

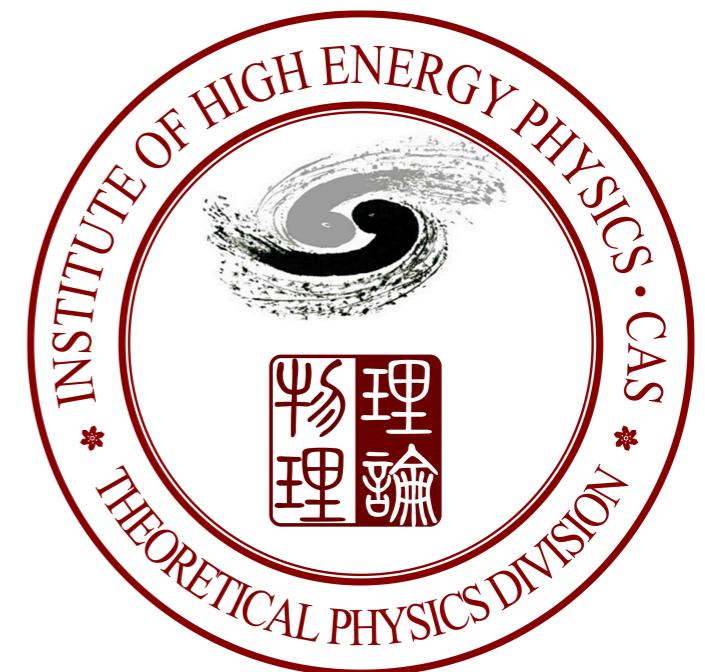
Positivity Bounds on VBS and aQGC

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Based on JHEP 1906 (2019) 137 with Q. Bi and S.-Y. Zhou



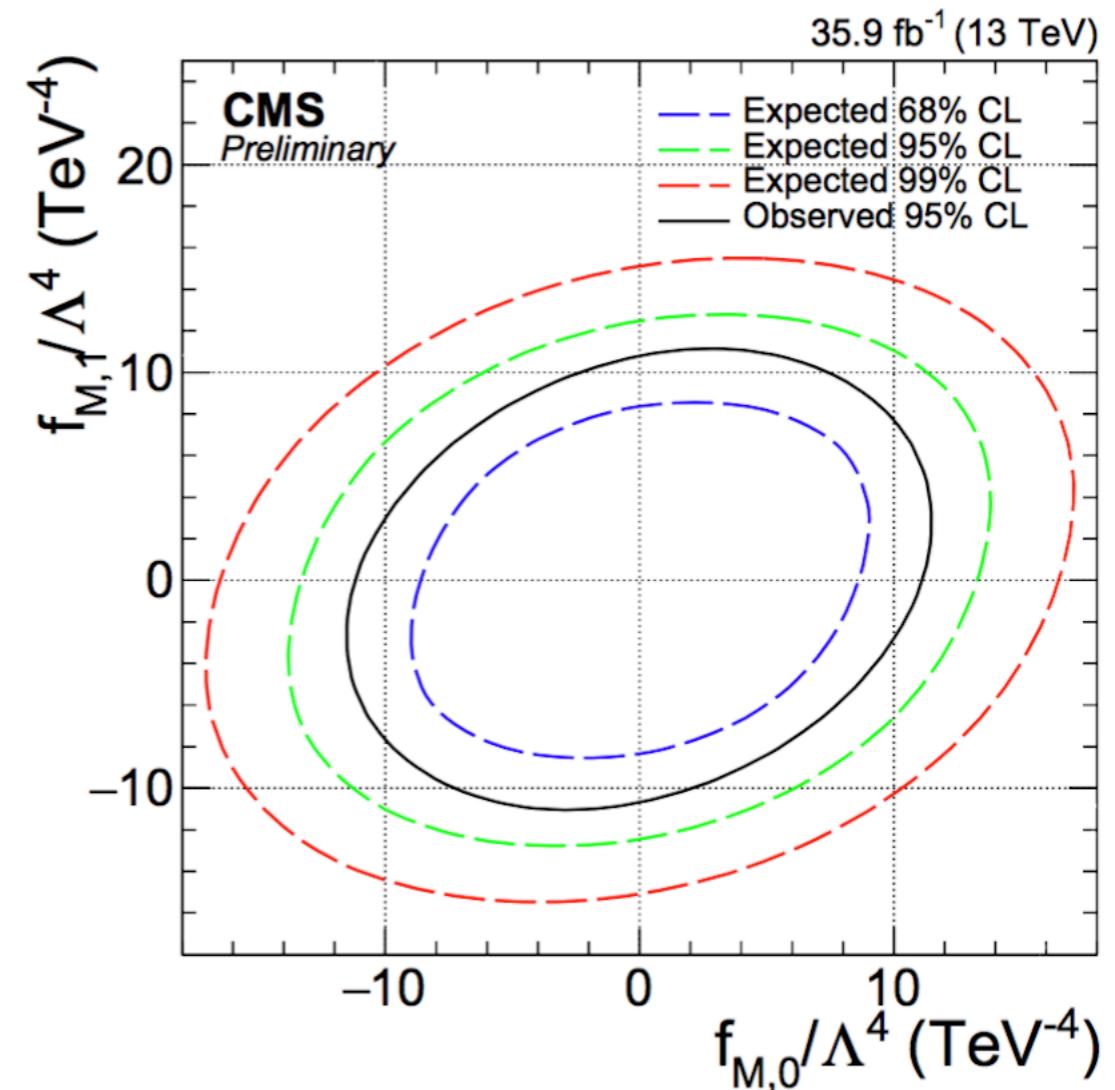
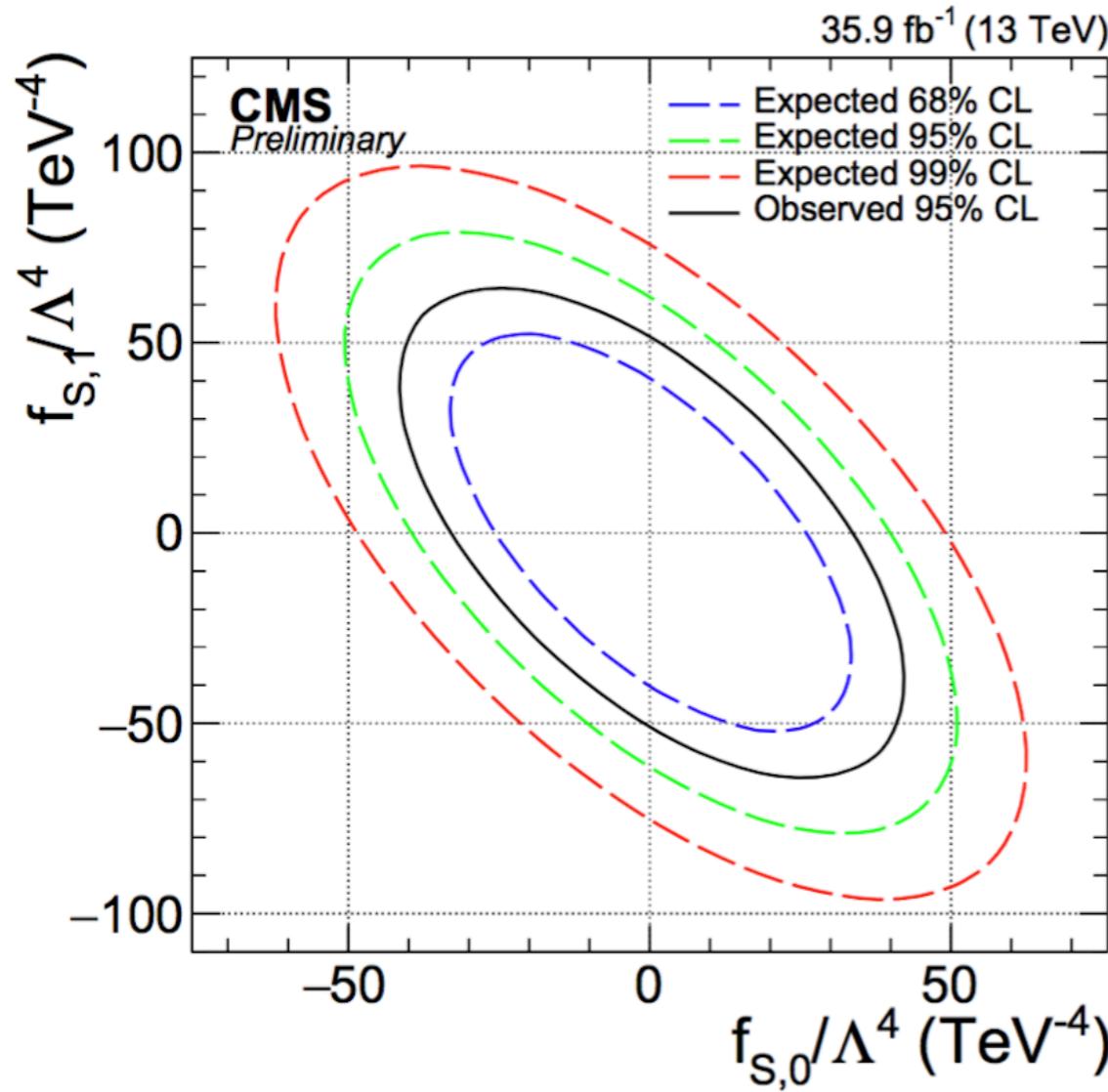
What are these bounds

- Positivity bounds: a set of **theoretical** bounds on dim-8 SMEFT operators, coming from the **fundamental properties of the QFT** (analyticity, unitarity, Lorentz...)
- **Certain linear combinations of Dim-8 operators must be positive**

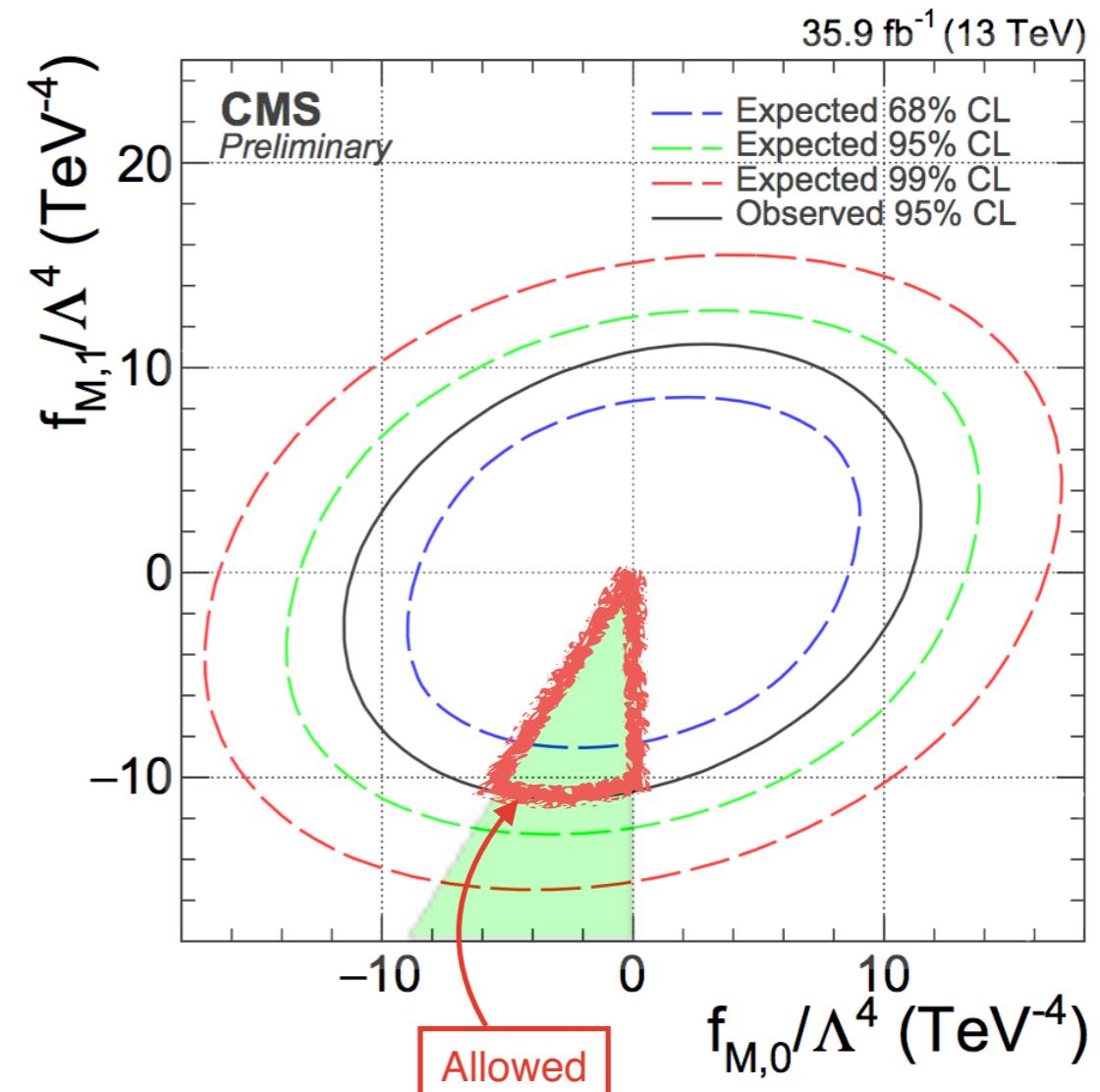
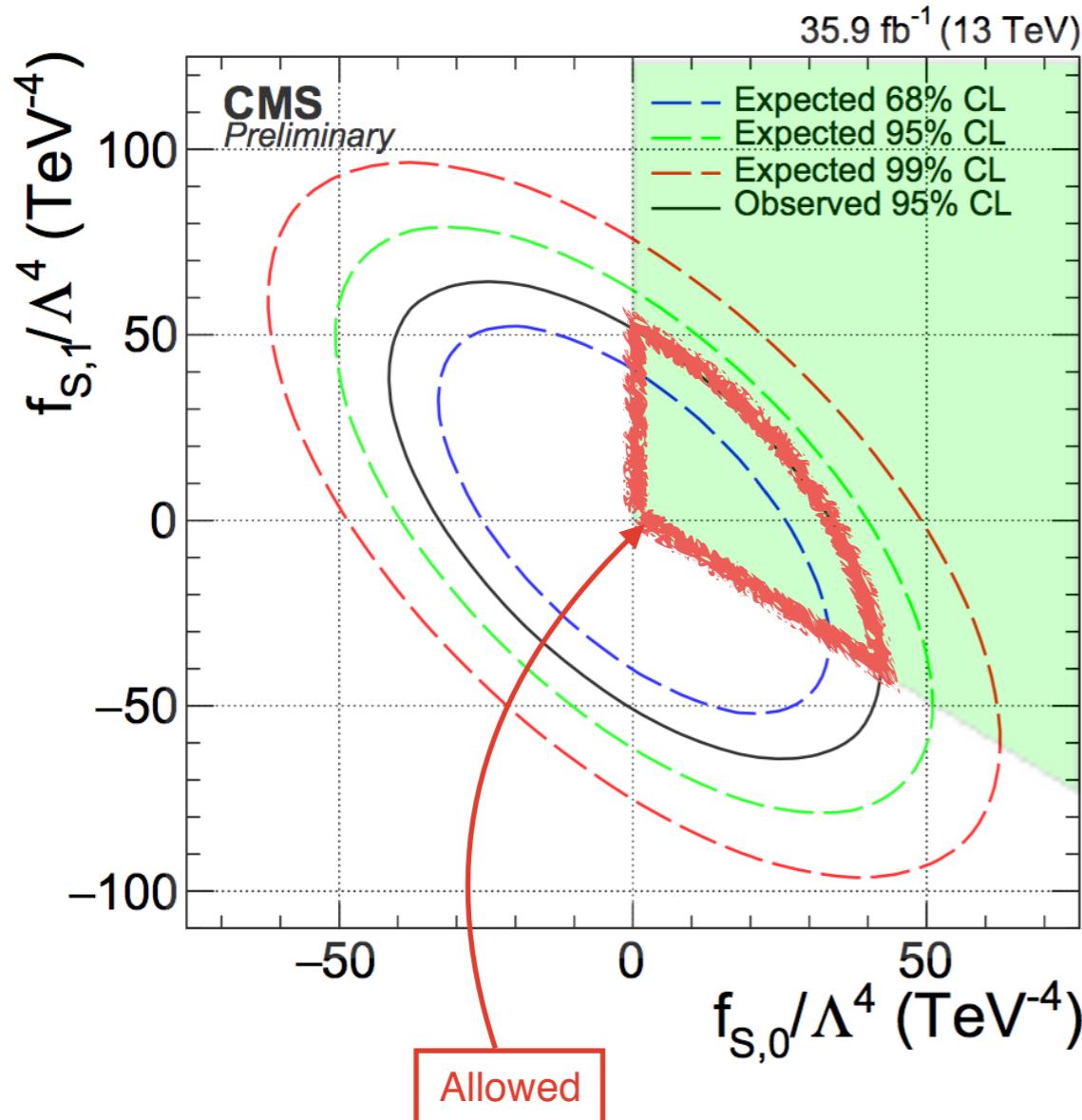
$$\sum_i c_i^{(8)} x_i \geq 0 \quad \text{or} \quad \vec{c} \cdot \vec{x} \geq 0$$

- Applies to Dim-8 operators, independent of
 - the presence of Dim-6 operators
 - Unitarization

E.g. WZjj (CMS-PAS-SMP-18-001)



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Positivity restricts the directions in which SM deviation is possible

Outline

- How to derive the bounds
- How is the aQGC parameter space constrained
- How does it affect VBS measurements

aQGC parametrization

[Eboli, Gonzalez-Garcia, Mizukoshi, PRD 06]

[C. Degrande et al. Snow Mass Proceedings 13]

[Eboli, Gonzalez-Garcia, PRD 16]

$$O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi]$$

$$O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi]$$

$$O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi]$$

$$O_{M,0} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times [(D_\beta \Phi)^\dagger D^\beta \Phi]$$

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$$O_{M,4} = \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\mu \Phi \right] \times \hat{B}^{\beta\nu}$$

$$O_{M,5} = \frac{1}{2} \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\nu \Phi \right] \times \hat{B}^{\beta\mu} + h.c.$$

$$O_{M,7} = \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\mu} D^\nu \Phi \right]$$

$$O_{T,0} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \text{Tr} \left[\hat{W}_{\alpha\beta} \hat{W}^{\alpha\beta} \right]$$

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S-type: longitudinal only

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T-type: transversal only

M-type: both longitudinal & transversal

Deriving the bounds

Positivity bounds

- First established in 06, based on dispersion relation and optical theorem on a forward 2-to-2 scattering.

[A. Adams, A. Arkani-Hamed, S. Dubovsky, A. Nicolis, R. Rattazzi, JHEP 06]

There are similar positivity conditions in more familiar effective field theories in particle physics. Consider for instance the $SU(2)$ chiral Lagrangian, parametrized by the unitary field $U = e^{i\pi^a \sigma^a}$,

$$\mathcal{L} = f^2 \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + L_4 [\text{tr}(\partial_\mu U^\dagger \partial^\mu U)]^2 + L_5 [\text{tr}(\partial_\mu U^\dagger \partial_\nu U)]^2 + \dots \quad (52)$$

$$L_{4,5} > 0$$

Of course the pion chiral Lagrangian follows from QCD which is a local quantum field theory, so these conditions must necessarily be satisfied. The situation is perhaps more interesting for the electroweak chiral Lagrangian governing the dynamics of the longitudinal components of the W/Z bosons. While it is most likely, given precision electroweak constraints, that the UV completion involves Higgses and a linear sigma model, there may also be more exotic possibilities, including in the extreme case a low fundamental scale close to the electroweak scale. This physics should manifest itself through the higher-dimension operators in the effective Lagrangian, and assuming custodial $SU(2)$ is a good approximate symmetry, the constraint on the electroweak chiral Lagrangian is the same $L_{4,5} > 0$ (with the derivatives covariantized for the $SU(2) \times U(1)$ gauge symmetry $\partial_\mu \rightarrow D_\mu$). These operators are not associated with the well-known constraints of precision electroweak physics— instead, in unitary gauge $U = 1$, they represent anomalous quartic couplings for the W/Z , which must be positive.

