

NLO EW/QCD corrections for W^+Z scattering at the LHC

$p p \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X @ \mathcal{O}(\alpha_s \alpha^6)$ and $\mathcal{O}(\alpha^7)$ for $\sqrt{s} = 13 \text{ TeV}$

arXiv:1904.00882

Christopher Schwan

with:

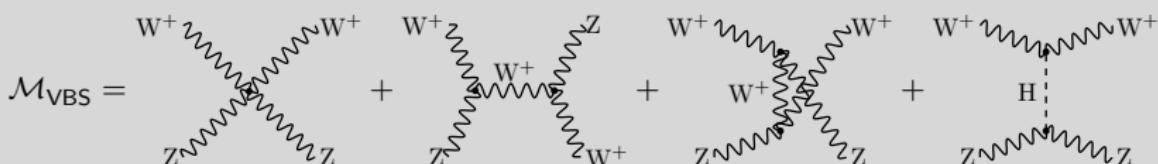
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Philipp Maierhöfer, Mathieu Pellen

MBI 2019, Thessaloniki, 26 August



Vector-boson scattering in a nutshell

$$W^+ Z \rightarrow W^+ Z$$



Physics opportunities of Vector-boson scattering (VBS):

- Constrain anomalous quartic gauge couplings
- Probe EW symmetry breaking (QGC+TGC vs. HVV couplings) → **Giovanni's talk**
- **Precise prediction** of the SM cross section needed

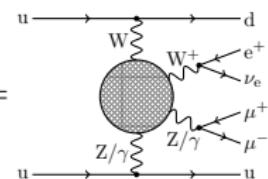
Scattering of two (massive) vector-bosons, e.g.:

$$\rightarrow W^\pm Z \rightarrow W^\pm Z$$

$$pp \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X:$$

- $W^\pm W^\pm \rightarrow W^\pm W^\pm$ ("like-sign W scattering")
- $ZZ \rightarrow ZZ$ (and $W^+ W^- \rightarrow ZZ$)

$$\mathcal{M}_{\text{VBS@LHC}} =$$

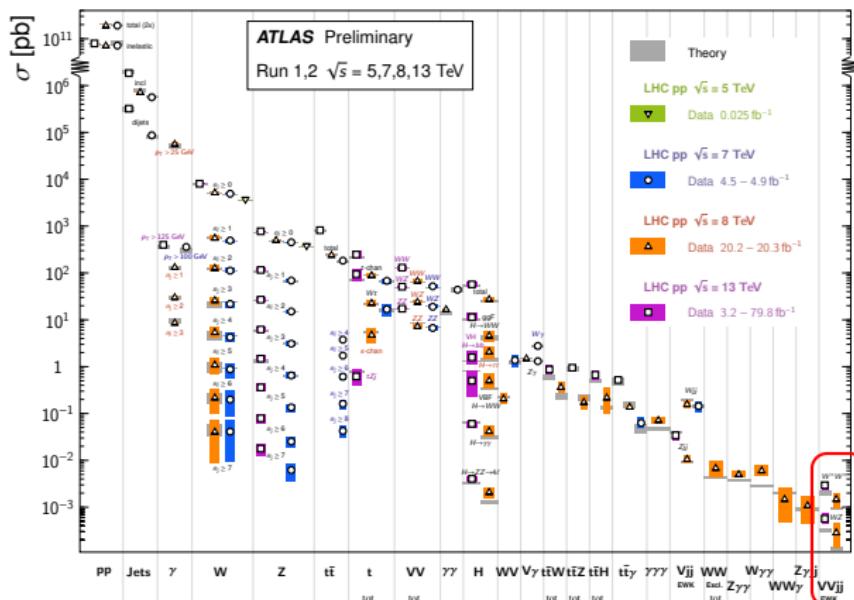


VBS at the LHC, $pp \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$

VBS: $\sigma \approx \mathcal{O}(1\text{ fb}) \rightarrow$ need large \sqrt{s} and \mathcal{L} : new class of rare processes **accessible in run II**

Standard Model Production Cross Section Measurements

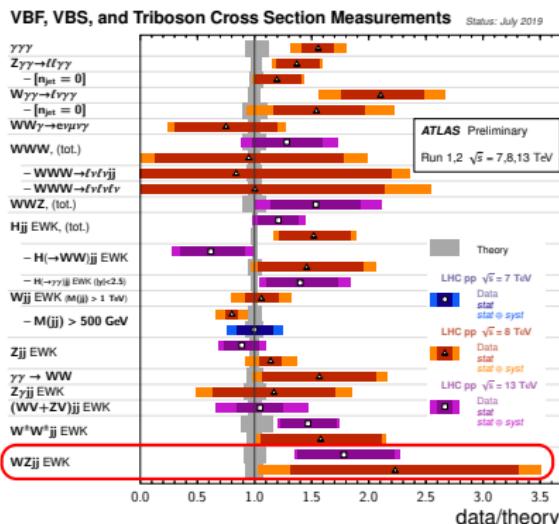
Status: July 2019



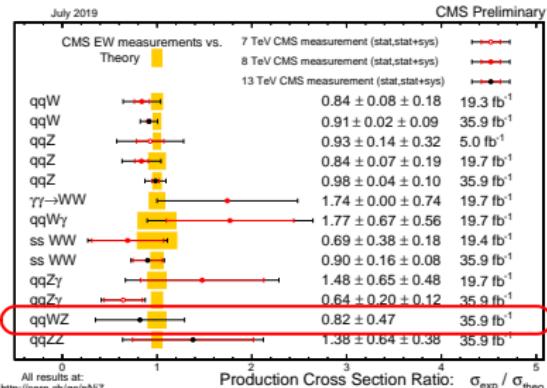
(staircase plot from the [ATLAS Collaboration])

- Largest VBS channel is $W^+W^+ \rightarrow W^+W^+$, full NLO corrections available [Biedermann, Denner, Pellen]
- Second largest channel: $W^+Z \rightarrow W^+Z$

Experiment: $pp \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$



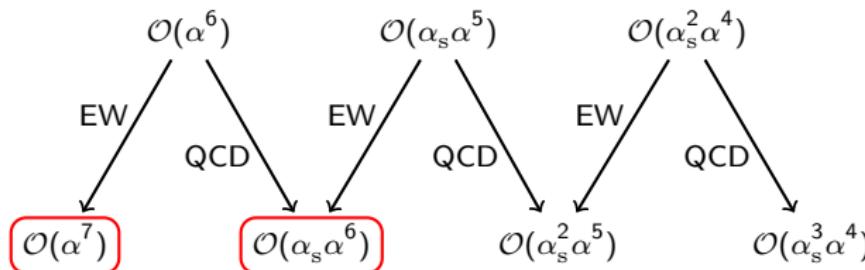
- ATLAS 8 TeV: [CERN-EP-2016-017]
- ATLAS 13 TeV: Obsers. with 5.6σ sig.
($\mathcal{L} = 36.1 \text{ fb}^{-1}$) [ATLAS-CONF-2018-033]
- ATLAS 13 TeV: Obsers. with 5.3σ sig.
($\mathcal{L} = 36.1 \text{ fb}^{-1}$) [CERN-EP-2018-286]



- CMS 13 TeV: Meas. with 1.9σ sig.
($\mathcal{L} = 35.9 \text{ fb}^{-1}$) [CMS-PAS-SMP-18-001]
- CMS 13 TeV: Meas. with 2.2σ sig.
($\mathcal{L} = 35.9 \text{ fb}^{-1}$) [CMS-SMP-18-001]

$$pp \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$$

- Two quark-lines → 3 LOs, 4 NLOs:



- All LOs in Sec. V.3 of the SM Les Houches 2017 report [Bendavid et al.]
- Approx. $\mathcal{O}(\alpha_s \alpha^6)$: +4% [Bozzi, Jäger, Oleari, Zeppenfeld]
- $\mathcal{O}(\alpha_s^3 \alpha^4)$ calculation available [Campanario, Kerner, Ninh, Zeppenfeld]
- Parton-shower effects of the EW production [Jäger, Karlberg, Scheller]
- $\mathcal{O}(\alpha_s^2 \alpha^5)$: unknown, probably small (?)
 - calculated full $\mathcal{O}(\alpha^7)$ EW corrections: like-sign VBS receives large (-16%) corrections
 - calculated full $\mathcal{O}(\alpha_s \alpha^6)$ for an updated setup

Fiducial phase space volume and parameters

Cuts are exactly the “loose fiducial” cuts defined by CMS [CMS-SMP-18-001]:

- At least two $R = 0.4$ anti- k_t jets with $p_T > 30 \text{ GeV}$, $|\eta| < 4.7$, and $\Delta R_{j\ell} > 0.4$
- $M_{j_1 j_2} > 500 \text{ GeV}$, $\Delta\eta_{j_1 j_2} > 2.5$
- $p_{T,\ell} > 20 \text{ GeV}$ and $|y_\ell| < 2.5$
- $|M_{\mu\bar{\mu}} - M_Z| < 15 \text{ GeV}$
- $M_{\ell\ell} > 4.0 \text{ GeV}$ and $M_{3\ell} > 100.0 \text{ GeV}$

Other:

- Photons recombined with charged particles using anti- k_t algorithm with $R = 0.4$
- PDFs: NNPDF31_nlo_as_0118_luxqed
- $\sqrt{s} = 13 \text{ TeV}$

Complex mass scheme [Denner, Dittmaier, Roth, Wackeroth][Denner, Dittmaier, Roth, Wieders], input parameters:

- $G_\mu = 1.6638 \times 10^{-5} \text{ GeV}^{-2}$
- $M_W = 80.3530 \text{ GeV}$, $\Gamma_W = 2.0843 \text{ GeV}$
- $M_Z = 91.1535 \text{ GeV}$, $\Gamma_Z = 2.4943 \text{ GeV}$
- $M_H = 125.0 \text{ GeV}$, $\Gamma_H = 4.07 \times 10^{-3} \text{ GeV}$

with EW coupling calculated as:

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right)$$

Scale choice:

- $\mu = \sqrt{p_{T,j_1} \cdot p_{T,j_2}}$ [Denner, Hošeková, Kallweit]
- 7-point scale variation to estimate pert. uncertainty

Checks and validation

We performed **two independent calculations** for all leading- and next-to-leading orders:

BONSAY+OPENLOOPS

- General purpose Monte Carlo [CS]
- MEs from OPENLOOPS 1 [Cascioli, Maierhöfer, Pozzorini]
- Loops evaluated with DD (COLI fallback) from COLLIER
- Dipole subtraction [Catani, Seymour] to regularize IR singularities
- PDFs from LHAPDF 6 [Buckley, et. al.]

