Higher Dimensional Operators, SSB and consistency of Reg's

Fotis Koutroulis

Notation

- Higher Dimensional Operator (HDO)
- Higher Derivative Operator (HDOp)
- Spontaneous Symmetry Breaking (SSB)
- Reparameterization Ghosts (Reg's)
- Ostrogradsky Ghosts (Og's)
- Dimensional Regularization (DR)
- degrees of freedom (d.o.f.)

- Fotis Koutroulis

Motivation

- Higgs Hierarchy Problem(Quadratic divergencies)
- Many solutions (SUSY, Composite Higgs, Extra dimensions,…)
- Focus on Non-Perturbative Gauge-Higgs Unification(Hint from: Nikos Irges and F.K. Nucl. Phys. B 937 (2018) 135-195)

- Fotis Koutroulis

HDO and SSB

- Current work in progress with Nikos Irges (to appear)
- Main goal: SSB and Higgs mechanism including only HDOp's
- Toy model: ϕ^4-theory plus dim=(6,8) operators

HDO and SSB

- The Lagrangian is

$$\mathscr{L}^{(6,8)} = -\frac{1}{2}\left(\phi_0 \Box \phi_0 + \phi_0 m_0^2 \phi_0\right) - \frac{\lambda_0}{4!}\phi_0^4$$

$$+\frac{c_{1,0}^{(6)}}{\Lambda^2}\phi_0^2 \Box \phi_0^2 + \frac{c_{2,0}^{(6)}}{\Lambda^2}\phi_0 \Box^2 \phi_0 + \frac{c_{3,0}^{(6)}}{\Lambda^2}\phi_0^6$$

$$+\frac{c_{1,0}^{(8)}}{\Lambda^4}\phi_0^3 \Box \phi_0^3 + \frac{c_{2,0}^{(8)}}{\Lambda^4}\phi_0^2 \Box^2 \phi_0^2 + \frac{c_{3,0}^{(8)}}{\Lambda^4}\phi_0 \Box^3 \phi_0 + \frac{c_{4,0}^{(8)}}{\Lambda^4}\phi_0^8$$

- Performing a general field redefinition

HDO and SSB

$$\phi_0 \to \phi_0 + \frac{x}{\Lambda^2}\Box\phi_0 + \frac{y}{\Lambda^2}\phi_0^3 + \frac{z}{\Lambda^4}\Box^2\phi_0 + \frac{u}{\Lambda^4}\phi_0\Box\phi_0^2 + \frac{w}{\Lambda^4}\phi_0^5$$

- The redefined Lagrangian is

$$\mathscr{L}^{(6,8)} = -\frac{1}{2}\phi_0\Box\phi_0 - \frac{1}{2}F_1(x)m_0^2\phi_0^2 - F_2(y)\frac{\lambda_0}{4!}\phi_0^4 + F_3(x,y,u)\frac{c_{1,0}^{(6)}}{\Lambda^2}\phi_0^2\Box\phi_0^2$$

$$+F_4(x,z)\frac{c_{2,0}^{(6)}}{\Lambda^2}\phi_0\Box^2\phi_0 + F_5(y,w,x)\frac{c_{3,0}^{(6)}}{\Lambda^2}\phi_0^6 + F_6(w,x,y,u)\frac{c_{1,0}^{(8)}}{\Lambda^4}\phi_0^3\Box\phi_0^3$$

$$+F_7(x,y,u,z)\frac{c_{2,0}^{(8)}}{\Lambda^4}\phi_0^2\Box^2\phi_0^2 + F_8(x,z)\frac{c_{3,0}^{(8)}}{\Lambda^4}\phi_0\Box^3\phi_0 + F_9(y,w)\frac{c_{4,0}^{(8)}}{\Lambda^4}\phi_0^8$$

- Freedom to eliminate any kind of operator

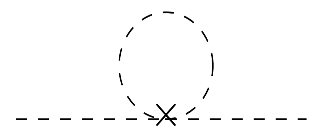
HDO and SSB

- Keep only HDO's without derivatives (ϕ^6, ϕ^8)
- Focus on dim=6 case with $(x = c_{1,0}^{(6)}, y = c_{1,0}^{(6)} - \frac{\lambda}{6} c_{1,0}^{(6)})$

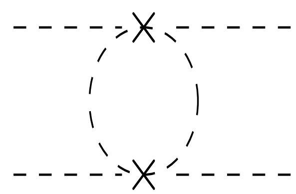
$$\mathscr{L}^{(6)} = -\frac{1}{2}\phi_0 \Box \phi_0 - \frac{1}{2}\tilde{m}_0^2 \phi_0^2 - \frac{\tilde{\lambda}_0}{4!}\phi_0^4 + \frac{\tilde{c}_{3,0}^{(6)}}{6!\Lambda^2}\phi_0^6$$

