Reduction of Couplings in Finite Unified Theories

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The ad hoc Yukawa and Higgs sectors of the Standard Model induce $\sim$ 20 free parameters. How can they be related to the gauge sector in a more *fundamental* level?

The straightforward way to induce relations among parameters is to add more symmetries.

$\rightarrow$ i.e. GUTs.

Another approach is to look for renormalization group invariant (RGI) relations among couplings at the GUT scale that hold up to the Planck scale.

$\rightarrow$ less free parameters $\rightarrow$ more predictive theories

Reduction of Couplings

About *dimensionless* couplings: an RGI expression among couplings

$$\mathcal{F}(g_1, \dots, g_N) = 0$$

must satisfy the pde

$$\mu \frac{d\mathcal{F}}{d\mu} = \sum_{a=1}^{N} \beta_a \frac{\partial \mathcal{F}}{\partial g_a} = 0$$

There are $(N-1)$ independent $\mathcal{F}s$ and finding them is equivalent to solve the ode

$$\beta_g \left(\frac{dg_a}{dg} \right) = \beta_a, \qquad a = 1, \dots, N$$

where g is considered the primary coupling. The above equations are the so-called *reduction equations (RE)*. Zimmermann (1985)

Using all $(N-1)$ $\mathcal{F}s$ to impose RGI relations, all other couplings can be expressed in terms of one (primary) coupling g.

Ansatz: assume power series solutions to the REs (which are motivated by and preserve perturbative renormalizability):

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1}$$

- Examining in one-loop sufficient for uniqueness to all loops

Oehme, Sibold, Zimmermann (1984); (1985)

For some models the *complete reduction* can prove to be too restrictive $\rightarrow$ use fewer $\mathcal{F}s$ as RGI constraints (*partial reduction*).

The reduction of couplings scheme is necessary for finiteness!

Finiteness

SM $\rightarrow$ quadratic divergences
SUSY $\rightarrow$ only logarithmic divergences
Finite theories $\rightarrow$ no divergences

For a chiral, anomaly free, $N = 1$ theory the superpotential is:

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k$$

$N = 1$ non-renormalization theorem $\rightarrow$ no mass and cubic-interaction-terms infinities $\rightarrow$ only wave-function infinities.

The one-loop gauge β-functions are given by

$$\beta_g^{(1)} = \frac{g^3}{16\pi^2} \left[\sum_i T(R_i) - 3C_2(G) \right]$$

The Yukawa β-functions are related to the anomalous dimensions of the matter fields:

$$\beta_{ijk}^{(1)} = C_{ijl} \gamma_k^l + C_{ikl} \gamma_j^l + C_{jkl} \gamma_i^l \qquad \gamma_j^{i(1)} = \frac{1}{32\pi^2} \left[C^{ikl} C_{jkl} - 3g^2 C_2(R) \delta_j^i \right]$$

In one-loop, all β-functions of the theory vanish if the one-loop gauge β-functions and the anomalous dimensions of all superfields vanish, imposing the conditions:

$$\sum_i T(R_i) = 3C_2(G) \quad , \qquad C_{ikl}C^{jkl} = 2\delta^i_j g^2 C_2(R_i)$$

$\rightarrow$ The gauge and Yukawa sectors of the theory are now related (Gauge-Yukawa Unification - GYU).

- One-loop finiteness is sufficient for two-loop finiteness *Parkes, West (1984)*
  - 2-loop corrections for matter fields vanish if one-loop finite $\rightarrow$ sufficient for $\beta_g^{(2)} = 0 = \beta_{ijk}^{(2)}$
- $C_2[U(1)] = 0 \rightarrow$ finiteness cannot be achieved in the MSSM $\rightarrow$ GUT
- $C_2[singlet] = 0 \rightarrow$ supersymmetry can be broken only softly.

All-loop Finiteness

Theorem — *Lucchesi, Piguet, Sibold (1988)*

Consider an N=1 supersymmetric Yang-Mills theory with simple gauge group. If:

1. There is no gauge anomaly
2. The gauge β-function vanishes at one-loop $\beta_g^{(1)} = 0$
3. All superfield anomalous dimensions vanish at one-loop $\gamma_j^{i(1)} = 0$
4. The *RE*s admit uniquely determined power series solution that in lowest order is a solution of the vanishing anomalous dimensions
   - $C_{ijk} = \rho_{ijk} g$
   - these solutions are *isolated and non-degenerate* when considered as solutions of vanishing one-loop Yukawa β-functions

then the associated Yang-Mills models depend on the single coupling constant g with a β-function which *vanishes at all orders*.

