A new mechanism for dynamical nonperturbative generation of elementary fermion masses

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List of publications

- **S. Capitani, P.D., M. Garofalo, R. Frezzotti, B. Kostrzewa, F. Pittler, G.C. Rossi, C. Urbach**
A novel mechanism for dynamical generation of elementary fermion mass: lattice evidence
sub. to PRL, arXiv:1901.09872 [hep-th]

- **R. Frezzotti and G.C. Rossi**
Nonperturbative mechanism for elementary particle mass generation
PRD 92 (2015) 054505

- **R. Frezzotti. M. Garofalo and G.C. Rossi**
Nonsupersymmetric model with unification of electroweak and strong interactions
PRD 93 (2016) 105030

- **S. Capitani, G.M. de Divitiis, P.D., M. Garofalo** *et al.*
Non-perturbative generation of elementary fermion masses: a numerical study
PoS LATTICE2018, arXiv:1811.10327 [hep-lat]

- **S. Capitani, G.M. de Divitiis, P.D., M. Garofalo** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup
EPJ Web Conf. 175 (2018) 08008

- **S. Capitani, G.M. de Divitiis, P.D., M. Garofalo** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: Numerical results
EPJ Web Conf. 175 (2018) 08009

- **S. Capitani, P.D., M. Garofalo, R. Frezzotti** *et al.*
Check of a new non-perturbative mechanism for elementary fermion mass generation
PoS LATTICE2016 (2016) 212

- **R. Frezzotti and G.C. Rossi**
Dynamical mass generation
PoS LATTICE2013 (2014) 354

Outline

- Motivation
- Dynamical generation of elementary fermion masses
- A toy-model
- Lattice study of the toy-model
- Numerical evidence giving support to the mechanism
- Conclusions & Outlook

Motivation

- The double face of the **SM**:
  - **successful**, renormalisable quantum field theory, for description and predictions in the EW and strong interaction physics.
  - **incomplete**: several fundamental phenomena are not satisfactorily or not at all described by/within it:

  - neutrino masses and mixings
  - matter-antimatter asymmetry and need for stronger CP violation
  - dark matter & energy

  - Higgs mechanism trades masses for Yukawa couplings
  - masses are accomodated by fitting to experimental data
  - unnaturalness of the higgs mass
  - huge mass hierarchy e.g. $m_e \sim 10^{-6} m_t$
  - large number of input parameters ($O(20)$ + neutrino sector)
- **our focus**: a deeper insight into the elementary particle mass origin.

Dynamical generation of elementary fermion mass

- Mechanism occurs in strongly coupled gauge fermion interactions.
- Based on dynamical χSB effects similar to those generating $\langle \bar{q}q \rangle \neq 0$.
- Under assumption of symmetry enhancement chiral breaking effects that are naively suppressed (irrelevant) trigger non-perturbative (NP) dynamical mass generation.

⋆ Alternative to the Higgs mechanism.

[R. Frezzotti, G.C. Rossi PRD (2015)]

Toy-model

- **A simple toy-model where the mechanism can be realised**:
  - $SU(N_f = 2)$ doublet of strongly ($SU(3)$) interacting fermions coupled to scalars via Yukawa and Wilson-like terms.
  - Enlarged chiral symmetry acting on fermions and scalars. Fermionic chiral symmetry explicitly broken.
  - Physics depends crucially on the phase (Wigner or NG).
  - Enhancement of symmetry (refers to naturalness concept by 't Hooft).
  - leads to elementary fermion mass: $m_Q = O(\alpha_s)\Lambda_s$, ($\Lambda_s$ RGI strong scale).
- Intrinsic NP character of the mechanism:

→ Numerical investigation employing lattice methods is necessary.

→ The proposed BSM mechanism can be falsified.

Toy-model

- **QCD$_{N_f=2}$ + Scalar field + Yukawa + $(d=6)$ Wilson-like**

$$\mathbf{L_{toy}} = L_{kin}(Q,A,\Phi) + V(\Phi) + L_Y(Q,\Phi) + L_W(Q,A,\Phi) :$$

$$L_{kin}(Q,A,\Phi) = \frac{1}{4}(F\cdot F) + \bar{Q}_L\gamma_\mu D_\mu Q_L + \bar{Q}_R\gamma_\mu D_\mu Q_R + \frac{1}{2}\mathrm{Tr}\left[\partial\Phi^\dagger\partial\Phi\right]$$

$$V(\Phi) = \frac{1}{2}\mu_0^2\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{1}{4}\lambda_0\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

$$L_Y(Q,\Phi) = \eta\left(\bar{Q}_L\Phi Q_R + \bar{Q}_R\Phi Q_L\right)$$

$$L_W(Q,A,\Phi) = \rho\frac{b^2}{2}\left(\bar{Q}_L\overleftarrow{D}_\mu\Phi D_\mu Q_R + \bar{Q}_R\overleftarrow{D}_\mu\Phi^\dagger D_\mu Q_L\right)$$

$$[\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + ig_s\lambda^a A^a_\mu,\ D_\mu = \partial_\mu - ig_s\lambda^a A^a_\mu]$$