- Renormalization procedure, with ε-expansion, in DR

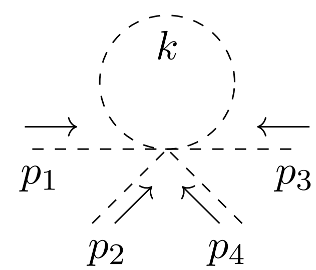
HDO and SSB



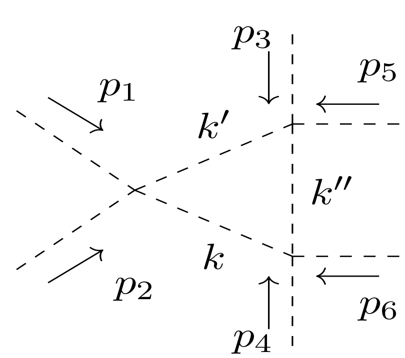
$$\frac{\lambda m^2}{16\pi^2}$$

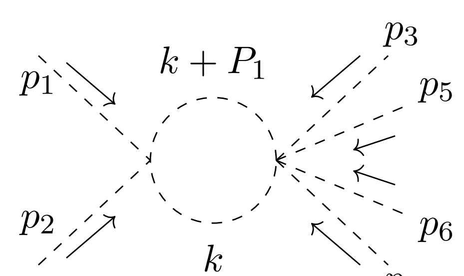


$$\frac{3\widetilde{\lambda}^2}{16\pi^2}$$



$$-\frac{3\widetilde{c}_3^{(6)}}{16\pi^2}\frac{\widetilde{m}^2}{\Lambda^2}$$





$$\frac{5\widetilde{\lambda}\widetilde{c}_3^{(6)}}{16\pi^2}$$

1-loop corrections to the $\mathscr{L}^{(6)}$

HDO and SSB

- In massless limit:

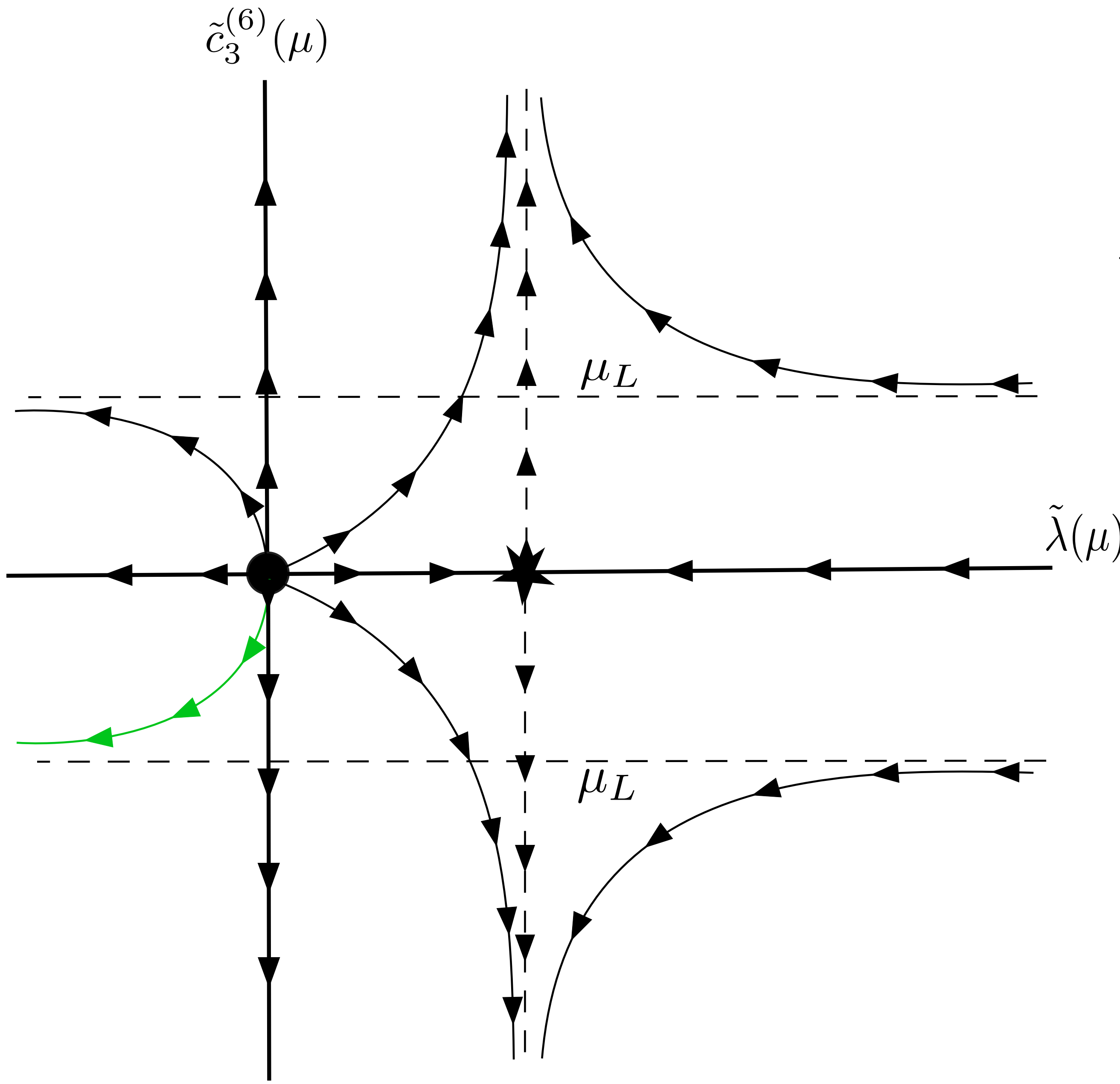
$$\beta_{\tilde{\lambda}} = -\varepsilon\tilde{\lambda} + \frac{3\tilde{\lambda}^2}{16\pi^2} \quad \text{and} \quad \beta_{\tilde{c}_3^{(6)}} = -2\varepsilon\tilde{c}_3^{(6)} + \frac{5\tilde{c}_3^{(6)}\tilde{\lambda}}{16\pi^2}$$

$$V(\phi) = \frac{\tilde{\lambda}}{4!}\phi^4 - \frac{\tilde{c}_3^{(6)}}{6!\Lambda^2}\phi^6$$