MoCANLO+RECOLA

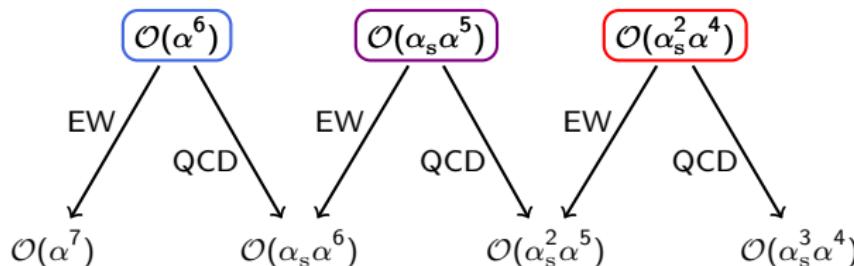
- MoCANLO [Feger] used by M. Pellen
- MEs from RECOLA [Actis, Denner, Hofer, Scharf, Uccirati]
- Loops evaluated with COLI (and DD) from COLLIER [Denner, Dittmaier, Hofer]
- CS dipole subtraction with α -dependent dipoles [Nagy]
- PDFs from LHAPDF 6

Checks

- NLO virtuals checked against each other for 1000 PS points passing the cuts
- Integrated cross sections
- Each bin of 26 differential distributions within stat. unc., ca. 8000 bins for each order

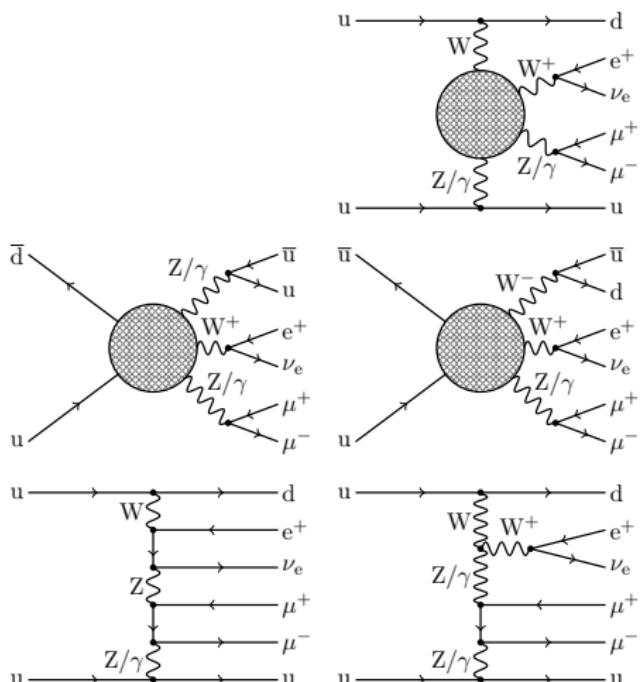
Overview: Leading orders

$p p \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$ has three LOs:



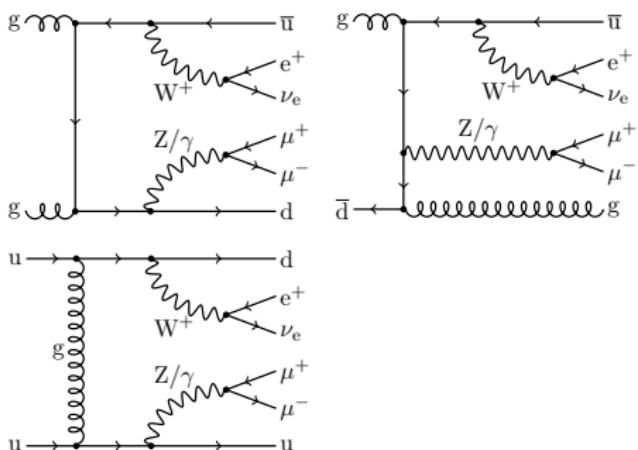
We divided them into five (mutually exclusive) classes:

- ① $\mathcal{O}(\alpha^6)$ **electroweak production** with quark-quark initial state (but without bottom-quarks)
- ② $\mathcal{O}(\alpha_s^2 \alpha^4)$ **strong production** (without bottom-quarks)
- ③ $\mathcal{O}(\alpha_s \alpha^5)$ quark-quark **interference**
- ④ $\mathcal{O}(\alpha^6)$ double-**photon** initiated and $\mathcal{O}(\alpha_s \alpha^6)$ single-**photon** initiated
- ⑤ $\mathcal{O}(\alpha^6)$ and $\mathcal{O}(\alpha_s^2 \alpha^4)$ with **bottom-quarks**

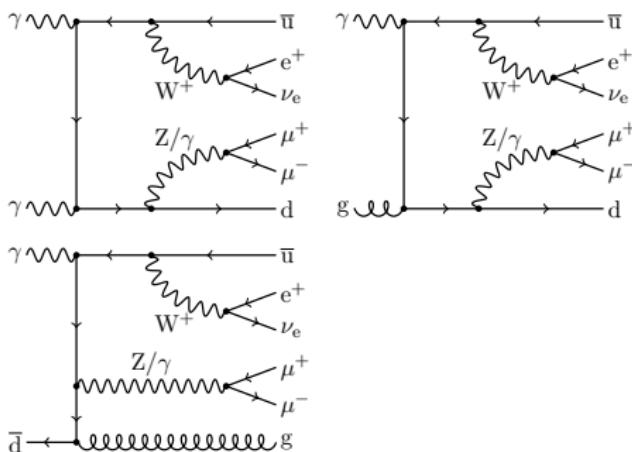
Electroweak production: $\mathcal{O}(\alpha^6)$ 

- 40 different partonic channels at $\mathcal{O}(\alpha^6)$
- contain the vector-boson scattering subdiagrams,
- and “semi-leptonic triple-gauge-boson production” processes ($W^\pm ZZ$, W^+W^-Z), suppressed at LO by $M_{jj} > 500$ GeV
- and other double-, single, non-resonant diagrams

Strong production: $\mathcal{O}(\alpha_s^2 \alpha^4)$

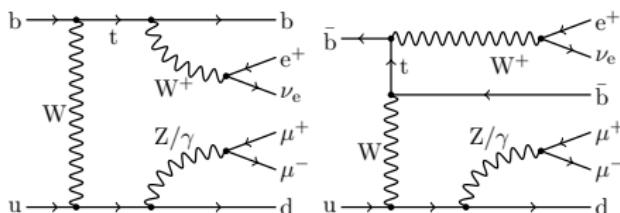


- In comparison to like-sign W-scattering gluons are possible at LO (charge)
- 8 additional MEs with **two gluons**, making up 66 % of the cross section
- in total the $\mathcal{O}(\alpha_s^2 \alpha^4)$ is **4.3 times larger** than the electroweak LOs

Photon-initiated: $\mathcal{O}(\alpha^6)$, $\mathcal{O}(\alpha_s \alpha^5)$ 

- Every channel with an initial-state photon
 - 2 **double photon** MEs at $\mathcal{O}(\alpha^6)$ (tiny contribution)
 - 12 **photon-gluon** MEs at $\mathcal{O}(\alpha_s \alpha^5)$ (very small contribution)
 - remember: no final state photons at LO because of $n_j \geq 2$

Bottom-quark LOs: $pp \rightarrow tZj$



12 MEs with bottom-quarks:

- ① $bu \rightarrow e^+ \nu_e \mu^+ \mu^- bd$
- ② $\bar{b}u \rightarrow e^+ \nu_e \mu^+ \mu^- \bar{b}d$
- ③ $b\bar{d} \rightarrow e^+ \nu_e \mu^+ \mu^- b\bar{u}$
- ④ $\bar{b}\bar{d} \rightarrow e^+ \nu_e \mu^+ \mu^- \bar{b}\bar{u}$
- ⑤ $u\bar{d} \rightarrow e^+ \nu_e \mu^+ \mu^- \bar{b}b$
- ⑥ $b\bar{b} \rightarrow e^+ \nu_e \mu^+ \mu^- \bar{u}d$
- + MEs with 2nd gen. quark line

- also contain VBS, but dominant contributions are ...
- “top-Z-jet production” for $b(u/c) \rightarrow e^+ \nu_e \mu^+ \mu^- b(d/s)$
- no resonant anti-tops because of $W^+ \rightarrow$ up-bottom contribution dominates over all others (90 %)
- separable with b-tagging in principle, except for n. 6 (very small)
- contribution comparable in size with the EW LOs

top-Z-jet analyses:

- ATLAS: [\[arXiv:1710.03659\]](https://arxiv.org/abs/1710.03659)
- CMS: [\[arXiv:1712.02825\]](https://arxiv.org/abs/1712.02825)

LO integrated cross sections

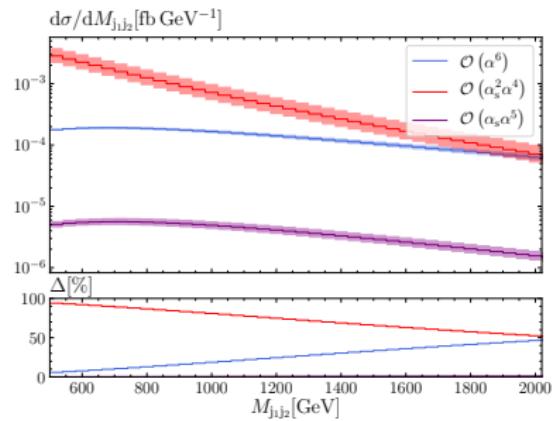
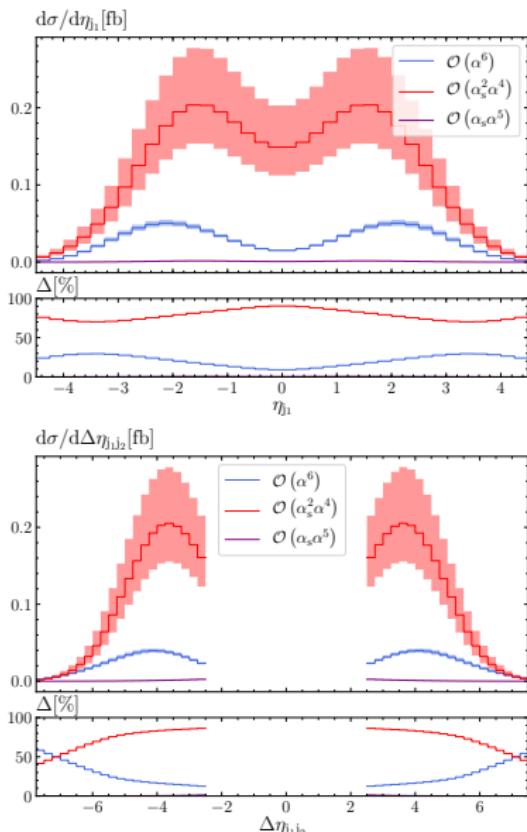
Integrated xs for $\text{pp} \rightarrow e^+ \nu_e \mu^+ \mu^- jj$ @ $\sqrt{s} = 13 \text{ TeV}$:

Sum [fb]	EW [fb]	QCD [fb]	Int. [fb]
1.55	$0.255^{+9.03\%}_{-7.75\%}$	$1.10^{+37.0\%}_{-24.9\%}$	$0.00682^{+18.4\%}_{-14.4\%}$
100 %	16.4 %	70.6 %	0.439 %

Photons [fb]	Bottom-quarks [fb]
$0.000988^{+11.5\%}_{-9.47\%}$	$0.195^{+3.59\%}_{-7.22\%}$
0.0636 %	12.5 %