- an SU(3) gauge field $A^a_\mu, (a=1,\ldots,8)$.
- one Dirac fermion doublet Q transforming as a colour triplet under SU(3).
- one complex scalar doublet (singlet under SU(3)) $\Phi = \phi_0 + i\phi_j\tau_j$.
- **Q coupled to gauge and scalar through Yukawa & $(\mathbf{d=6})$ Wilson-like terms.**
- $b^{-1} \equiv \Lambda_{UV}$: UV cutoff.

Toy-model - symmetries

- Enlarged global chiral $\chi_L \times \chi_R$ transformations are symmetry of $\mathbf{L_{toy}}$:

$$\chi_L : \tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) \qquad \chi_R : \tilde{\chi}_R \otimes (\Phi \to \Omega_R \Phi)$$

$$\tilde{\chi}_L : \begin{array}{l} Q_L \to \Omega_L Q_L, \\ \bar{Q}_L \to \bar{Q}_L \Omega_L^\dagger \\ \Omega_L \in SU(2)_L \end{array} \qquad \tilde{\chi}_R : \begin{array}{l} Q_R \to \Omega_R Q_R, \\ \bar{Q}_R \to \bar{Q}_R \Omega_R^\dagger \\ \Omega_R \in SU(2)_R \end{array}$$

- **Exact symmetry** $\chi \equiv \chi_L \times \chi_R$ acting on fermions and scalars
  $\Longrightarrow$ $(\bar{Q}_L Q_R + \bar{Q}_R Q_L)$ operator NOT invariant under $\chi \equiv \chi_L \times \chi_R$
  $\Longrightarrow$ *NO* power divergent mass terms .

- Purely fermionic $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations are *not* a symmetry for *generic* (non-zero) η and ρ:

  $\Longrightarrow$ $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ **broken explicitly** by Yukawa and Wilson-like terms.

- P, C, T, gauge invariance are symmetries & power counting renormalisation.

Toy-model - properties

- The shape of $V(\Phi)$ determines crucially the physics of the model.
- When the scalar potential $V(\Phi)$ has one minimum
  - $\chi \equiv \chi_L \times \chi_R$ is realized à la Wigner
  - No spontaneous χ-symmetry breaking
- The fermionic $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations generate Schwinger-Dyson Equations.
- They can be renormalised taking in due account the operator mixing procedure.

Critical Model

- In fact the renormalised SD equation reads:

  $\partial_\mu \langle \tilde{Z}_{\tilde{J}} \tilde{J}^{L,i}_\mu(x) O(0) \rangle = (\eta - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0)) \langle [\bar{Q}_L \tau^i \Phi Q_R - h.c.](x) O(0) \rangle + O(b^2)$

  (SDE renrm/tion here analogous to chiral SDE renormalisation as in [Bochicchio *et al.* NPB 1985])

- **Critical Model**: $\tilde{\chi}$-symmetry restoration occurs at $\eta = \eta_{cr}$ where the Yukawa term "compensates" the Wilson term.

▶ We get the current conservation at $\eta - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0) = 0$
$\Longrightarrow \quad \eta_{cr}(g_s^2, \rho, \lambda_0)$.

- η_{cr} is dimensionless parameter. Dependence on renormalised scalar mass only through cutoff effects $O(b^2 \mu_\Phi^2)$.
  Hence η_{cr} identical in Wigner and Nambu-Goldstone (NG) phases.

Critical Model

- Low-Energy Effective Lagrangian (Wigner phase)

$$\Gamma^{Wig}_{\mu^2_\Phi>0} = \frac{1}{4}(F\cdot F) + \bar{Q}\,\not{D}Q + (\eta - \eta_{cr})(\bar{Q}_L \Phi Q_R + \text{h.c.})$$
$$+\frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \hat{\mathrm{V}}_{\mu^2_\Phi>0}(\Phi)$$

- In the critical theory, $\eta = \eta_{cr}$, scalars decouple from quarks and gluons.

- Fermionic $\tilde{\chi}$ become a symmetry (up to $O(b^2)$ cutoff effects).
  - Need to determine η_{cr}.
  - We expect (and verify) no fermionic mass generation.

▶ **Employ lattice methods.**

Lattice computation

- Lattice discretization, $L_{latt.}$ with Dirac fermions and $d = 6$ Wilson term: unbroken χ-symmetry.
- We limit our first study to the quenched approximation
- Quenching allows independent generation of gauge (U) and scalar (Φ) configurations.
  $\Rightarrow$ The mechanism under investigation, if exists, survives quenching.
- To avoid "exceptional configurations" ($\rightarrow$ due to fermions zero modes) introduce twisted mass IR regulator $L_{latt.} + i\mu\bar{Q}\gamma_5\tau^3 Q$.
  (Frezzotti, Grassi, Sint and Weisz, JHEP 2001)
  $\Rightarrow$ at a cost of soft breaking of $\chi_L \times \chi_R$, symmetry recovered after an extrapolation to $\mu \rightarrow 0$.