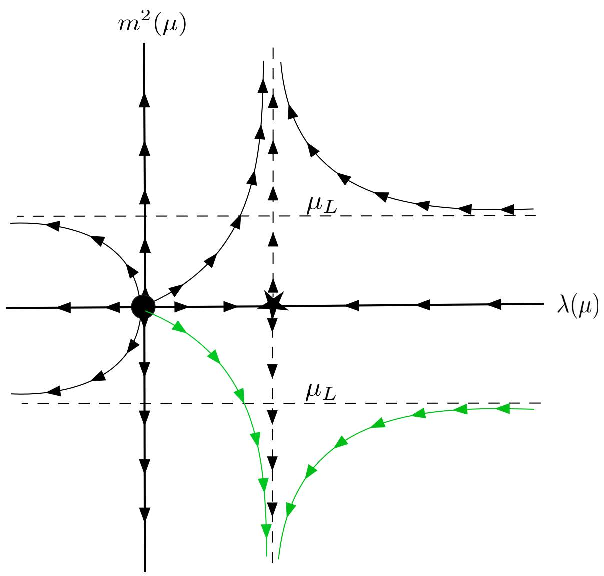
- The mass after SSB as a function of scale
- Phase diagram of the theory in $d = 3,4,5$

- Fotis Koutroulis

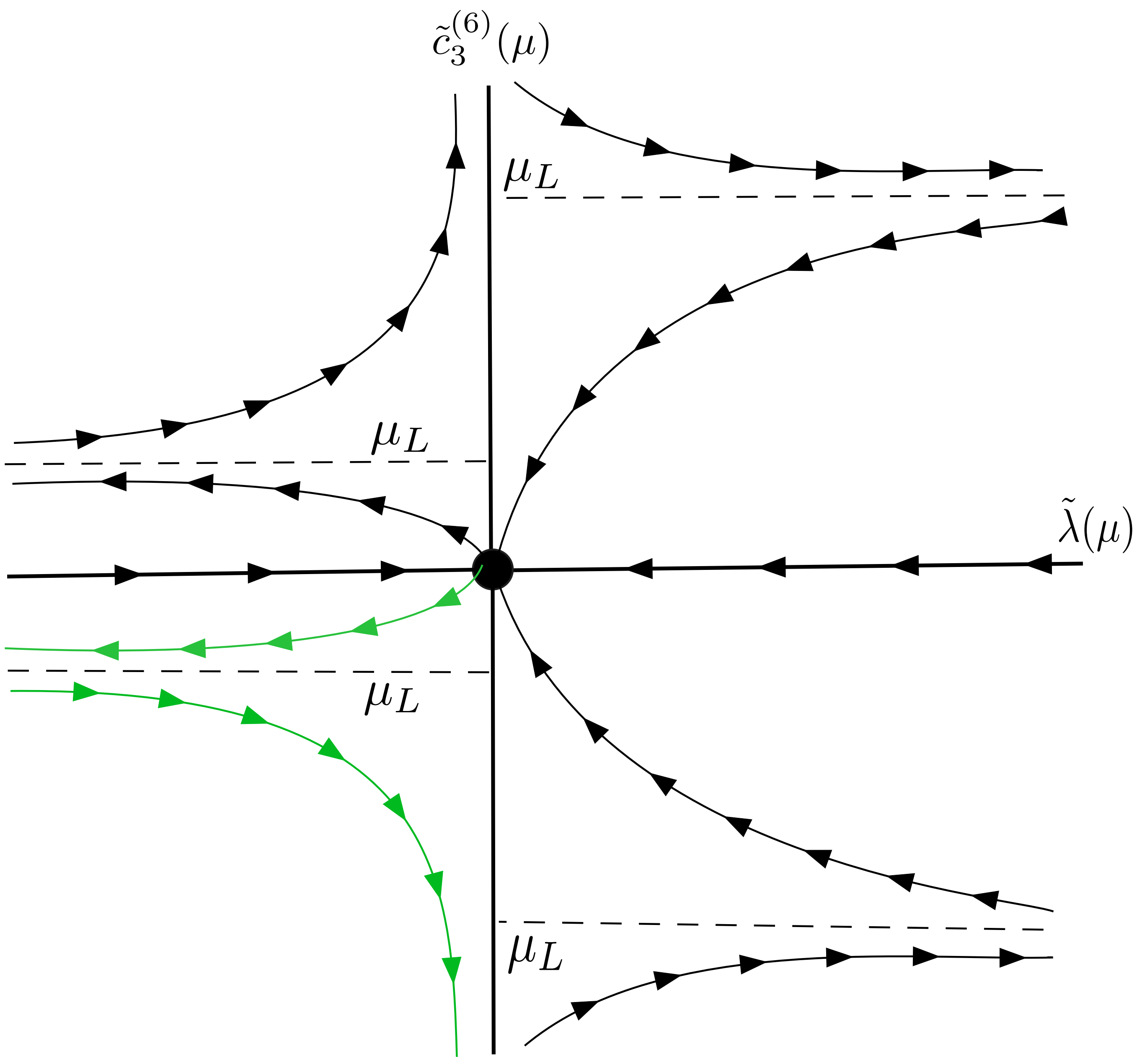
HDO and SSB

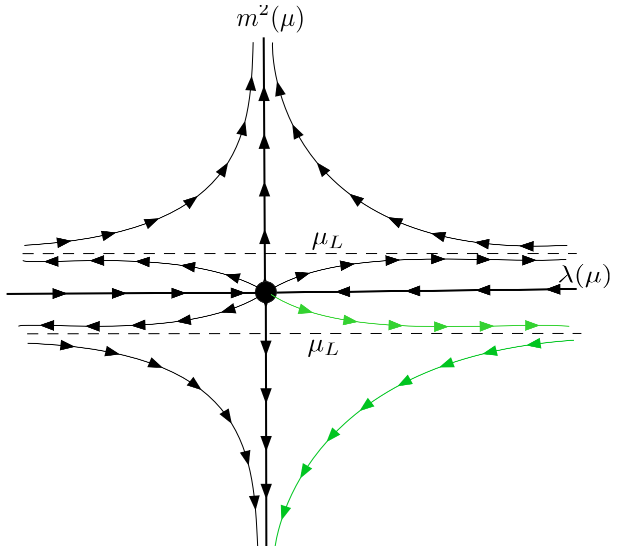