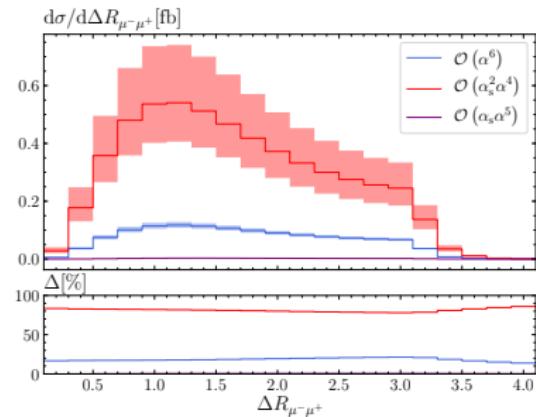
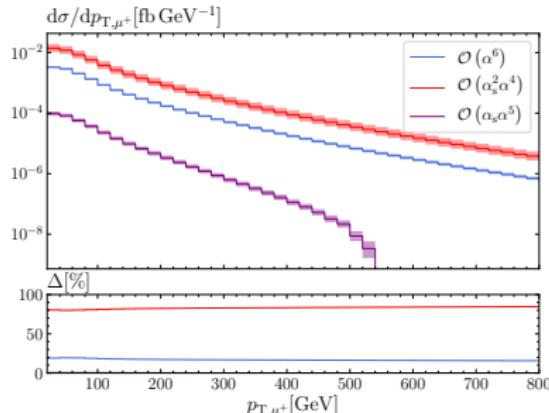
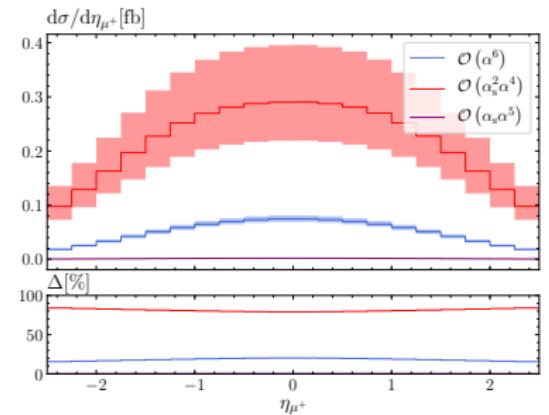
- very large QCD contributions mainly due to gluon-PDF
- small interference (colour and kinematical suppression)
- smaller EW contribution compared to like-sign VBS ($\rightarrow Z$ -boson)
- photon contributions completely irrelevant \rightarrow leave out photon-initiated at NLO
- important: bottom-quark contributions

Jet observables



- Δ : percentage of each contr. to the sum
- Typical behaviour for VBF/VBS: jets are typically in the forward region
- Crossover between EW and QCD at $M_{j_1,j_2} \approx 2100 \text{ GeV}$
- large QCD uncertainty band due to α_s^2 vs. α_s^0 in the EW

Leptonic observables

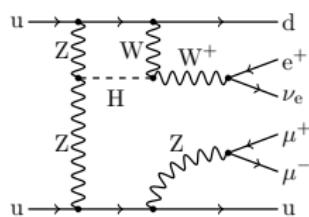
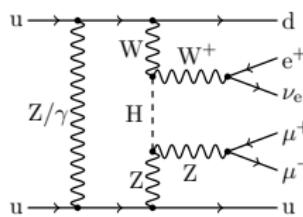
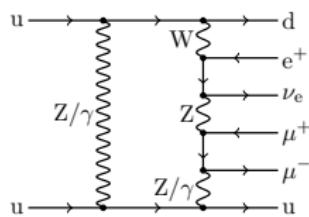
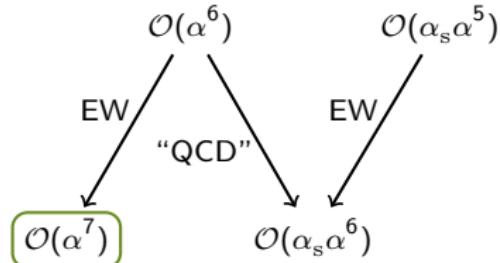
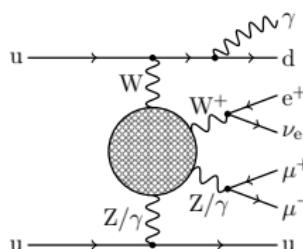
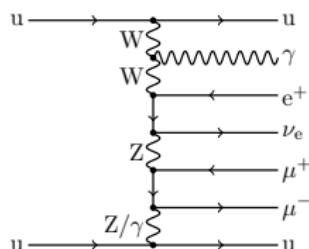


- Leptons are central, small p_T
- $d\sigma/dp_{T,\mu}$ becomes negative
- $M_{\bar{\mu}\mu} < M_Z + 15 \text{ GeV}$ and $p_{T,\ell} > 20 \text{ GeV}$
limit $\Delta\eta_{\bar{\mu}\mu} < 3.4$

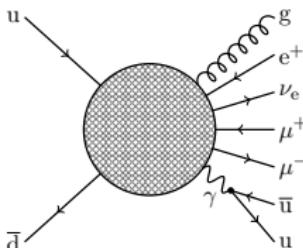
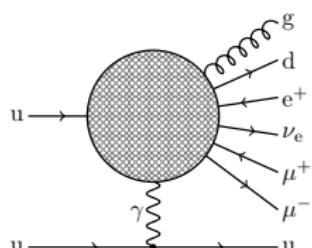
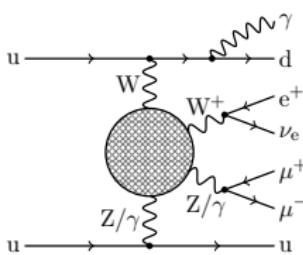
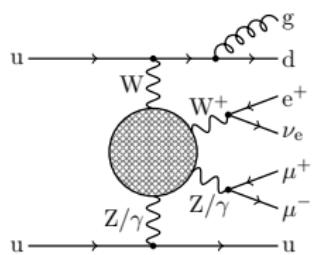
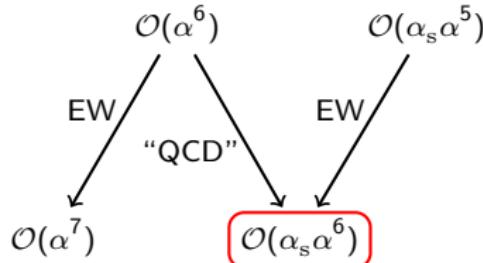
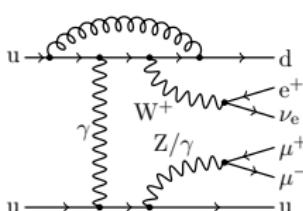
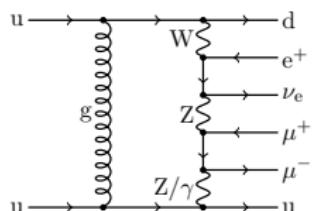
$$\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$$

$$\cosh \Delta\eta_{\ell_1 \ell_2} = \frac{M_{\ell_1 \ell_2}^2}{2 p_{T,\ell_1} p_{T,\ell_2}} + \cos \phi_{\ell_1 \ell_2}$$

$\mathcal{O}(\alpha^7)$ real and virtual correction diagrams



- No b-quark contributions (b-tagging)
- No $\gamma q/\gamma\gamma$ contributions (expectation 1–2 %) at NLO
- Loops with 8-point functions, different complex masses
- More diagrams with Higgs bosons!
- Up to **83,000 diagrams** per partonic channel

$\mathcal{O}(\alpha_s \alpha^6)$ mixed corrections: “QCD”


QED singularities in “QCD”:

- initial state: cancelled with collinear counterterm (PDFs)
- final state: **photon-to-jet conversion function** [Denner, Dittmaier, Pellen, C.S.]
→ Correction is neither purely QCD/EW, it is mixed

Integrated cross section

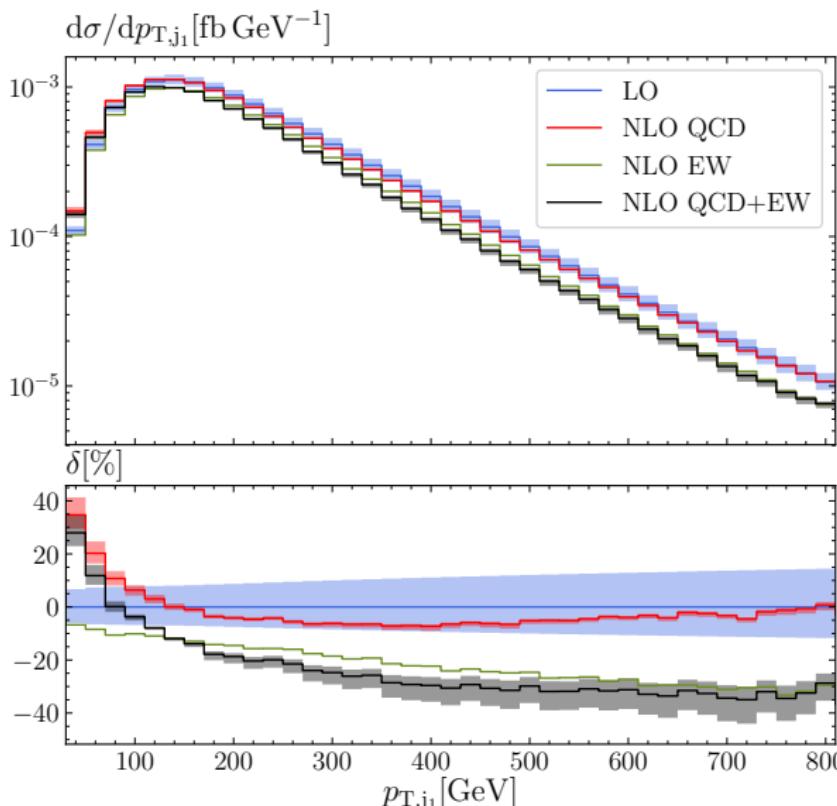
Integrated xs for $\text{pp} \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$ @ $\sqrt{s} = 13 \text{ TeV}$:

LO ² [fb]	NLO EW [fb]	NLO QCD [fb]	NLO EW+QCD [fb]
$0.2551^{+9.0\%}_{-7.8\%}$	$0.2142^{+8.5\%}_{-7.3\%}$	$0.2506^{+1.0\%}_{-1.0\%}$	$0.2097^{+1.3\%}_{-2.2\%}$
100.0 %	-16.0 %	-1.8 %	-17.8 %

- Large corrections on the integrated cross section, very similar to like-sign scattering
- QCD corrections small → additive combination of NLOs

²only $\mathcal{O}(\alpha^6)$

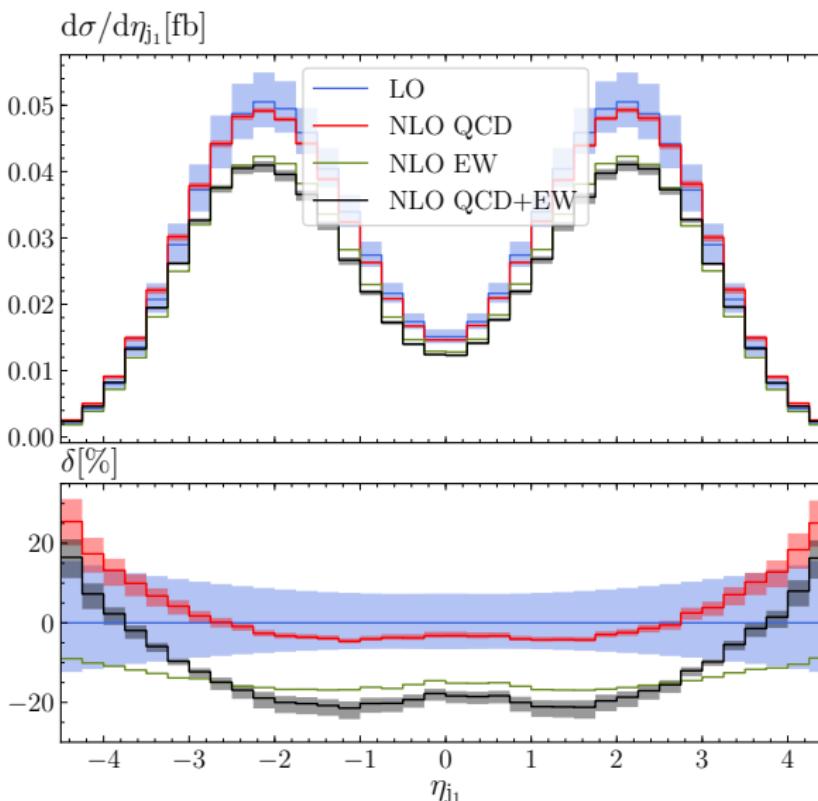
Jet observables (I)