Lattice computation

- ⋆ We performed simulations at **three** values of the lattice spacing
- ⋆ $\beta = 5.75$ ($b = 0.15$ fm), $\beta = 5.85$ ($b = 0.12$ fm) & $\beta = 5.95$ ($b = 0.10$ fm)
  Several lattice volumes corresponding to physical size of
  $L \sim 2.0 - 2.4$ fm, $T \sim 4.8$ fm.
  Extrapolation to the continuum limit exploiting $O(b^{2n})$ cutoff effects.
- ⋆ Use the Sommer lattice scale $r_0 = 0.5$ fm (motivated from QCD, for illustration) to determine the gauge coupling.
- ⋆ ρ: to check the existence of the mechanism it is sufficient to set it to some reasonable value $\neq 0$.

  (Most of our numerical results obtained at $\rho = 1.96$ both in the Wigner & NG phase but also at $\rho = 2.94$.)
- ⋆ Keep fixed physical scalar field parameters at all lattice spacings.

Determination of η_{cr} in the Wigner phase

- SD equation (similar for $\tilde{J}_\mu^{R,i}$):

$$\partial_\mu \langle \tilde{Z}_{\tilde{J}} \tilde{J}_\mu^{L,i}(x) O(0)\rangle = (\eta - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0))\langle [\bar{Q}_L \tau^i \Phi Q_R - h.c.](x) O(0)\rangle + O(b^2)$$

- Employ Schwinger-Dyson eqs on the lattice and consider the ratio of correlation functions:

$$r_{AWI}(\eta) \equiv \frac{\partial_\mu \sum_{\mathbf{x}} \langle \tilde{J}_\mu^{A,i}(\mathbf{x}, x_0) \tilde{D}^{P,i}(0)\rangle}{\sum_{\mathbf{x}} \langle \tilde{D}^{P,i}(\mathbf{x}, x_0) \tilde{D}^{P,i}(0)\rangle},$$

$$\tilde{J}_\mu^{A,i}(x) = \tilde{J}_\mu^{R\,i}(x) - \tilde{J}_\mu^{L\,i}(x), \quad \tilde{D}^{P,i}(y) = \bar{Q}_L(y)\left[\Phi, \frac{\tau^i}{2}\right] Q_R(y) - \bar{Q}_R(y)\left[\frac{\tau^i}{2}, \Phi^\dagger\right] Q_L(y)$$

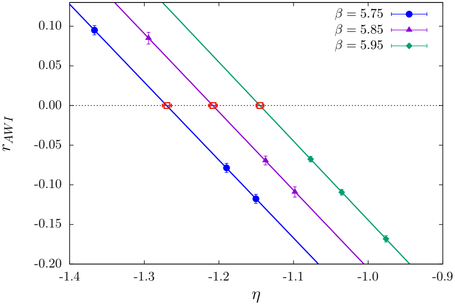
Compute $r_{AWI}(\eta)$ at several values of η before interpolating to η_{cr}.

- We expect:

$$r_{AWI}(\eta) \propto (\eta - \bar{\eta}(\eta; g_s^2, \lambda_0, \rho)) = (\eta - \eta_{cr}(g_s^2, \lambda_0, \rho))(1 - \frac{\partial \bar{\eta}}{\partial \eta}|_{\eta_{cr}}) + \dots$$

- and find ...

Determination of η_{cr} in the Wigner phase



β	ρ	λ_0	η_{cr}
5.75	1.96	0.5807	-1.271(10)
5.85	1.96	0.5917	-1.207(8)
5.95	1.96	0.6022	-1.145(6)

- Red squares denote the values η_{cr} at which $r_{AWI} = 0$.
- Less than 1% error on η_{cr} determination.

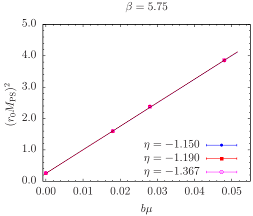
Further investigation in the Wigner phase

- IR tm μ-regulator breaks softly χ-symmetry ($\rightarrow$ at all η.)
- PS-bosons have mass vanishing linearly in μ (up to cutoff effects)

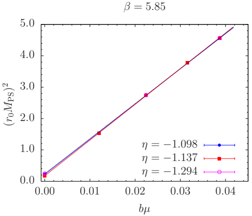
& in the C.L. PS-mass $\rightarrow$ 0

- In the Wigner phase no fermion mass generation - no seed for D$\tilde{\chi}$SB.

