Pohlmeyer Reduction, Dressing Method and Classical String Solutions on $\mathbb{R} \times S^2$

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based on arXiv:1805.09301 [hep-th], arXiv:1806.07730 [hep-th], arXiv:1903.01408 [hep-th] and arXiv:1903.01412 [hep-th]

in collaboration with D. Katsinis and I. Mitsoulas

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DEMOKRITOS

Section 1

Introduction

- Classical string solutions have shed light to several aspects of the holographic duality.
- The dispersion relations of the classical strings are related to the anomalous dimensions of operators in the dual CFT. [1] [2] [3]
- They also serve to develop some intuition on the dynamics of the classical system whose quantum version is the only known mathematically consistent theory of quantum gravity

[1] S. Frolov and A. A. Tseytlin, Nucl. Phys. B 668, 77 (2003) [hep-th/0304255]
[2] N. Beisert, J. A. Minahan, M. Staudacher and K. Zarembo, JHEP 0309, 010 (2003) [hep-th/0306139]
[3] S. Frolov and A. A. Tseytlin, Phys. Lett. B 570, 96 (2003) [hep-th/0306143]

In this work

- We focus on strings propagating on $\mathbb{R} \times \mathrm{S}^2$, which are Pohlmeyer reducible to the sine-Gordon equation.
- We invert Pohlmeyer reduction and construct systematically the solutions with elliptic SG counterparts
- Then we perform a Bäcklund transformation on the side of the SG equation and find new "dressed" string solutions

The new solutions have several interesting features

- The dressed solution have interacting spikes.
- There are interesting interrelations between properties of the strings and their SG counterparts.
- The dressed solutions reveal the stability properties of their seeds.
- The energy and angular momenta of the dressed solutions have several qualitative features that could be detectable on the side of the boundary CFT.

Section 2

Elliptic and Dressed Elliptic String Solutions

The String action

The action for strings propagating on $\mathbb{R} \times \mathrm{S}^2$, written as a Polyakov action is

$$S = T \int d\xi^+ d\xi^- \left((\partial_+ X) \cdot (\partial_- X) + \lambda \left(\vec{X} \cdot \vec{X} - R^2 \right) \right).$$

The equations of motion are non linear and difficult to treat.

$$\partial_+ \partial_- X^0 = 0 \Rightarrow X^0 = f_+ \left(\xi^+ \right) + f_- \left(\xi^- \right),$$
$$\partial_+ \partial_- \vec{X} = - \frac{1}{R^2} \left(\left(\partial_+ \vec{X} \right) \cdot \left(\partial_- \vec{X} \right) \right) \vec{X}.$$

Additionally, the solution must satisfy the Virasoro constraints

$$\left(\partial_\pm \vec{X} \right) \cdot \left(\partial_\pm \vec{X} \right) = \left({f_\pm}' \right)^2.$$

The Pohlmeyer Reduction

The system is integrable. A signature of the system's integrability is the fact that can be reduced to an SSSG (symmetric space sine-Gordon), in our case the sine-Gordon equation itself [4]. We first take advantage of the diffeomorphism invariance and we select a linear gauge

$$f_{\pm}\left(\xi^{\pm}\right) := m_{\pm}\xi^{\pm}$$

and then we define as reduced field the angle between the vectors $\partial_+\vec{X}$ and $\partial_-\vec{X}$

$$\left(\partial_+\vec{X}\right)\cdot\left(\partial_-\vec{X}\right) := {f_+}'{f_-}'\cos\varphi .$$

It is easy to show that the Pohlmeyer field obeys the sine-Gordon equation

$$\partial_+\partial_-\varphi = \mu^2 \sin\varphi ,$$

where $\mu^2 := -m_+m_-/R^2$.

[4] K. Pohlmeyer, Commun. Math. Phys. 46, 207 (1976)

Inversion of Pohlmeyer Reduction for Elliptic Solutions

The Pohlmeyer reduction is a non-local and many-to-one mapping, making its inversion a non-trivial task. However,

- There is an advantage in finding a string solution given a solution of the reduced system; the equations of motion assume the form of *linear* differential equations.