- Leading jet p_T peaks around 140 GeV
- Note that
$$p_{T,j_1} > p_{T,j_2} > p_{T,j_3} > 30 \text{ GeV}$$
- Large positive QCD corrections for small p_{T,j_1} (*all jets have small transv. momentum*)
- EW corr. become increasingly negative; Sudakov logs
- QCD uncertainty band small for large p_{T,j_1} due to

$$\mu = \sqrt{p_{T,j_1} \cdot p_{T,j_2}}$$

Jet observables (II)

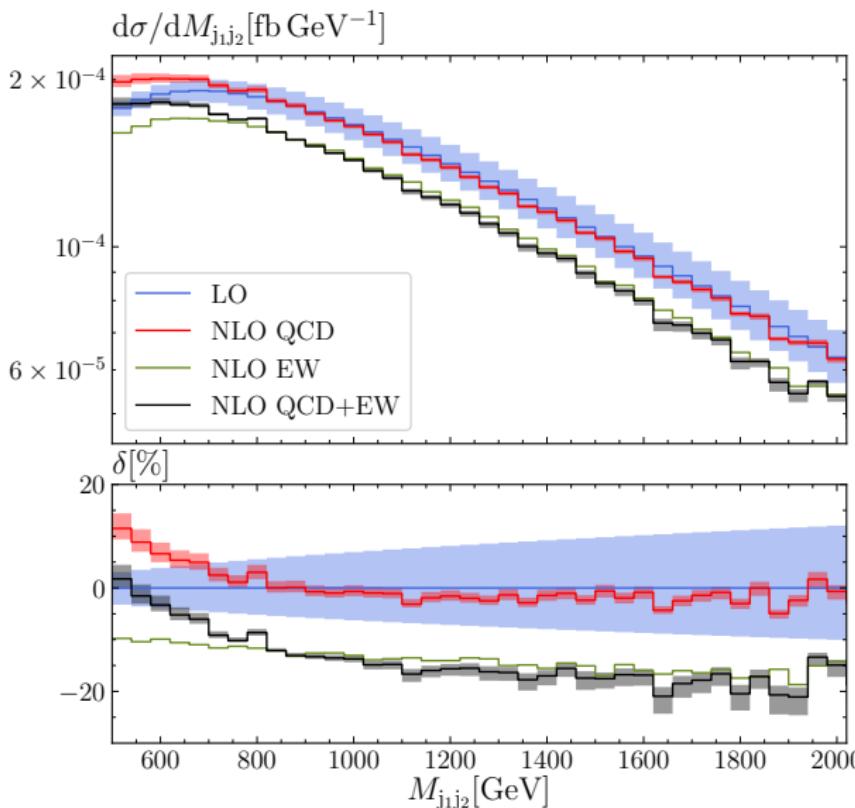


- EW corr. mostly flat
- QCD corr. positive in the forward region:
 - large pseudorapidity drives energy up:

$$\begin{aligned} E_{j_1} &= p_{T,j_1} \cosh \eta_{j_1} \\ &> 30 \text{ GeV} \cosh 4 \\ &\approx 820 \text{ GeV} \end{aligned}$$

- small p_{T,j_1} in the forward region

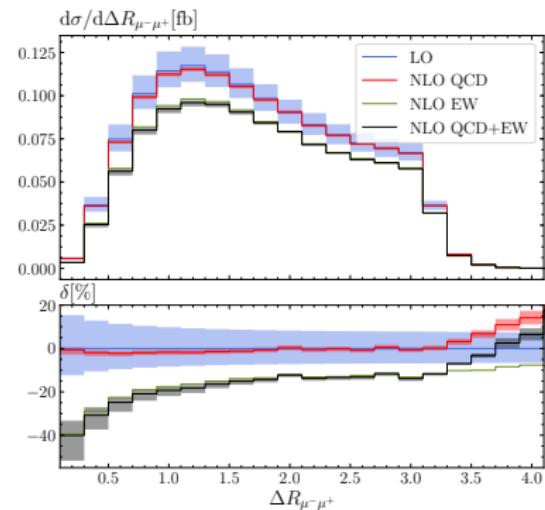
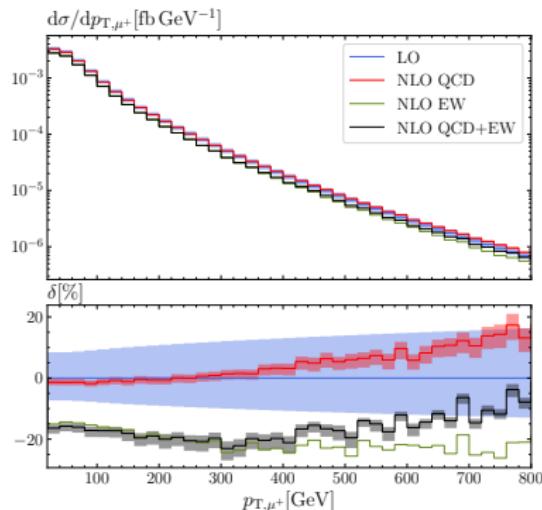
Jet observables (III)



- Small $M_{j_1 j_2}$ requires small $p_{T,j}$ due to $\Delta\eta_{j_1 j_2} > 2.5$ cut
→ large pos. QCD corrections
- EW corr. become increasingly negative
- accidental cancellation of QCD and EW for small $M_{j_1 j_2}$

$$\begin{aligned} M_{j_1 j_2} &\approx \sqrt{2p_{T,j_1} \cdot p_{T,j_2}} \\ &\cosh \Delta\eta_{j_1 j_2} \\ M_{j_1 j_2} &\leq 550 \text{ GeV} \Rightarrow \\ &\sqrt{p_{T,j_1} \cdot p_{T,j_2}} \leq 60 \text{ GeV} \end{aligned}$$

Leptonic observables



- Transv. momentum: Sudakov logs

- ΔR : Significant shape distortions, at small ΔR boosted Z

Why are the EW corrections so large?

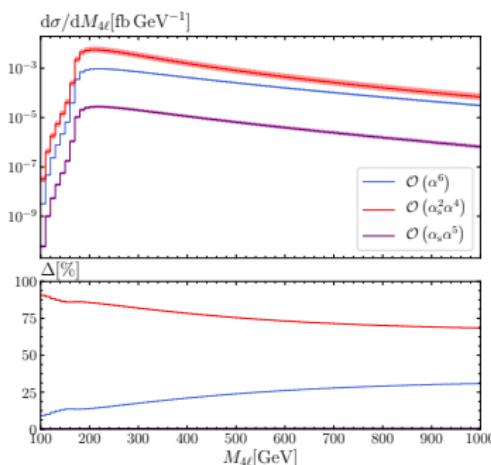
Approximate EW correction using most important EW logs [Denner, Pozzorini] [Denner, Pozzorini] to the VBS subprocess:

$$d\sigma_{LL} = d\sigma_{LO}(1 + \delta_{EW,LL})$$

with

$$\delta_{EW,LL} = \frac{\alpha}{4\pi} \left(-\frac{8}{s_w^2} \log^2 \left(\frac{Q^2}{M_W^2} \right) + \frac{19}{3s_w^2} \log \left(\frac{Q^2}{M_W^2} \right) \right)$$

with a characteristic scale Q chosen as $M_{4\ell} := M_{e\nu_e\bar{\mu}\mu}$ (s -invariant of the VBS):



- $Q = \langle M_{4\ell} \rangle \approx 413$ GeV $\Rightarrow \delta_{EW,LL} = -17.5\%$
- $Q = M_{4\ell}$, binwise with distribution on the left
 $\Rightarrow \delta_{EW,LL} = -16.4\%$

For comparison [Biedermann, Denner, Pellen]:

- like-sign-W VBS: $Q \approx 390$ GeV
- diboson production: $Q \approx 250$ GeV

Summary

- Vector-boson scattering: new class of processes explored in run II
 - After W^+W^+ , W^+Z scattering is the next important channel for VBS
 - Large EW corrections for a realistic setup: -16%
 - Integrated $\mathcal{O}(\alpha_s \alpha^6)$ correction small: -1.7%
 - Typical Sudakov logs that drive the EW corrections of some p_T observables
 - Large QCD corrections in PS regions where p_{T,j_1} small
- More distributions on the backup slides

Acknowledgments

Thank you!

- We acknowledge support by the state of Baden-Württemberg through bwHPC and the German Research Foundation (DFG) through grant no INST 39/963-1 FUGG and grant DI 784/3.
- This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 740006.

Longitudinal VBS and Tree-Level Unitarity (I)

- ① EW symmetry breaking makes W^\pm, Z massive
- ② Massive vector boson have one additional polarisation: **longitudinal**
- ③ In the high energy limit (Goldstone-Boson equivalence theorem), longitudinal polarisation $\varepsilon_L(p)$ behave as $\varepsilon_L(p) \rightarrow p$
- ④ Longitudinal VBS: Four longitudinal polarisations, does \mathcal{M}_{LLLL} blow up in the high energy limit (pert. unitarity violation)?

For $W^+(p_1)Z(p_2) \rightarrow W^+(p_3)Z(p_4)$ and $t = (p_1 - p_3)^2$ choose

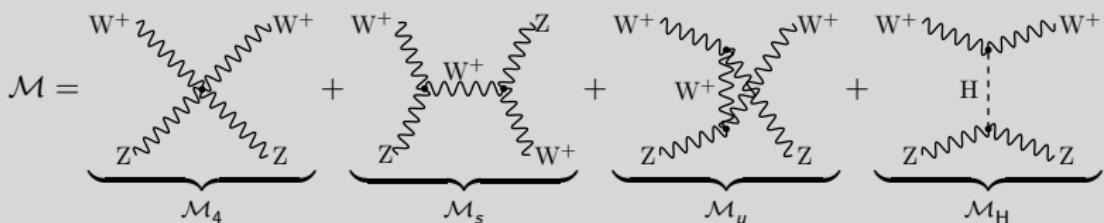
$$\varepsilon_L^\mu(p_i) = \frac{1}{N} \left(\frac{p_i^\mu}{M_i} - \frac{2M_i}{t - 2M_i^2} p_j^\mu \right), \quad j = (i+2) \bmod 4$$

such that $p \cdot \varepsilon_L(p) = 0$, with normalisation N so that $\varepsilon_L^* \cdot \varepsilon_L = -1$.