Positivity bounds in VBS

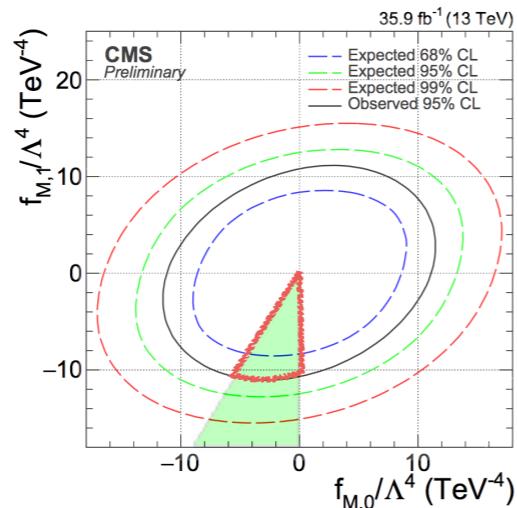
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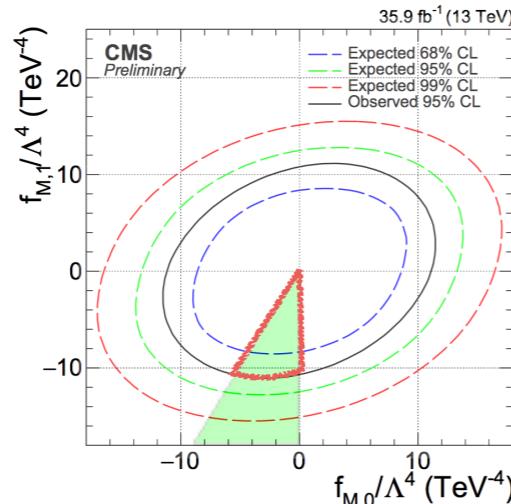


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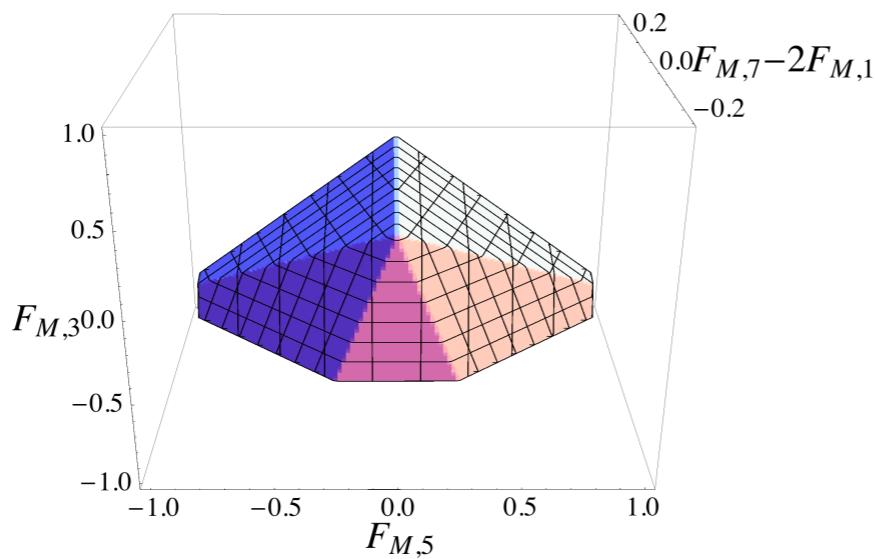
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- 3 OPs:

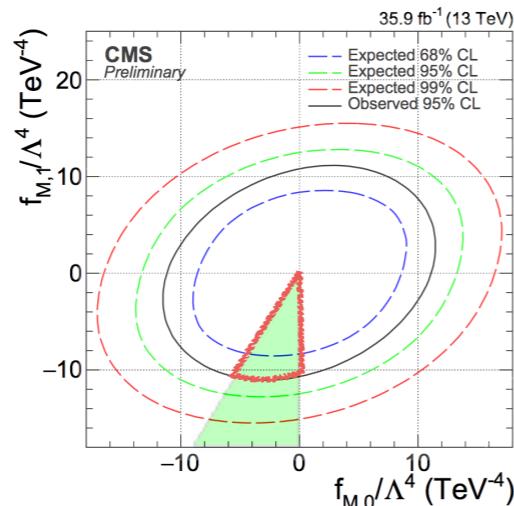


Positivity bounds in VBS

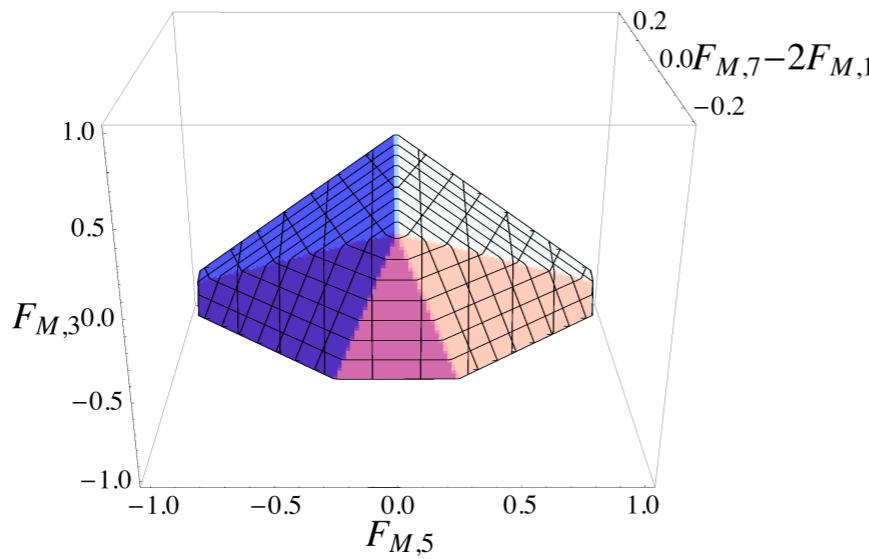
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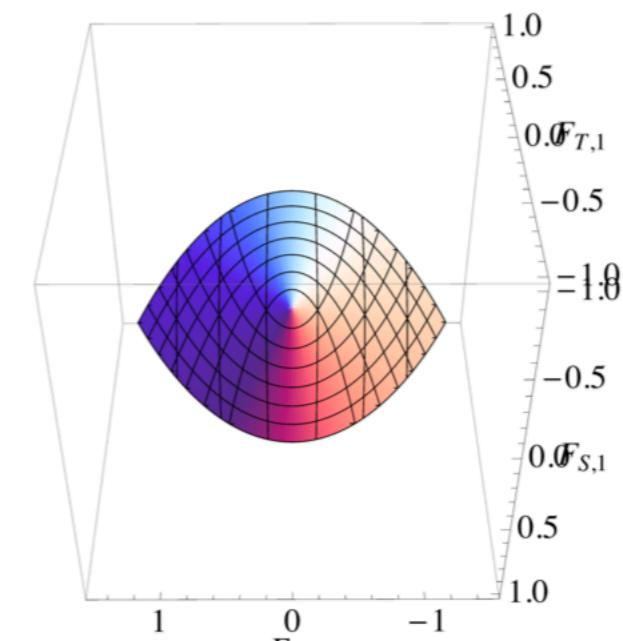
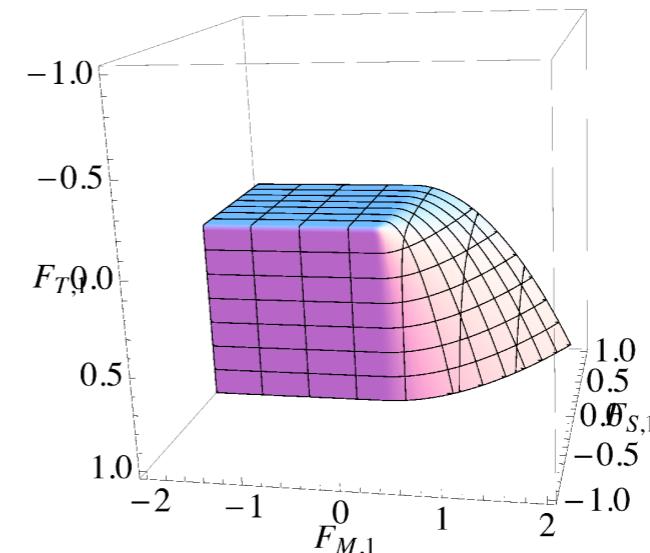
- 2 OPs:



- 3 OPs:



- Adding polarization:



Positivity bounds (simple version)

[C. Cheung, G. Remmen JHEP 16]

- Assume theory only has one particle, mass = m
- Consider 2-to-2 forward elastic scattering amplitude, $A(s, t=0)$
- **A is analytic** apart from poles and branch points on real axis.
- **A satisfies Froissart unitarity bounds:**

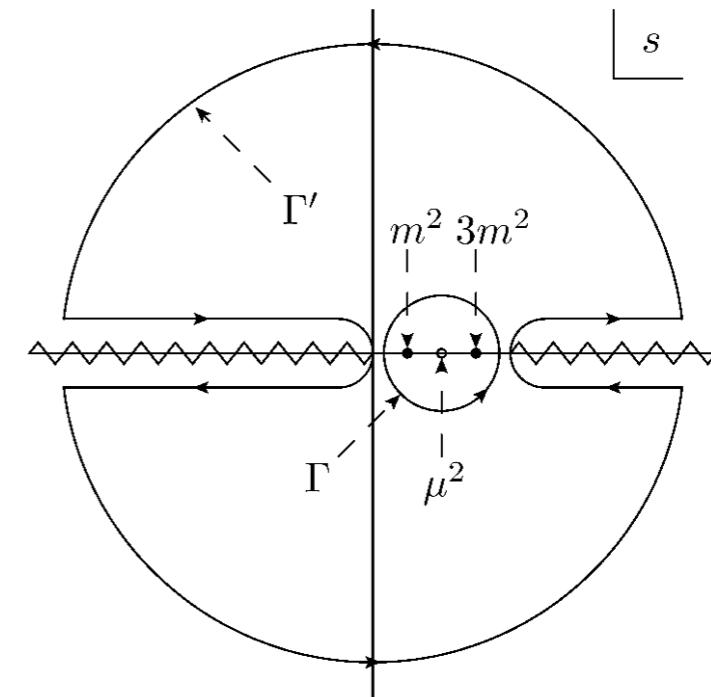
$$A(s, 0) < \mathcal{O}(s \ln^2 s)$$

Positivity bounds (simple version)

- Consider a contour integral

$$f = \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{A(s, 0)}{(s - \mu^2)^3}$$

- Deform Γ to Γ' . Boundary contributions vanish due to Froissart bound.



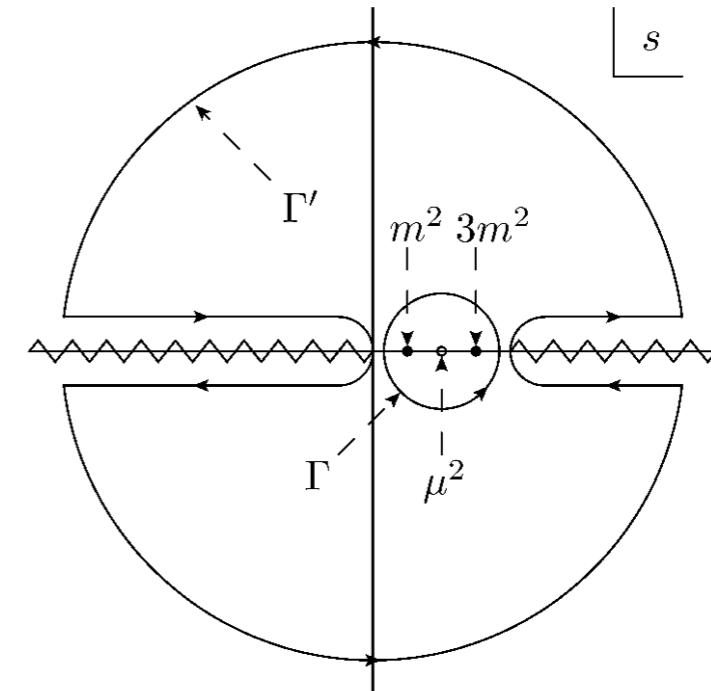
$$f = \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{A(s, 0)}{(s - \mu^2)^3} = \frac{1}{2\pi i} \left(\int_{-\infty}^0 + \int_{4m^2}^{\infty} \right) ds \frac{\text{Disc}A(s, 0)}{(s - \mu^2)^3}$$

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IR ↑

Can be calculated in EFT,
which gives $C_8 + C_6^2$

↑ UV

We will show the Disc
is positive

Positivity bounds (simple version)

- For the discontinuity in the + real axis:
use optical theorem

$$\text{Disc}A(s, 0) = 2i\text{Im}A(s, 0)$$

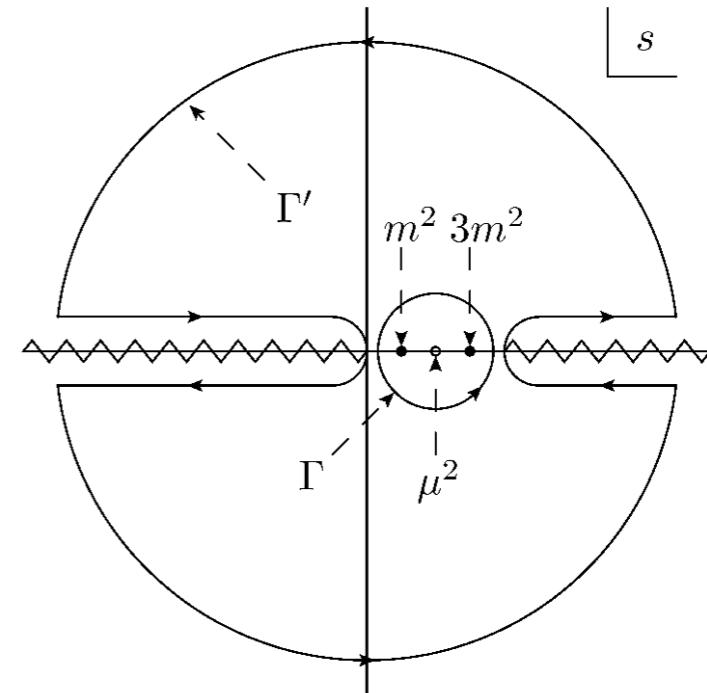
$$\text{Im}A(s, 0) = s\sigma(s)\sqrt{1 - 4m^2/s} > 0$$

- For the discontinuity in the - real axis:
use crossing then optical theorem

$$A(s, 0) \rightarrow A'(u, 0) = A'(4m^2 - s, 0)$$

- (Disc at large s is where NP enters)
- This implies

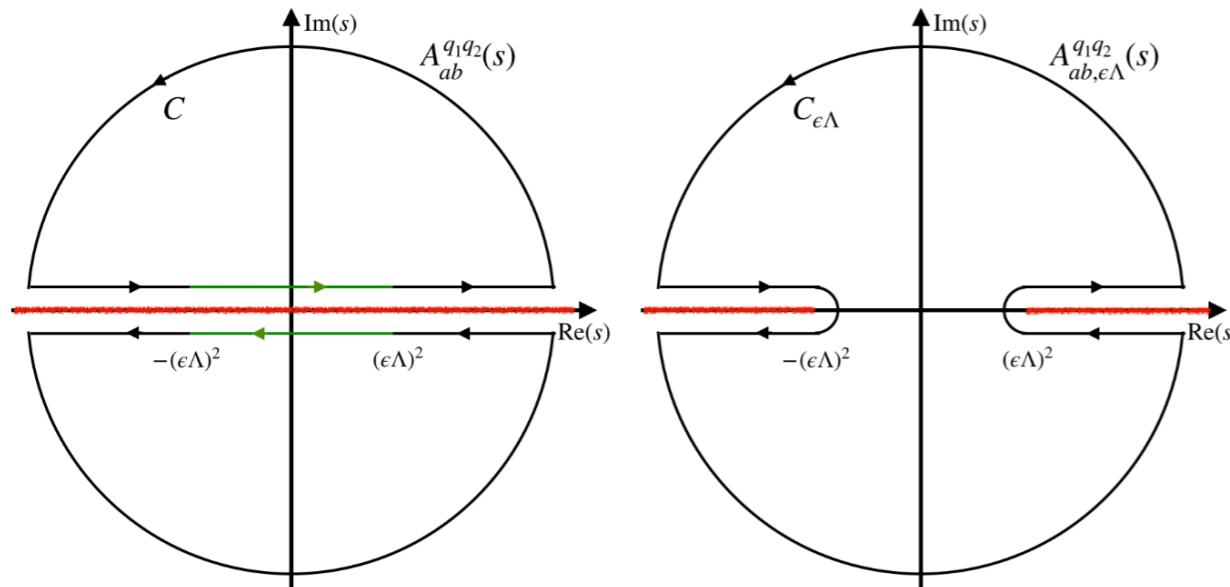
$$f \approx \frac{d^2A(\mu^2)}{ds^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} > 0$$



Positivity bounds (for SMEFT)

- In SM, branch cuts cover the entire real axis. Define a new amplitude with the same disc. above scale $\sim \Lambda$ but subtracted below $\sim \Lambda$ (i.e. the “improved positivity”)

[C. de Rham, S. Melville, A. J. Tolley and S.-Y. Zhou JHEP 17]



$$\begin{aligned}
 B_{ab,\epsilon\Lambda}^{q_1 q_2}(s) &\equiv A_{ab}^{q_1 q_2}(s) - \frac{1}{2\pi i} \int_{-(\epsilon\Lambda)^2}^{+(\epsilon\Lambda)^2} ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{s' - s} \\
 &= \frac{1}{2\pi i} \oint_C ds' \frac{A_{ab}^{q_1 q_2}(s')}{s' - s} - \frac{1}{2\pi i} \int_{-(\epsilon\Lambda)^2}^{+(\epsilon\Lambda)^2} ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{s' - s} \\
 f_{ab,\epsilon\Lambda}^{q_1 q_2}(s) &\equiv \frac{1}{2} \frac{d^2 B_{ab,\epsilon\Lambda}^{q_1 q_2}(s)}{ds^2} \\
 &= \frac{1}{2\pi i} \left(\int_{-\infty}^{-(\epsilon\Lambda)^2} + \int_{+(\epsilon\Lambda)^2}^{\infty} \right) ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{(s' - s)^3}
 \end{aligned}$$

- SM loop contributions: need to be estimated, and the subtraction term helps. Negligible compared to the best experimental precision.
E.g. $f_{\epsilon\Lambda}^{00,WW} = 0.038 \text{ TeV}^{-4}$ compared with $\mathcal{O}(1) \text{ TeV}^{-4}$ with current bounds
- Can be completely removed in weakly coupled UV, by using the LO amplitude.

Positivity bounds: remove dim-6

$$\sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} > 0 \quad \text{or} \quad \sum_i c_i^{(8)} x_i \geq - \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

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- By explicit calculation the RHS is positive definite

- For example WZ:

$$\text{R.H.S} \propto a_3^2 b_3^2 [e^2 C_{DW} - s_W^2 c_W^2 C_{\varphi D} - 4 s_W^3 c_W C_{\varphi WB}]^2 + 36(a_1 b_1 + a_2 b_2)^2 e^2 s_W^2 c_W^2 C_W^2$$

- WW:
External polarization

$$\text{R.H.S} \propto a_3^2 b_3^2 s_W^2 (e^2 C_{DB} + c_W^2 C_{\varphi D})^2 + e^2 c_W^2 [6(a_1 b_1 + a_2 b_2) s_W C_W + a_3 b_3 e C_{DW}]^2$$

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$$\sum_i c_i^{(8)} x_i \geq 0 \quad \text{or} \quad \vec{c} \cdot \vec{x} \geq 0$$

Positivity bounds: polarization dependence

- With FS, FM, FT operators, the spin of VV' can take **any direction, leading to (continuously) many different positivity bounds.**
- Amplitude depends on external polarization. We will parametrize by

$$\epsilon^\mu(V_1) = \sum_{i=1}^3 a_i \epsilon_{(i)}^\mu = \left(a_3 \frac{p_1}{m_{V_1}}, a_1, a_2, a_3 \frac{E_1}{m_{V_1}} \right),$$

$$\epsilon^\mu(V_2) = \sum_{i=1}^3 b_i \epsilon_{(i)}^\mu = \left(b_3 \frac{p_2}{m_{V_2}}, b_1, b_2, b_3 \frac{E_2}{m_{V_2}} \right),$$

- For example, from WW channel: (Similar for all other channels)

Positivity bound

$$\begin{aligned} & 2A_1(8F_{T,0} + 12F_{T,1} + 5F_{T,2}) + 6A_2F_{T,2} + (A_3 + A'_3)(-2F_{M,1} + F_{M,7}) \\ & + 2A_4(8F_{T,1} + F_{T,2}) + 2A'_4(8F_{T,0} + 4F_{T,1} + F_{T,2}) + A'_5(4F_{M,0} - F_{M,1} + F_{M,7}) \\ & + 4A_6(2F_{S,0} + F_{S,1} + F_{S,2}) > 0 \end{aligned}$$

Spin dependence

$$\begin{aligned} A_1 &\equiv |a_1|^2|b_1|^2 + |a_2|^2|b_2|^2, & A_4 &\equiv a_1 a_2^* b_1 b_2^* + c.c., \\ A_2 &\equiv |a_1|^2|b_2|^2 + |a_2|^2|b_1|^2, & A'_4 &\equiv a_1 a_2^* b_1^* b_2 + c.c., \\ A_3 &\equiv (|b_1|^2 + |b_2|^2)|a_3|^2, & A_5 &\equiv (a_1 b_1 + a_2 b_2)a_3^* b_3^* + c.c., \\ A'_3 &\equiv (|a_1|^2 + |a_2|^2)|b_3|^2, & A'_5 &\equiv -(a_1 b_1^* + a_2 b_2^*)a_3^* b_3 + c.c. \\ A''_3 &\equiv |b_1|^2|a_3|^2 & A_6 &\equiv |a_3|^2|b_3|^2, \end{aligned}$$