$$-\partial_0^2 \vec{X} + \partial_1^2 \vec{X} = \mu^2 \cos\varphi \vec{X},$$

- Using a solution of the reduced system that depends on only one world-sheet coordinate provides an extra advantage; these linear differential equations are solvable using separation of variables [5],

$$X^i(\xi^0, \xi^1) := \Sigma^i(\xi^1) \mathrm{T}^i(\xi^0).$$

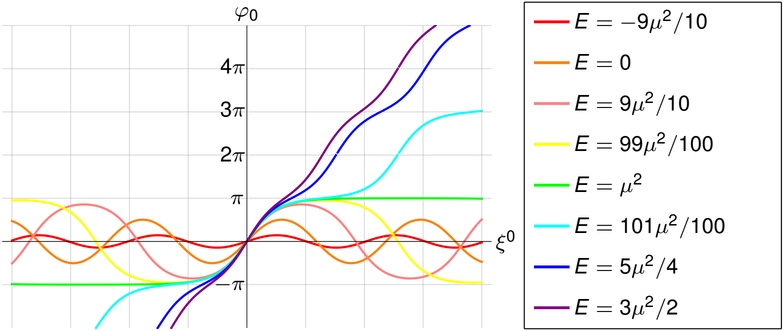
$$-\Sigma^{i\prime\prime} + \left(2\wp\left(\xi^1 + \omega_2\right) + x_1\right)\Sigma^i = \kappa^i \Sigma^i,$$
$$-\ddot{\mathrm{T}}^i = \kappa^i \mathrm{T}^i.$$

[5] I. Bakas and G. Pastras, JHEP 1607 (2016) 070 [arXiv:1605.03920 [hep-th]]

Elliptic Solutions of the SG Equation

The solutions of the sine-Gordon equation that depend solely on one of the two variables are its elliptic solutions. They can be understood as the solutions of the equation of motion of the simple pendulum

$$\cos\varphi_0\left(\xi^0;E\right)=-\frac{1}{\mu^2}\left(2\wp\left(\xi^0-\tau_0+\omega_2;g_2\left(E\right),g_3\left(E\right)\right)+\frac{E}{3}\right).$$



The Elliptic String Solutions - Periodicity

Without posting the details of the derivation, the Pohlmeyer reduction can be inverted in this case. In polar coordinates, the elliptic string solutions assume the form

$$\begin{aligned} t_{0/1} &= R\sqrt{x_2 - \wp(a)}\xi^0 + R\sqrt{x_3 - \wp(a)}\xi^1, \\ \cos\theta_{0/1} &= \sqrt{\frac{x_1 - \wp\left(\xi^{0/1} + \omega_2\right)}{x_1 - \wp(a)}}, \\ \varphi_{0/1} &= -\mathrm{sgn}(\mathrm{Im}a)\sqrt{x_1 - \wp(a)}\xi^{1/0} - \Phi\left(\xi^{0/1}; a\right), \end{aligned}$$

where the quasi-periodic function Φ is defined as

$$\Phi(\xi; a) = -\frac{i}{2}\ln\frac{\sigma(\xi + \omega_2 + a)\,\sigma(\omega_2 - a)}{\sigma(\xi + \omega_2 - a)\,\sigma(\omega_2 + a)} + i\zeta(a)\,\xi.$$

The Elliptic String Solutions - Rigid Rotation - Spikes

- The elliptic string solutions can be written in the form

$$f\left(\theta, \varphi - \omega t\right) = 0.$$

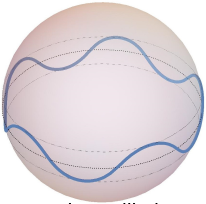
  Thus, the elliptic strings do not change shape with time. They just rotate as a rigid body.
- Writing down the Virasoro constraints in terms of the Pohlmeyer field, yields

$$\left|\partial_0 \vec{X}\right|^2 = R^2 \mu^2 \cos^2 \frac{\varphi}{2}, \quad \left|\partial_1 \vec{X}\right|^2 = R^2 \mu^2 \sin^2 \frac{\varphi}{2}.$$

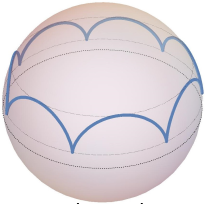
  Thus, whenever the Pohlmeyer field equals an integer multiple of 2π, the derivative $\partial_1 \vec{X}$ gets inverted and spikes emerge.
- The elliptic strings can be classified to four classes [6], depending on
  - which worldsheet coordinate the SG counterpart depends
  - whether the SG counterpart is an oscillatory or librating pendulum solution

[6] K. Okamura and R. Suzuki, Phys. Rev. D 75 (2007) 046001 [hep-th/0609026]

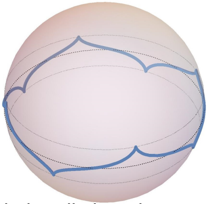
The Elliptic String Solutions - Classification