Soft supersymmetry breaking terms

What about *dimensionful* parameters?

The soft supersymmetry breaking sector introduces more than 100 new free parameters.

Reduction can be *extended* to the dimensionful sector.

Kubo, Mondragon, Zoupanos (1996)

→ Consider a $N=1$ supersymmetric gauge theory with soft terms:

$$-\mathcal{L}_{SSB} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)^j_i\phi^{*i}\phi_j + \frac{1}{2}M\lambda\lambda + h.c.$$

In addition to $\beta_g^{(1)} = 0 = \gamma_j^{i(1)}$, one-loop finiteness can be achieved if we demand:

$$h^{ijk} = -MC^{ijk} \qquad (m^2)^j_i = \frac{1}{3}MM^*\delta^j_i$$

Jones, Mezincescu, Yao (1984)

Like in the dimensionless case, the above one-loop conditions are also sufficient for two-loop finiteness.

Jack, Jones (1994)

However, the soft scalar masses universal rule leads to phenomenological problems:

- charge and colour breaking vacua
- Incompatible with radiative electroweak breaking

Assuming

- one-loop finiteness in the dimensionless sector $\beta_g^{(1)} = \gamma_j^{i(1)} = 0$
- the REs $\beta_C^{ijk} = \beta_g \frac{dC^{ijk}}{dg}$ admit power series solutions $C^{ijk} = g \sum \rho_{(n)}^{ijk} g^{2n}$
- the soft scalar masses satisfy the diagonality relation $(m^2)^i_j = m_j^2 \delta^i_j$

Kobayashi, Kubo, Mondragon, Zoupanos (1998)

based on Martin, Vaughn, Yamada, Jack, Jones (1994)

then the universal rule can be "relaxed" to a (*two-loop*) soft scalar mass sum rule,

$$(m_i^2 + m_j^2 + m_k^2)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)}$$

where the two-loop correction

$$\Delta^{(2)} = -2 \sum_i \left[(m_i^2/MM^\dagger) - (1/3) \right] T(R_i)$$

vanishes for the $N = 1$ $SU(5)$ FUTs.

All-loop Finiteness in the soft sector

- It is possible to find all-loop RGI relations between the β-functions of dimensionless and soft parameters (in both finite and non-finite theories).

  Hisano, Shifman (1997); Kazakov (1999); Jack, Jones, Pickering (1998)

  Assuming that

  - the REs $\beta_C^{ijk} = \beta_g \frac{dC^{ijk}}{dg}$ hold
  - an RGI surface exists on which $h^{ijk} = -M \frac{dC(g)^{ijk}}{d \log g}$ hold in all orders

  then, since the dimensionless sector is already finite to all loops, the soft sector is also all-loop finite.
- The following relations are also shown to hold to all loops:

$$M = M_0 \frac{\beta_g}{g}$$

$$h^{ijk} = -M_0 \beta_C^{ijk}$$

$$b^{ij} = -M_0 \beta_\mu^{ij}$$

$$(m_i^2 + m_j^2 + m_k^2) = MM^\dagger$$

Kobayashi, Kubo, Zoupanos (1998); Kobayashi, Kubo, Mondragon, Zoupanos (2000)

The $SU(5)$ Finite Model

1+2-loop finite — Hamidi, Schwarz; Jones, Raby (1984); Leon, Perez-Mercader, Quiros (1985);
all-loop finite — Kapetanakis, Mondragon, Zoupanos (1993)

We study an all-loop finite $N = 1$ supersymmetric $SU(5)$ model with content:

$$3\ \mathbf{\bar{5}} + 3\ \mathbf{10} + 4\ (\mathbf{5} + \mathbf{\bar{5}}) + \mathbf{24}$$

Heinemeyer, Mondragon, Zoupanos (2008)

Under GUT scale: broken $SU(5) \rightarrow$ MSSM; no longer finite.