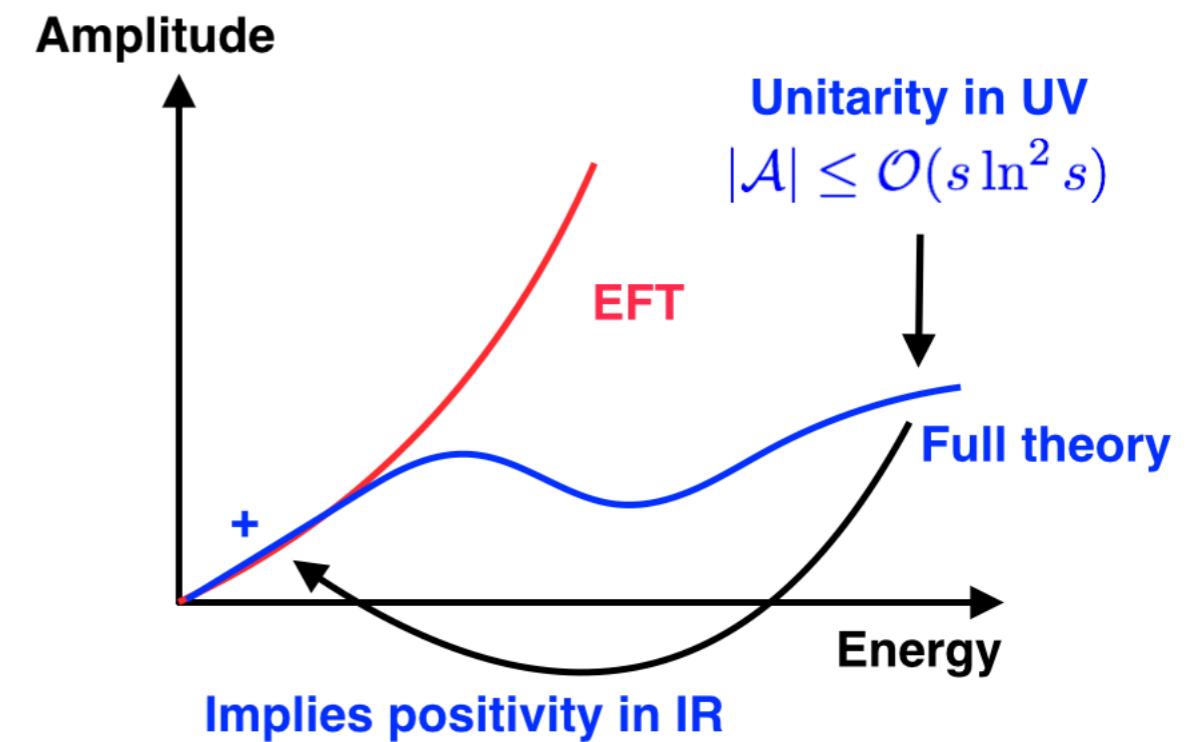
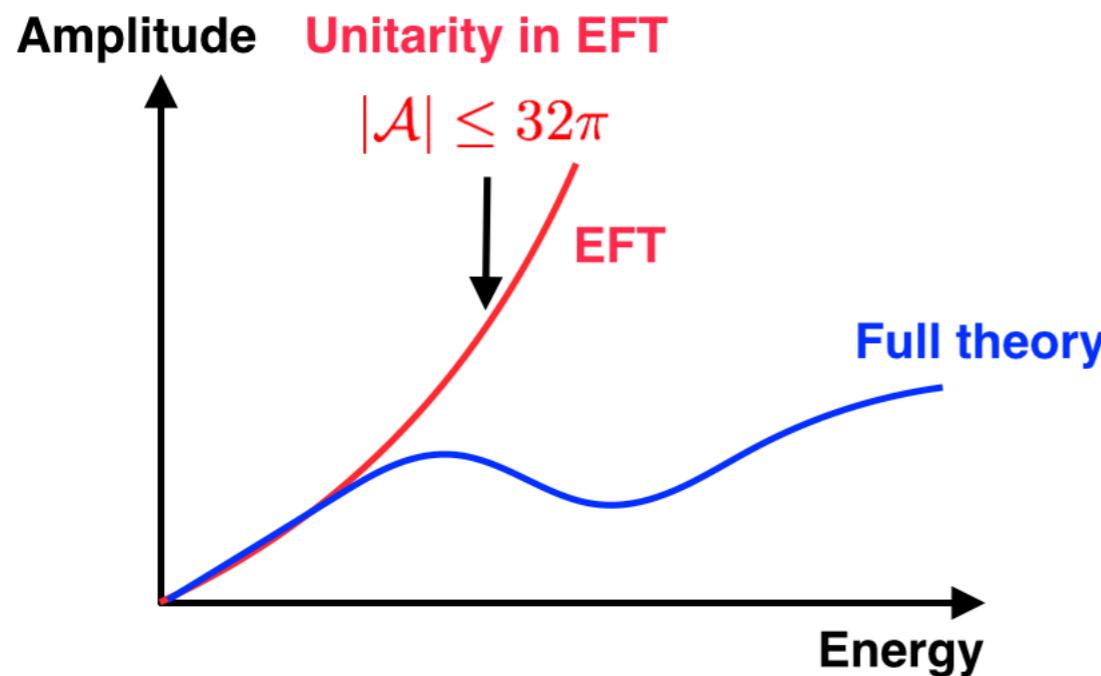
Positivity bounds: polarization dependence

- These results are complete, but difficult to use, because the polarization (a,b) show up as free parameters.
- To understand better the parameter space, need remove them:
 - Go through all possible (a,b) polarization, combine all the resulting bounds.
 - Or equivalently, we “solve” these conditions, identify a set of (a,b) values that are the most crucial (e,g, once positivity bounds are satisfied for them, they are guaranteed to be satisfied for the rest.)
- The result will be 19 linear, 3 quadratic and 1 quartic polynomial inequalities that describe the allowed parameter space.

- Unitarity bounds

\neq

- Positivity bounds



- This is assuming the 18 QGC parameters form a complete set for $VV \rightarrow VV$. This is why only $VV \rightarrow VV$ is used to derive bounds.
- G. N. Remmen and N. L. Rodd, in progress.

Understanding of the parameter space

1D case: individual limits

$f_{S,0}$	$f_{S,1}$	$f_{S,2}$	$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
+	+	+	X	-	O	-	O	X

$f_{M,7}$	$f_{T,0}$	$f_{T,1}$	$f_{T,2}$	$f_{T,5}$	$f_{T,6}$	$f_{T,7}$	$f_{T,8}$	$f_{T,9}$
+	+	+	+	X	+	X	+	+



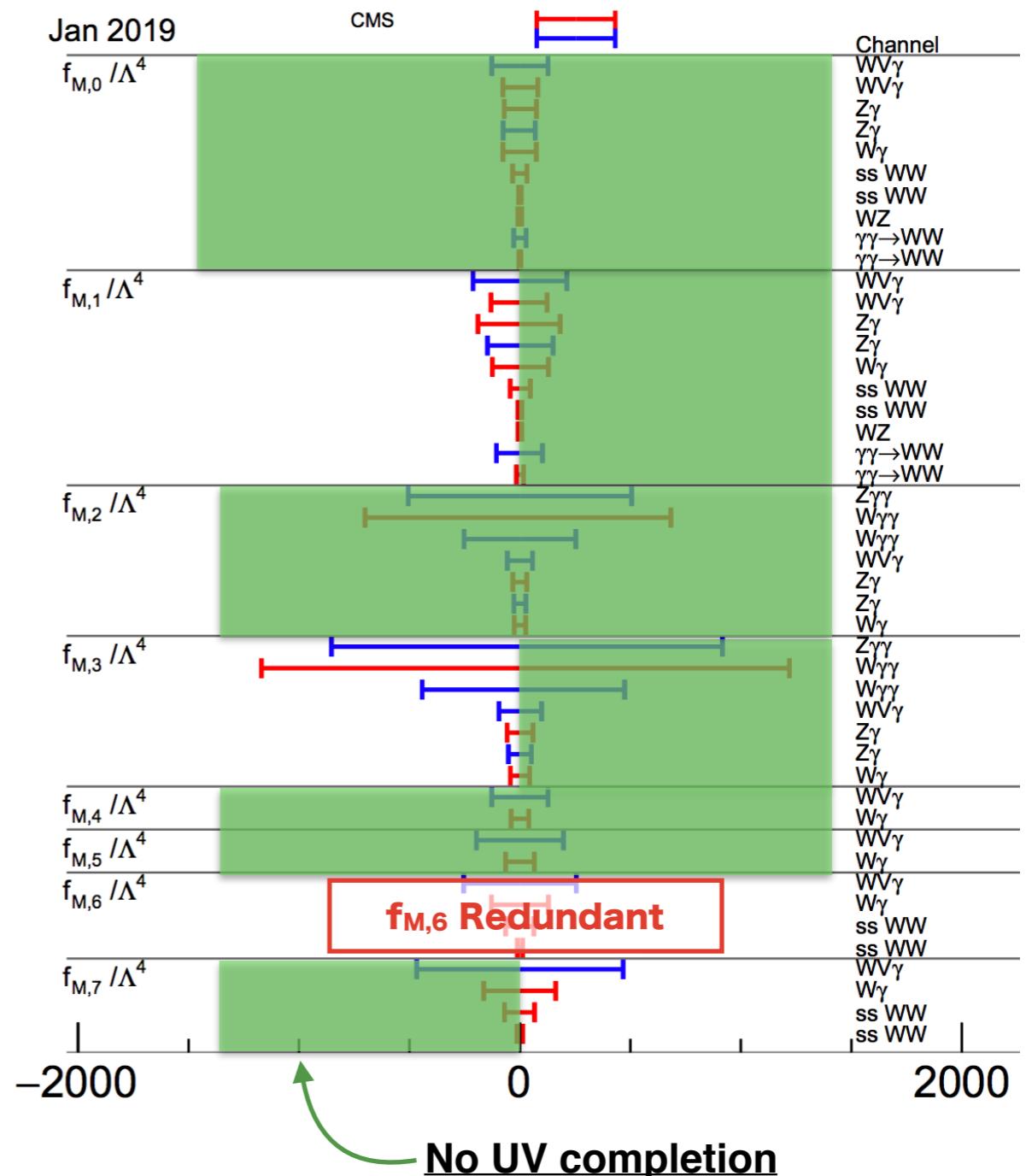
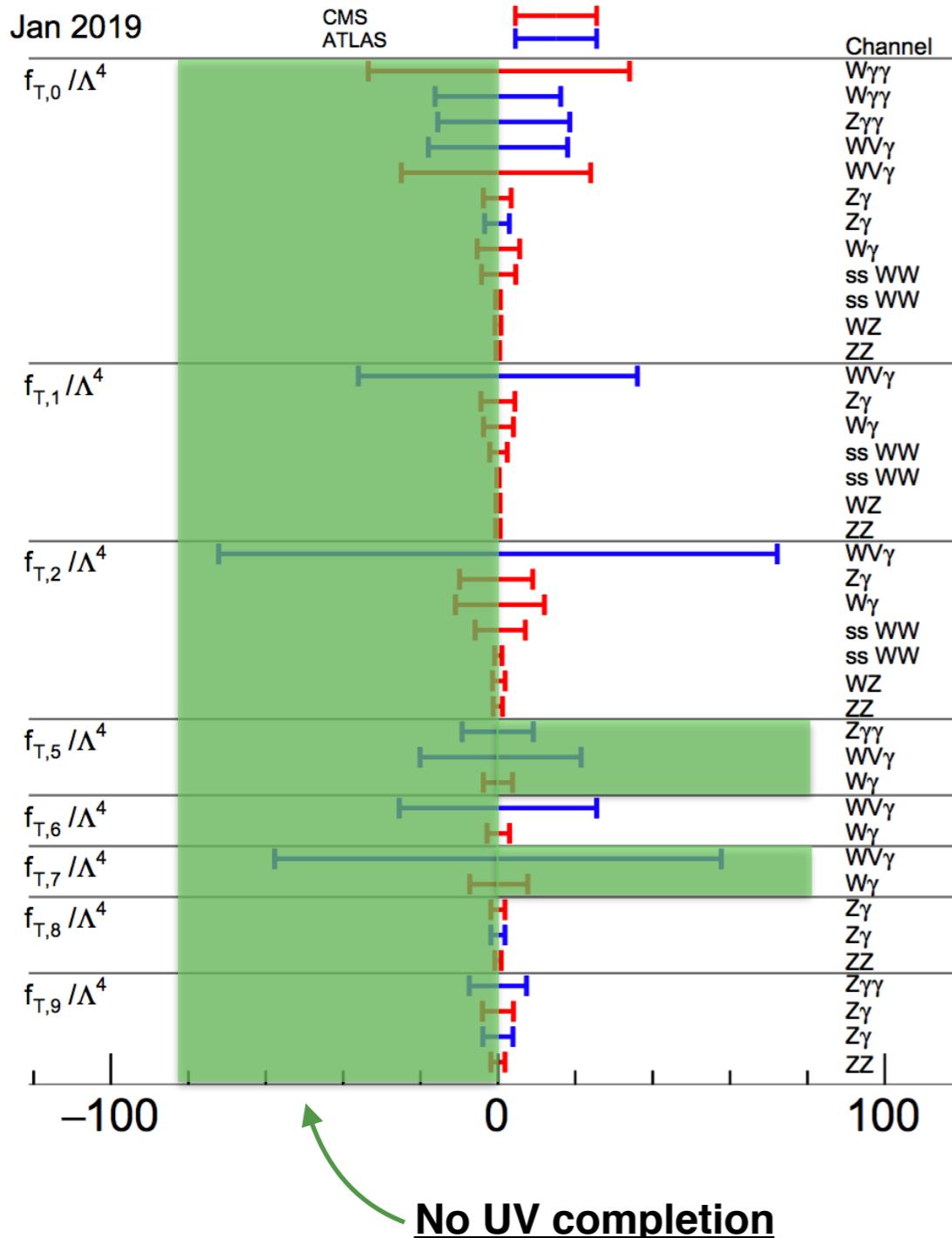
$F_{S,0}$	$F_{S,1}$	$F_{S,2}$	$F_{M,0}$	$F_{M,1}$	$F_{M,2}$	$F_{M,3}$	$F_{M,4}$	$F_{M,5}$
+	+	+	X	-	X	-	X	X

$F_{M,7}$	$F_{T,0}$	$F_{T,1}$	$F_{T,2}$	$F_{T,5}$	$F_{T,6}$	$F_{T,7}$	$F_{T,8}$	$F_{T,9}$
+	+	+	+	X	+	X	+	+

arXiv:1808.00010
polarization vectors are real

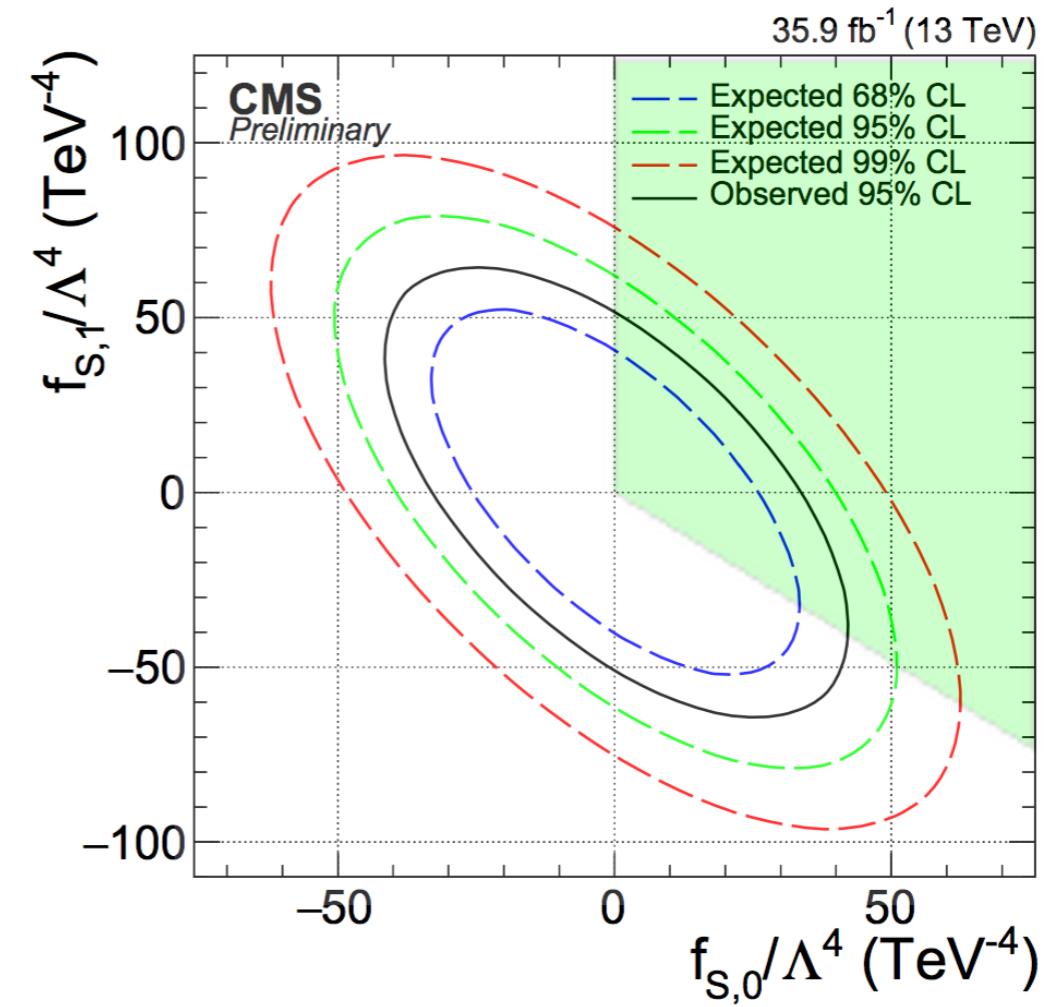
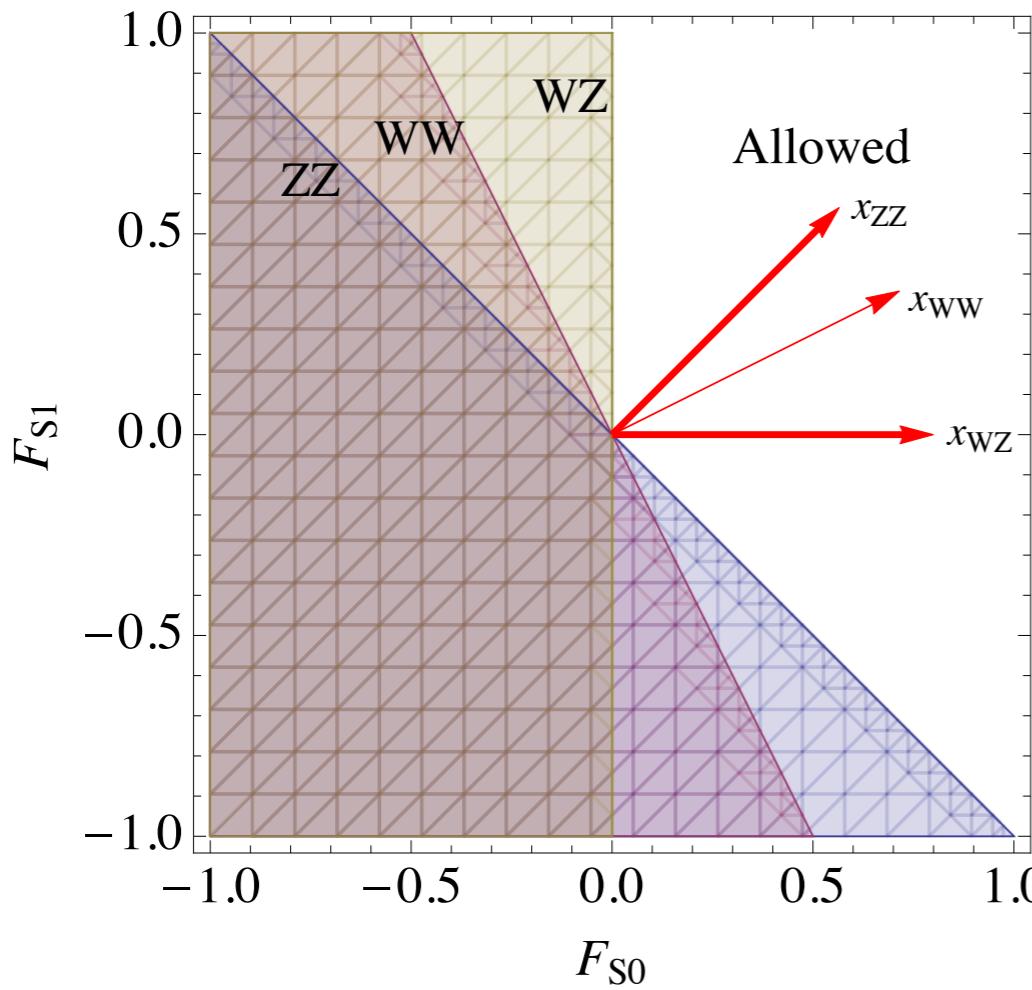
JHEP 1906 (2019) 137
Improvement by
allowing for complex polarization