Phase diagram in d=3



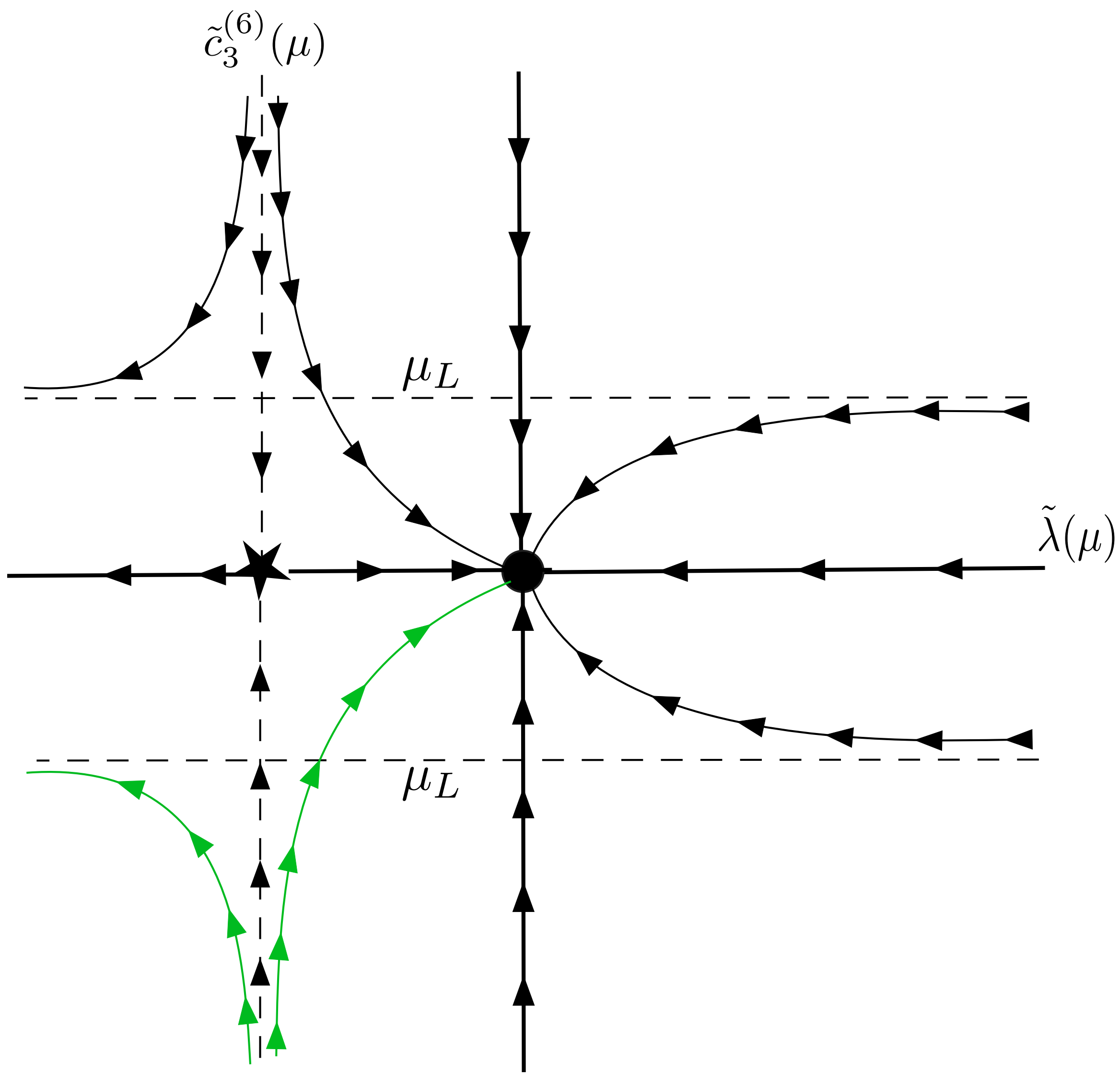
HDO and SSB

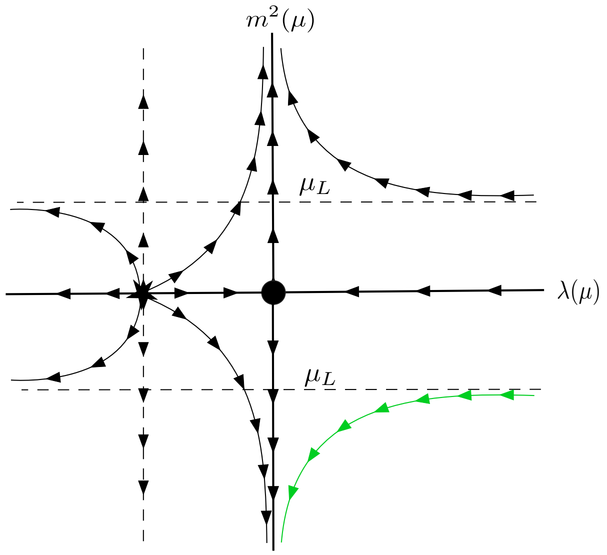




Phase diagram in d=4

HDO and SSB

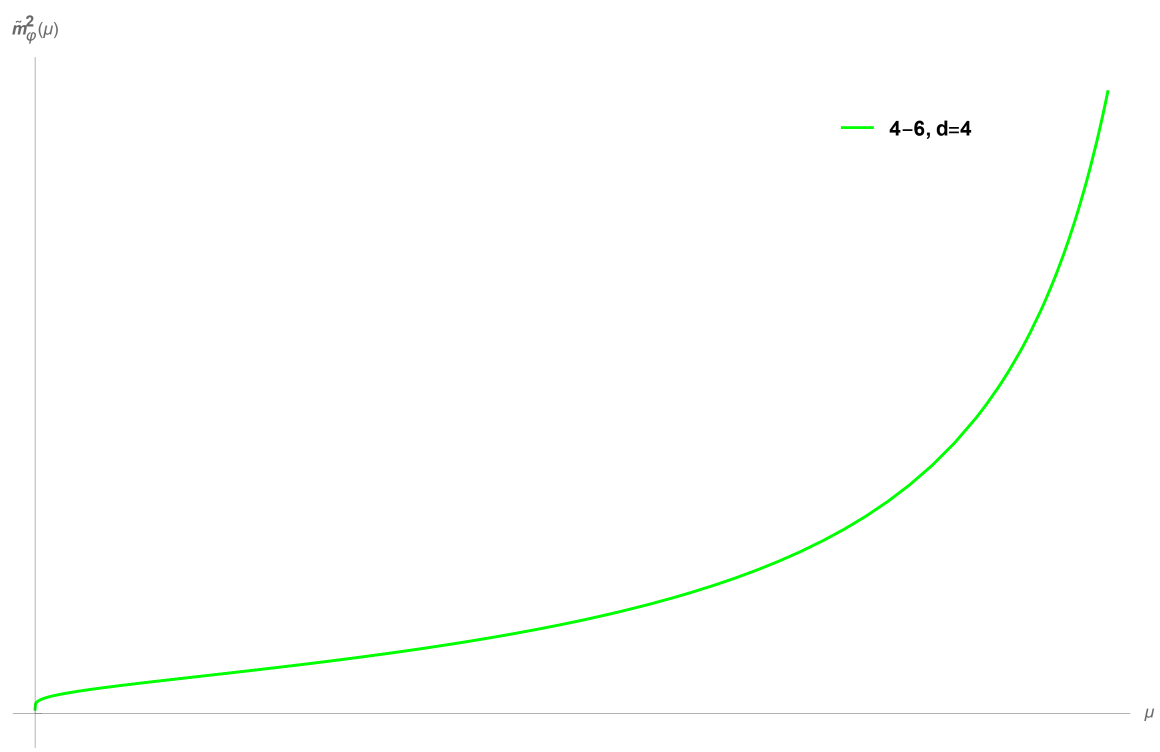




Phase diagram in d=5

HDO and SSB

- The scalar mass in this case runs as



-

Running of the physical scalar mass. There is LP

HDO and SSB

- Same procedure for the case of $\dim = (6,8)$ in the massless limit with $\tilde{\lambda} \to 0$