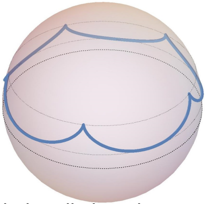
static oscillating counterpart



static rotating counterpart



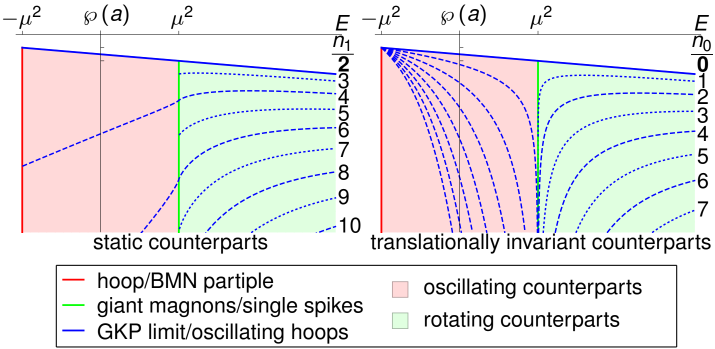
translationally invariant oscillating counterpart



translationally invariant rotating counterpart

The Elliptic String Solutions - Moduli Space

There are several interesting limits of the generic solution that include many well known solutions on the sphere [7] [8] [9]. They are summarized in the following diagram of the moduli space of solutions



[7] D. E. Berenstein, J. M. Maldacena and H. S. Nastase, JHEP 0204 (2002) 013 [hep-th/0202021]

[8] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Nucl. Phys. B 636 (2002) 99 [hep-th/0204051]

[9] D. M. Hofman and J. M. Maldacena, J. Phys. A 39 (2006) 13095 [hep-th/0604135]

The Dressing Method

- The theories emerging after the Pohlmeyer reduction of the non-linear sigma models, which the propagation of classical strings in symmetric spaces, possess autoBäcklund transformations, which connect pairs of solutions. These transformations are a manifestation of the model's integrability.
- The dressing method [10] [11] is the direct analogue of the Bäcklund transformations in the NLSM.
- The simple formulation of the elliptic string solutions that emerged naturally via the inversion of the Pohlemeyer reduction facilitates the application of the dressing method, although the seeds are highly non-trivial.

[10] J. P. Harnad, Y. Saint Aubin and S. Shnider, Commun. Math. Phys. 92 (1984) 329

[11] T. J. Hollowood and J. L. Miramontes, JHEP 0904 (2009) 060 [arXiv:0902.2405 [hep-th]]

The Dressed Elliptic Strings

The application of the dressing method to the elliptic seeds is highly technical. Without posting the details of the derivation, the dressed solution with the simplest dressing factor, which corresponds to the application of a single Bäcklund transformation, is

$$X' = U\sqrt{\frac{1}{2X_+^T X_-}} \sin\theta_1 \left(X_+ + X_-\right) + \cos\theta_1 X_0 := U\left(\sin\theta_1 X_1 + \cos\theta_1 X_0\right),$$

where

$$X_+ = \hat{\Psi}\left(\lambda_1\right)\theta p, \quad X_- = \theta\hat{\Psi}\left(\lambda_1\right)\theta p$$

and the matrix Ψ is composed by the three vectors

$$\begin{aligned} E_1 &:= \cos\left(\sqrt{\Delta}\xi^0 - \Phi\left(\xi^1;\tilde{a}\right)\right) e_1 + \sin\left(\sqrt{\Delta}\xi^0 - \Phi\left(\xi^1;\tilde{a}\right)\right) e_2, \\ E_2 &:= -\cos\left(\sqrt{\Delta}\xi^0 - \Phi\left(\xi^1;\tilde{a}\right)\right) e_2 + \sin\left(\sqrt{\Delta}\xi^0 - \Phi\left(\xi^1;\tilde{a}\right)\right) e_1, \\ E_3 &:= e_3 \end{aligned}$$

The Auxiliary System for an Elliptic Seed - Solution Moduli

The matrix θ is simply $\mathrm{diag}\{1, 1, -1\}$, the vector p is any vector obeying $p^T p = 0$ and $\bar{p} = \theta p$. The vectors e_i are simply X_0, $X_0 \times X$ and $X_0 \times (X_0 \times X)$, where X_0 equals (001). The parameter $\lambda = e^{i\theta_1}$ determines the position of the poles of the dressing factor and it is directly connected to the Bäcklund parameter. The two important parameters Δ and $\tilde{a}$ that determine the behaviour of the dressed solution are given by

$$\Delta = \frac{E}{2} + \frac{m_+^2}{4}\left(\frac{1-\lambda}{1+\lambda}\right)^2 + \frac{m_-^2}{4}\left(\frac{1+\lambda}{1-\lambda}\right)^2,$$

$$\wp\left(\tilde{a}\right) = -\frac{E}{6} - \frac{m_+^2}{4}\left(\frac{1-\lambda}{1+\lambda}\right)^2 - \frac{m_-^2}{4}\left(\frac{1+\lambda}{1-\lambda}\right)^2.$$

When $\lambda = e^{i\theta_1}$ which will be the case of interest, they obey the following properties

- Δ is real. When the seed is oscillatory it is always negative, whereas when the seed is rotating there is a range of θ_1 that sets Δ positive.

