monitoring by HOU)

Status of project

- System hardware is complete for input and output borads
- Enough processing boards for pattern matching and track fitting to full barrel of ATLAS. Boards for end-caps coming along.
- System in commissioning now:
 - Special high pile-up μ = 82 commissioning run collected October 2018
- One of the 64 geometrical towers equiped and ran stably for ~2 hours and outputted tracks to ATLAS special data stream for trigger development and rate predictions
- Collected ~0.5M FTK tracks





- 1st tracking stage is outputting tracks
- Validation of AUX firmware

Summary & Conclusions

- Full programme of ZZ and WZ measurements by AUTH/HOU
 - SM diboson production is often a background to BSM physics searches
- Shown here: electroweak diboson production (WZ,ZZ)
 - Fully-leptonic final states are the first measurements we do of these processes
- These measurements have challenged theorists to compute predictions to NNLO and beyond
 - So far, theorists (and the Standard Model) have risen to that challenge
- No evidence yet of enhancement of these processes from BSM physics
 - Targeting high s-hat regions we have continued to set limits on anomalous Triple Gauge boson Coupling and Quartic Gaugeboson Couplings, in the EFT framework now.
- Trigger: tracking with hardware

Summary & Conclusions

- Full programme of ZZ and WZ measurements by AUTH/HOU
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Thank you

ZZ production cross section

$$N_{sigal}(pp \rightarrow ZZ \rightarrow 4l) = L \cdot \sigma^{tot}(pp \rightarrow ZZ) \cdot BR(ZZ \rightarrow 4l) \cdot AC$$
Total cross section

Measured in a reduced phase-space (geometrical & kinematic requirements on the decay products) $4l = \{4e \text{ or } 4\mu \text{ or } 2e2\mu\}$

Total cross section for ZZ production

Acceptance correction $A = \frac{Fiducial\ events}{Total\ events}$ for the geometrical & kinematic criteria.

 $C = \frac{Reconstructed\ events}{Generated\ fiducial\ events}$

Efficiency correction for detector ability to reconstruct these objects

1. We measure a "fiducial cross section", which corresponds to the reduced phase-space of the actual measurement. * This is a fraction of the total:

$$\sigma^{fiducial}(pp \rightarrow ZZ \rightarrow 4l) = \frac{N_{obs} - N_{bkg}}{L \cdot C}$$

$$\sigma^{fiducial}(pp \rightarrow ZZ \rightarrow 4l) = \sigma^{tot}(pp \rightarrow ZZ) \cdot BR(ZZ \rightarrow 4l) \cdot A$$

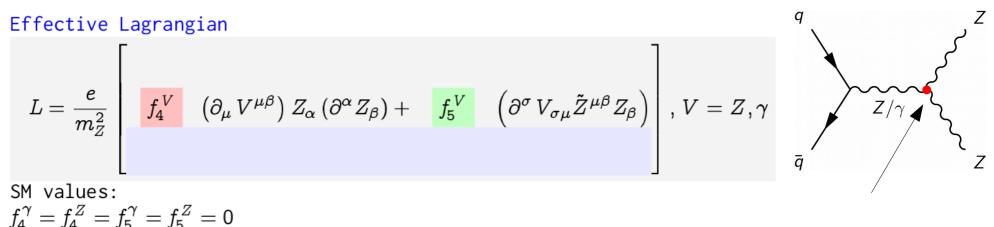
2. We then extrapolate to the "total cross section" for ZZ production by extrapolating the leptons to the full phase-space,

$$\sigma^{tot}(pp \rightarrow ZZ \rightarrow 4l) = \frac{N_{obs} - N_{bkg}}{L(BR(ZZ \rightarrow 4l) \cdot A) \cdot C}$$

and correcting for the BR(ZZ \rightarrow 4l) \sim 4 * (3.4% * 3.4%) for 4e, 4 μ and 2e2 μ together

Contribution of neutral aTGCs to the cross section

 Traditionally, effective Lagrangian used to include effect of aTGCs: e.g., G.L.Gounaris et al: PRD61 073013; Bauer, Reiwater: PRD 62, 113011



ZZ cross section enhanced by aTGCs with ~quadratic dependence on them

With one aTGC active: $d\sigma_{SM+TGC} = F_0 + fF_1 + f^2F_2$

$$d\sigma_{SM+TGC} = \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline F_{00} & +f_4^{\gamma} & F_{01} & +f_4^{Z} & F_{02} & +f_5^{\gamma} & F_{03} & +f_5^{Z} & F_{04} \\ & + & \left(f_4^{\gamma}\right)^2 & F_{11} & +f_4^{\gamma} f_4^{Z} & F_{12} & +f_4^{\gamma} f_5^{\gamma} & F_{13} & +f_4^{\gamma} f_5^{Z} & F_{14} \\ & + & \left(f_4^{Z}\right)^2 & F_{22} & +f_4^{Z} f_5^{\gamma} & F_{23} & +f_4^{Z} f_5^{Z} & F_{24} \\ & + & \left(f_5^{\gamma}\right)^2 & F_{33} & +f_5^{\gamma} f_5^{Z} & F_{34} \\ & + & \left(f_5^{Z}\right)^2 & F_{44} \end{array}$$