Longitudinal VBS and Tree-Level Unitarity (II)

$W^+Z \rightarrow W^+Z$

@ $M_H = 125$ GeV



$$\mathcal{M}_4 \propto -s^2 - u^2 - 4su + 2(M_W^2 + M_Z^2) \frac{s^2 + 6su + u^2}{s + u} + \dots$$

$$\mathcal{M}_s \propto s^2 + 2su - 2M_W^2 \frac{3su + u^2}{s + u} - 2M_Z^2 \frac{2u^2 + 3su - s^2}{s + u} - \frac{M_Z^4}{M_W^2} s + \dots$$

$$\mathcal{M}_u \propto u^2 + 2su - 2M_W^2 \frac{3su + s^2}{s + u} - 2M_Z^2 \frac{2s^2 + 3su - u^2}{s + u} - \frac{M_Z^4}{M_W^2} u + \dots$$

$$\mathcal{M}_H \propto -\frac{M_Z^4}{M_W^2} \frac{t^2(t - 4M_W^2)(t - 4M_Z^2)}{(t - M_H)(t - 2M_W^2)(t - 2M_Z^2)} = \frac{M_Z^4}{M_W^2} (s + u) + \dots$$

$$\mathcal{M} = \mathcal{M}_4 + \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_H \propto 0 + \dots$$

Calculation from [Schwartz]

- SM VVH vertex crucial to the cancellation

- different Higgs sector: enhancements in intermediate regions possible

Challenges of multi-leg NLO calculations: $\text{pp} \rightarrow e^+ \nu_e \mu^+ \mu^- jj + X$

Although NLO calculations are (basically) straightforward today, we discovered some “unexplored territory”:

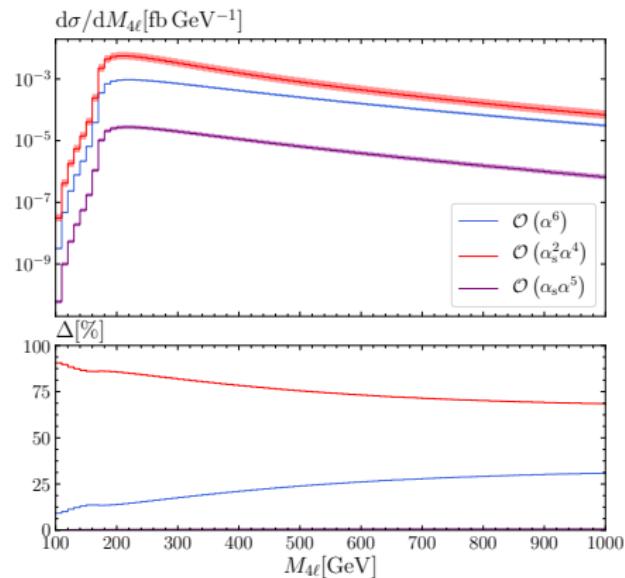
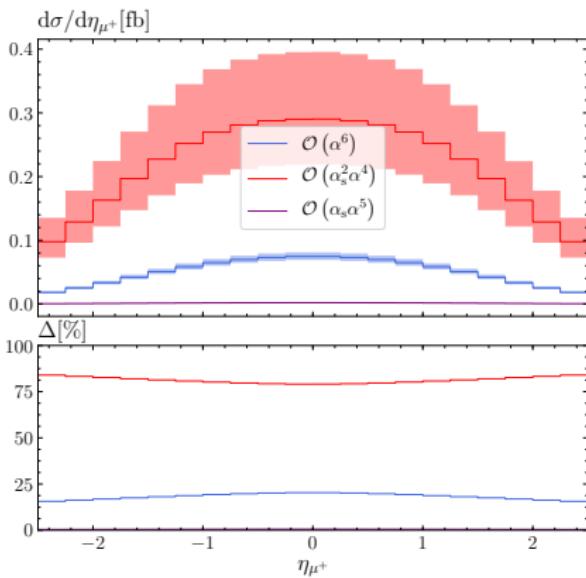
Virtuals:

- COLI used about 10 GB during cache training (fixed in COLLIER 1.2.3) → running in parallel hits the memory limit
- **ME evaluation expensive**: evaluation of each partonic process takes about ten seconds → parallelisation with MPI [[MPI Forum](#)]

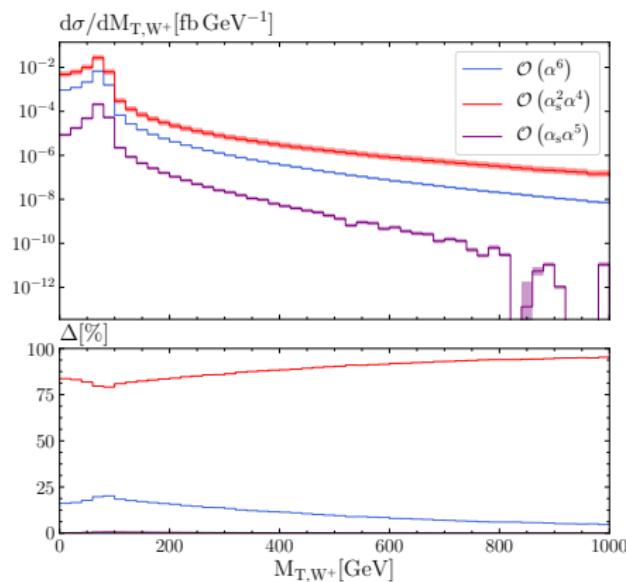
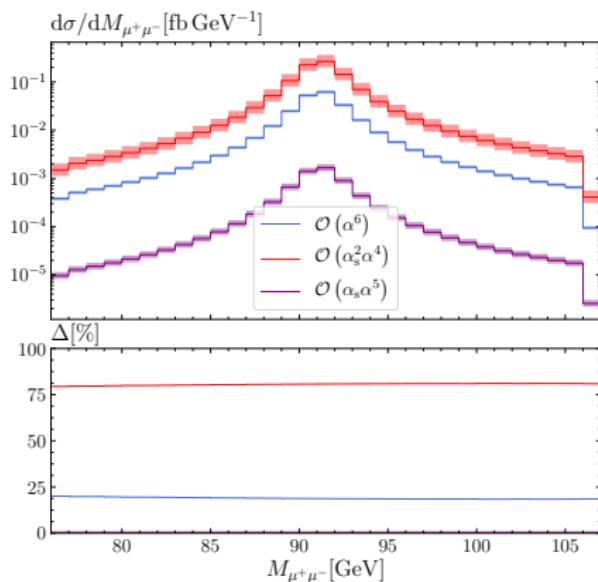
Reals:

- Example: $\mathcal{O}(\alpha_s \alpha^6)$
 - 40 qq partonic processes for the QCD reals, each has 12–14 dipoles
 - 14 qg partonic processes for the QCD reals, each has 10–11 dipoles
 - 16 qg partonic processes for the EW reals, each has 42 dipoles
- around **1500 dipoles**: automation is key → most complicated NLO computation to date
- **phase space integration** complicated: 5×10^{10} points before cuts (eff.: 14 %) → MPI

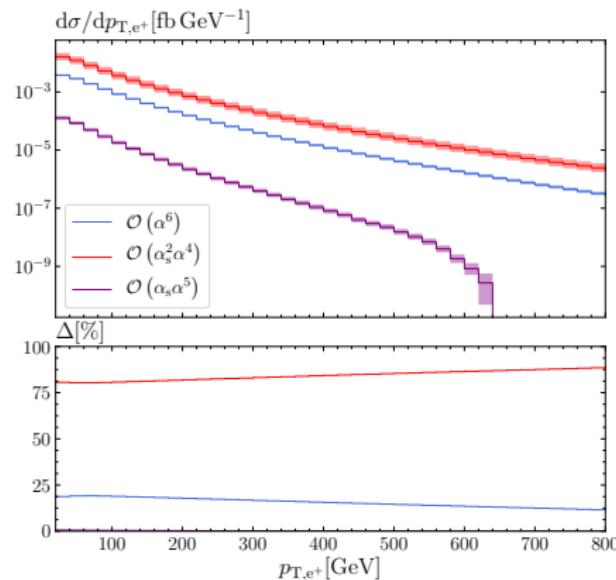
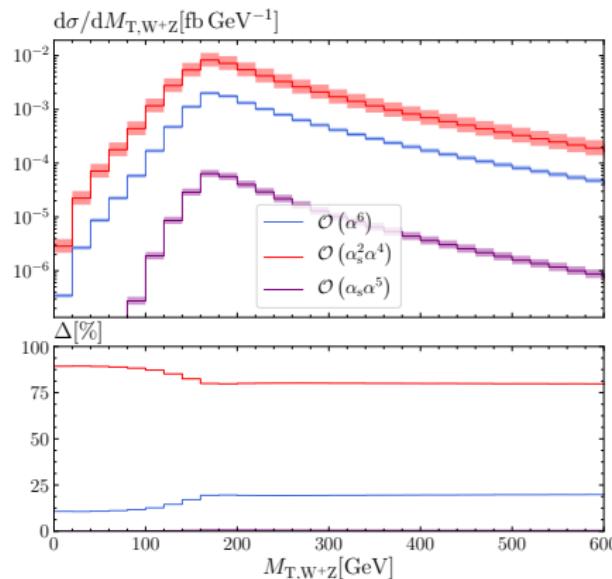
LO distributions (VI)



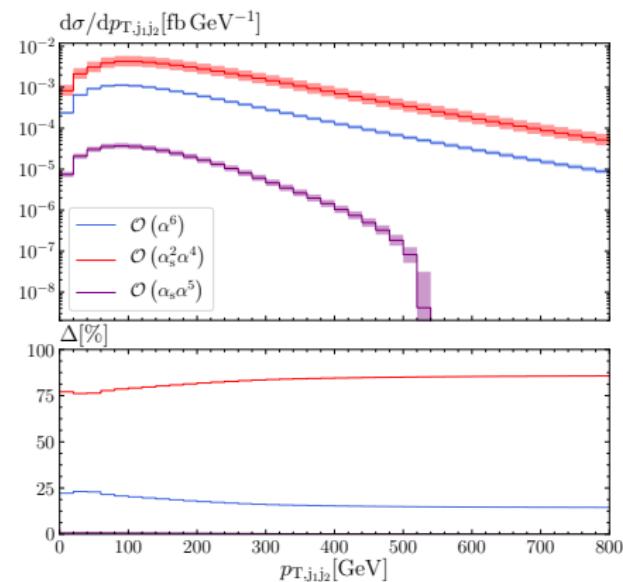
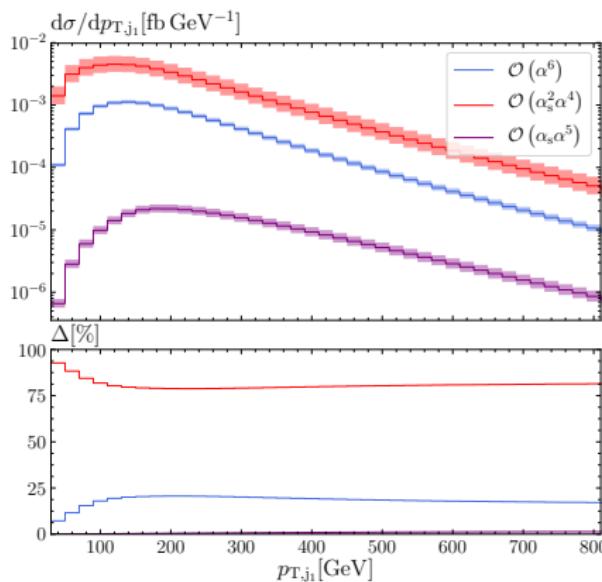
LO distributions (VIII)