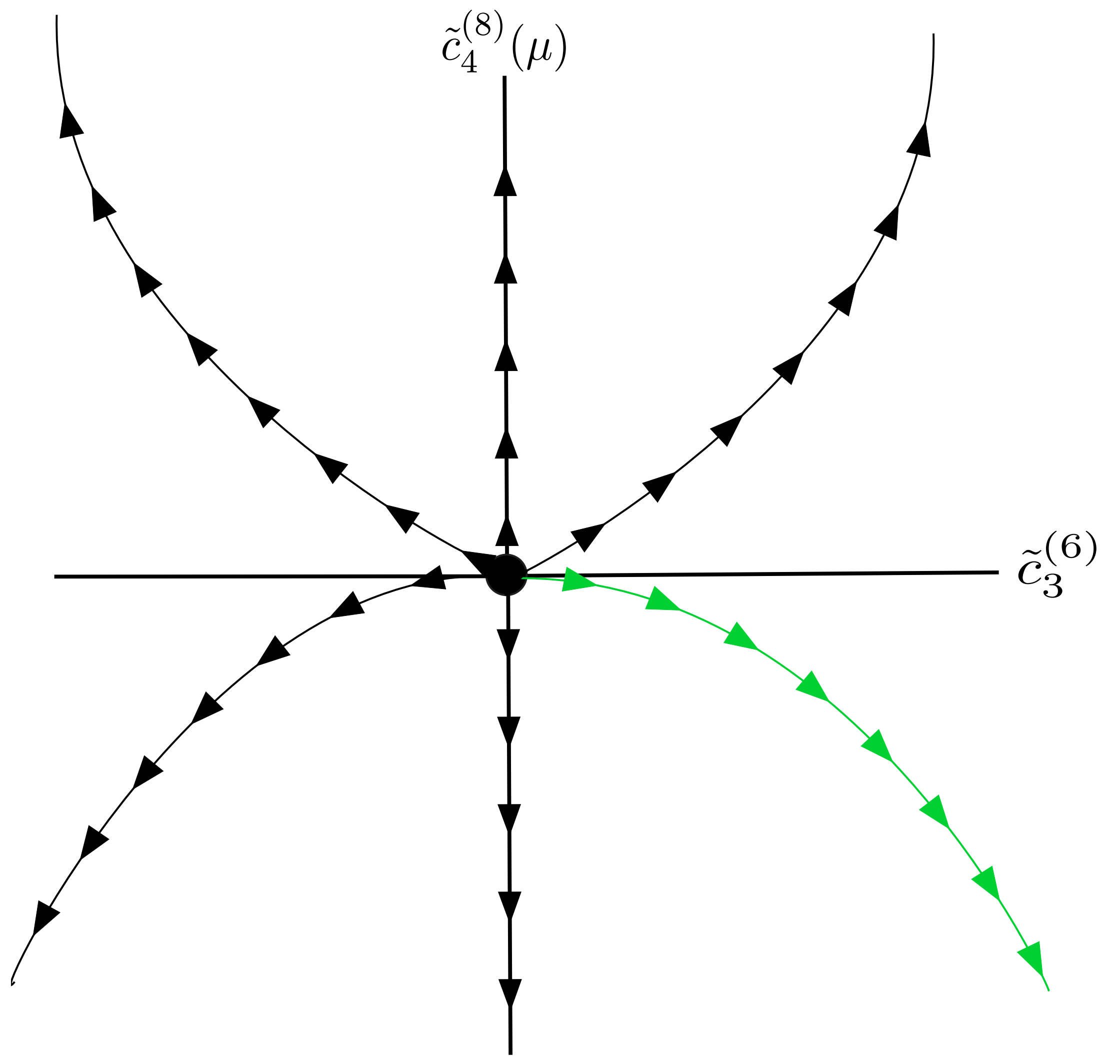
- The Lagrangian is

$$\mathscr{L}^{(6,8)} = -\frac{1}{2}\phi_0 \Box \phi_0 + \frac{\tilde{c}^{(6)}_{3,0}}{6!\Lambda^2}\phi_0^6 + \frac{\tilde{c}^{(8)}_{4,0}}{8!\Lambda^4}\phi_0^6$$

- The new potential is $V(\phi) = -\frac{\tilde{c}^{(6)}_{3,0}}{6!\Lambda^2}\phi_0^6 - \frac{\tilde{c}^{(8)}_{4,0}}{8!\Lambda^4}\phi_0^6$

- $\beta_{\tilde{c}^{(6)}_3} = -2\varepsilon\tilde{c}^{(6)}_3 \quad \beta_{\tilde{c}^{(6)}_3} = -3\varepsilon\tilde{c}^{(8)}_4 + \frac{7(\tilde{c}^{(6)}_3)^2}{16\pi^2}$