1D case: individual limits



2D cases

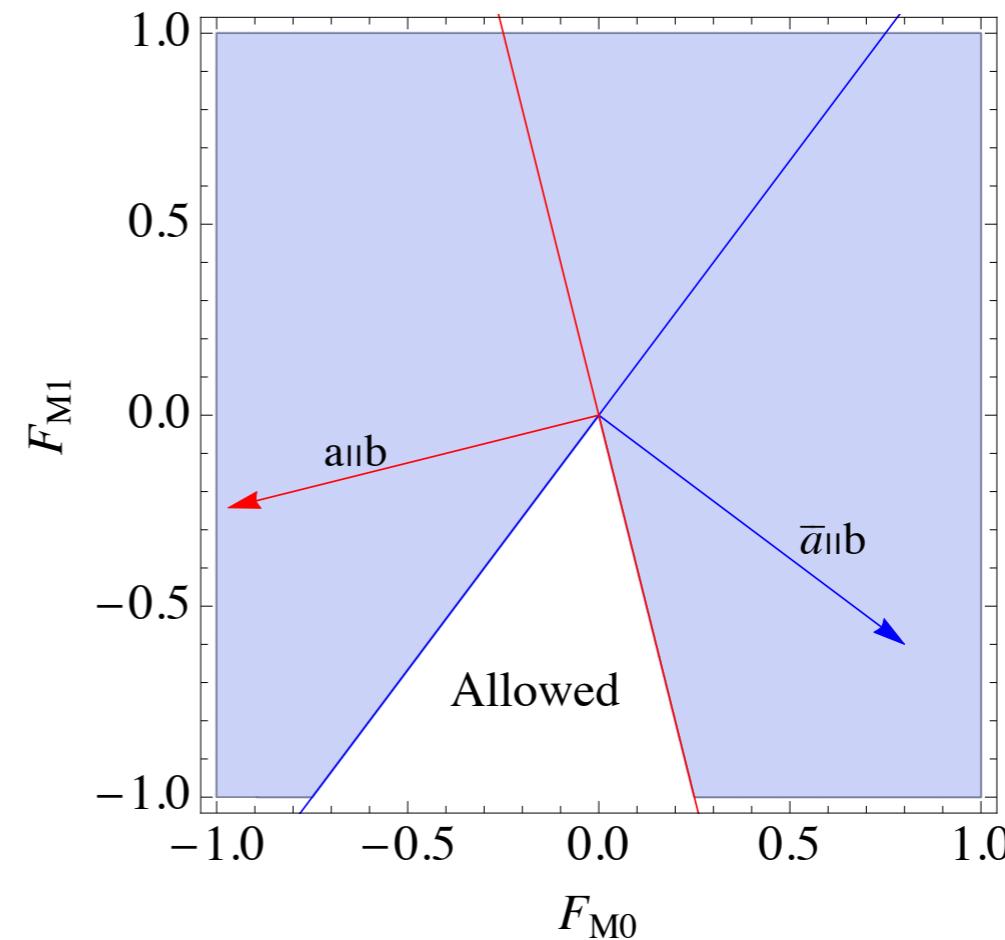
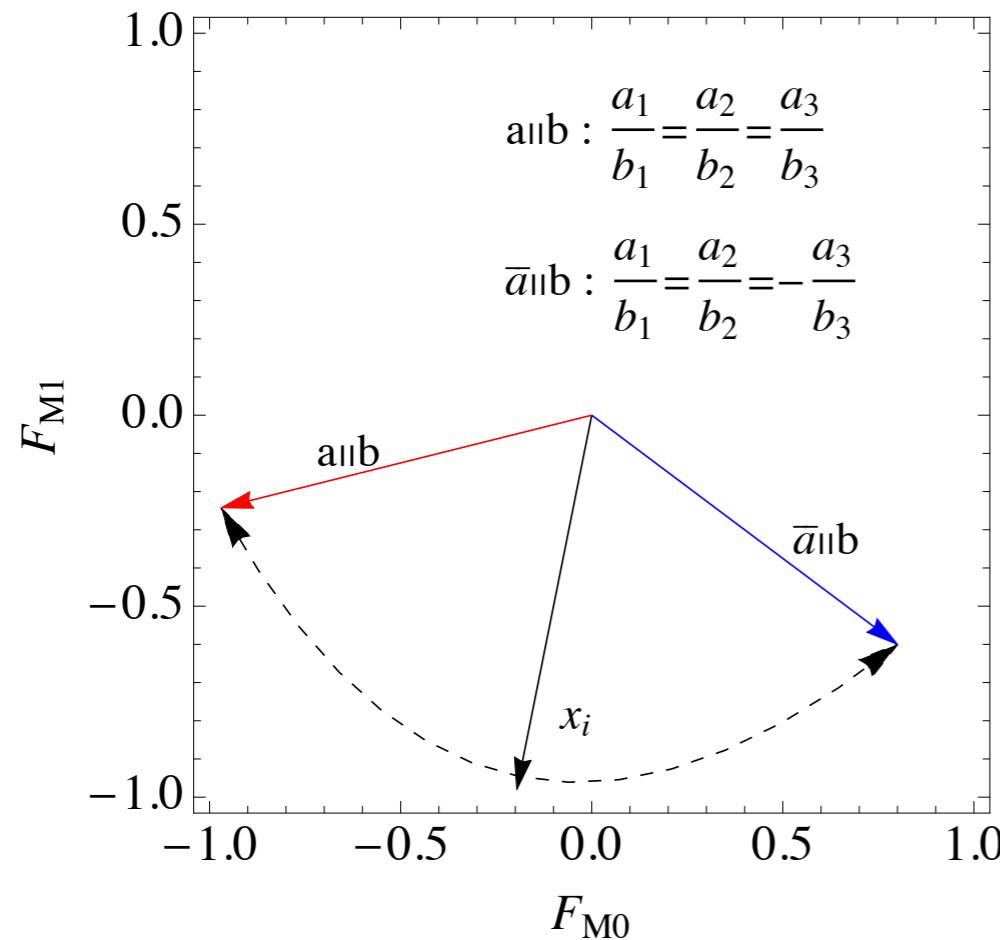
- Consider F_{S0} and F_{S1}
 - Think of positivity as $\vec{F} \cdot \vec{x} = f_i x_i > 0, \quad \vec{F} = (F_{S,0}, F_{S,1}, \dots)$
 - i.e. F vector is positive once projected on x direction



2D cases

- Consider F_{M0} and F_{M1}
 - Now we have polarization dependence...

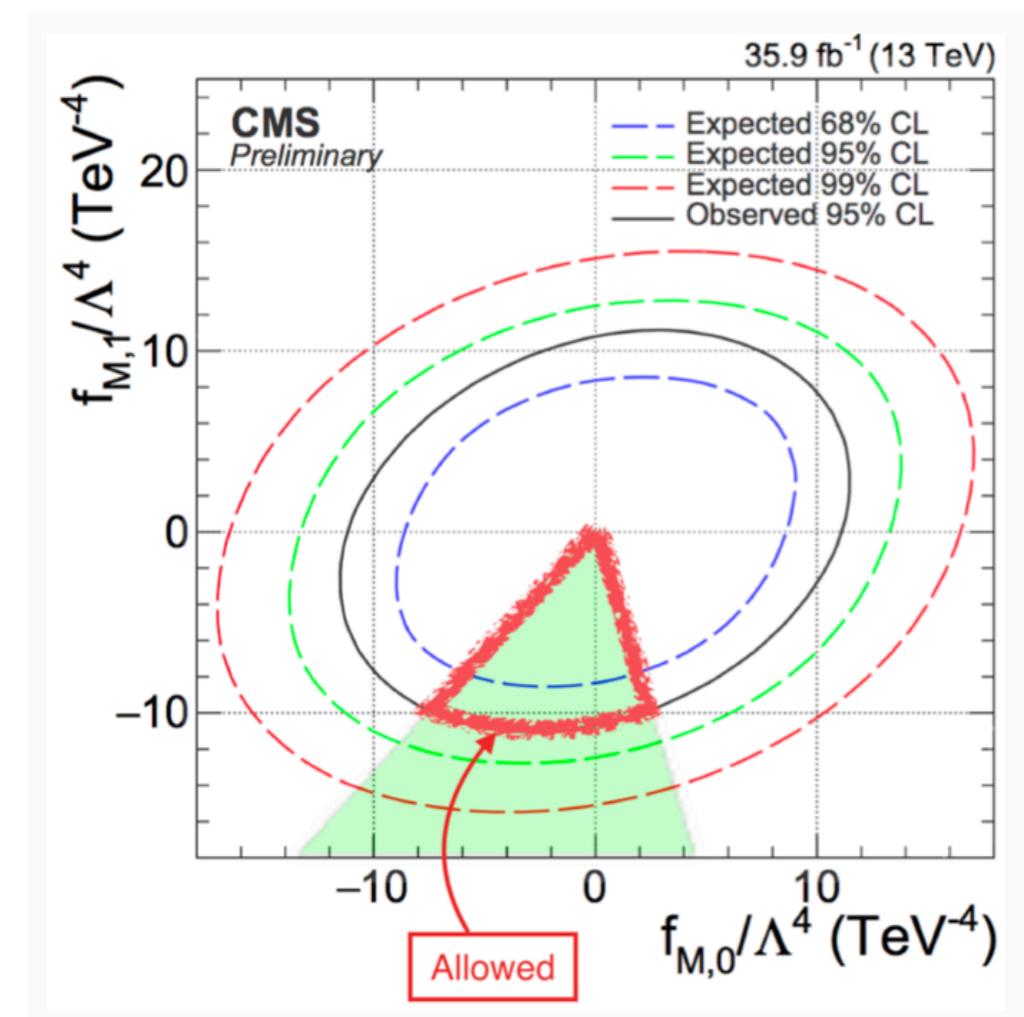
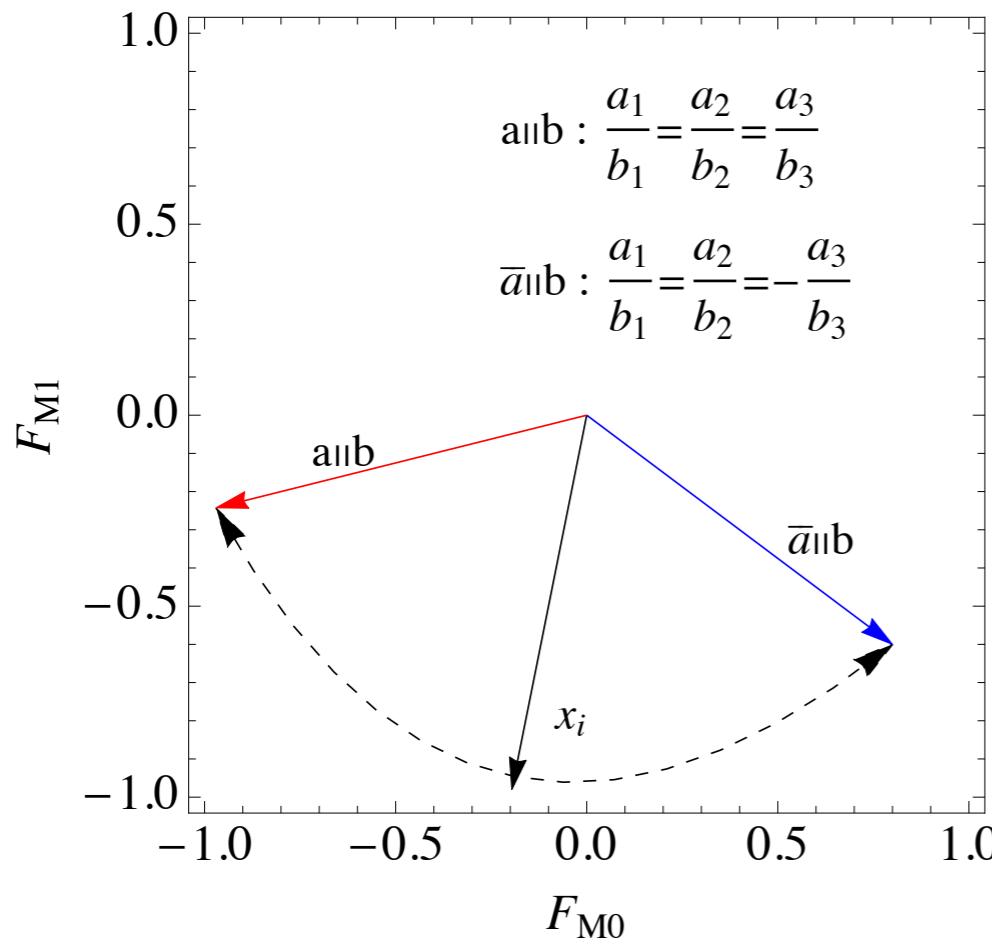
$$\vec{x}_{WW} = - (4(a_1 b_1 + a_2 b_2) a_3 b_3, (a_1^2 + a_2^2) b_3^2 - (a_1 b_1 + a_2 b_2) a_3 b_3 + (b_1^2 + b_2^2) a_3^2)$$



2D cases

- Consider F_{M0} and F_{M1}
 - Now we have polarization dependence...

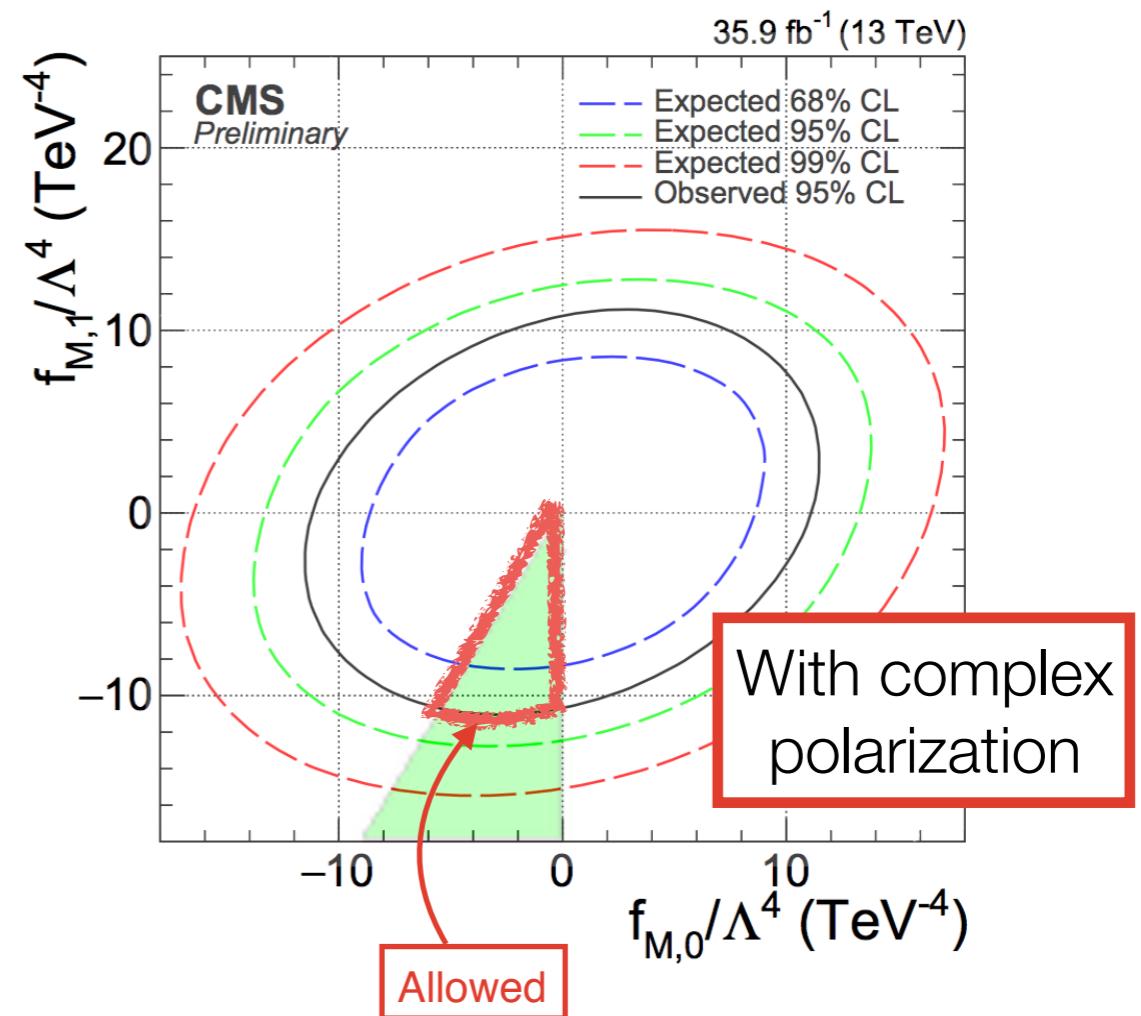
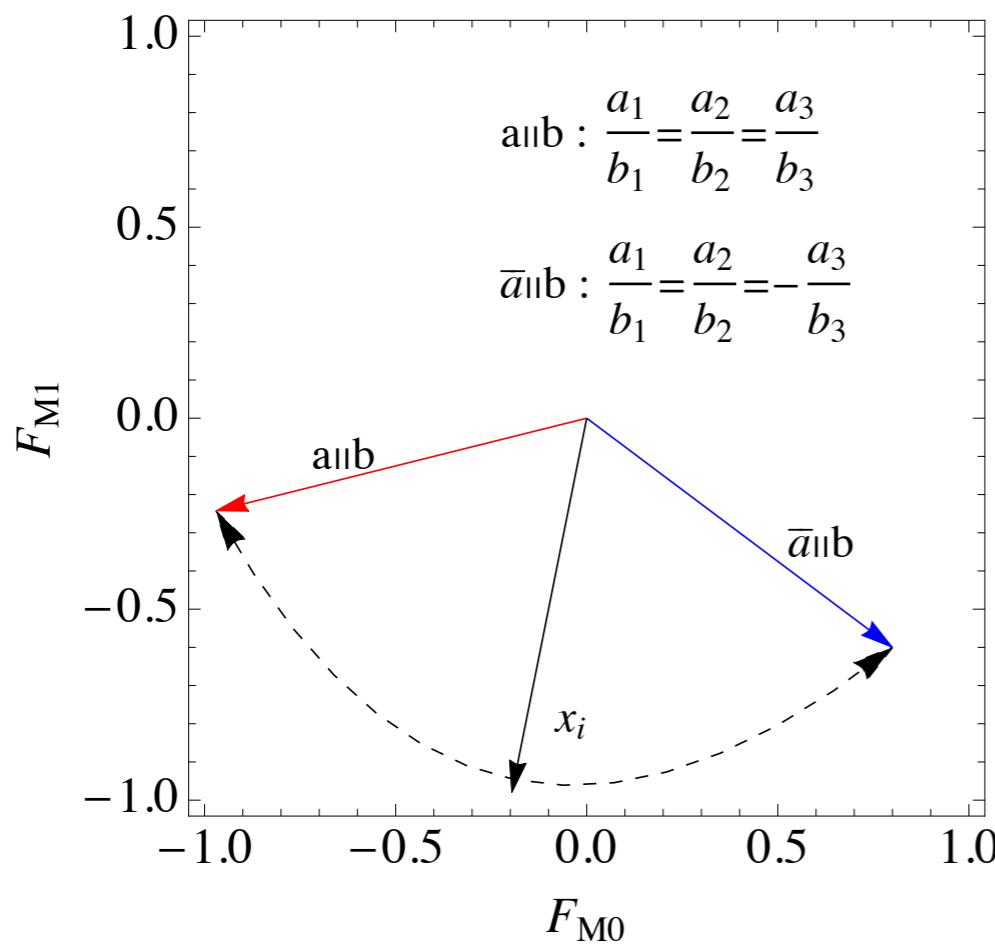
$$\vec{x}_{WW} = - (4(a_1 b_1 + a_2 b_2) a_3 b_3, (a_1^2 + a_2^2) b_3^2 - (a_1 b_1 + a_2 b_2) a_3 b_3 + (b_1^2 + b_2^2) a_3^2)$$



2D cases

- Consider F_{M0} and F_{M1}
 - Now we have polarization dependence...

$$\vec{x}_{WW} = - (4(a_1 b_1 + a_2 b_2) a_3 b_3, (a_1^2 + a_2^2) b_3^2 - (a_1 b_1 + a_2 b_2) a_3 b_3 + (b_1^2 + b_2^2) a_3^2)$$



3D cases: the pyramid case

- Consider F_{M0}, F_{M1}, F_{M5} .