Dressing vs Bäcklund Transformation

The sine-Gordon equation possesses the Bäcklund transformations

$$\partial_+ \frac{\varphi + \tilde{\varphi}}{2} = a\mu \sin \frac{\varphi - \tilde{\varphi}}{2},$$
$$\partial_- \frac{\varphi - \tilde{\varphi}}{2} = \frac{1}{a}\mu \sin \frac{\varphi + \tilde{\varphi}}{2},$$

The simplest dressing factor corresponds to a single Bäcklund transformation with parameter

$$a = \sqrt{-\frac{m_+}{m_-}} \tan \frac{\theta_1}{2}.$$

The Dressed Elliptic Strings SG Counterparts

It is not difficult to perform this single Bäcklund transformation to find the sine-Gordon counterparts of the dressed elliptic solutions. They are

$$\tilde{\varphi} = \hat{\varphi} + 4 \arctan \left[\frac{A + B}{D} \tanh \frac{D\xi^1 + i\Phi\left(\xi^0; \tilde{a}\right)}{2} \right],$$

where

$$\hat{\varphi} = 2 \arctan \left(\frac{a - a^{-1}}{a + a^{-1}} \tan \frac{\varphi}{2} \right) + (2k - 1)\pi + \operatorname{sgn}\left(a^2 - 1\right) 2\pi \left\lfloor \frac{\varphi}{2\pi} + \frac{1}{2} \right\rfloor, \quad (1)$$

$$A = s_c \frac{\mu}{2} \sqrt{a^2 + a^{-2} + 2\cos\varphi}, \quad (2)$$

$$B = -\partial_0 \frac{\varphi}{2}. \quad (3)$$

The quantity $\tilde{a}$ coincides with the one from the dressed strings when calculated at $\lambda = e^{i\theta}$ and D^2 coincides with $-\Delta$ in the same case.

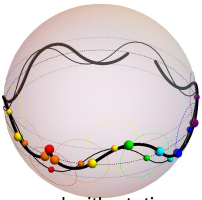
Section 3

Features of the Dressed Elliptic Strings

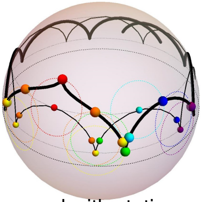
The Dressed Elliptic Strings - Epicycle Picture

- The vector X_1 is a unit vector, which is perpendicular to X_0. Thus, the arc connecting the endpoints of the vectors X and X' is equal to θ_1. In other words, *the dressed string solution can be visualized as being drawn by a point in the circumference of an epicycle of arc radius θ_1, which moves so that its center lies on the seed string solution*.
- The form of the new solution provides a nice geometric visualization of the action of the dressing on the shape of the string.
- It is the outcome of the form of the dressing factor in the case it has only two poles and *not* a specific property of the dressed elliptic solutions, but a *generic* property that holds whenever the simplest dressing factor is adopted.
- A further implication of the above is the fact that at the limit $\theta_1 \to 0$ the dressed solution tends to the seed, whereas as $\theta_1 \to \pi$ the dressed solution tends to the reflection of the seed with respect to the origin of the enhanced space.

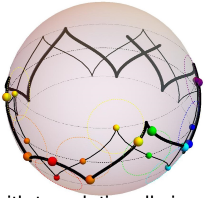
The Dressed Elliptic Strings - Epicycle Picture



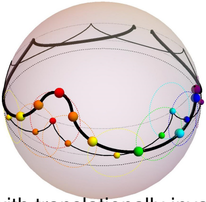
seed with static
oscillating counterpart



seed with static
rotating counterpart



seed with translationally invariant
oscillating counterpart

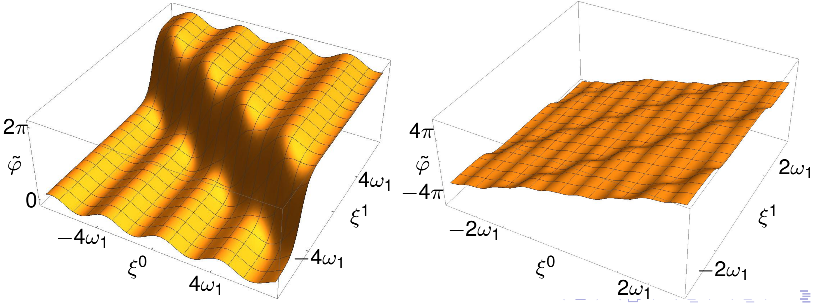


seed with translationally invariant
rotating counterpart

The Sine-Gordon Counterparts - Classification

From the form of the SG counterparts, it is evident that there is a qualitative difference between the solution with $D^2 > 0$ and $D^2 < 0$.