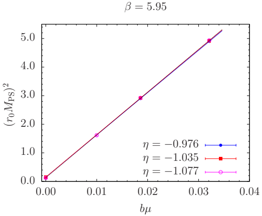
$[\beta = 5.75]$



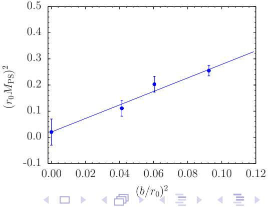
$[\beta = 5.85]$



$[\beta = 5.95]$



C.L.



NG phase:

dynamical mass generation

Properties of the toy-model in NG-phase

- $V(\Phi)$ has a mexican hat shape - $< \Phi^\dagger \Phi > \equiv v^2 \neq 0$.
- $\chi_L \times \chi_R$ realised à la NG; it is natural to use the parametrisation: $\Phi = v + \sigma + i\vec{\tau}\vec{\pi}$
- Similar to the LQCD case (Wilson fermions):
  $L_W(Q, A, \Phi) = \frac{\rho b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \text{h.c.} \right) \overset{r \leftrightarrow b v \rho}{\sim} L_W^{QCD}(Q, A) = -\frac{br}{2} \left(\bar{Q}_L D^2 Q_R + \text{h.c.} \right)$.

  $\eta(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L) \quad \overset{<\Phi>=v}{\longrightarrow} \quad \eta v \bar{Q} Q$
- In the *critical* theory, $\eta = \eta_{cr}$, where Yukawa term is compensated by the Wilson-like term:
  - Yukawa mass term, $v\bar{Q}Q$, gets cancelled;
  - hence, no "Higgs-like" fermion mass.
- **Conjecture**: $\tilde{\chi}-$ breaking due to residual $O(b^2 v)$ effects are expected to trigger dynamical χSB.

Dynamically generated mass

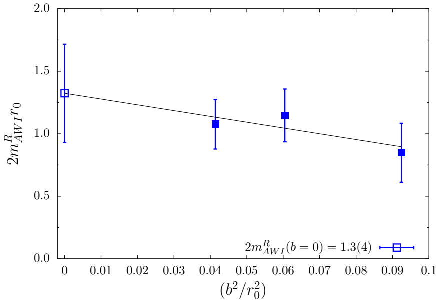
Compute

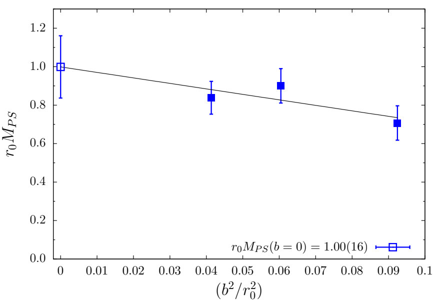
- WTI quark mass: $m_{AWI}(\eta) = \frac{\sum_{\mathbf{x}} \partial_0 \langle \tilde{J}_0^{A,i}(x) P^i(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^i(x) P^i(0) \rangle}$, $P^i = \bar{Q}\gamma_5 \tau^i Q$
- Mass M_{ps} of the lowest PS-meson contributing to $\sum_{\mathbf{x}} \langle P^i(x) P^i(0) \rangle$

Dynamically generated mass

Compute

- WTI quark mass: $m_{AWI}(\eta) = \frac{\sum_{\mathbf{x}} \partial_0 \langle \tilde{J}_0^{A,i}(x) P^i(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^i(x) P^i(0) \rangle}$, $P^i = \bar{Q}\gamma_5 \tau^i Q$
- Mass M_{ps} of the lowest PS-meson contributing to $\sum_{\mathbf{x}} \langle P^i(x) P^i(0) \rangle$





- Continuum limit (linear) extrapolation of m_{AWI} and M_{PS}.
- (Very) small cutoff effects towards the C.L.
- At η_{cr} the C.L. estimates of quark mass and PS-meson mass do not vanish!

The toy-model in NG-phase

- **Symmetries dictate the form of Γ^{NG}.**

- **NP** mass term has to be $\chi_L \times \chi_R$ invariant.
  Note that a term like $m[\bar{Q}_L Q_R + \bar{Q}_R Q_L]$ is not $\chi_L \times \chi_R$ invariant.
- At *generic η*, two $\tilde{\chi}$-breaking operators are expected to arise:

Yukawa induced + dynamically generated (← conjecture)

- $\Gamma^{NG} = \ldots + (\eta - \eta_{cr})(\bar{Q}_L \Phi Q_R + \text{h.c.}) + c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.})$

  where $$\mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v+\sigma)\mathbb{1} + i\vec{\tau}\vec{\varphi}}{\sqrt{v^2 + 2v\sigma + \sigma^2 + \vec{\varphi}\vec{\varphi}}} \simeq \mathbb{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \ldots$$

  and $\Lambda_s \equiv$ RGI NP mass scale.
- $\mathcal{U}$ is a non-analytic function of Φ, but transforms like Φ under $\chi_L \times \chi_R$; obviously $\mathcal{U}$ can not be defined in the Wigner phase ($\langle \Phi \rangle = 0$)
- Note that (from the χ-inv. term):