$$\vec{x}_{WW} = (-4T_a T_b \cos \phi, -T_a^2 + T_a T_b \cos \phi - T_b^2, 0)$$

$$\vec{x}_{ZZ} = (0, -2c_W^2, s_W^2)$$

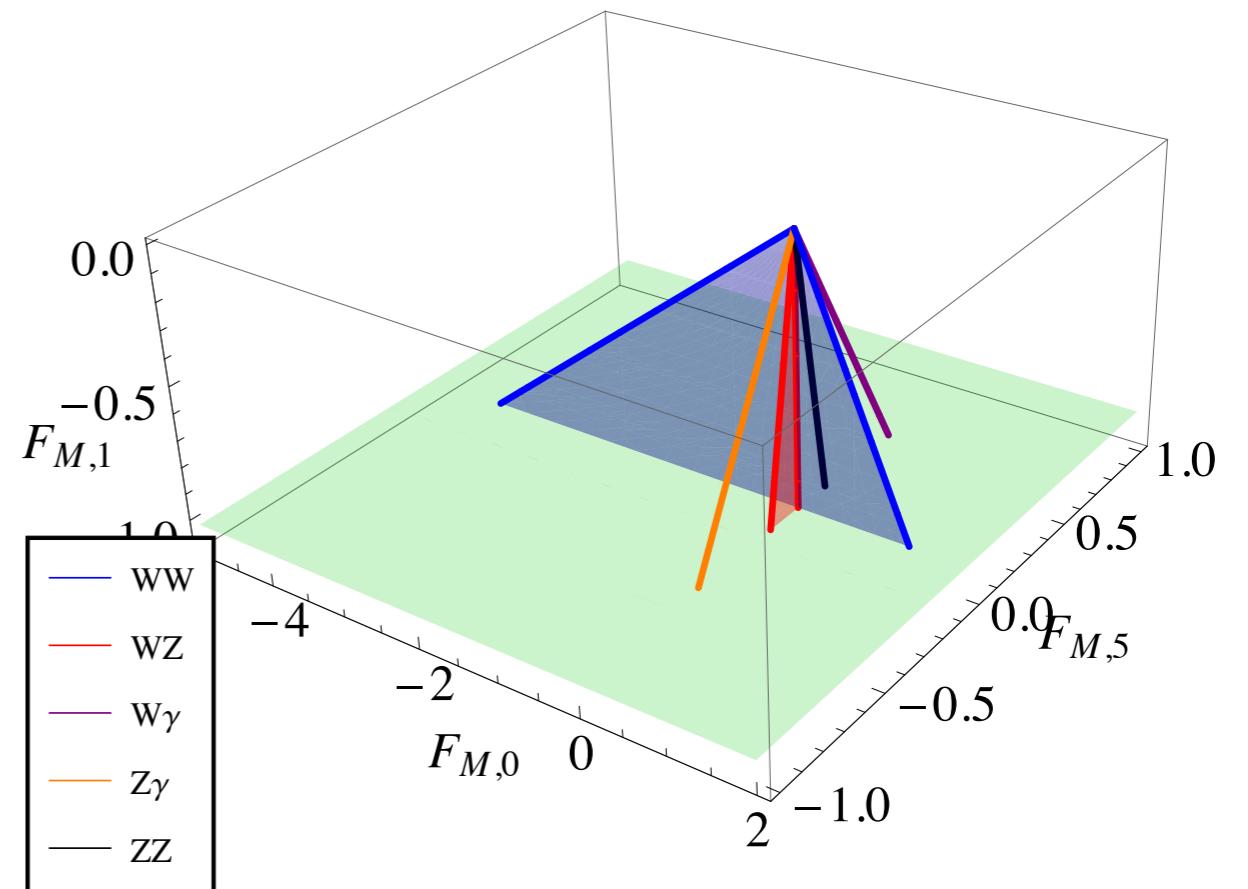
$$\vec{x}_{WZ} = (0, -2c_W^2 T_a^2 - 2T_b^2, -s_W^2 T_a^2)$$

$$\vec{x}_{W\gamma} = (0, -2, 1)$$

$$\vec{x}_{Z\gamma} = (0, -2, -1)$$

$$T_a = \frac{a_3}{\sqrt{a_1^2 + a_2^2}}, \quad T_b = \frac{b_3}{\sqrt{b_1^2 + b_2^2}}.$$

$$\cos \phi = \frac{a_1 b_1 + a_2 b_2}{\sqrt{a_1^2 + a_2^2} \sqrt{b_1^2 + b_2^2}}$$



3D cases: the pyramid case

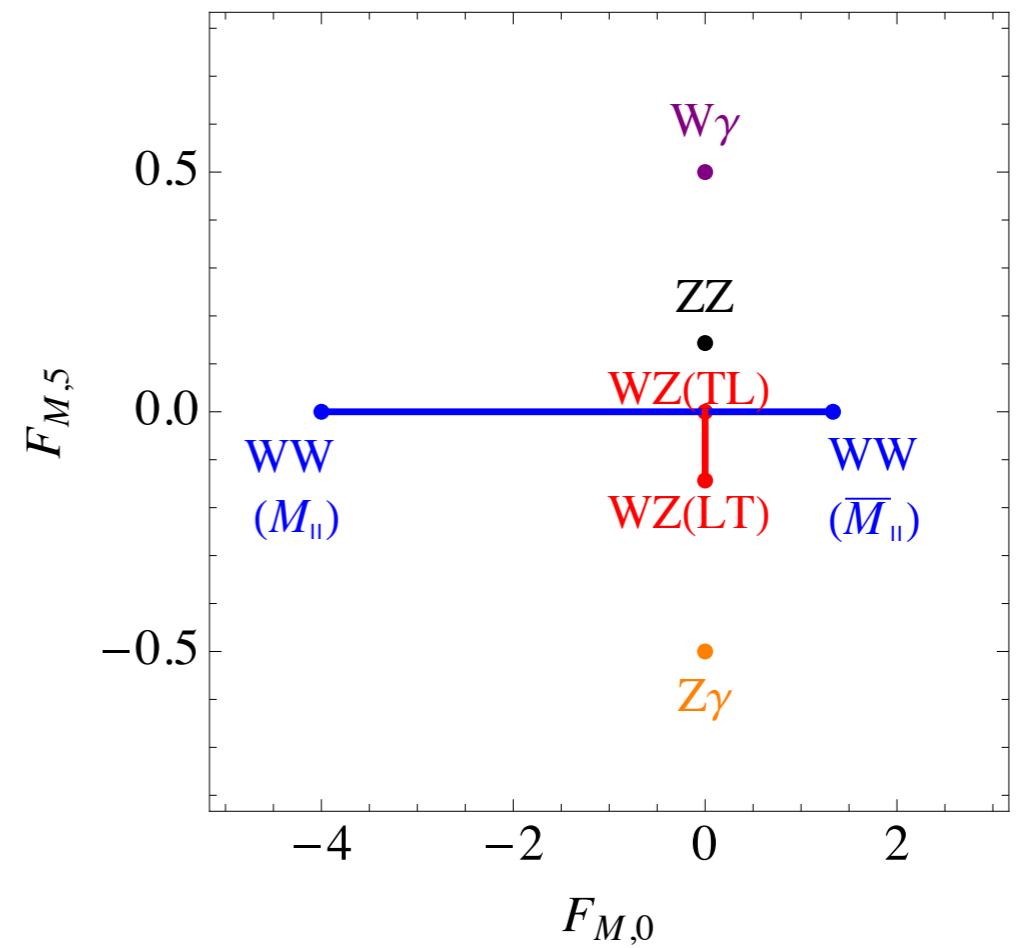
- Consider F_{M0}, F_{M1}, F_{M5} .

- Useful fact: if some x is a positive linear combination of some other x 's

$$\vec{x} = \sum_{i,j} c_{i,j} \vec{x}_i(\vec{a}_j, \vec{b}_j), \quad c_{i,j} \geq 0$$

Then it gives no new exclusion:

$$\vec{F} \cdot \vec{x} = \sum_{i,j} c_{i,j} \vec{F} \cdot \vec{x}_i(\vec{a}_j, \vec{b}_j)$$



3D cases: the pyramid case

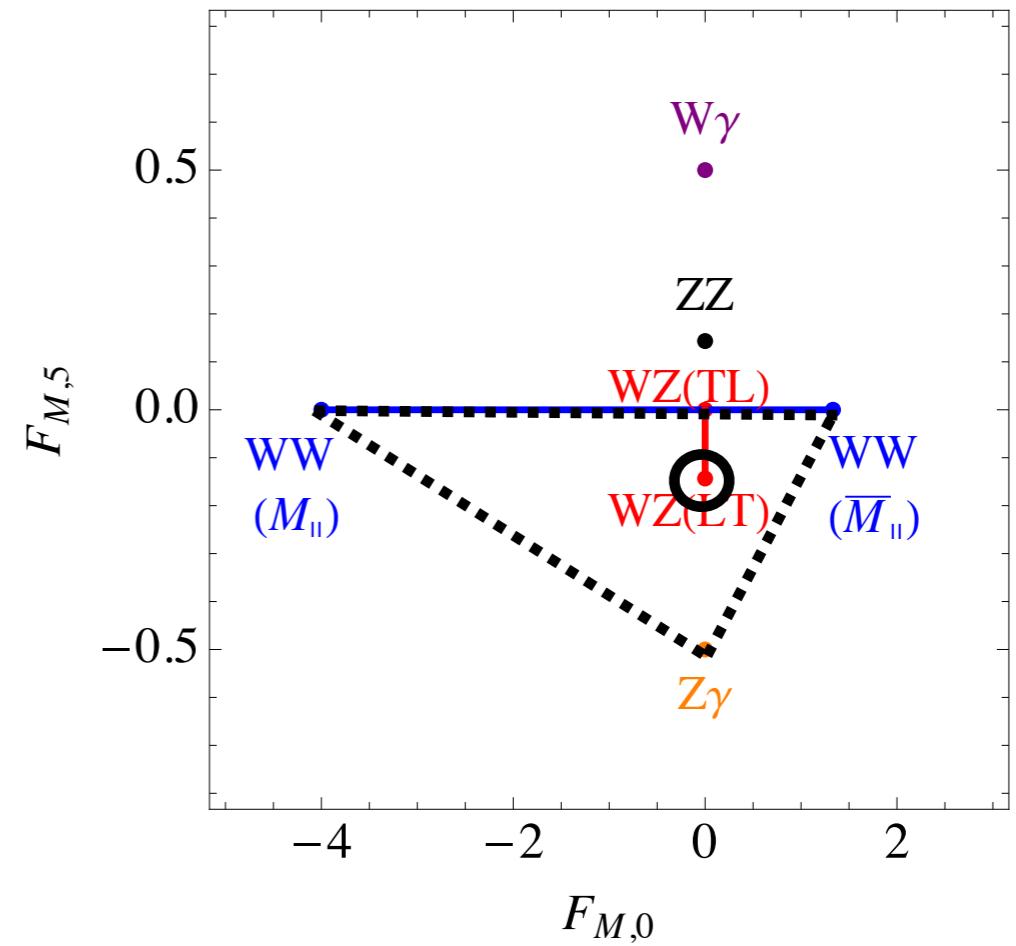
- Consider F_{M0}, F_{M1}, F_{M5} .

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$$\vec{x} = \sum_{i,j} c_{i,j} \vec{x}_i(\vec{a}_j, \vec{b}_j), \quad c_{i,j} \geq 0$$

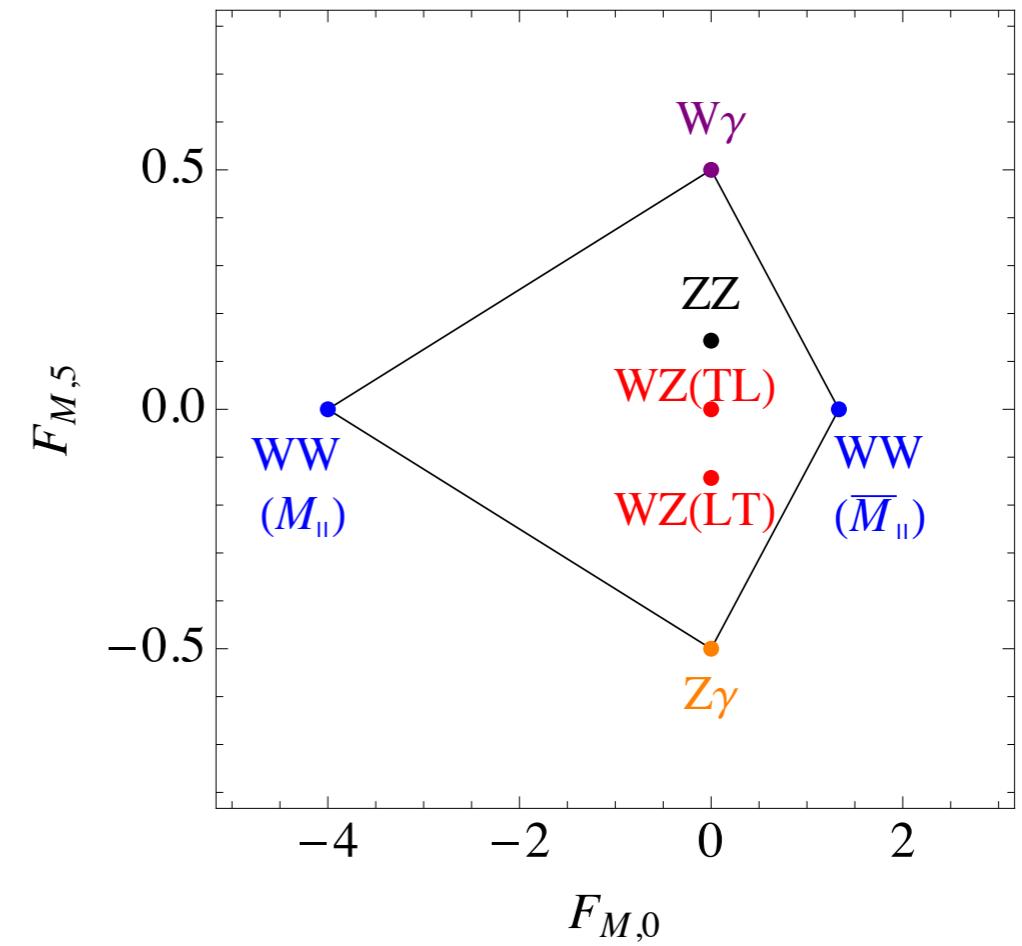
Then it gives no new exclusion:

$$\vec{F} \cdot \vec{x} = \sum_{i,j} c_{i,j} \vec{F} \cdot \vec{x}_i(\vec{a}_j, \vec{b}_j)$$



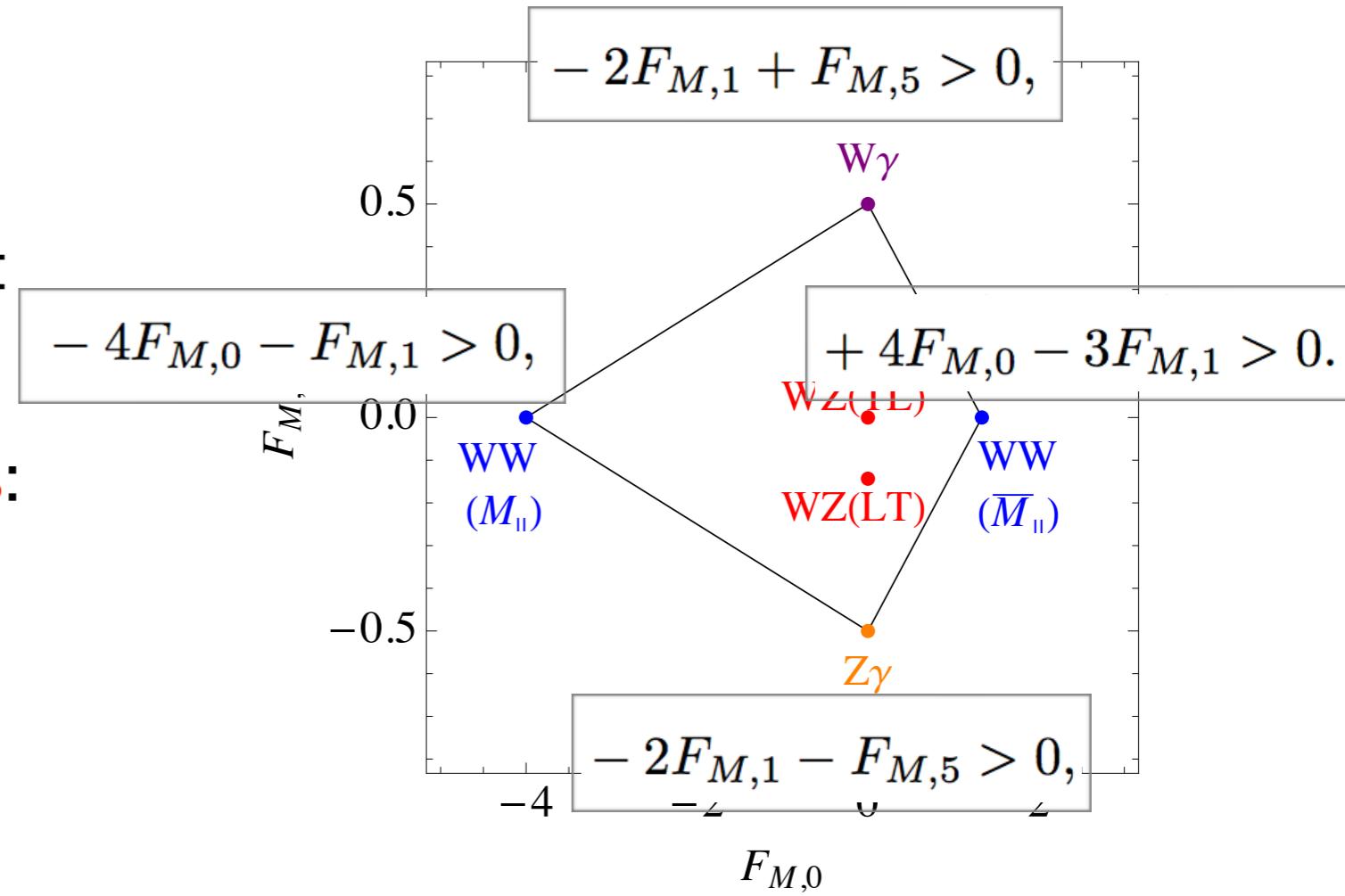
3D cases: the pyramid case

- Consider F_{M0} , F_{M1} , F_{M5} .
- Points inside polygons formed by other points can be removed: but the largest polygon is the **convex hull**
- Now we have 4 key vectors: $W\gamma$, $Z\gamma$, WW with parallel polarization, and WW with “anti-parallel” polarization. They are all we need to characterize the parameter space.