- The former describe a localised kink-like disturbance propagating on top of an elliptic background, whereas the latter are a non-localised disturbance of the elliptic background.
- The former are (quasi-)periodic under translations by a single vector, whereas the latter under two vectors forming a lattice.



The Sine-Gordon Counterparts - $D^2 > 0$ - Kink-Background Interaction

In the case $D^2 > 0$, the two moduli parametrising the dressed solution acquire a simple physical interpretation. The first has to do with the asymptotic behaviour of the solutions. In the region far away from the kink

$$\lim_{D\xi^1 + i\Phi(\xi^0;\tilde{a}) \to \pm\infty} s_d \tilde{\varphi} = \varphi\left(\xi^0 \pm \tilde{a}\right) + s_d\left((2k-1) \pm s_c\right)\pi .$$

Therefore, the passage of the kink effectively causes a delay to the translationally invariant motion of the system equal to

$$\Delta\xi^0 = 2\left|\tilde{a}\right| .$$

Similar is the picture in the case of a static background. In this case the kink causes a displacement of the background.

The Sine-Gordon Counterparts - $D^2 > 0$ - Kink Energy and Momentum

The other modulus D determines the energy and momentum of the kink. It turns out that

$$\begin{aligned} E_{0\mathrm{kink}} &= 8D, \quad P_{0\mathrm{kink}} = 8D\bar{v}_0, \\ E_{1\mathrm{kink}} &= 8D/\bar{v}_1, \quad P_{1\mathrm{kink}} = 8D \end{aligned}$$

The energy of the kink, the energy density of the background and the effect of the passage of the kink of the phase of the background are connected via an equation of state, in principle experimentally verifiable in systems that realize the sine-Gordon equation

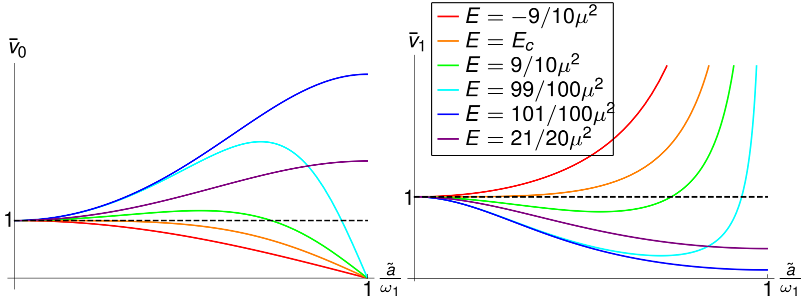
$$\begin{aligned} \frac{E_{\mathrm{kink}}^2}{64} &= \wp\left(\frac{\Delta\xi^0}{2}; \frac{E^2}{3} + \mu^4, \frac{E}{3}\left(\frac{E^2}{9} - \mu^4\right)\right) - \frac{E}{3}, \\ \frac{P_{\mathrm{kink}}^2}{64} &= \wp\left(\frac{\Delta\xi^1}{2}; \frac{E^2}{3} + \mu^4, \frac{E}{3}\left(\frac{E^2}{9} - \mu^4\right)\right) - \frac{E}{3}, \end{aligned}$$

The Sine-Gordon Counterparts - $D^2 > 0$ - Kink Velocity

The mean velocity of the kink is given by

$$\bar{v}_0 = \frac{\zeta\left(\tilde{a}\right)\omega_1 - \zeta\left(\omega_1\right)\tilde{a}}{\omega_1 D}, \quad \bar{v}_1 = \frac{\omega_1 D}{\zeta\left(\tilde{a}\right)\omega_1 - \zeta\left(\omega_1\right)\tilde{a}}.$$

It may be subluminal or superluminal depending on the moduli of the solution



The Asymptotics of the Dressed Strings with $D^2 > 0$

The asymptotics of the dressed strings are closely related to the asymptotics of their SG counterparts,

$$\lim_{\tilde{\Phi}\to\pm\infty} \theta_{0/1}\left(\sigma^0,\sigma^1\right) = \theta_{\mathrm{seed}}\left(\sigma^0,\sigma^1 \mp \frac{\tilde{a}}{2\omega_1}\delta\sigma_0\right),$$