In order for the model to become predictive, it should also have the following properties:

- Fermions do not couple to the adjoint rep **24**
- The two Higgs doublets of the MSSM are mostly made out of a pair of Higgs $(\mathbf{5} + \mathbf{\bar{5}})$ which couple to the third generation

We can enhance the symmetry so that the superpotential will be:

$$W=\sum_{i=1}^{3}\Big[\,\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_iH_i+g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\,\overline{H}_i\,\Big]+g_{23}^u\,\mathbf{10}_2\mathbf{10}_3H_4$$
$$+\,g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4+g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4+g_2^f\,H_2\,\mathbf{24}\,\overline{H}_2+g_3^f\,H_3\,\mathbf{24}\,\overline{H}_3+\frac{g^\lambda}{3}\,(\mathbf{24})^3$$

The isolated and non-degenerate solutions to $\gamma_i^{(1)}=0$ then give:

$$(g_1^u)^2=\frac{8}{5}\,g^2\,,\;(g_1^d)^2=\frac{6}{5}\,g^2\,,\;(g_2^u)^2=(g_3^u)^2=\frac{4}{5}\,g^2\,,$$
$$(g_2^d)^2=(g_3^d)^2=\frac{3}{5}\,g^2\,,\;(g_{23}^u)^2=\frac{4}{5}\,g^2\,,\;(g_{23}^d)^2=(g_{32}^d)^2=\frac{3}{5}\,g^2\,,$$
$$(g^\lambda)^2=\frac{15}{7}g^2\,,\;(g_2^f)^2=(g_3^f)^2=\frac{1}{2}\,g^2\,,\;(g_1^f)^2=0\,,\;(g_4^f)^2=0$$

Since our theory is supersymmetric, we could remove terms that are not needed by hand in order to obtain the solutions. This method is equivalent to imposing the extra symmetry.

From the sum rule we obtain:

$$m_{H_u}^2+2m_{\mathbf{10}}^2=M^2\,,\qquad m_{H_d}^2-2m_{\mathbf{10}}^2=-\frac{M^2}{3}\,,\qquad m_{\overline{\mathbf{5}}}^2+3m_{\mathbf{10}}^2=\frac{4M^2}{3}$$

Only two free parameters ($m_{\mathbf{10}}$ and M) in the dimensionful sector.

Phenomenology

Gauge symmetry broken → MSSM → boundary conditions at M_{GUT} remain of the form:

(a) $C_i = \rho_i g$

(b) $h = -MC$

(c) sum rule

One-loop β-functions for the soft sector, everything else in two loops.

Input: The only value fixed is the one of m_τ.

Output:

- solutions that satisfy m_t , m_b , m_h experimental constraints
- solutions that satisfy B physics observables
- neutral LSP
- no fast proton decay
- SUSY breaking scale and full SUSY spectrum

Flavour Constraints

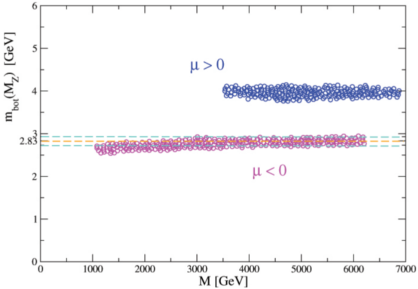
Four types of flavour constraints, where supersymmetry has significant impact:

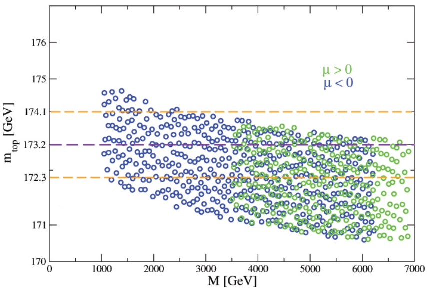
- $\frac{BR(b \to s\gamma)^{\mathrm{exp}}}{BR(b \to s\gamma)^{\mathrm{SM}}} = 1.089 \pm 0.27$
- $\frac{BR(B_u \to \tau\nu)^{\mathrm{exp}}}{BR(B_u \to \tau\nu)^{\mathrm{SM}}} = 1.39 \pm 0.69$
- $BR(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9}$
- $\frac{\Delta M_{B_s}^{\mathrm{exp}}}{\Delta M_{B_s}^{\mathrm{SM}}} = 0.97 \pm 0.2$

Uncertainties: linear combination of experimental error and twice the theoretical MSSM uncertainty.

Bottom and Top Quark Mass

$\hat{m}_t$, $m_b(M_Z)$ as a function of the unified gaugino mass M for $\mu < 0$ and $\mu > 0$.

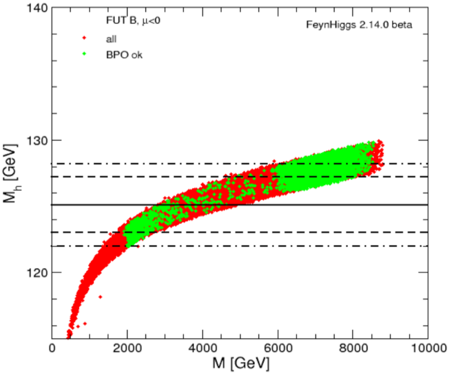




Only $\mu < 0$ phenomenologically acceptable choice.