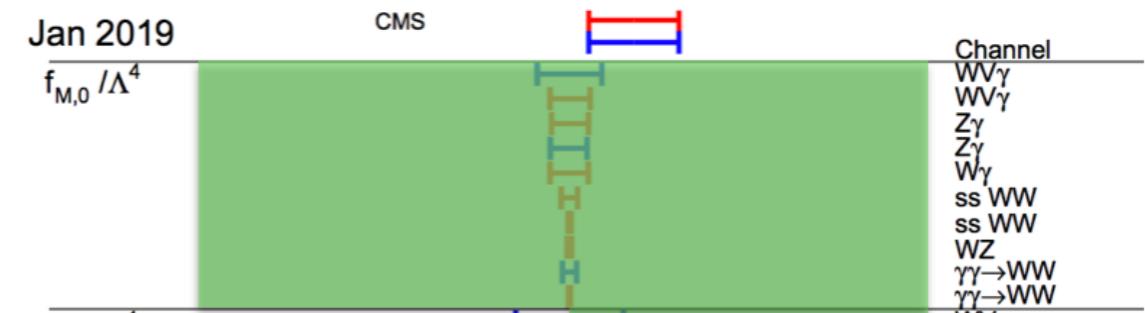
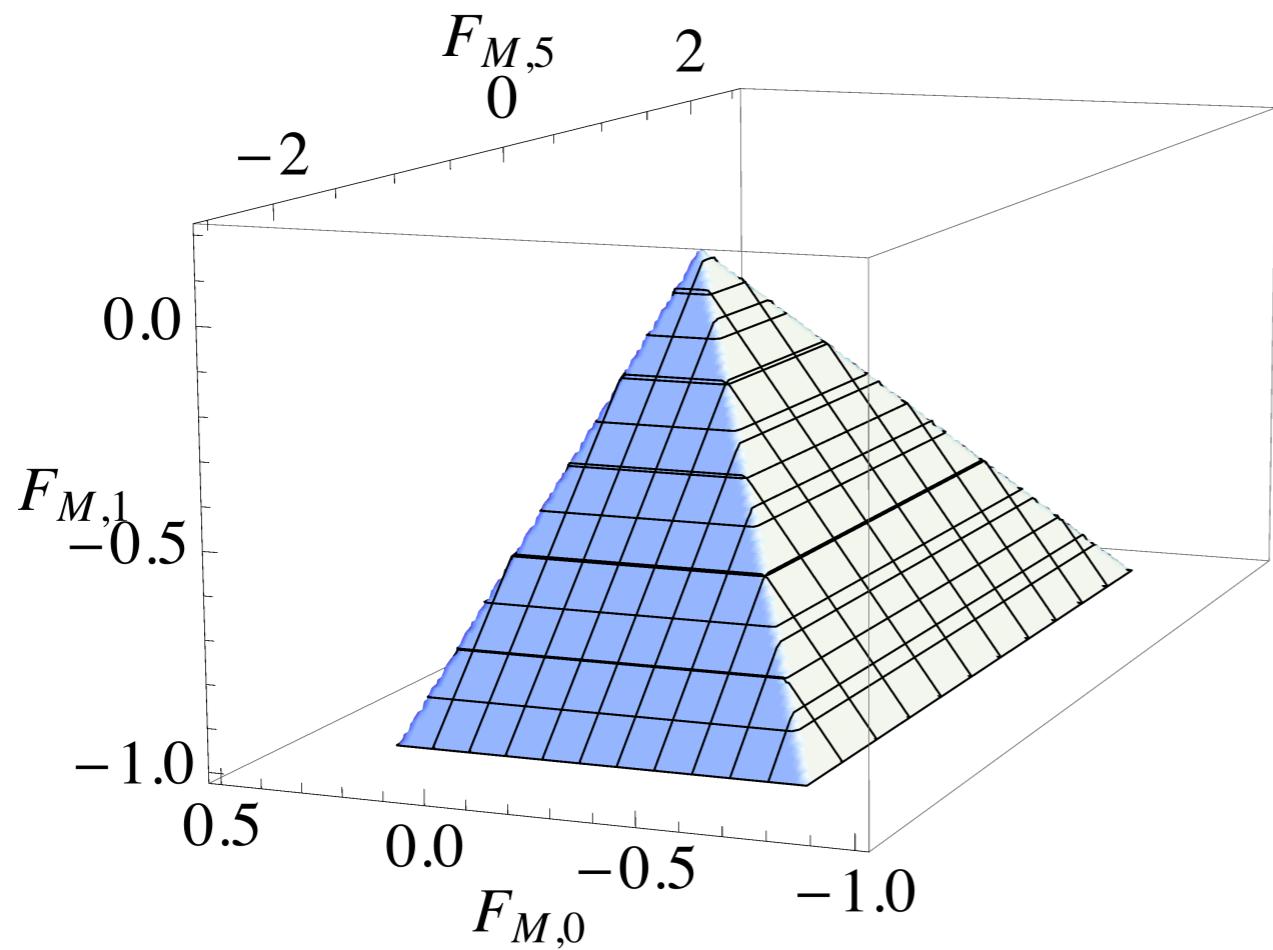
3D cases: the pyramid case

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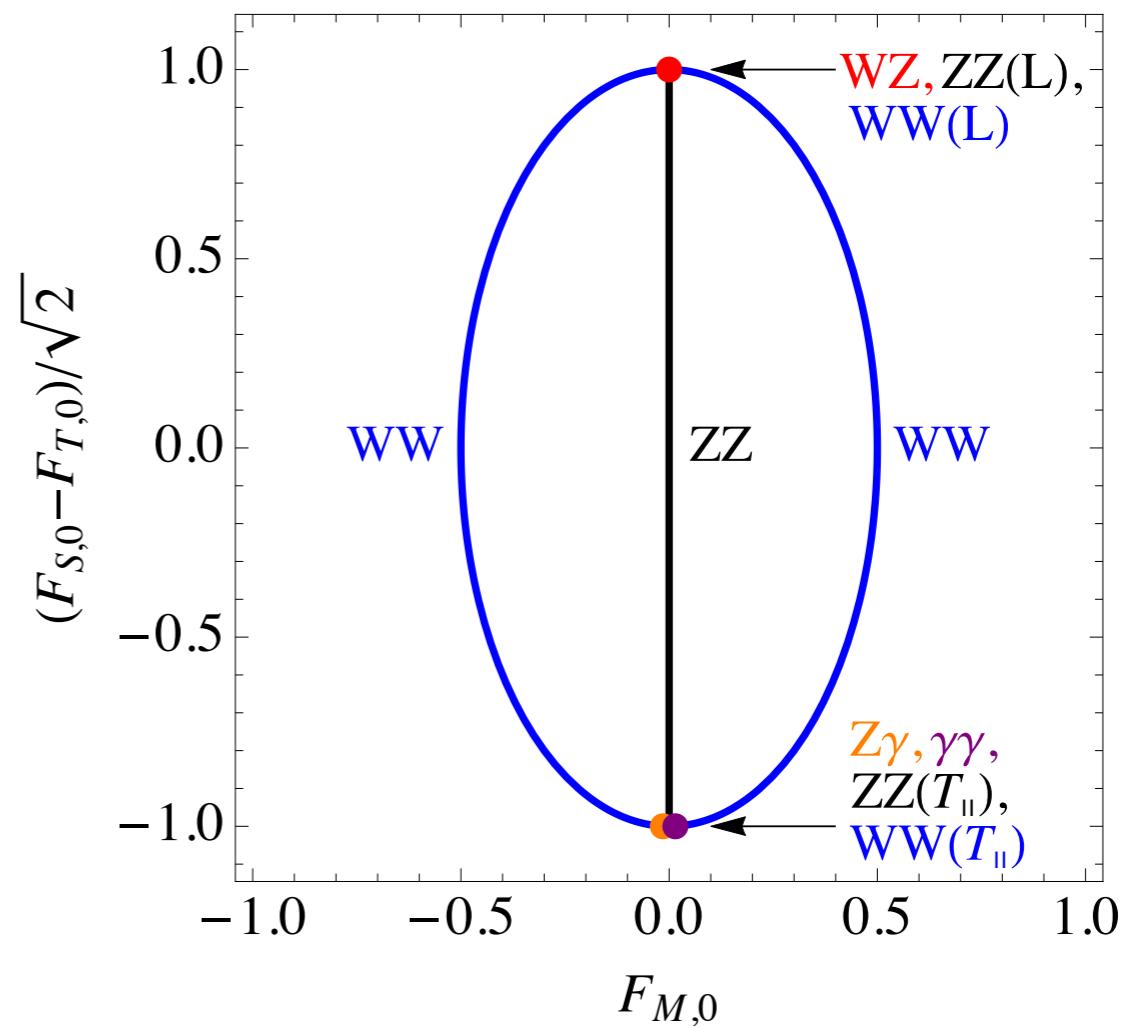
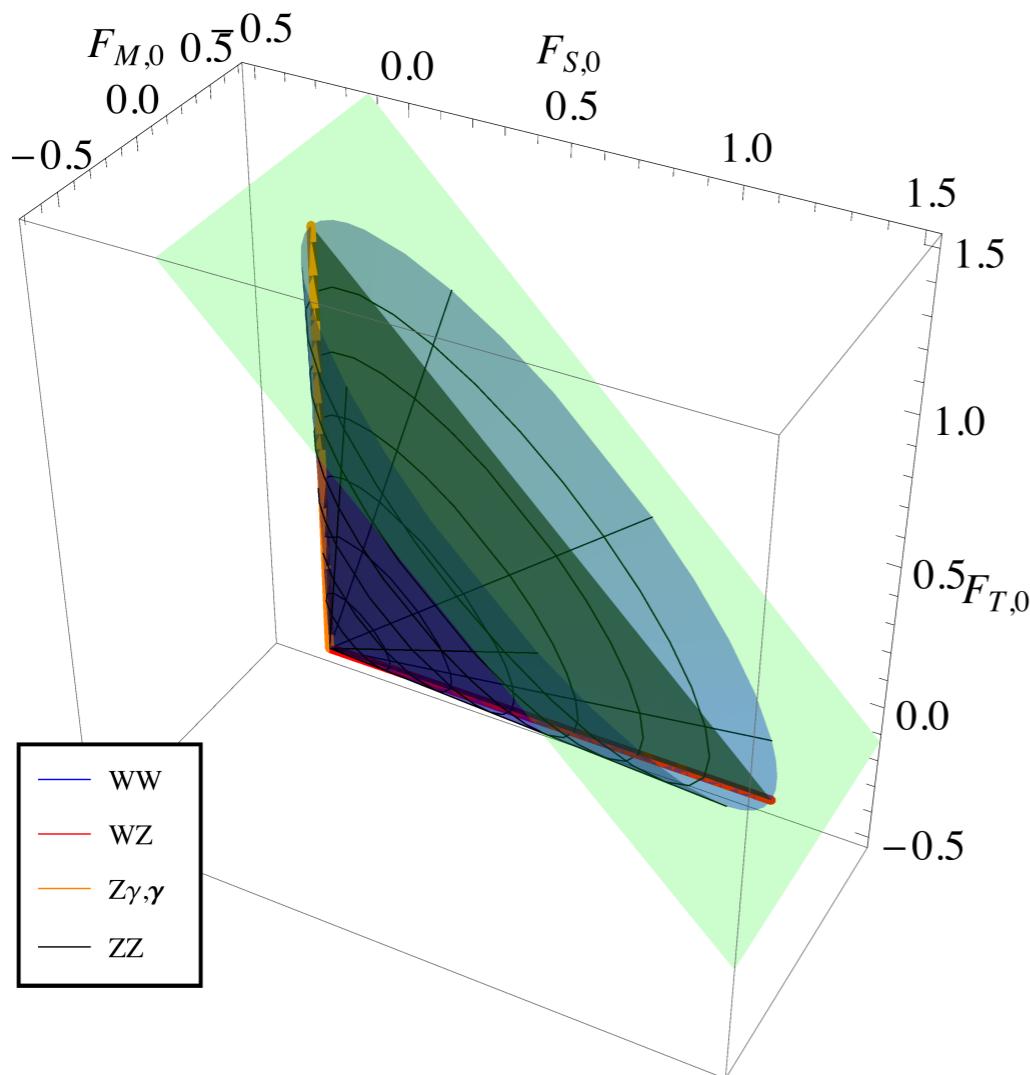
3D cases: the pyramid case

$$\begin{aligned} -2F_{M,1} + F_{M,5} &> 0, & -4F_{M,0} - F_{M,1} &> 0, \\ -2F_{M,1} - F_{M,5} &> 0, & +4F_{M,0} - 3F_{M,1} &> 0. \end{aligned}$$



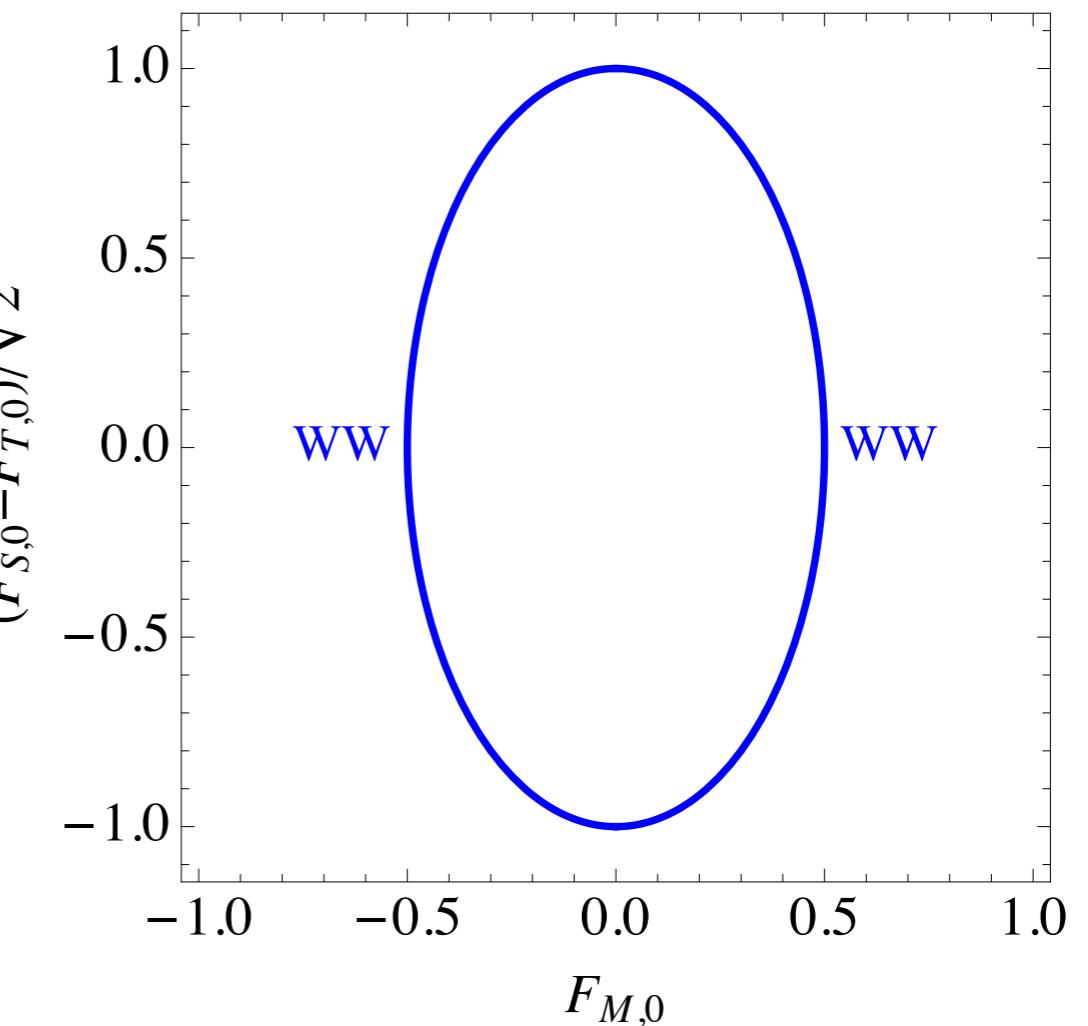
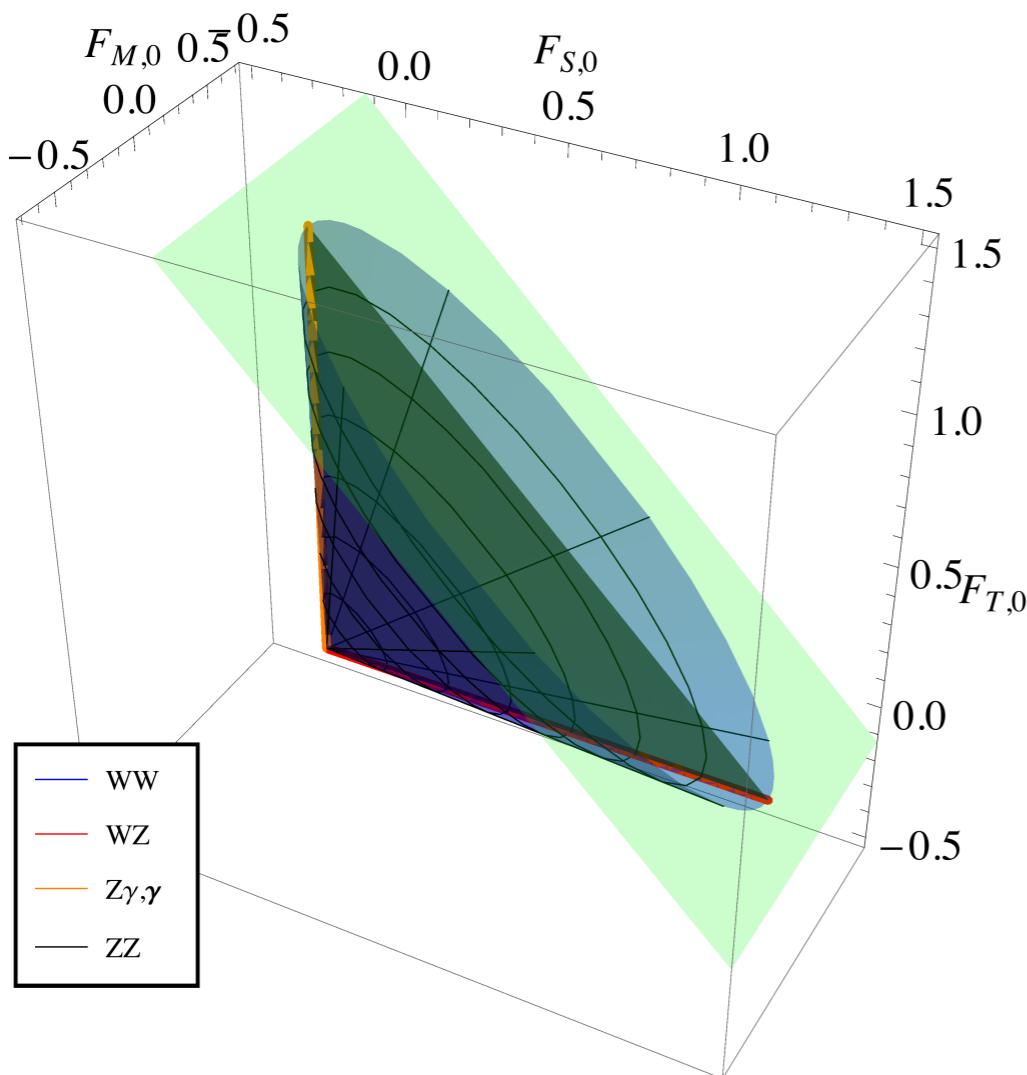
3D cases: the cone case

- Consider F_{S0} , F_{M0} , F_{T0} .



3D cases: the cone case

- Consider F_{S0} , F_{M0} , F_{T0} .

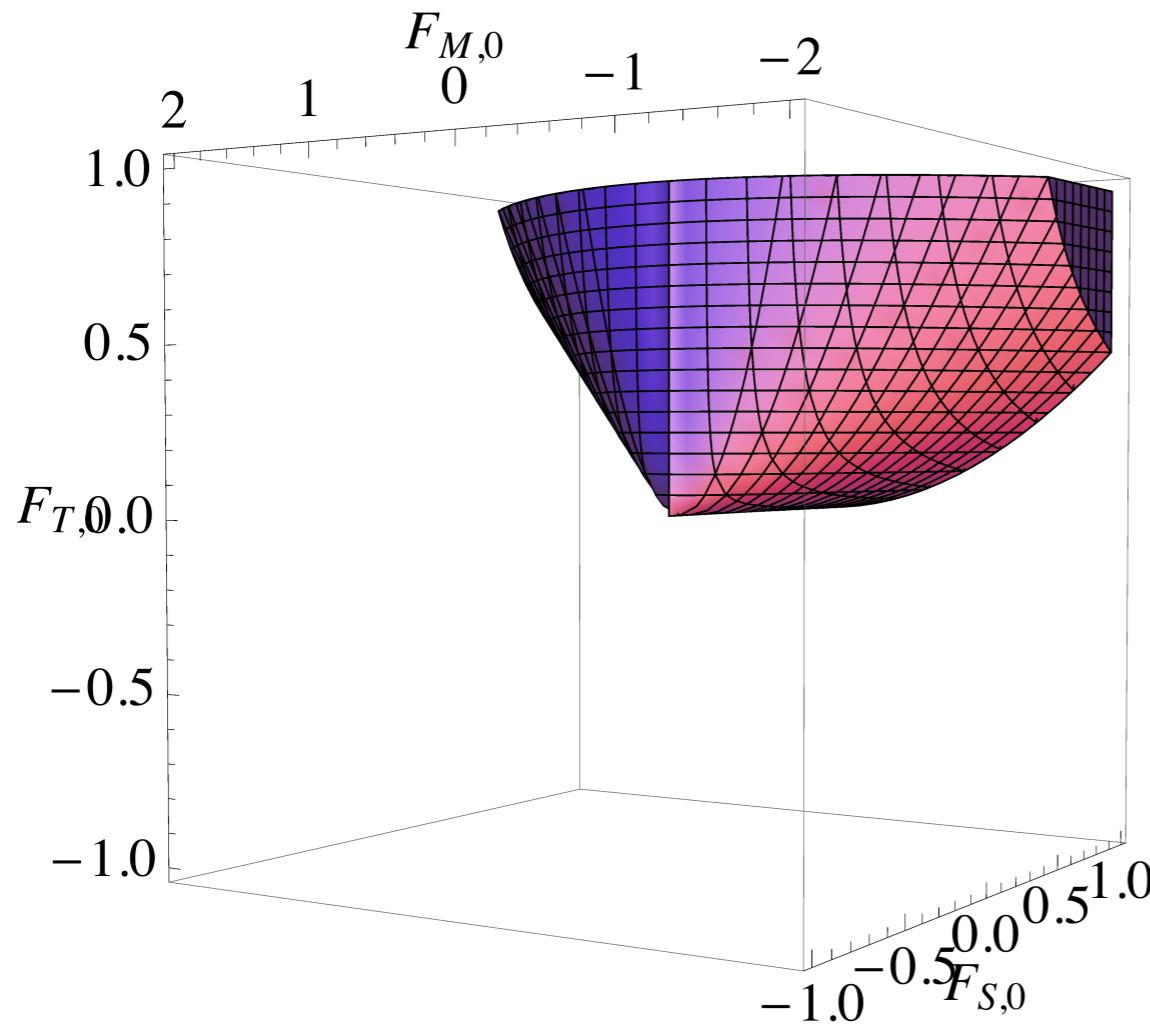


$$F_{S,0}(T_a T_b / \cos \phi)^2 - F_{M,0}(T_a T_b / \cos \phi) + 2F_{T,0}$$

$$F_{S,0} > 0 \quad \text{and} \quad 8F_{S,0}F_{T,0} - F_{M,0}^2 > 0$$

3D cases: the cone case

$$F_{S,0} > 0 \quad \text{and} \quad 8F_{S,0}F_{T,0} - F_{M,0}^2 > 0$$



- In this case the allowed area is a cone.
- The inequality becomes **quadratic**.
- This is because we “marginalize” over the external polarization directions of WW. This “upgrades” the degree of resulting inequality.