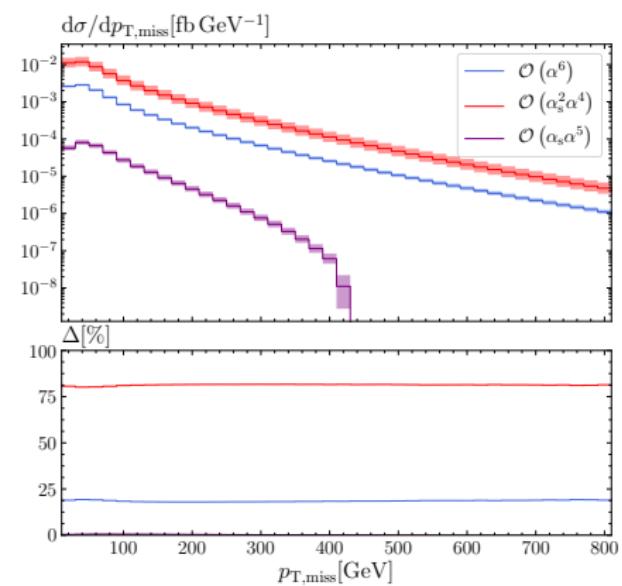
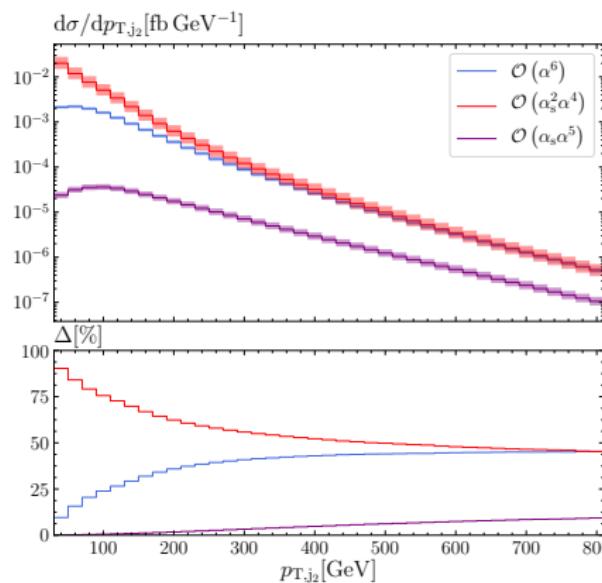
LO distributions (IX)



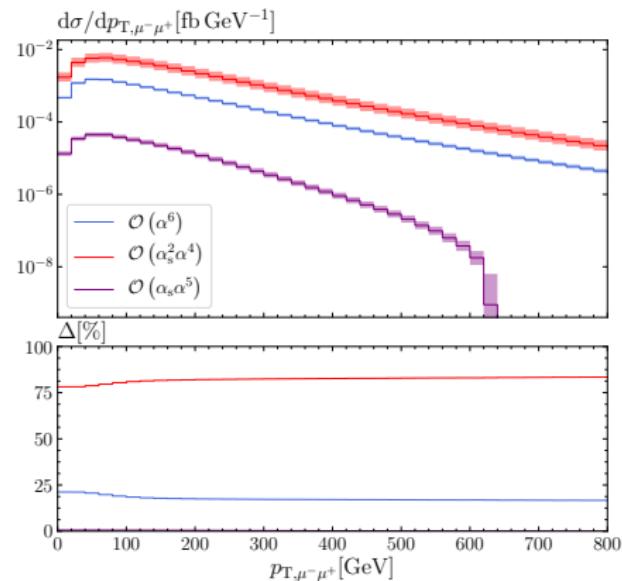
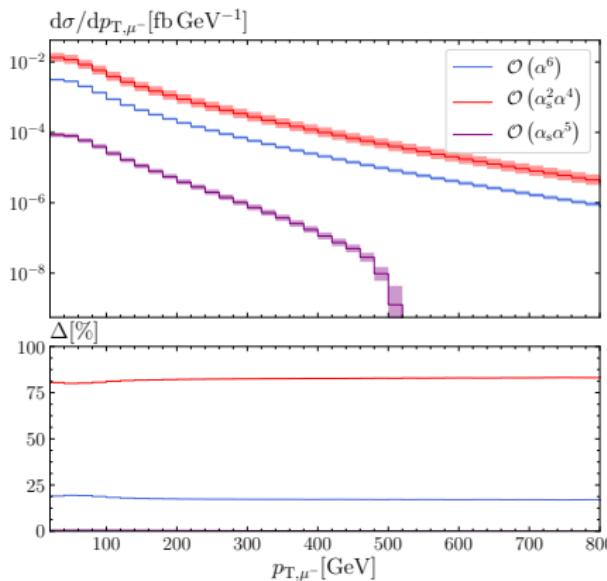
LO distributions (X)



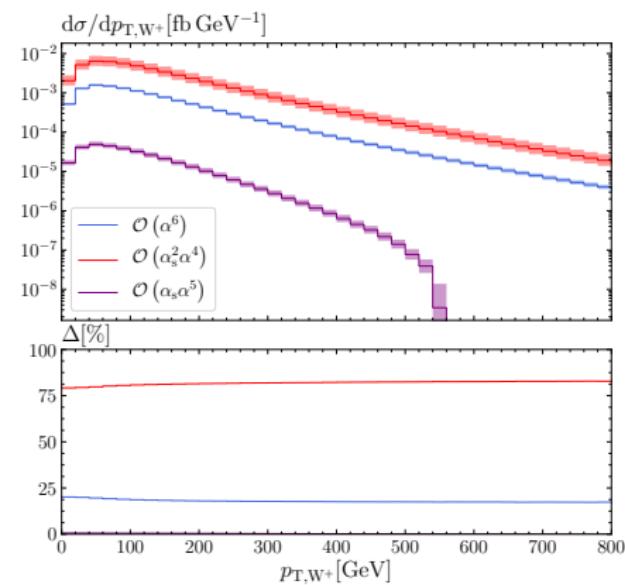
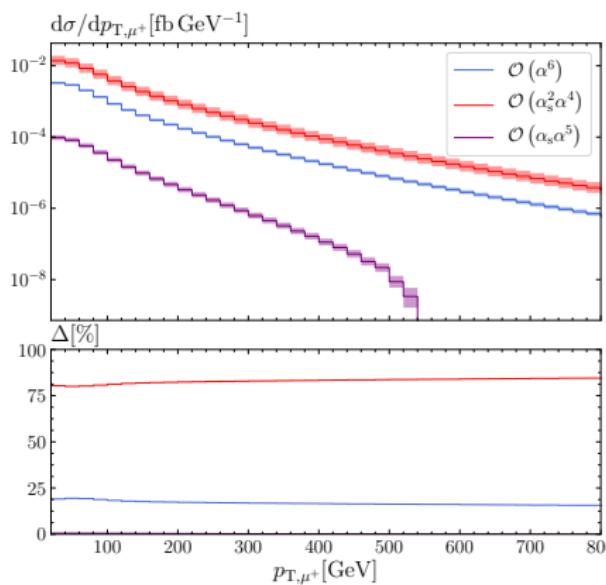
LO distributions (XI)



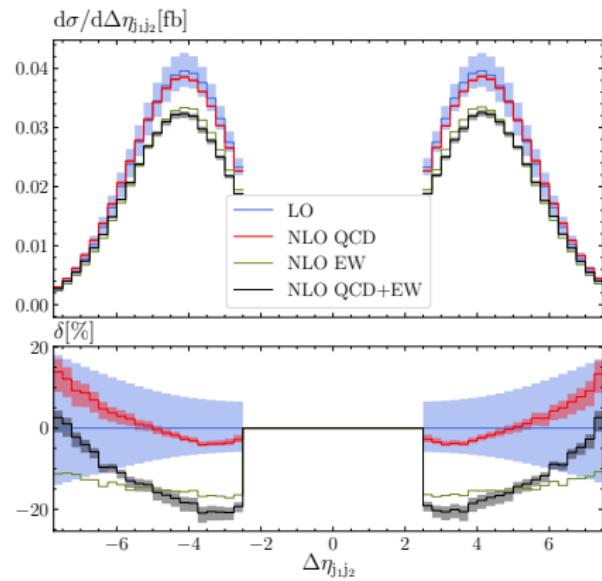
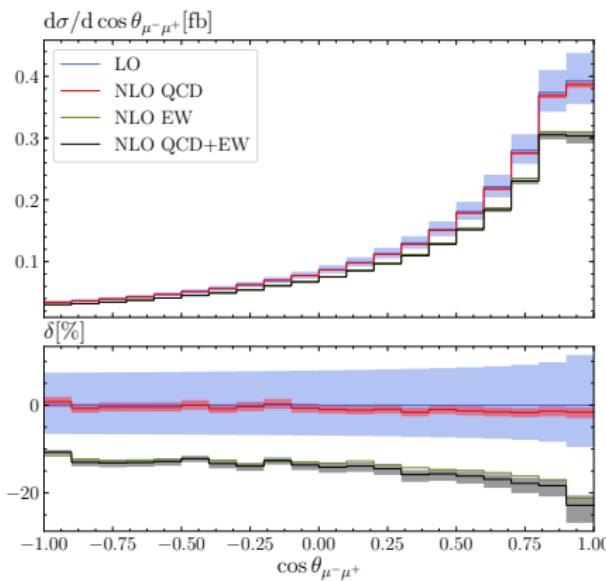
LO distributions (XII)



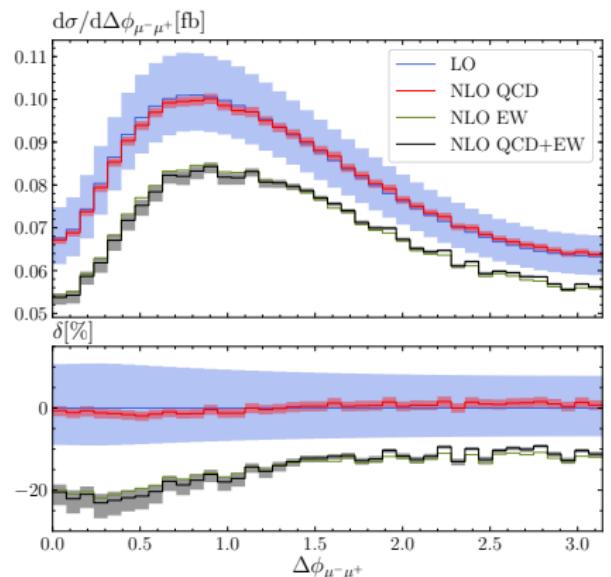
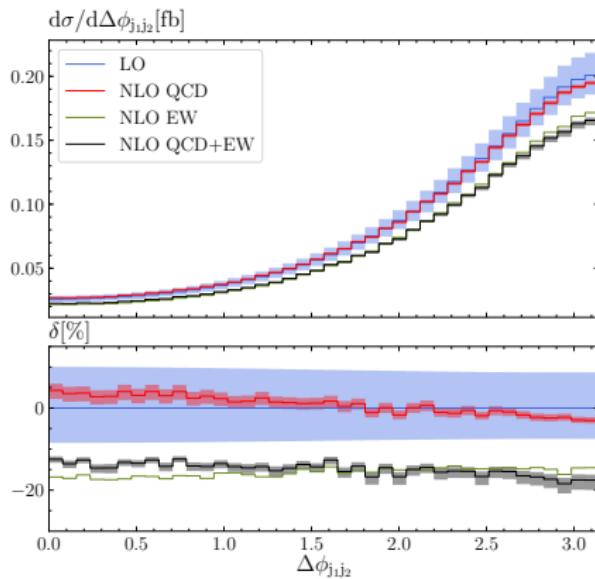
LO distributions (XIII)