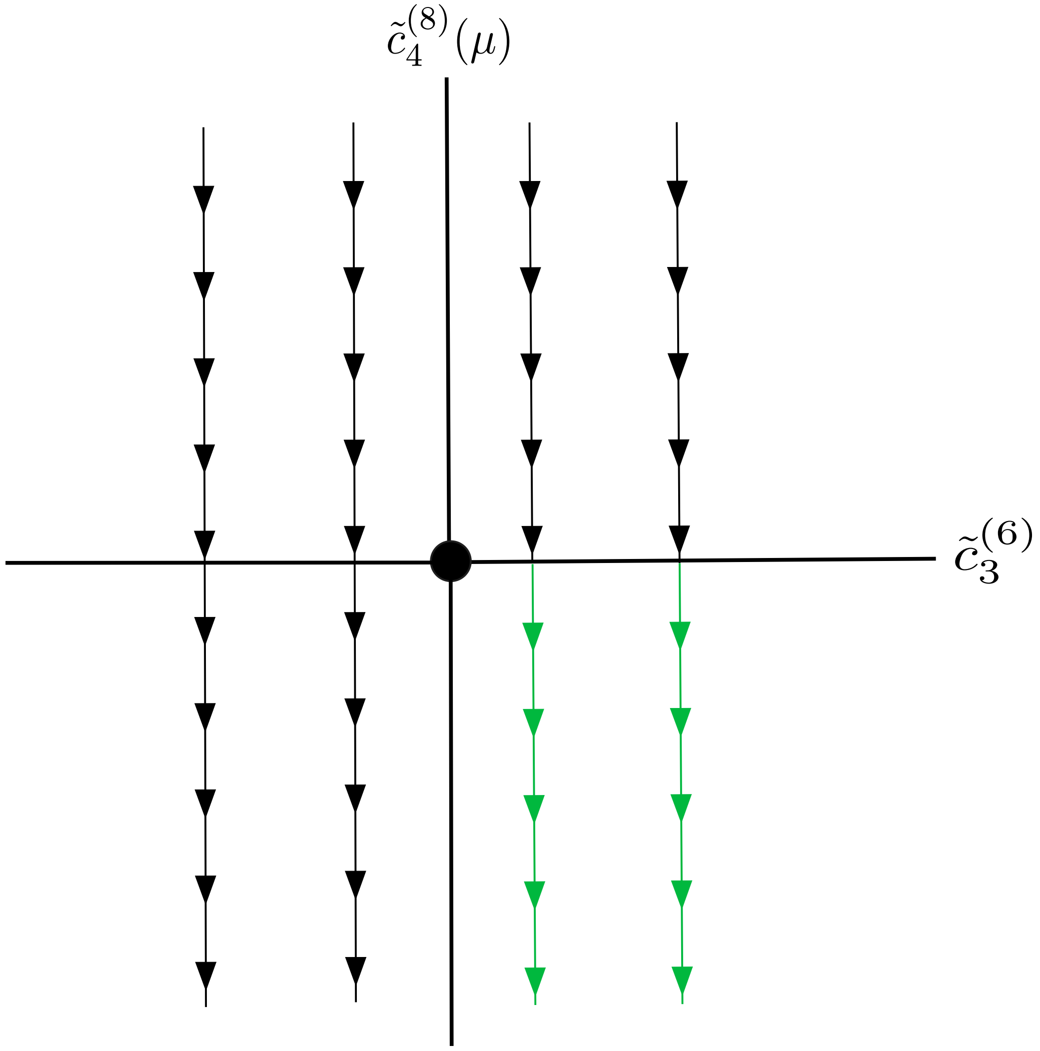
HDO and SSB



Phase diagram in d=3

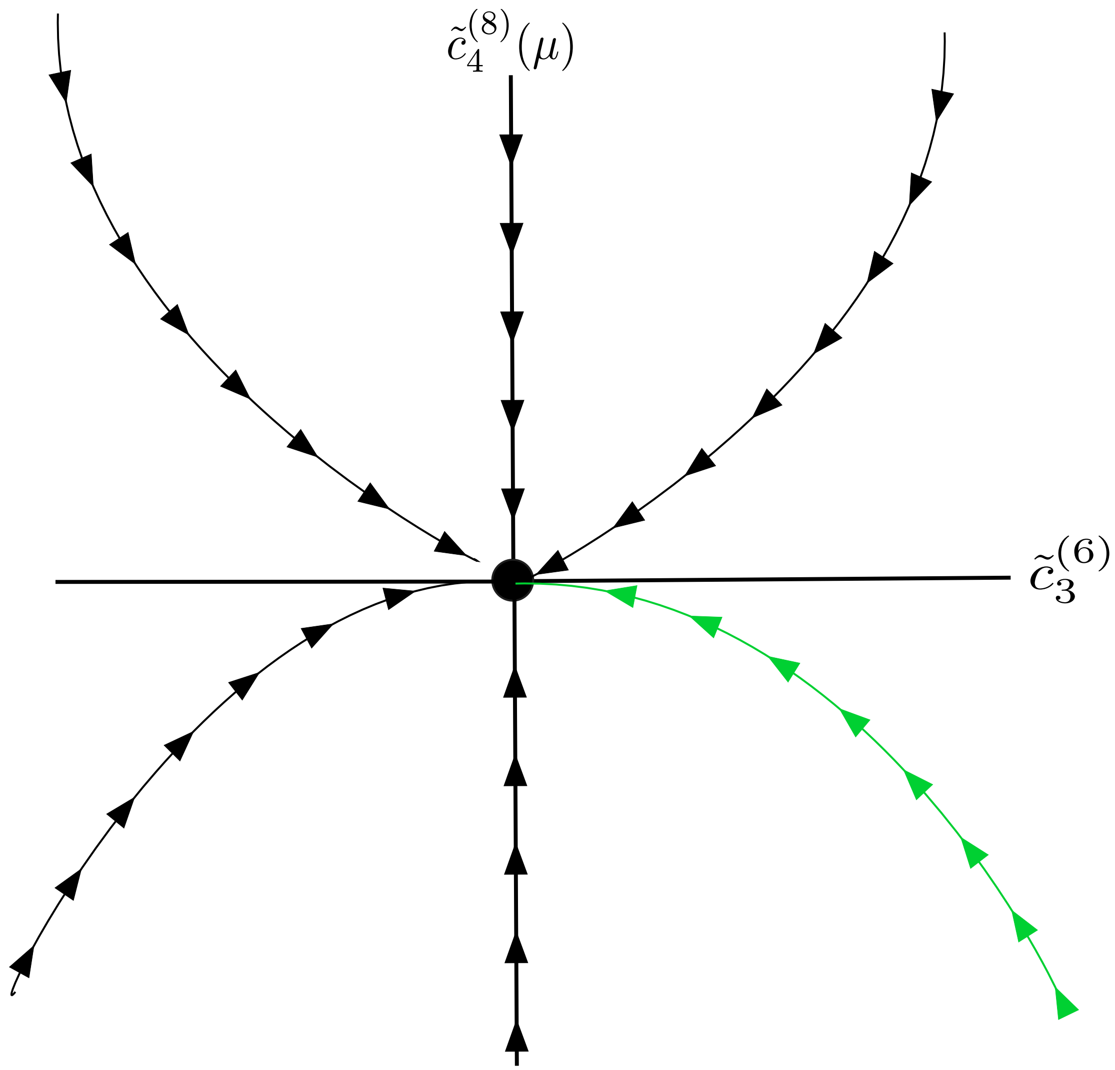
◆ Fotis Koutroulis

HDO and SSB



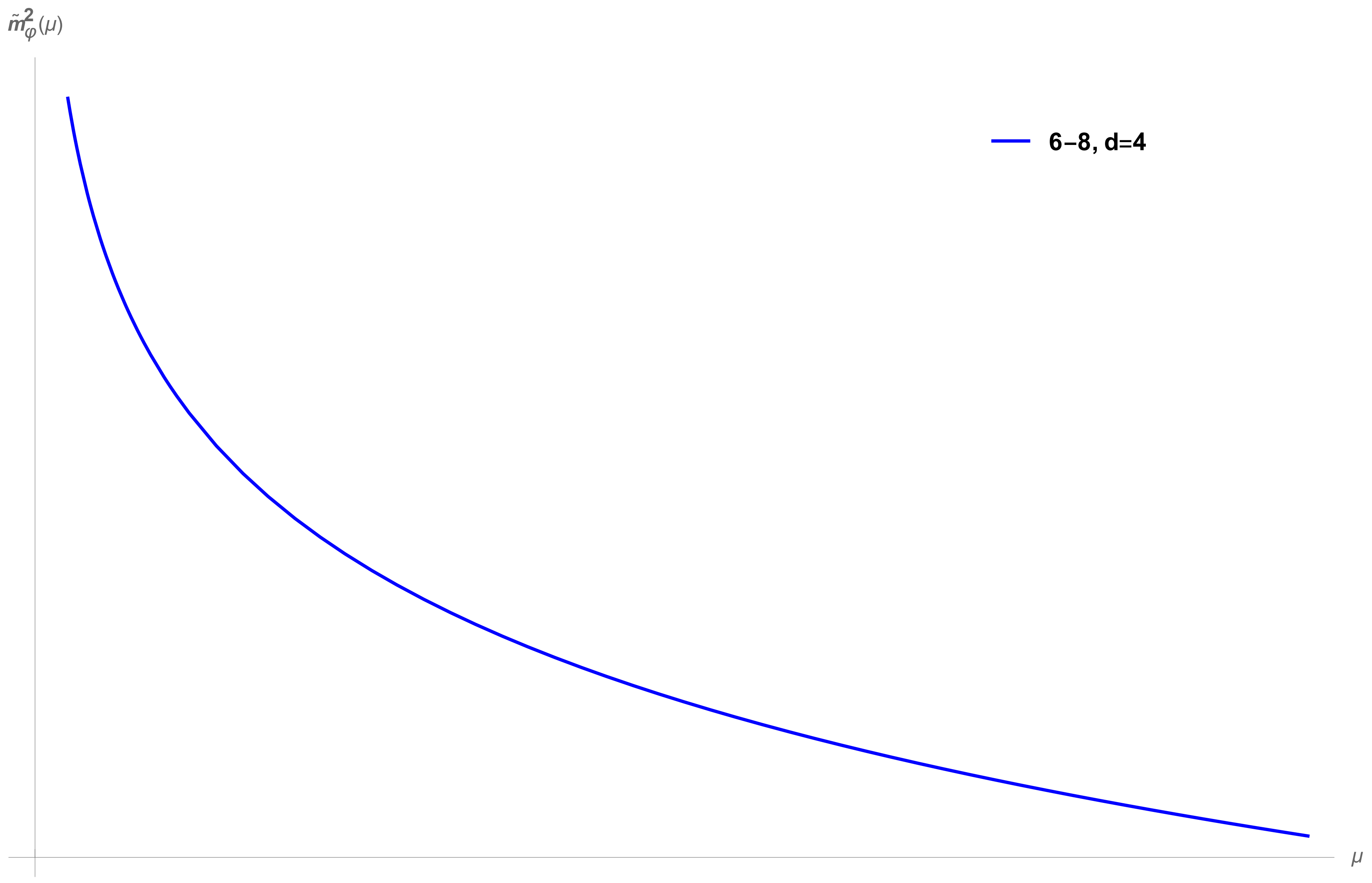
Phase diagram in d=4

HDO and SSB



Phase diagram in d=5

HDO and SSB



Running of the physical mass. Inverse LP

Reg's

- Give polynomial HDO and HDOp's equivalent picture?

$$\mathscr{L}^{(6)} = -\frac{1}{2}\phi_0 \Box \phi_0 - \frac{1}{2}\tilde{m}_0^2 \phi_0^2 - \frac{\tilde{\lambda}_0}{4!}\phi_0^4 + \frac{\tilde{c}_{3,0}^{(6)}}{6!\Lambda^2}\phi_0^6$$

- The answer through the Phase diagram
- An other field redefinition to go to the original basis is $\phi_0' = \phi_0 + \frac{x}{\Lambda^2}\Box \phi_0 + \frac{y}{\Lambda^2}\phi_0^3$

- Fotis Koutroulis

Reg's

- Illegal step unless considering the path integral

$$\mathcal{Z}'[0] = \int \mathcal{D}\phi_0 \frac{d\phi_0'}{d\phi_0} e^{iS[\phi_0 + \frac{x}{\Lambda^2}\Box\phi_0 + \frac{y}{\Lambda^2}\phi_0^3]}$$