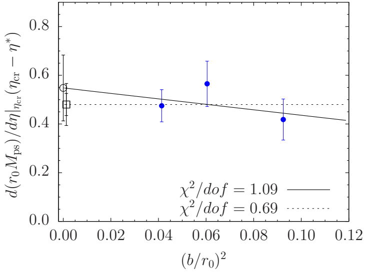
$$c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.}) \simeq c_1 \Lambda_s \bar{Q} Q + O(v^{-1})$$

Further checks - I

- For generic value of η: $\boxed{m_{AWI} = (\eta - \eta_{cr})v + c_1\Lambda_s \xrightarrow{\eta=\eta_{cr}} m_{AWI} = c_1\Lambda_s}$
- Moreover there is (and can be determined numerically) a certain $\eta = \eta^*$ where m_{AWI} vanishes. Namely $\boxed{\eta^* = \eta_{cr} - c_1\Lambda_s/v \Rightarrow \eta_{cr} \neq \eta^* \leftrightarrow c_1\Lambda_s \neq 0}$
- Hence check whether $(\eta_{cr} - \eta^*) \neq 0$ in the C.L.
- But $(\eta_{cr} - \eta^*)$ has to be renormalised:
- define the renormalised quantity:

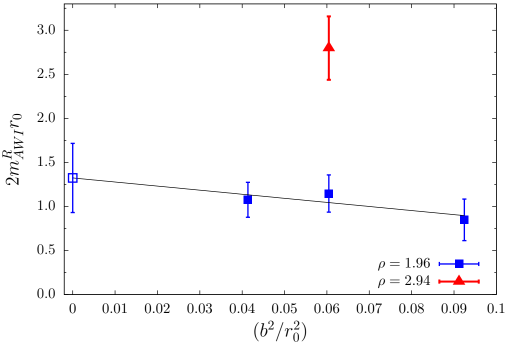
$$\mathcal{D}_\eta \equiv \frac{d(r_0 M_{PS})}{d\eta}\Big|_{\eta_{cr}}(\eta_{cr} - \eta^*) \equiv Z_\eta(\eta_{cr} - \eta^*)$$

and evaluate it (through extrapolation) in the C.L.



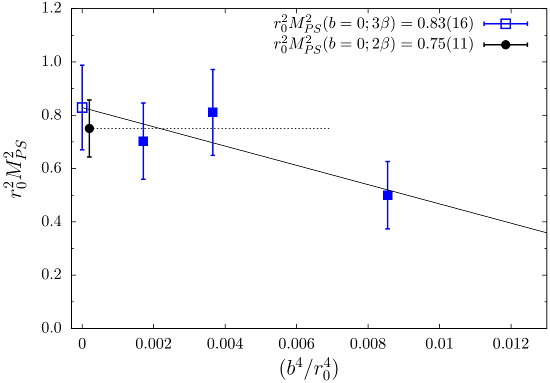
Further checks - II

- Check behaviour of the fermion mass breaking $\tilde{\chi}_L \times \tilde{\chi}_R$ as ρ varies.
- For $\rho \to 0$ also $\eta_{cr} \to 0$ for which . . .
- $\tilde{\chi}_L \times \tilde{\chi}_R$ transformations become exact symmetry: $m_{AWI}, M_{PS} \to 0$.
- Hence it is expected that m_{AWI}, M_{PS} should be increasing functions of ρ.
- Compute m_{AWI} at $\rho = 2.94$ and compare with $\rho = 1.96$.



Test of no mechanism hypothesis

- It can be shown that in case of no mechanism i.e. $c_1\Lambda_s = 0$, then $M_{\mathrm{PS}} \sim O(b^4)$.



No mechanism hypothesis is not supported by the data
($5\sigma - 7\sigma$ away from zero).

Conclusions

- We have presented a toy-model where a *novel* NP mechanism for elementary fermion mass generation, alternative to the Higgs mechanism is in action.

- The **toy model** is a non-Abelian gauge model with an SU($N_f = 2$)-doublet of strongly interacting fermions coupled to scalars through Yukawa and Wilson-like terms: at the *critical point*, where (fermion) $\tilde{\chi}$ invariance is recovered the model gives rise in the NG phase to a dynamical $\tilde{\chi}$-SSB and the generation of a non-perturbative elementary fermion mass.

- This pattern has been shown to occur in explicit numerical simulations.