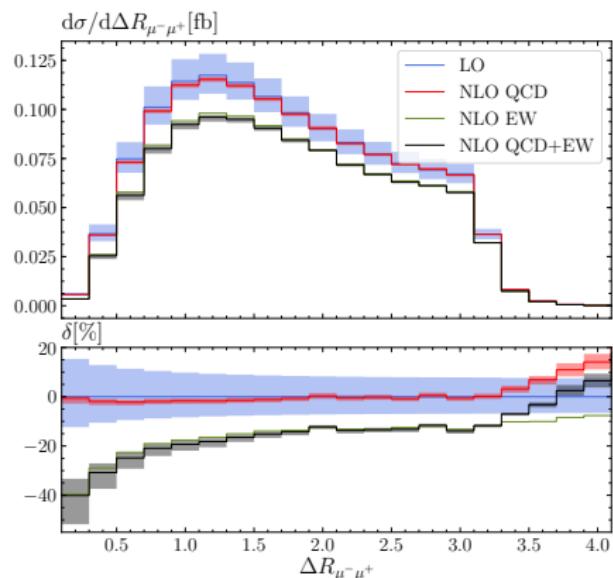
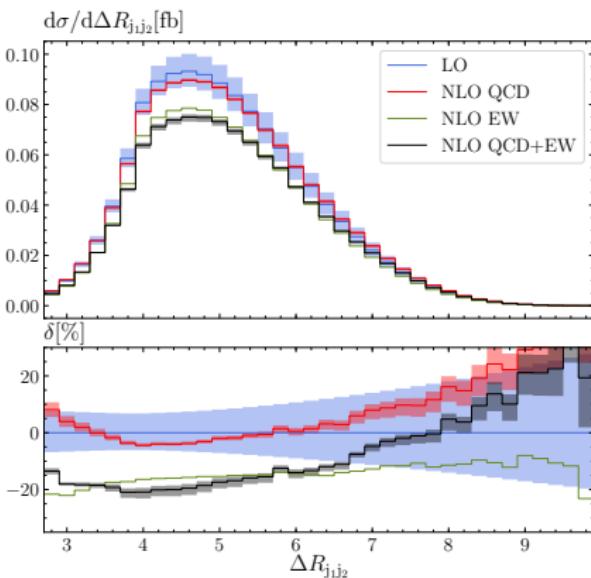
NLO distributions (I)



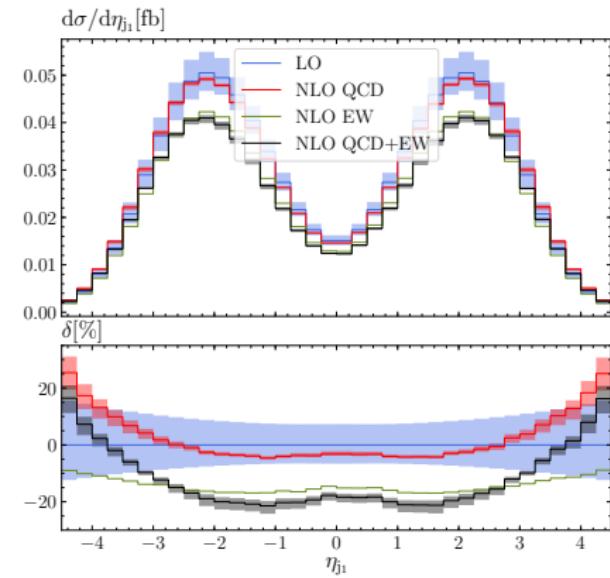
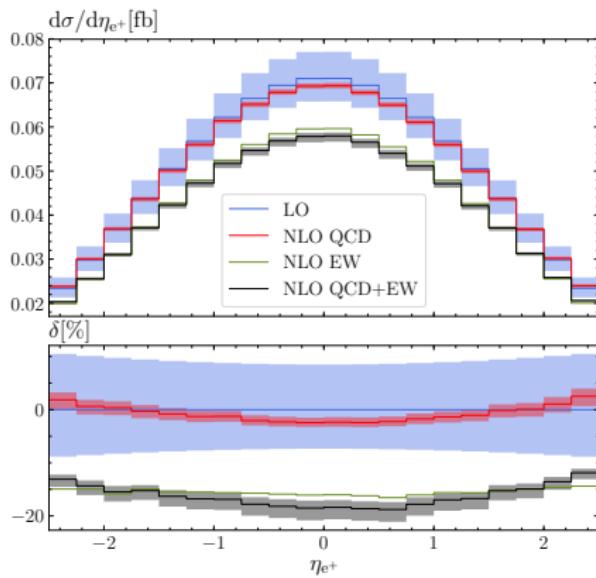
NLO distributions (II)



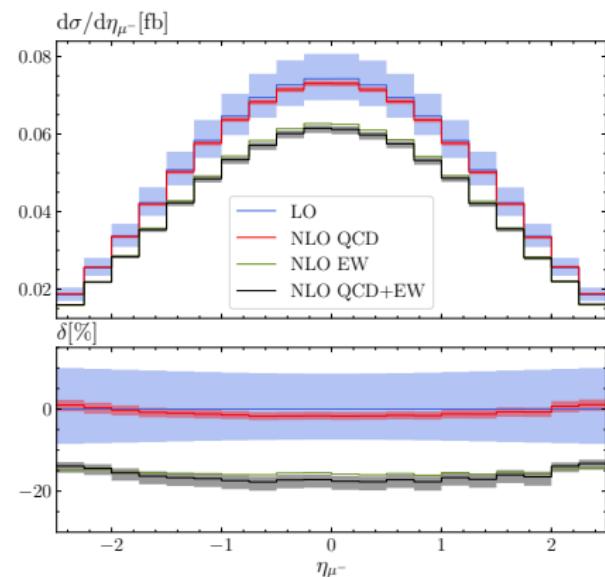
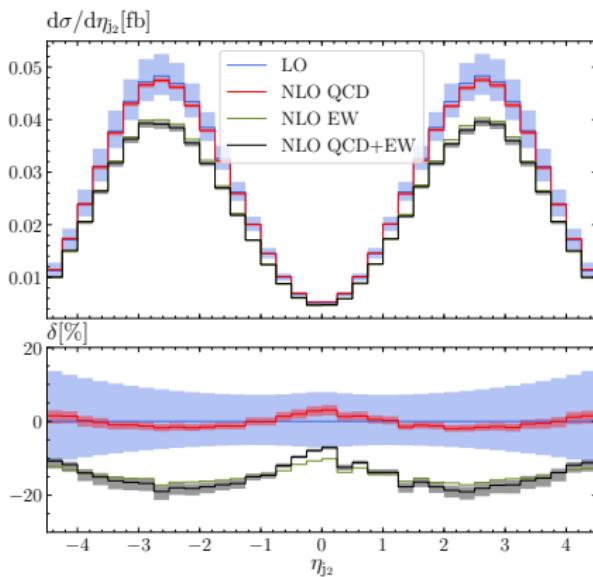
NLO distributions (III)



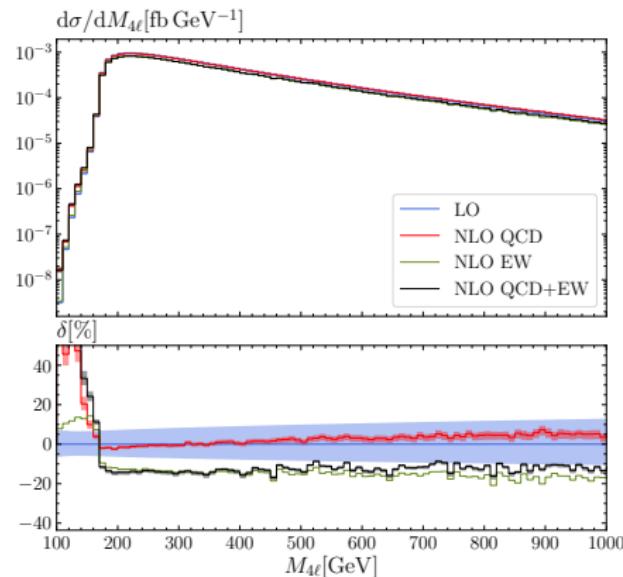
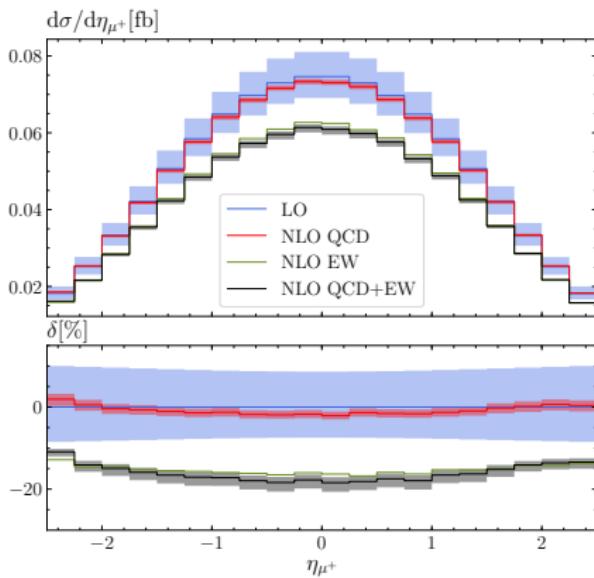
NLO distributions (IV)



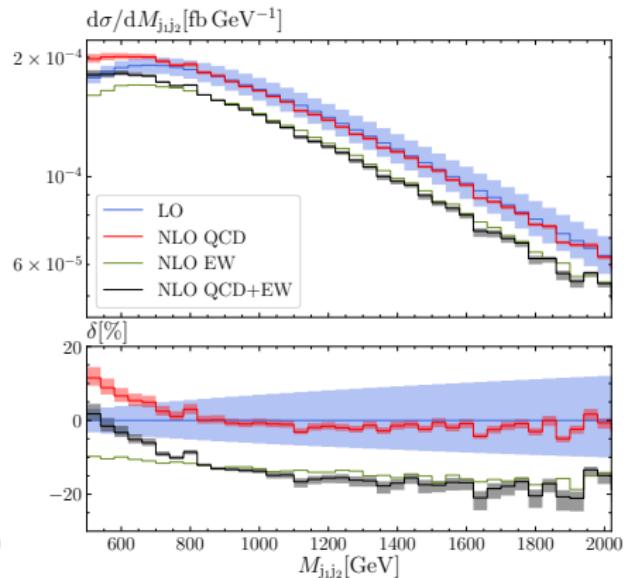
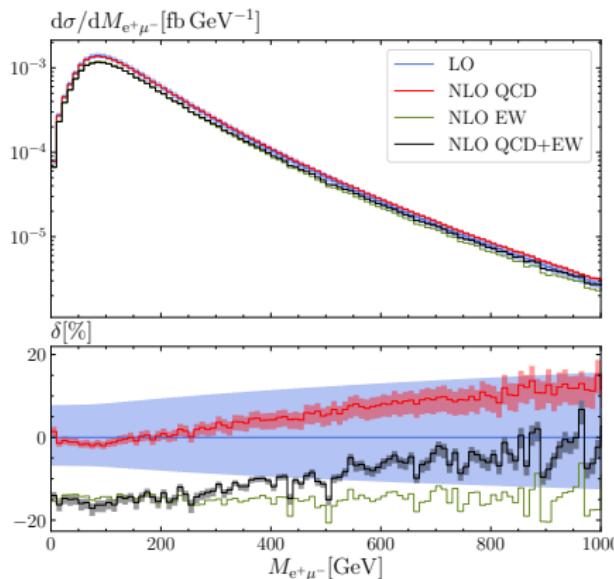
NLO distributions (V)