$$\mathcal{Z}'[0] = \int \mathcal{D}\phi_0 \mathcal{D}\bar{\chi}_0' \mathcal{D}\chi_0' \, e^{iS[\phi_0 + \frac{x}{\Lambda^2}\Box\phi_0 + \frac{y}{\Lambda^2}\phi_0^3] - i\int d^4x \bar{\chi}_0' F(\phi_0)\chi_0'}$$

- Insertions of Reg's and Og's

- Fotis Koutroulis

Reg's

$$\mathscr{L}^{(6)} = -\frac{1}{2}\phi_0 \Box \phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4 + \frac{c_{1,0}^{(6)}}{4!\Lambda^2}\phi_0^2 \Box \phi_0^2 + \frac{c_{2,0}^{(6)}}{2\Lambda^2}\phi_0 \Box^2 \phi_0 + \frac{c_{3,0}^{(6)}}{6!\Lambda^2}\phi_0^6 - \bar{\chi}_0 \Box \chi_0 + \frac{2\Lambda^2}{c_{2,0}^{(6)}}\bar{\chi}\chi - \frac{\lambda_\chi}{2}\bar{\chi}\chi\phi^2$$

- Renormalization gives $\delta\phi, \delta\chi, \delta m, \delta c^{(6)}_{(1,2,3)}, \delta\lambda_\chi$
- Unphysical d.o.f. cancel each others $(\delta\phi, \delta\chi)$
- What about different basis with HDO in Standard Model? (Warsaw-basis…)

Outlook

- Alternative solution to Higgs Hierarchy through HDOp's
- Is it true that HDOp's ≡ HDO's?
- Are the HDO-basis (Warsaw-basis, etc) in Standard Model consistent with ghost insertions?

THANK YOU

- Fotis Koutroulis