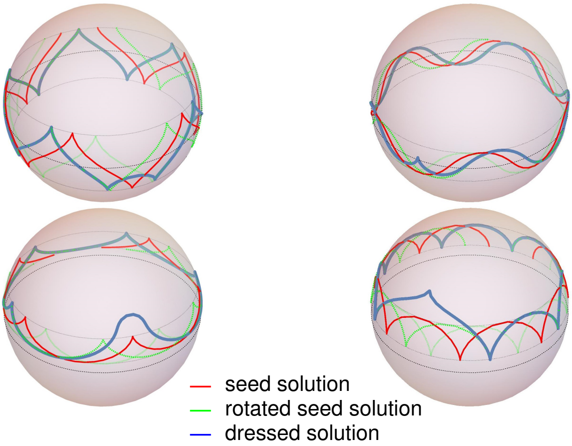
$$\lim_{\tilde{\Phi}\to\pm\infty} \varphi_{0/1}\left(\sigma^0,\sigma^1\right) = \varphi_{\mathrm{seed}}\left(\sigma^0,\sigma^1 \mp \frac{\tilde{a}}{2\omega_1}\delta\sigma_0\right) \pm \Delta\varphi_{0/1},$$

$$\Delta\varphi_{0/1} = \arg\left(\ell + iD\right) + \arg\sigma\left(\tilde{a}+a\right) + i\left(\zeta\left(a+\omega_{x_{3/2}}\right) - \zeta\left(\omega_{x_{3/2}}\right)\right)\tilde{a}.$$

As long as the characteristic length of the exponential damping of the kink is much smaller that the number of periods appearing in the seed solution, we can claim that we may adjust the periodicity conditions in order to find a string solution that is not exactly a closed finite string, but nevertheless an exponentially good approximation of such a solution.
Such solutions should obey

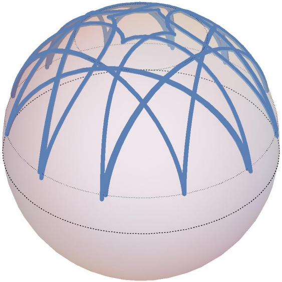
$$\left(n_1\delta\varphi + 2s_\Phi\Delta\varphi\right)n_2 = 2\pi, \quad n_1, n_2 \in \mathbb{Z}.$$

Approximate Finite Closed Strings with $D^2 > 0$



Exact Infinite Closed Strings with $D^2 > 0$

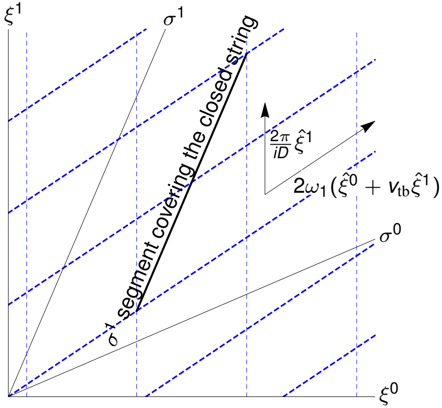
Had we not restricted to finite length strings, we could form infinite strings that obey appropriate and exact periodicity conditions in the same sense as the single spike solution.



For this purpose, both $\delta\varphi$ and $\Delta\varphi$ should be a rational fraction of 2π.

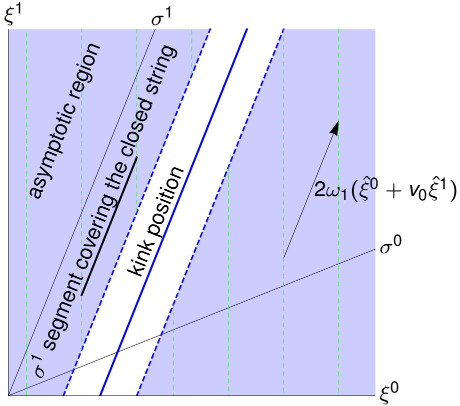
Finite Closed Strings with $D^2 < 0$

In the case $D^2 < 0$, one can show that there are finite closed dressed strings, whenever the σ^1 axis (the axis perpendicular to the physical time) coincides to a direction of the periodicity lattice of the sine-Gordon counterpart.



Finite Closed Strings with $D^2 > 0$

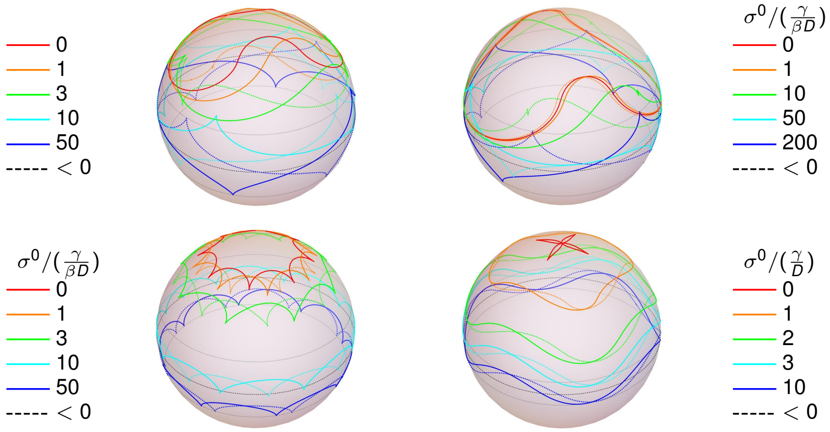
In a similar manner there is a special case one can construct finite closed dressed strings with $D^2 > 0$. In this case, the σ^1 axis should coincide with the periodicity vector of the counterpart. This may happen *only when the kink is superluminal*.