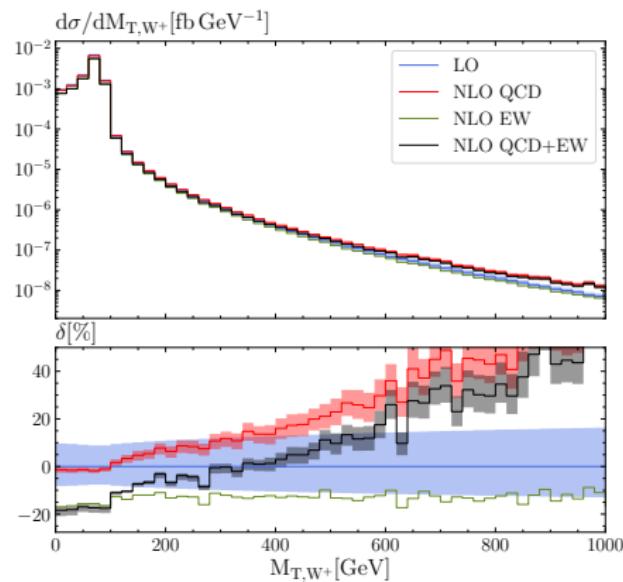
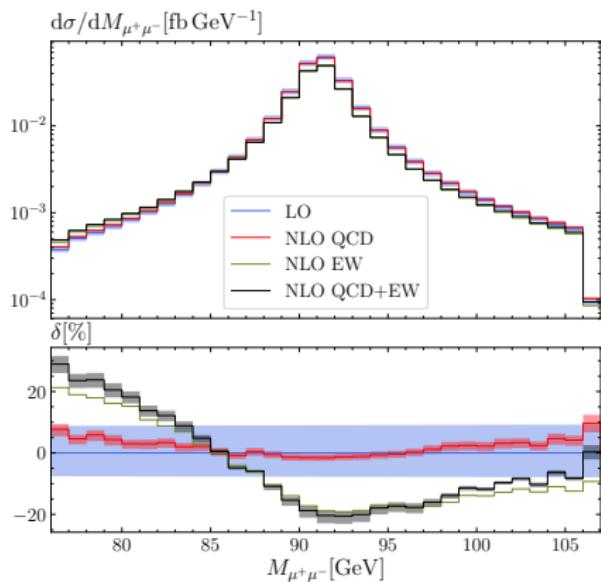
NLO distributions (VI)



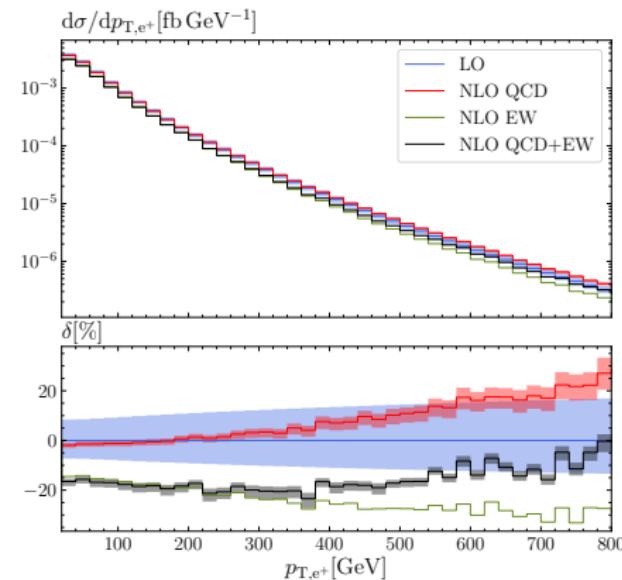
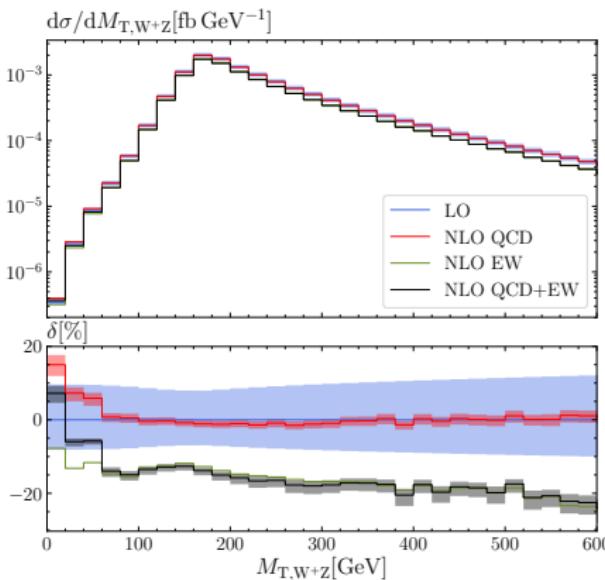
NLO distributions (VII)



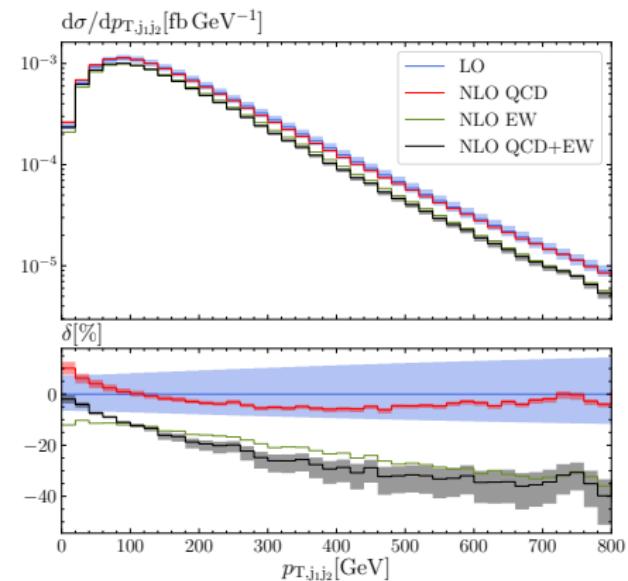
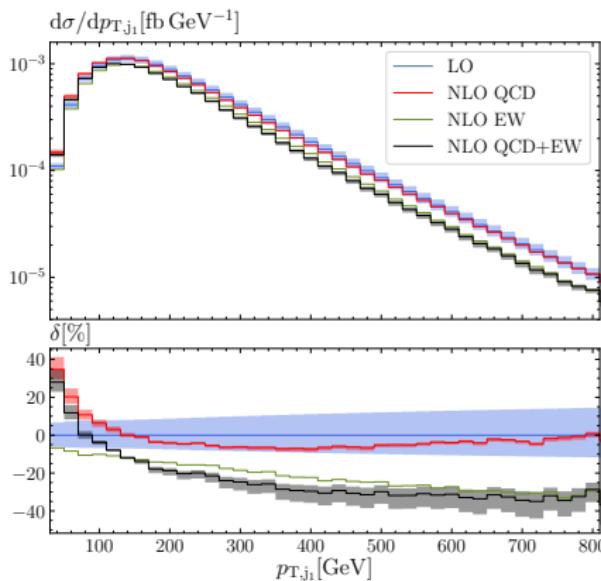
NLO distributions (VIII)

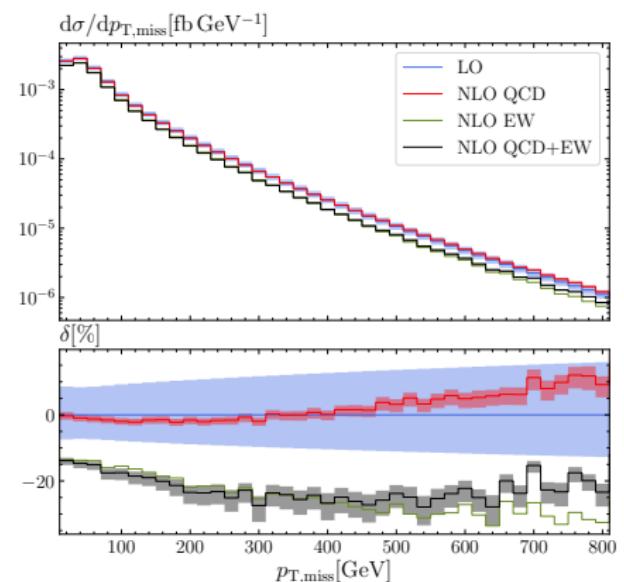
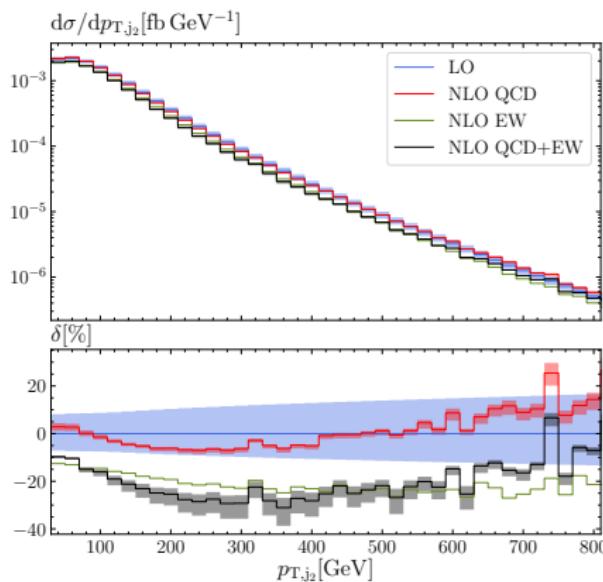


NLO distributions (IX)

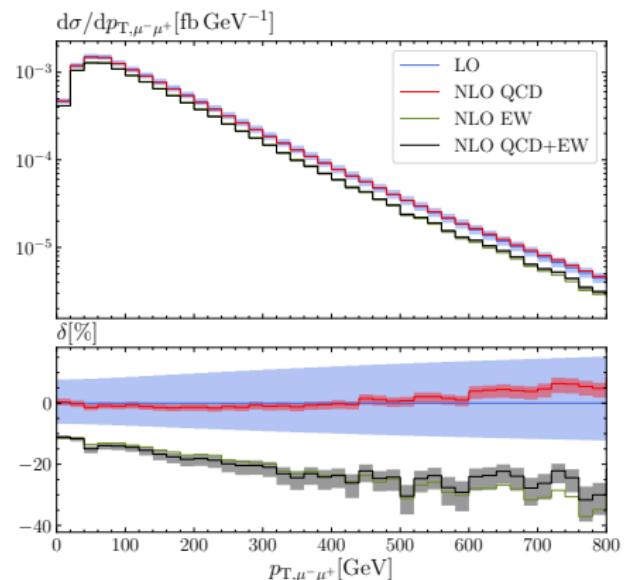
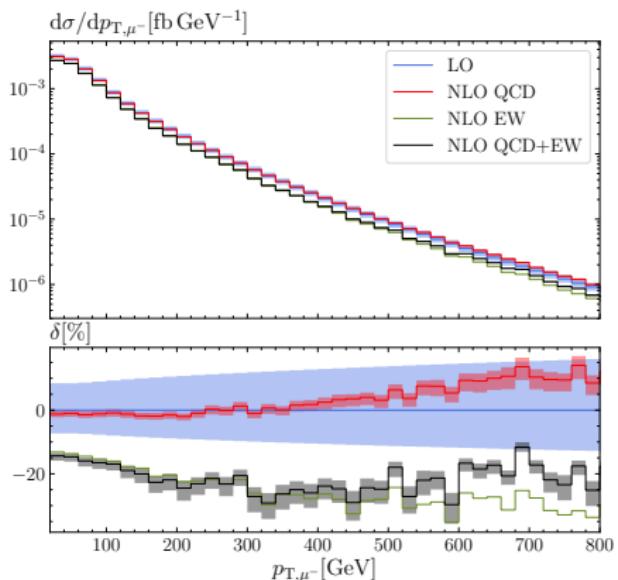


NLO distributions (X)



NLO distributions (χ^2)

NLO distributions (XII)



NLO distributions (XIII)