Lightest Higgs Mass

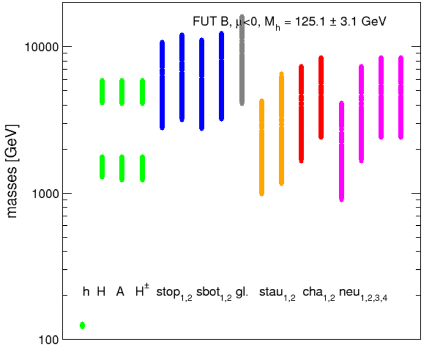
FeynHiggs: Hybrid approach of fixed-order diagrammatic calculations and EFT resummation of large logarithmic contributions.

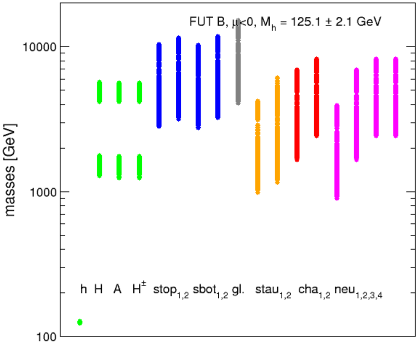


FeynHiggs 2.14.0 $\longrightarrow$ downward shift $\sim$ 2 *GeV*

Supersymmetric Spectrum

SUSY spectrum for $M_h = 125.1 \pm 3.1$ GeV (left) and $M_h = 125.1 \pm 2.1$ GeV (right)





- $\tan\beta \sim 44 - 46$
- SUSY spectrum > 600 GeV

Supersymmetric Spectrum

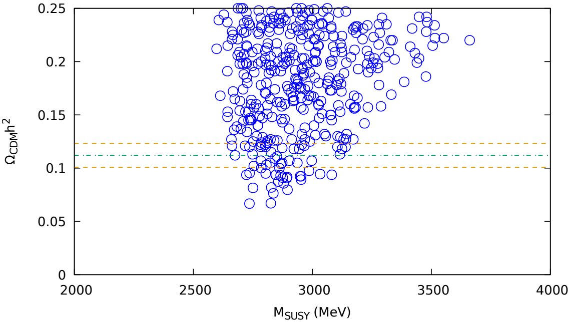
$\delta M_h = 2.1$	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	123.1	1533	1528	1527	2800	3161	2745	3219	4077
heaviest	127.2	4765	4737	4726	10,328	11,569	10,243	11,808	15,268
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	983	1163	1650	2414	900	1650	2410	2414	45
heaviest	4070	5141	6927	8237	3920	6927	8235	8237	46
$\delta M_h = 3.1$	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	122.8	1497	1491	1490	2795	3153	2747	3211	4070
heaviest	127.9	4147	4113	4103	10,734	12,049	11,077	12,296	16,046
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	1001	1172	1647	2399	899	647	2395	2399	44
heaviest	4039	6085	7300	8409	4136	7300	8406	8409	45

CDM Relic Density Constraints

We have a neutralino LSP $\rightarrow$ CDM candidate.

Most sensitive constraint $\rightarrow$ Relic Density

Calculation with MicrOMEGAs 5.0 $\rightarrow$ large exclusions (but still viable results)



Our LSP is $\sim$ 97% Bino-like, $\sim$ 3% Higgsino-like.

Ways to get $N = 1$ $SU(3) \otimes SU(3) \otimes SU(3)$

- We start from a $10D$, $N = 1$ theory with a E_8 gauge group on a $M^4 \times B$ space.

  If we reduce over the nearly-Kaehler space $B = SU(3)/U(1) \times U(1)$,
  then the surviving gauge group is $E_6 \times U(1)_A \times U(1)_B$.

  Kapetanakis, Zoupanos (1992); Manousselis, Zoupanos (2002)

  This can be further reduced via the Wilson flux mechanism if instead we consider a reduction over $B = SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$.
  Then, the surviving group will be $SU(3)_c \times SU(3)_L \times SU(3)_R$

  Hosotani (1983); Irges, Zoupanos (2011)

- We start from a $4D$, $N = 4$ theory with a $SU(3N)$ gauge group on M^4.

  Orbifolding by embedding $\mathbb{Z}_3$ in the $SU(3)$ subgroup of $SU(4)_R \to$ the projected theory is a $N = 1$ $SU(N)^3$.