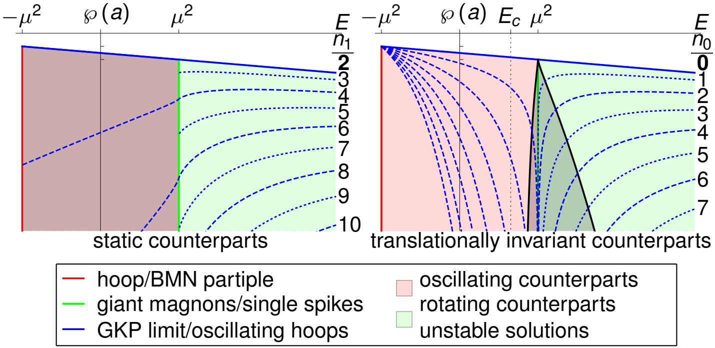
Stability of the Seeds

- The latter solutions can be used for an unconventional approach to discover instabilities of the seed elliptic string solutions. These solutions tend asymptotically in time to an elliptic string solution, but they are not a small perturbation around the latter. Such solutions are the analog, for example, in the case of the simple pendulum, to the trajectories connecting asymptotically two consecutive unstable vacua. The existence of such a solution reveals that the elliptic solution, which is the asymptotic limit of the latter, is unstable.
- The special solutions of this kind emerge only when the kink propagating on top of an elliptic background in the sine-Gordon counterpart of the solution is superluminal.
- The dressing method analysis gives identical results to those of a small perturbation analysis, encouraging the use of the dressing method as a general tool for the study of string solution stability.

Stability of the Seeds

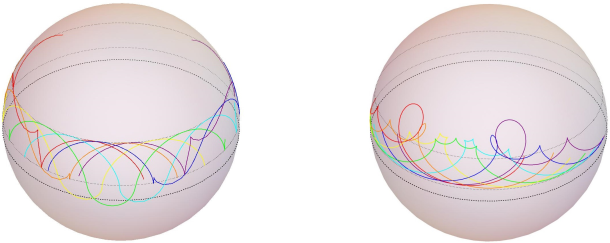


Stability of the Seeds - Moduli Space



Spike Interactions

- An interesting feature of the elliptic string solutions is the presence of singular points, i.e. *the spikes*[12]. However these string solutions do not change shape, and thus, the spikes never interact.
- Interacting spikes emerge in higher genus solutions. The simplest possible such solutions are those presented here.



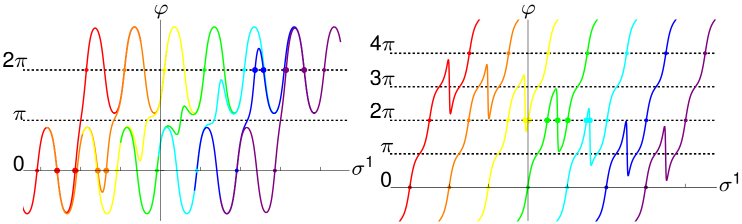
We may observe two kinds of spike interactions.

- Two spikes annihilate and regenerate at a different position.
- A loop dissolves to two spikes and vice versa.

[12] K. Okamura and R. Suzuki, Phys. Rev. D 75 (2007) 046001 [hep-th/0609026]

Spike Interactions - SG side

The above processes are quite simple to understand in the language of the sine-Gordon equation. Spike appear only at positions where the Pohlmeyer field $\varphi = 2n\pi$. The shape of the kink alters periodically as it advances in the elliptic background. As the shape changes, it is possible that the solution ceases to cross a $\varphi = 2n\pi$ horizontal line, or on the opposite may start crossing such a line. Continuity ensures that whenever this happens, two points where the solution crosses a $\varphi = 2n\pi$ line appear or disappear. It follows that spikes *interact in pairs*.

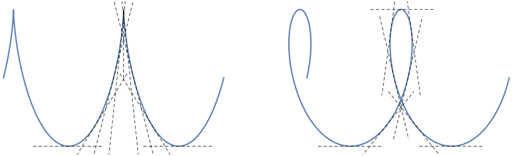


Spike Interactions - Topological Charge

The closed dressed string solutions are characterized by a topological number N,

$$2\pi N = \int_{\text{string}} d\sigma \partial_\sigma \varphi, \quad N \in \mathbb{Z}.$$

In the case of the elliptic strings this is equal to the number of spikes.
The form of the spike interactions suggests that N is mapped to a conserved quantity, which receives ± 1 contributions from each spike and ± 2 from each loop.
The turning number of the closed string cannot be defined due to the spikes. However, the string contains only this kind of non-smooth points. Therefore, the unoriented tangent to the string is continuous and an unoriented turning number t can be defined. This is a member of the fundamental group of the mappings from $\mathbb{S}^1$ to $\mathbb{R}P^1$, i.e. $\pi_1\left(\mathbb{R}P^1\right) = \mathbb{Z}$ and receives the appropriate contributions.