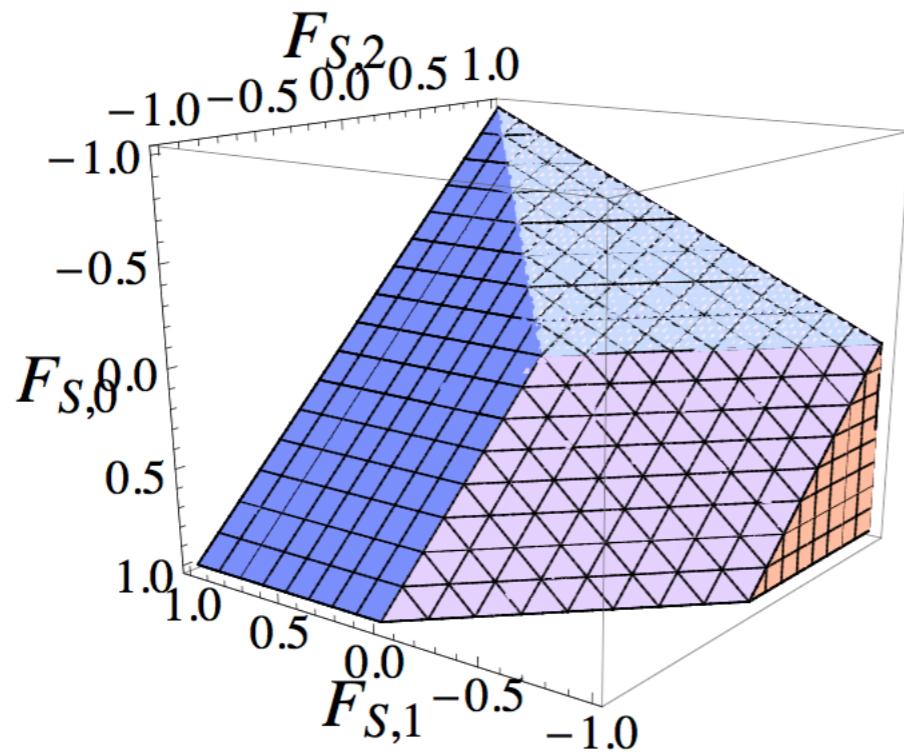
18D case: linear bounds

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{S,0} \\ F_{S,1} \\ F_{S,2} \end{pmatrix} > 0 \quad \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} F_{M,0} \\ F_{M,1} \\ F_{M,2} \\ F_{M,3} \\ F_{M,4} \\ F_{M,5} \\ F_{M,7} \end{pmatrix} > 0$$

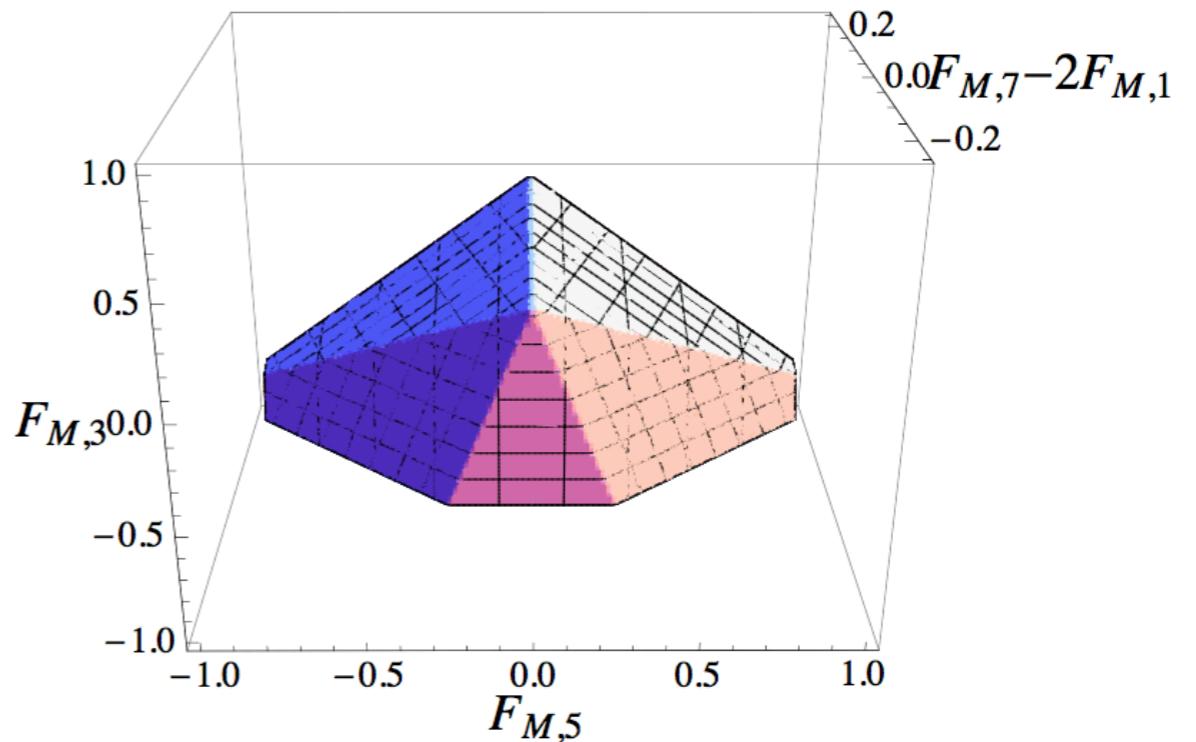
$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} F_{T,0} \\ F_{T,1} \\ F_{T,2} \\ F_{T,5} \\ F_{T,6} \\ F_{T,7} \\ F_{T,8} \\ F_{T,9} \end{pmatrix} > 0$$

18D case: S and M subspaces

- Bounds are convex multidimensional pyramids or prisms



3D triangular pyramid



3D pentagonal pyramid

- T subspace: 8D pyramid with 11 edges...

18D case: quadratic and quartic bounds

$WW, M_{+-} \& \bar{M}_{+-}$

$$32(2F_{S,0} + F_{S,1} + F_{S,2})(2F_{T,0} + F_{T,1} + F_{T,2}) \\ - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0 \quad (3.107)$$

$WW, M_{||} \& \bar{M}_{||}$

$$8(2F_{S,0} + F_{S,1} + F_{S,2})(8F_{T,0} + 12F_{T,1} + 5F_{T,2}) \\ - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0 \quad (3.108)$$

$ZZ, M_I \& \bar{M}_I$

$$8(F_{S,0} + F_{S,1} + F_{S,2}) [4c_W^8(2F_{T,0} + 2F_{T,1} + F_{T,2}) + 2c_W^4 s_W^4(2F_{T,5} + 2F_{T,6} + F_{T,7}) \\ + s_W^8(2F_{T,8} + F_{T,9})] - \max [0, 2(2c_W^4 F_{M,0} + F_{M,2}s_W^4 - F_{M,4}s_W^4 + F_{M,4}s_W^2), \\ -c_W^4(4F_{M,0} - 2F_{M,1} + F_{M,7}) - 2c_W^2 F_{M,4}s_W^2 - s_W^4(2F_{M,2} - F_{M,3}) - F_{M,5}(s_W^2 - s_W^4)]^2 > 0 \quad (3.109)$$

$WZ, M'_I \& \bar{M}'_I$

$$16(F_{S,0} + F_{S,2}) [4c_W^4(4F_{T,1} + F_{T,2}) + s_W^4(4F_{T,6} + F_{T,7})] - \max [0, +2c_W^2 F_{M,7} \\ - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5}s_W^2 + F_{M,3}s_W^4)} + 4F_{M,4}s_W^2 + F_{M,5}s_W^2, \\ - 2c_W^2 F_{M,7} - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5}s_W^2 + F_{M,3}s_W^4)} \\ - 4F_{M,4}s_W^2 - F_{M,5}s_W^2]^2 > 0 \quad (3.110)$$

18D case: quadratic and quartic bounds

$WW, M_{+-} \& \bar{M}_{+-}$

$$32(2F_{S,0} + F_{S,1} + F_{S,2})(2F_{T,0} + F_{T,1} + F_{T,2}) - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0 \quad \xrightarrow{\text{red arrow}} \quad \left(\sum_i a_i F_{S,i} \right) \left(\sum_i b_i F_{T,i} \right) > \max \left(0, \sum_i c_i F_{M,i}, \sum_i d_i F_{M,i} \right)^2$$

$WW, M_{\parallel} \& \bar{M}_{\parallel}$

$$8(2F_{S,0} + F_{S,1} + F_{S,2})(8F_{T,0} + 12F_{T,1} + 5F_{T,2}) - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0 \quad (3.108)$$

$ZZ, M_I \& \bar{M}_I$

$$8(F_{S,0} + F_{S,1} + F_{S,2}) [4c_W^8(2F_{T,0} + 2F_{T,1} + F_{T,2}) + 2c_W^4 s_W^4(2F_{T,5} + 2F_{T,6} + F_{T,7}) + s_W^8(2F_{T,8} + F_{T,9})] - \max [0, 2(2c_W^4 F_{M,0} + F_{M,2}s_W^4 - F_{M,4}s_W^4 + F_{M,4}s_W^2), -c_W^4(4F_{M,0} - 2F_{M,1} + F_{M,7}) - 2c_W^2 F_{M,4}s_W^2 - s_W^4(2F_{M,2} - F_{M,3}) - F_{M,5}(s_W^2 - s_W^4)]^2 > 0 \quad (3.109)$$

$WZ, M'_I \& \bar{M}'_I$

$$16(F_{S,0} + F_{S,2}) [4c_W^4(4F_{T,1} + F_{T,2}) + s_W^4(4F_{T,6} + F_{T,7})] - \max [0, +2c_W^2 F_{M,7} - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5}s_W^2 + F_{M,3}s_W^4)} + 4F_{M,4}s_W^2 + F_{M,5}s_W^2, -2c_W^2 F_{M,7} - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5}s_W^2 + F_{M,3}s_W^4)} - 4F_{M,4}s_W^2 - F_{M,5}s_W^2]^2 > 0 \quad (3.110)$$

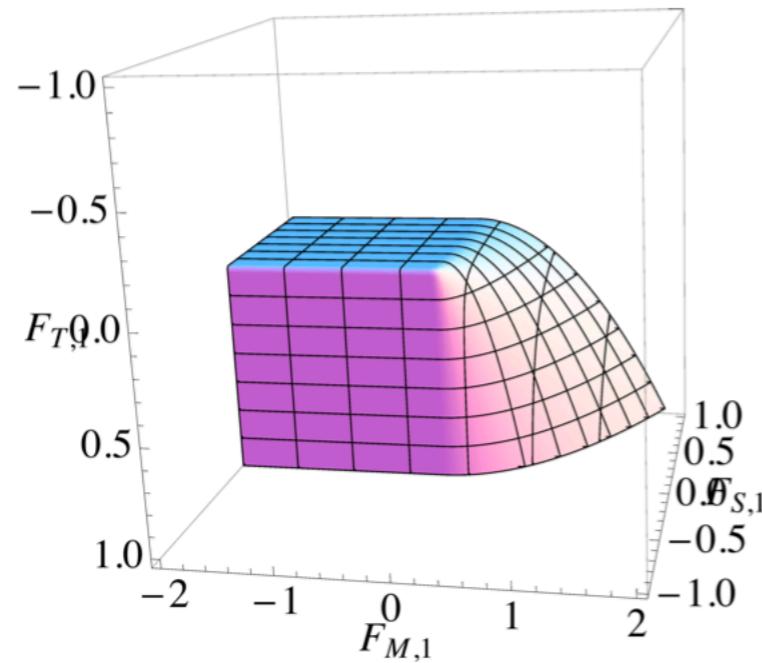


Figure 14. Positivity bound from the quadratic inequality in the WW channel, in the subspace spanned by $F_{S,1}$, $F_{M,1}$ and $F_{T,1}$.

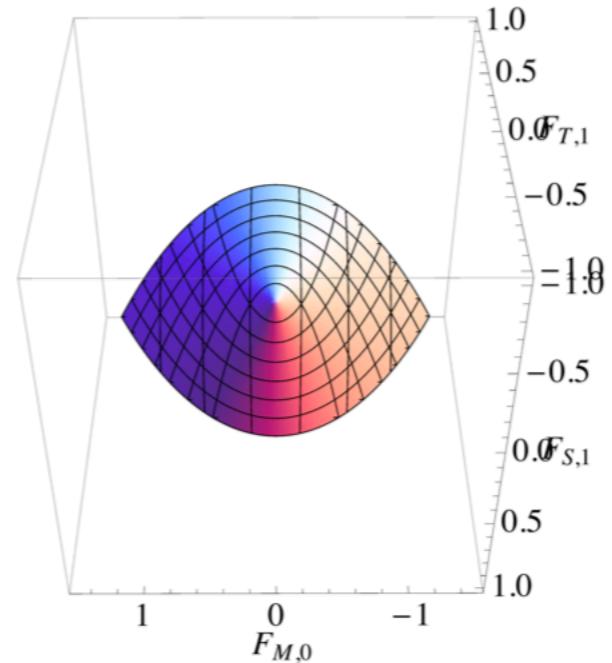


Figure 15. Same as Figure 14, but in the subspace spanned by $F_{S,1}$, $F_{M,0}$ and $F_{T,1}$.

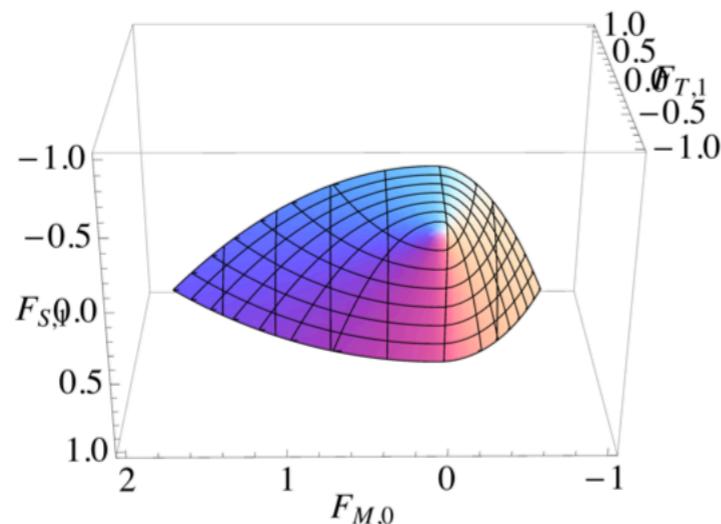


Figure 16. Similar to Figure 15, but imposing a restriction $F_{M,0} + F_{M,1} = 0$.

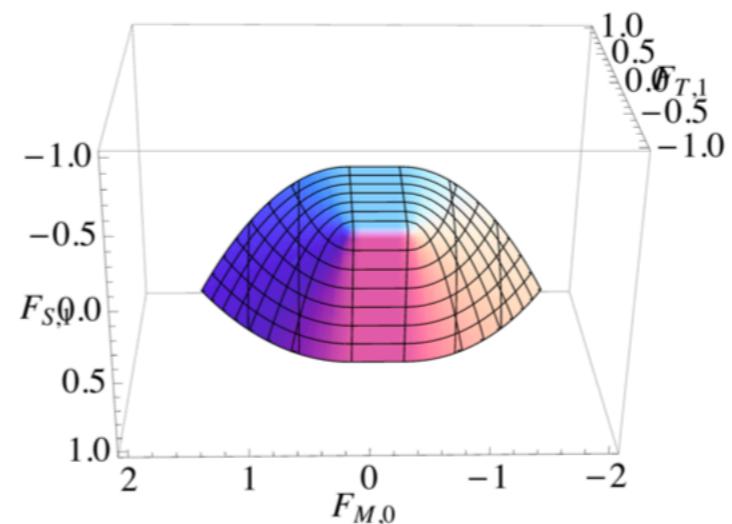
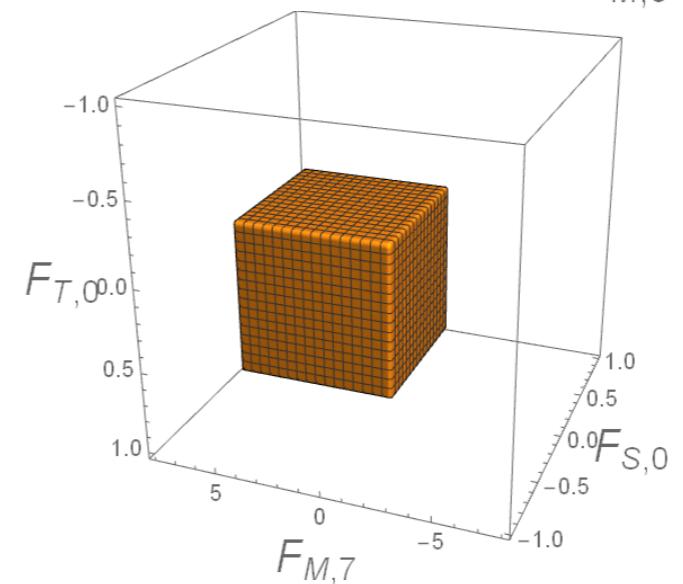
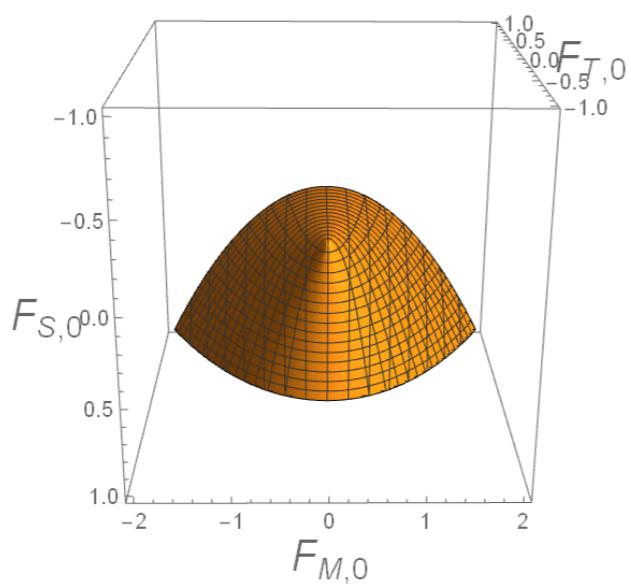
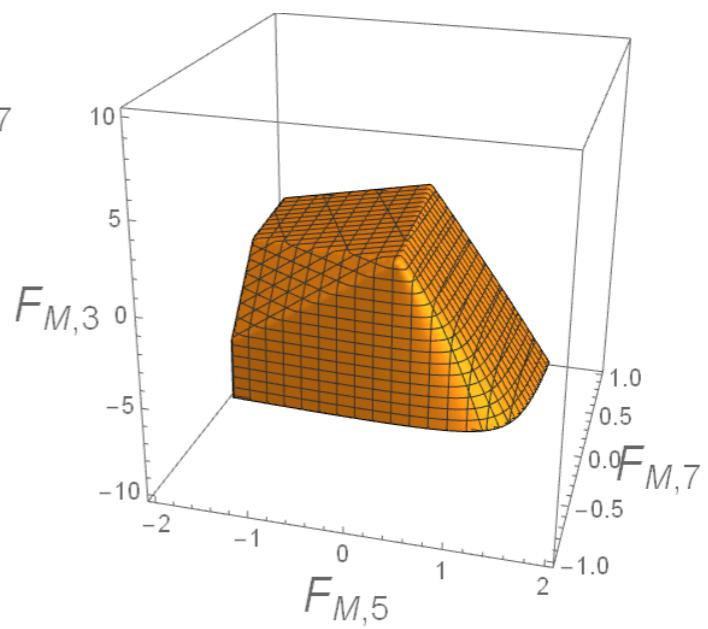
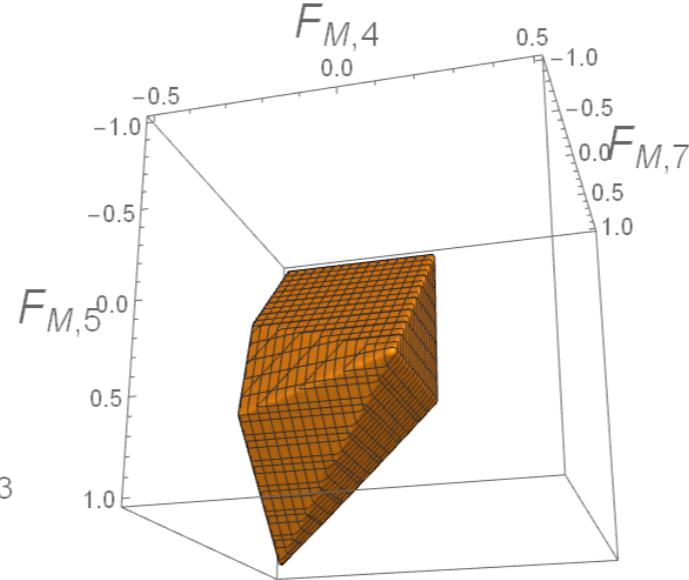
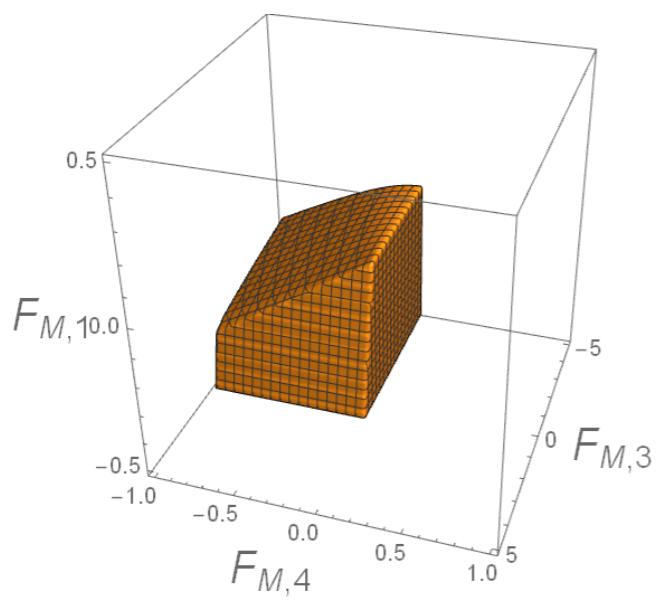
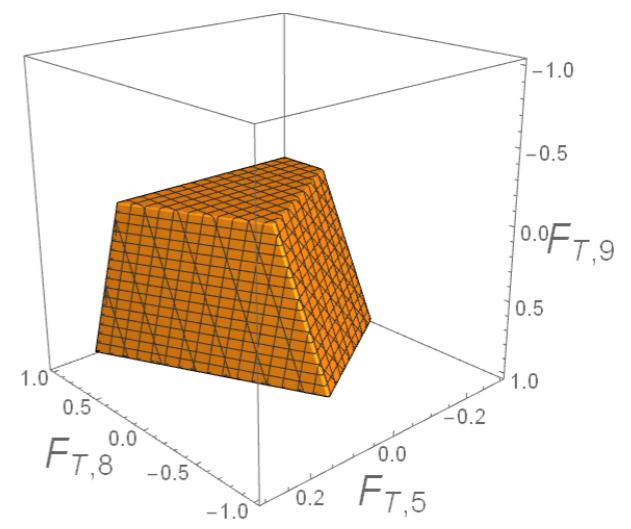
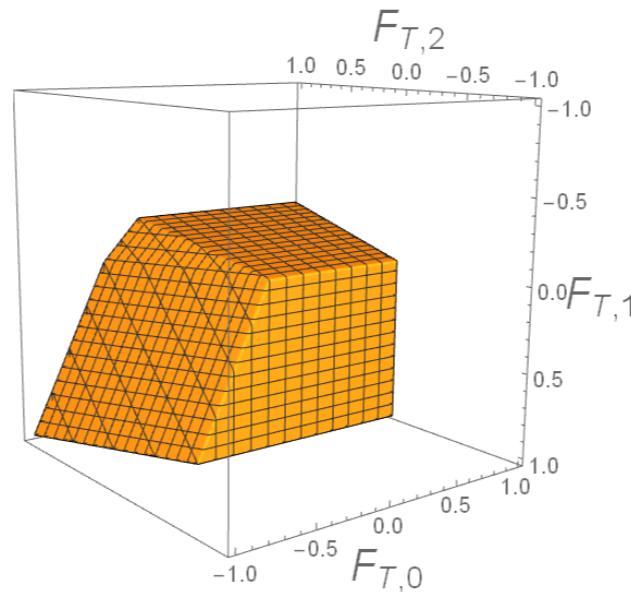
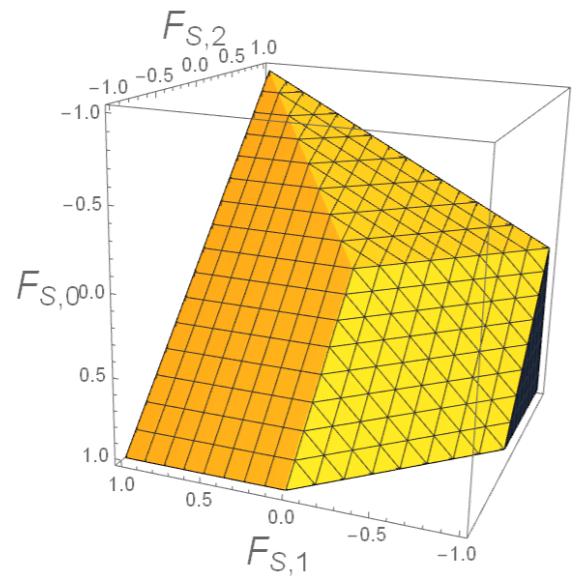