Conclusions

- A first principles study of the model has been performed at three values of the lattice spacing ($\sim$ 0.10, 0.12 and 0.15 fm).
- We have shown that the Yukawa coupling where the fermionic $\tilde{\chi}$ symmetry gets restored can be accurately determined.
- The effects of dynamical SSB of the (restored) $\tilde{\chi}$-symmetry in the NG phase look very well compatible with the generation of non-zero elementary fermion (quark) mass and PS-meson mass $\sim O(\Lambda_s)$.

Outlook

- These findings will be further checked with more simulations (finer lattice spacings, $\Lambda_s/v \to 0$ etc).
- Dynamically generated "NP anomaly" opens discussion on revised framework concerning the concept of universality.
- Towards a realistic model: since masses are conjectured to be parametrically of order of the RGI scale, this has to be much larger than Λ_{QCD} in order for the heavier particle masses (e.g. m_t) can be reproduced.
- This points to the existence of a new non-Abelian interaction with scale $\Lambda_T \gg \Lambda_{QCD}$ and to new elementary fermionic particles with NP mass of $O(\Lambda_T)$.
- The mechanism can be extended to include EW interactions (χ_L is gauged).

Thank you for your attention!

Extra slides

Wilson fermions case

- In (massless) LQCD with Wilson term Non-Perturbative contribution ($\propto \Lambda_{QCD}$) is accompanied by an $1/a$ divergent term.
  - Axial WTI: $\partial_\mu \langle \hat{J}_{5\mu}(x)\hat{O}(0)\rangle = 2(m_0 - \bar{M}(m_0))\langle \hat{P}(x)\hat{O}(0)\rangle + O(a)$
  - where: $\bar{M}(m_0) = \dfrac{c_0(1-d_1)}{a} + c_1(1-d_1)\Lambda_{\mathrm{QCD}} + d_1 m_0 + O(a)$
  - If we could set

    $$m_0 = c_0/a \quad \rightarrow \quad \partial_\mu \langle \hat{J}_{5\mu}(x)\hat{O}(0)\rangle = c_1(1-d_1)\Lambda_{\mathrm{QCD}}\langle \hat{P}(x)\hat{O}(0)\rangle + O(a)$$

- Separation of the two effects requires an infinite fine tuning ($\rightarrow$ *naturalness problem*).

Lattice formulation

$$S^{lat}_{toy} = b^4 \sum_x \left\{ \mathcal{L}^{YM}_{kin}[U] + \mathcal{L}^{scal}_{kin}(\Phi) + \mathcal{V}(\Phi) + \bar{q} D_{lat}[U;\Phi] q \right\}$$

$\mathcal{L}^{YM}_{kin}[U]$: SU(3) plaquette action

$$\mathcal{L}^{scal}_{kin}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\mathrm{Tr}\,[\Phi^\dagger(-\partial^*_\mu\partial_\mu)\Phi] + \frac{m_0^2}{2}\mathrm{Tr}\,[\Phi^\dagger\Phi] + \frac{\lambda_0}{4}(\mathrm{Tr}\,(\Phi^\dagger\Phi])^2$$

where in terms of the 2×2 matrix-field Φ and the 8×8 matrix-field F

$$\Phi = \varphi_0 \mathbb{1} + i\varphi_j\tau^j \qquad \text{and} \qquad F(x) \equiv [\varphi_0 \mathbb{1} + i\gamma_5\tau^j\varphi_j](x)$$

we have

- $(D_{lat}[U,\Phi]q)(x) = \gamma_\mu\widetilde{\nabla}_\mu q(x) + \eta F(x)q(x) - b^2\rho\frac{1}{2}F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu q(x) +$
  $-b^2\rho\frac{1}{4}\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu q(x+\hat{\mu}) + (\partial^*_\mu F)(x)U^\dagger_\mu(x-\hat{\mu})\widetilde{\nabla}_\mu q(x-\hat{\mu})\right]$
- $\widetilde{\nabla}_\mu f(x) \equiv \frac{1}{2}(\nabla_\mu + \nabla^*_\mu)f(x)$
- $b\nabla_\mu f(x) \equiv U_\mu(x)f(x+\hat{\mu}) - f(x)\,, \quad b\nabla^*_\mu f(x) \equiv f(x) - U^\dagger_\mu(x-\hat{\mu})f(x-\hat{\mu})$