It follows that t and N differ by an ever integer.

Energy and Angular Momentum

The variation of the energy and angular momentum imposed by the dressing is

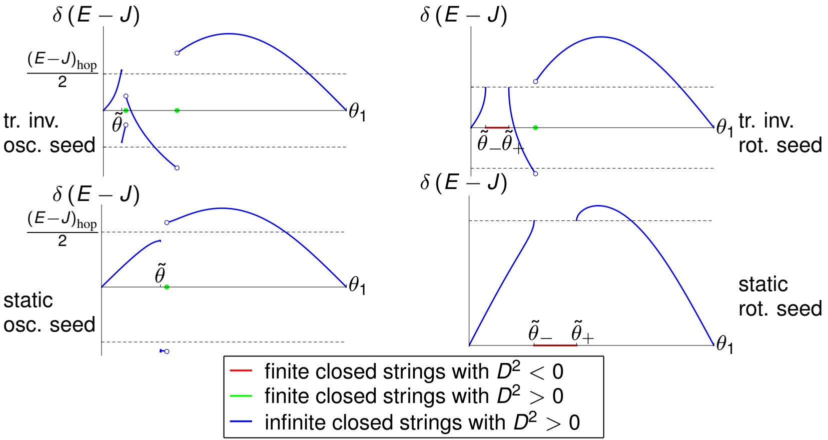
$$\Delta E_{0/1} = \pm 2 s_\Phi n_2 \frac{TR\mu^2 \tilde{a}}{\sqrt{x_{3/2} - \wp(a)}},$$

$$\Delta J_{0/1} = 2 s_\Phi n_2 \frac{1}{\ell} \left(\zeta(\tilde{a}) + x_{2/3} \tilde{a} - D \cos\theta_1 \right),$$

These vanish for the dressed solutions with $D^2 < 0$ and the exact finite solutions with $D^2 > 0$.

The exact infinite dressed strings with $D^2 > 0$ have obviously infinite energy and angular momentum. However, since they are the $n_1 \to \infty$ limit of the approximate solutions, thus, the difference of their energy and momentum to those of their elliptic seeds is well-defined. In other words the finite approximate closed dressed strings may serve as a regularization scheme for the exact infinite closed dressed strings.

Energy and Angular Momentum



Energy and Angular Momentum - Qualitative Characteristics

- There is an interesting bifurcation in the dispersion relation of the dressed string solution occurring at $D^2 = 0$. When considering dressed strings whose seeds have rotating counterparts, the dispersion relation is a rather peculiar function of the angle θ_1; there is a range for θ_1 where the dispersion relation does not depend on the latter.
- There is yet another interesting bifurcation of the form of the dispersion relation that has to do with the presence of the instabilities. When the seed is unstable, the quantity $\Delta E - \Delta J$ contains further discontinuities related with the existence of the instability.
- Although the dispersion relations of the dressed strings are too complicated expressions to be directly verifiable in a holographically dual theory, the above discontinuities in the behaviour of the dispersion relation could be in principle detectable.
- Whenever the moduli a and $\tilde{a}$ are a rational fraction of the corresponding half-period, there are closed algebraic relations between the charges.

Section 4

Future Extensions

- More complicated dressing factors can be applied, without further solving of differential equations, to find solutions whose Pohlmeyer counterparts are several kinks scattering on top of an elliptic background or even breather propagating on the latter.
- Similar techniques can be applied for strings propagating on other symmetric spaces, such as the dS, AdS or AdS$\times$S or minimal surfaces in hyperbolic spaces[13]. The latter are interesting in the framework of the RT conjecture.
- The nice geometric interpretation of the dressed string as being drawn by an epicycle of given radius whose center moves on the seed solution deserves further investigation in the case of strings propagating on other symmetric spaces.

[13] G. Pastras, arXiv:1612.03631 [hepth]

- The discovery of instabilities of the seed solutions through the dressing method is an interesting feature. Comparison with the results from linear perturbation implies that indeed the string without dressed instabilities are stable. Thus, the dressing method can be a more general tool for the study of string solution stability.
- The discovery of the qualitative behaviour of the dispersion relation of the dressed strings in the anomalous dimensions of operators of the boundary CFT is interesting.
- It can be shown that whenever the moduli a and $\tilde{a}$ are a rational fraction of the corresponding half-period, there are closed algebraic relations between the charges. These deserve investigation on the side of the CFT.

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Thank you for your attention!