  By adding SSB terms $\to$ we get vacua that can be described by 3 fuzzy spheres.
  Theory breaks down to $SU(3)_c \times SU(3)_L \times SU(3)_R$ + finite Kaluza-Klein towers

  Aschieri, Grammatikopoulos, Steinacker, Zoupanos (2006);
  Chatzistavrakidis, Steinacker, Zoupanos (2010)

The $N = 1$ $SU(3)_c \times SU(3)_L \times SU(3)_R$ Model

The supermultiplets are given as:

$$q^c = \begin{pmatrix} d_R^1 & u_R^1 & D_R^1 \\ d_R^2 & u_R^2 & D_R^2 \\ d_R^3 & u_R^3 & D_R^3 \end{pmatrix} \rightarrow (\overline{3}, 1, 3) \qquad q = \begin{pmatrix} d_L^1 & d_L^2 & d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix} \rightarrow (3, \overline{3}, 1)$$

$$L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R & e_R & S \end{pmatrix} \rightarrow (1, 3, \overline{3})$$

The 1-loop β-functions for each gauge group are given by:

$$\beta_i = (16\pi^2)^{-1} a_i g_i^3, \qquad a_i = T(R_1)d(R_2) - 3C_2(G_1)$$

However, $a_i = 3n_G - 9$, where n_G is the number of fermionic families.

$\rightarrow$ β_{g_i}=0 in 1-loop

Ma, Mondragon, Zoupanos (2007)

For one fermionic family, the trinilear invariant terms of the superpotential are:

$$fTr(Lq^c q) + \frac{1}{6} f' \epsilon_{ijk}\epsilon_{abc}(L_{ia}L_{jb}L_{kc} + q^c_{ia}q^c_{jb}q^c_{kc} + q_{ia}q_{jb}q_{kc})$$

Then, the vanishing condition of the anomalous dinemsions becomes:

$$\frac{1}{2}(3|f|^2 + 2|f'|^2) = 2\left(\frac{4}{3}g^2\right)$$

If f' vanish, then the above relation has an isolated solution:

$$f^2 = \frac{16}{9}g^2$$

and the model becomes finite in all loops.

However $\rightarrow$ without f' there are no lepton masses!

Alternatively: we keep 2-loop finiteness and allow lepton masses

$$f^2 = \frac{16}{9}g^2 \qquad f'^2 = (1-r)\frac{8}{3}g^2$$

Bottom and Top Quark Mass

Boundary conditions at M_{GUT}

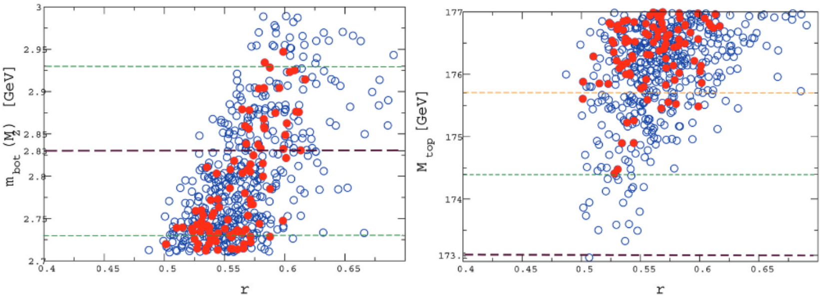
(a) $f^2 = 16/9g^2 \quad , \quad f'^2 = (1-r)8/3g^2$

(b) $h = -MC$

(c) sum rule

One-loop β-functions for the soft sector, everything else in two loops.

For $\mu < 0$



Summary

- Reduction of Couplings: powerful tool that implies Gauge-Yukawa Unification
- Finiteness: old dream of HEP, very predictive models
- completely finite theories $\rightarrow$ both in dimensionless and dimensionful sector

$SU(5)$:

- past analysis predicted the lightest Higgs boson mass
- Re-examined in two-loop (one-loop for the SSB sector) and with the new FeynHiggs code
- $\mu < 0$ survives phenomenological constraints (including relic density)
- heavy SUSY spectrum, probably eludes present and next-gen accelerators

$SU(3) \otimes SU(3) \otimes SU(3)$:

- "low-energy" remnant of larger symmetries
- 1-loop β functions vanish for 3 fermionic families
- $\mu < 0$ survives phenomenological constraints (at least for top and bottom quarks)
- promising analysis $\rightarrow$ to be extended to Higgs mass, SUSY spectrum and relic density