SDE mixing-renormalisation

- Bare $\tilde{\chi}_L \times \tilde{\chi}_R$ SDEs read [Bochicchio *et al.* 1985]
  - $\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}_L^i \hat{O}(0)\rangle \delta(x) - \eta\, \langle O_{Yuk}^{L,i}(x)\, \hat{O}(0)\rangle - b^2 \langle \underline{O_{Wil}^{L,i}}(x)\, \hat{O}(0)\rangle$
    - $\tilde{J}_\mu^{L,i} = \bar{q}_L \gamma_\mu \frac{\tau^i}{2} q_L - \frac{b^2}{2}\rho\left(\bar{q}_L \frac{\tau^i}{2} \Phi \mathcal{D}_\mu q_R - \bar{q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} q_L\right)$
    - $O_{Yuk}^{L,i} = \left[\bar{q}_L \frac{\tau^i}{2} \Phi q_R - \text{h.c.}\right]$
    - $\underline{O_{Wil}^{L,i}} = \frac{\rho}{2}\left[\bar{q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2} \Phi \mathcal{D}_\mu q_R - \text{h.c.}\right]$
- Mixing & Renormalization
  - $b^2 \underline{O_{Wil}^{L,i}} = (Z_{\tilde{J}} - 1)\partial_\mu \tilde{J}_\mu^{L,i} - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O_{Yuk}^{L,i} + \ldots + \mathrm{O}(b^2)$
  - $\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}_L^i \hat{O}(0)\rangle \delta(x) - (\eta - \bar{\eta}(\eta))\, \langle O_{Yuk}^{L,i}(x)\, \hat{O}(0)\rangle + \ldots + \mathrm{O}(b^2)$
- Critical theory $\to \eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0) = 0 \implies \eta_{cr}(g_s^2, \rho, \lambda_0) = \mathrm{O}(g_s^2)$
  - $\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, \hat{O}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{O}(0)\rangle_{\eta_{cr}} \delta(x) + \mathrm{O}(b^2) + \ldots$
- All the same with $[L \leftrightarrow R\ \&\ \Phi \leftrightarrow \Phi^\dagger]$

parameters listing

From several simulations of the $\lambda_0(\Phi^\dagger\Phi)^2$ theory with $12 < L/b < 24$ and $T = 2L$ $\Rightarrow$

scalar sector parameters matching renorm. conditions in NG phase & $\beta \leftrightarrow r_0/b$

[SU(3)-YM data for $\beta \leftrightarrow r_0/b$ from Necco and Sommer, Nucl.Phys. B622 (2002) 328-346]

β	r_0/b	$r_0^2 M_\sigma^2$	$r_0^2 v_R^2$	λ_{NP}	$b^2\mu_0^2$	λ_0	κ
5.75	3.29	1.278(4)	1.464(3)	0.437(2)	-0.5941	0.5807	0.132283
5.85	4.06	1.286(4)	1.459(3)	0.441(2)	-0.5805	0.5917	0.132000
5.95	4.91	1.290(5)	1.453(3)	0.444(2)	-0.5756	0.6022	0.131870

κ: code hopping parameter, s.t. $\kappa^{-1} - 2\kappa\lambda_0 - 8 = b^2 m_0^2$, $\eta_{code} = \eta\sqrt(2\kappa)$, $\rho_{code} = \rho\sqrt(2\kappa)$

Values of μ_{cr}^2 & μ_0^2, λ_0 parameters for simulations in Wigner phase at fixed $\mu_\Phi^2 r_0^2 > 0$

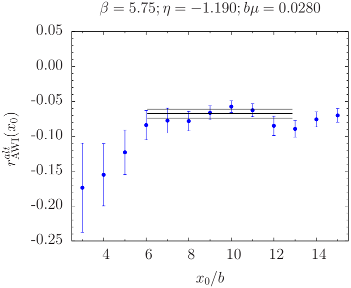
β	r_0/b	$(\mu_0^2 - \mu_{cr}^2)b^2$	$b^2\mu_{cr}^2$	$b^2\mu_0^2$	λ_0	κ
5.75	3.29	0.1119(12)	-0.5269(12)	-0.4150	0.5807	0.129280
5.85	4.06	0.0742(11)	-0.5357(11)	-0.4615	0.5917	0.130000
5.95	4.91	0.0504(10)	-0.5460(10)	-0.4956	0.6022	0.130521

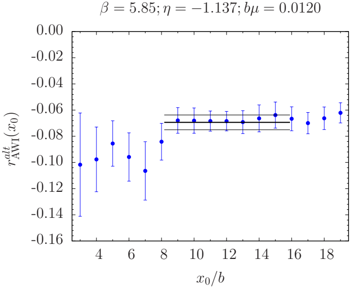
Alternative determination of η_{cr}

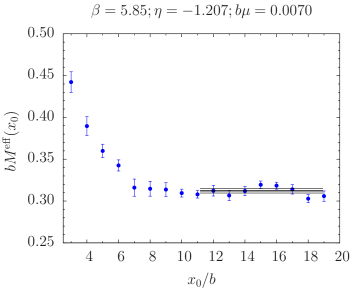
$$r^{alt}_{AWI}(\eta; g_s^2, \lambda_0, \rho, \mu) = \frac{\sum_{\mathbf{x}} \sum_{\mathbf{y}} \langle P^1(0)[\partial_0 \tilde{J}_0^{A,i}](x)\phi^0(y)\rangle}{\sum_{\mathbf{x}} \sum_{\mathbf{y}} \langle P^1(0) D^{P,i}(x)\phi^0(y)\rangle}$$

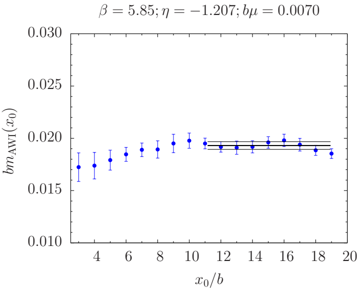
with: $\tilde{D}^{P,i}(x) = \bar{Q}_L(x)\{\Phi, \frac{\tau^i}{2}\}Q_R(x) - \bar{Q}_R(x)\{\frac{\tau^i}{2}, \Phi^\dagger\}Q_L(x)$, $P^i = \bar{Q}\gamma_5\tau^i/2Q$,

$\phi^0 = \frac{1}{4}\mathrm{Tr}[\Phi + \Phi^\dagger] = \frac{1}{2}\mathrm{Tr}[\Phi]$, $y_0 = x_0 + \tau$ (τ = fixed (in practice ~ 0.6 fm)).

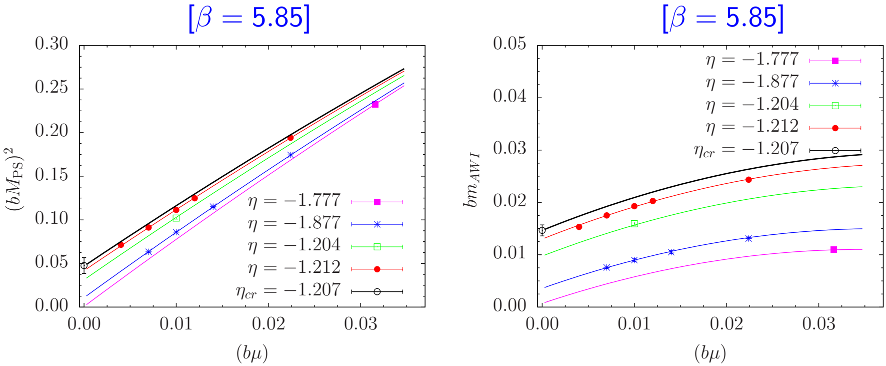





$\beta = 5.85; \eta = -1.207; b\mu = 0.0070$
$bM^{\rm eff}(x_0)$
0.50
0.45
0.40
0.35
0.30
0.25
4
6
8
10
12
14
16
18
20
x_0/b


$\beta = 5.85; \eta = -1.207; b\mu = 0.0070$
$bm_{\rm AWI}(x_0)$
0.030
0.025
0.020
0.015
0.010
4
6
8
10
12
14
16
18
20
x_0/b

- simultaneous polynomial fits for M_{PS}^2 and m_{AWI} in η and in μ (example: $\beta = 5.85$).
- extra/interpolation to η_{cr} (at $\mu \to 0$).



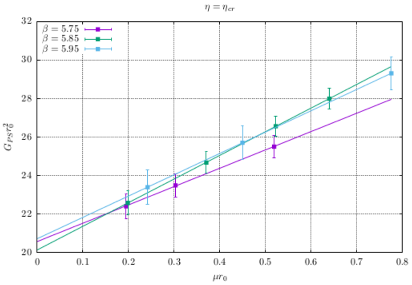
- similar for M_{PS}.

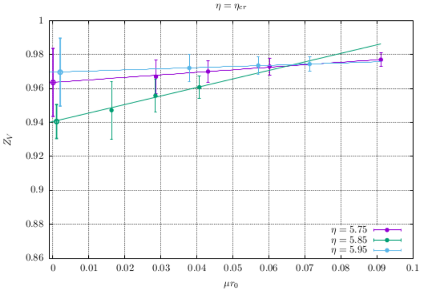
▶ Determination of m^{ren}_{AWI}

- RCs UV quantities: can be calculated either in Wigner of in NG phases
- $1/Z_P^{had} = \langle 0|\bar{Q}\gamma_5 \frac{\tau^1}{2} Q|P^1_{meson}\rangle|_{\eta_{cr},\mu\to 0+}\, r_0^2 \equiv G^{Wig}_{PS}\, r_0^2$ eval. in Wigner phase
- $Z_{\widetilde{V}}$: $Z_{\widetilde{V}}\langle 0|\partial_0 \widetilde{V}^2_0|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+} = 2\mu\langle 0|\bar{Q}\gamma_5 \frac{\tau^1}{2} Q|P^1_{\rm meson}\rangle|_{\eta_{cr},\mu\to 0+}$

  evaluated in NG phase
- $Z_{\widetilde{V}} = Z_{\widetilde{A}}$ (at η_{cr})
- $m^{ren}_{AWI} = \dfrac{Z_{\widetilde{V}}}{Z_P^{had}} m_{AWI}$

▶ Determination of m^{ren}_{AWI}





- **Check for finite size effects ($\beta = 5.85$)**