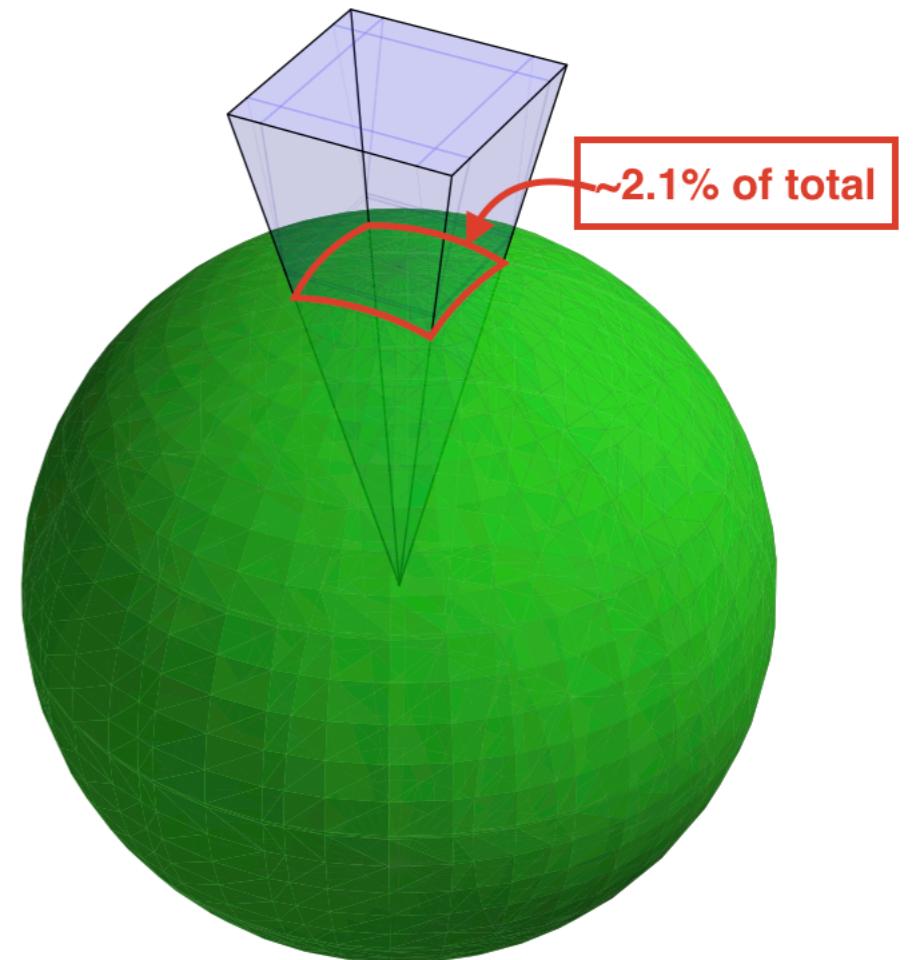
Figure 17. Similar to Figure 15, but imposing a restriction $F_{M,1} = -1/2$.



18D case: volume

- Volume measures “constraining power”. (like the “GDP”)
[Durieux, Grojean, Gu, Wang JHEP 17]
- However positivity does not constrain the magnitudes of deviation.
It only constraints possible directions. So the regular volume is ∞ .
- But we can measure the “solid angle”
 - Randomly throw points on 18D sphere.
 - Count how many of them satisfy a given positivity condition.

98% of the parameter space is redundant: no UV completion exists.



Simplified model

[J. de Blas et al. JHEP 18]

Names	scalar \mathcal{S}	scalar Ξ	scalar Ξ_1	vector \mathcal{L}_1
Irrep	$(1, 1)_0$	$(1, 3)_0$	$(1, 3)_1$	$(1, 2)_{\frac{1}{2}}$

$$\mathcal{L}_{int} = \mathcal{S}\mathcal{J}_{\mathcal{S}} + \Xi^i \mathcal{J}_{\Xi}^i + \left(\Xi_1{}^{i\dagger} \mathcal{J}_{\Xi_1}^i + h.c. \right) + \left(\mathcal{L}_1{}^{\mu\dagger} \mathcal{J}_{\mathcal{L}_1\mu} + h.c. \right)$$

$$\begin{aligned} \mathcal{J}_{\mathcal{S}} &= a_H (D_\mu \phi)^\dagger D^\mu \phi + a_W W_{\mu\nu}^i W^{i\mu\nu} + a_B B_{\mu\nu} B^{\mu\nu} \\ \mathcal{J}_{\Xi}^i &= b_H (D_\mu \phi)^\dagger \sigma^i D^\mu \phi + b_{WB} W_{\mu\nu}^i B^{\mu\nu} \\ \mathcal{J}_{\Xi_1}^i &= c_H \left(D_\mu \tilde{\phi} \right)^\dagger \sigma^i D_\mu \phi \\ \mathcal{J}_{\mathcal{L}_1}^\mu &= d_{HB} D_\nu \phi B^{\nu\mu} + d_{HW} \sigma^i D_\nu \phi W^{i\nu\mu} \end{aligned}$$

$$F_{S,0} = 2 \frac{c_H^2}{M_{\Xi_1}^2}$$

$$F_{S,1} = \frac{a_H^2}{2M_{\mathcal{S}}^2} - \frac{b_H^2}{2M_{\Xi}^2}$$

$$F_{S,2} = \frac{b_H^2}{M_{\Xi}^2}$$

$$F_{T,0} = \frac{\bar{a}_W^2}{2M_{\mathcal{S}}^2}$$

$$F_{M,0} = \frac{a_H \bar{a}_W}{M_{\mathcal{S}}^2}$$

$$F_{M,1} = -\frac{\bar{d}_{HW}^2}{M_{\mathcal{L}_1}^2}$$

$$F_{M,2} = \frac{a_H \bar{a}_B}{M_{\mathcal{S}}^2}$$

$$F_{M,3} = -\frac{\bar{d}_{HB}^2}{M_{\mathcal{L}_1}^2}$$

$$F_{T,5} = \frac{\bar{a}_W \bar{a}_B}{M_{\mathcal{S}}^2}$$

$$F_{T,6} = \frac{\bar{b}_{WB}^2}{M_{\Xi}^2}$$

$$F_{T,8} = \frac{\bar{a}_B^2}{2M_{\mathcal{S}}^2}$$

$$F_{M,4} = 2 \frac{b_H b_{WB}}{M_{\Xi}^2}$$

$$F_{M,5} = 2 \frac{\bar{d}_{HW} \bar{d}_{HB}}{M_{\mathcal{L}_1}^2}$$

$$F_{M,7} = -\frac{\bar{d}_{HW}^2}{M_{\mathcal{L}_1}^2}$$

Simplified model

$$M_S \cdot F_S = \begin{pmatrix} \frac{1}{2} \left(\frac{a_H^2}{M_S^2} + \frac{b_H^2}{M_{\Xi}^2} + \frac{8c_H^2}{M_{\Xi_1}^2} \right) \\ \frac{1}{2} \left(\frac{a_H^2}{M_S^2} + \frac{b_H^2}{M_{\Xi}^2} + \frac{4c_H^2}{M_{\Xi_1}^2} \right) \\ \frac{b_H^2}{M_{\Xi}^2} + \frac{2c_H^2}{M_{\Xi_1}^2} \end{pmatrix}$$

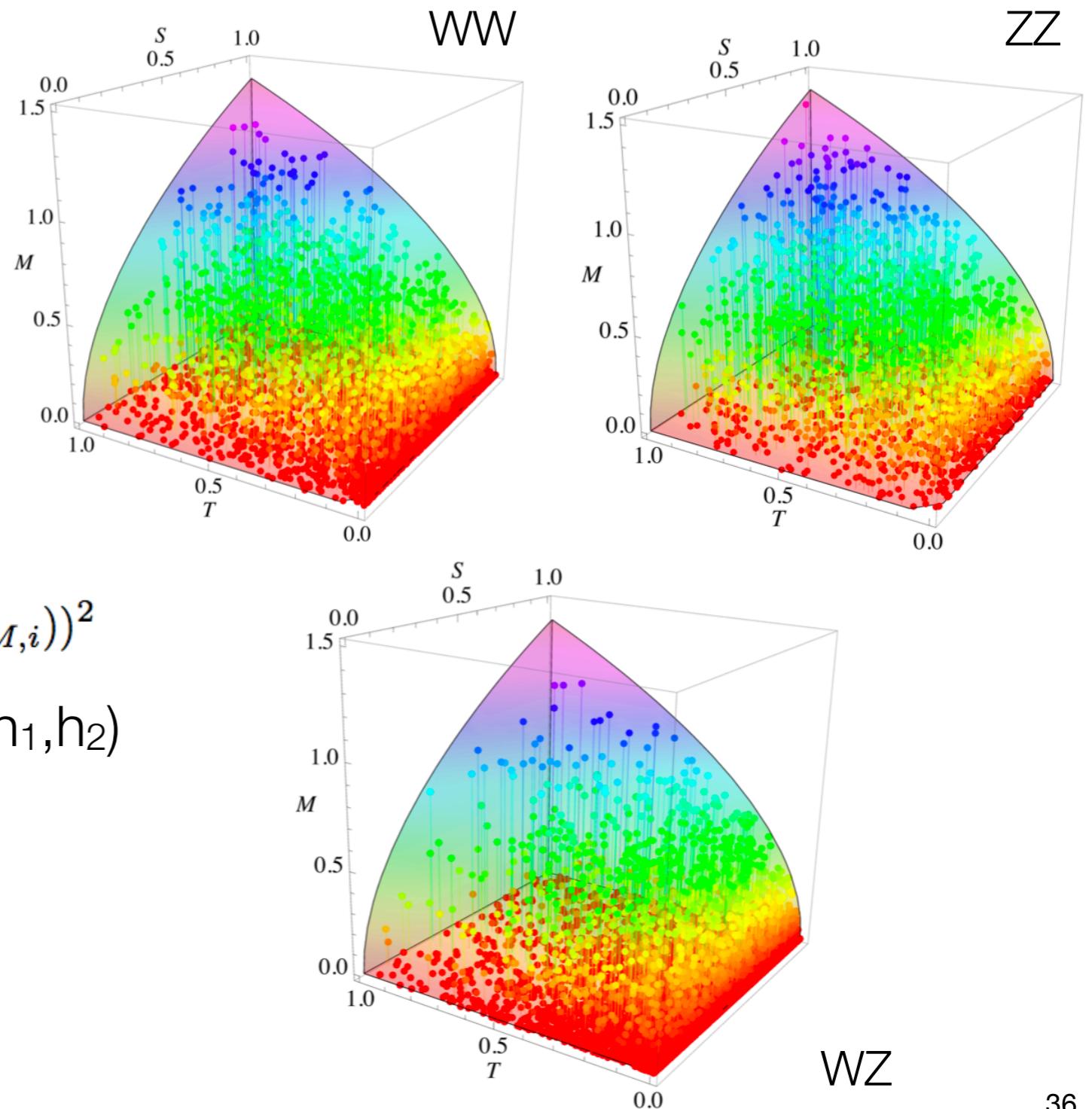
$$M_M \cdot F_M = \begin{pmatrix} \frac{2d_{HW}^2}{M_{\mathcal{L}_1}^2} \\ \frac{d_{HW}^2 c_W^4 + (d_{HW} c_W^2 + d_{HB} s_W^2)^2}{M_{\mathcal{L}_1}^2} \\ \frac{d_{HW}^2 c_W^4 + (d_{HW} c_W^2 - d_{HB} s_W^2)^2}{M_{\mathcal{L}_1}^2} \\ \frac{2d_{HW}^2}{M_{\mathcal{L}_1}^2} \\ \frac{(d_{HB} + d_{HW})^2 + d_{HW}^2}{M_{\mathcal{L}_1}^2} \\ \frac{(d_{HB} - d_{HW})^2 + d_{HW}^2}{M_{\mathcal{L}_1}^2} \end{pmatrix}$$

$$M_T \cdot F_T = \begin{pmatrix} 0 \\ 0 \\ \frac{a_W^2}{M_S^2} \\ \frac{4a_W^2}{M_S^2} \\ \frac{(a_B s_W^4 + 2a_W c_W^4)^2}{M_S^2} + \frac{4b_{WB}^2 s_W^4 c_W^4}{M_{\Xi}^2} \\ 0 \\ 0 \\ \frac{4b_{WB}^2 s_W^4}{M_{\Xi}^2} \\ 0 \\ \frac{4b_{WB}^2}{M_{\Xi}^2} \\ 0 \\ \frac{4b_{WB}^2 (c_W^2 - s_W^2)^2}{M_{\Xi}^2} + \frac{4(a_B s_W^2 - 2a_W c_W^2)^2}{M_S^2} \\ 0 \\ \frac{(a_B + 2a_W)^2}{M_S^2} + \frac{4b_{WB}^2}{M_{\Xi}^2} \end{pmatrix}$$

Simplified model

$$f(F_{S,i})g(F_{T,i}) > \max(0, h_1(F_{M,i}), h_2(F_{M,i}))^2$$

Parameter space in f , g , $\max(0, h_1, h_2)$

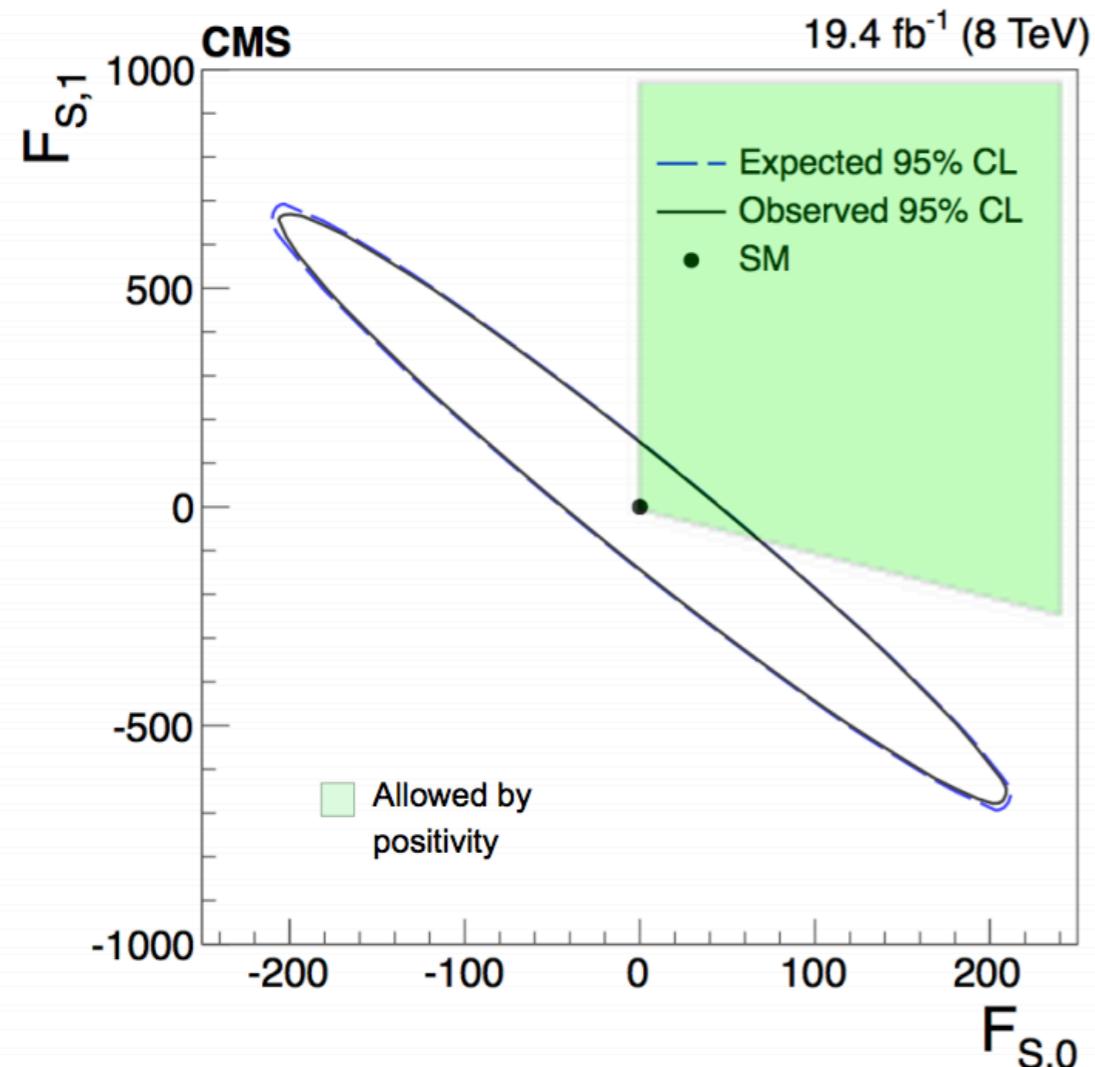


How does this affect the real measurements?

If you believe it

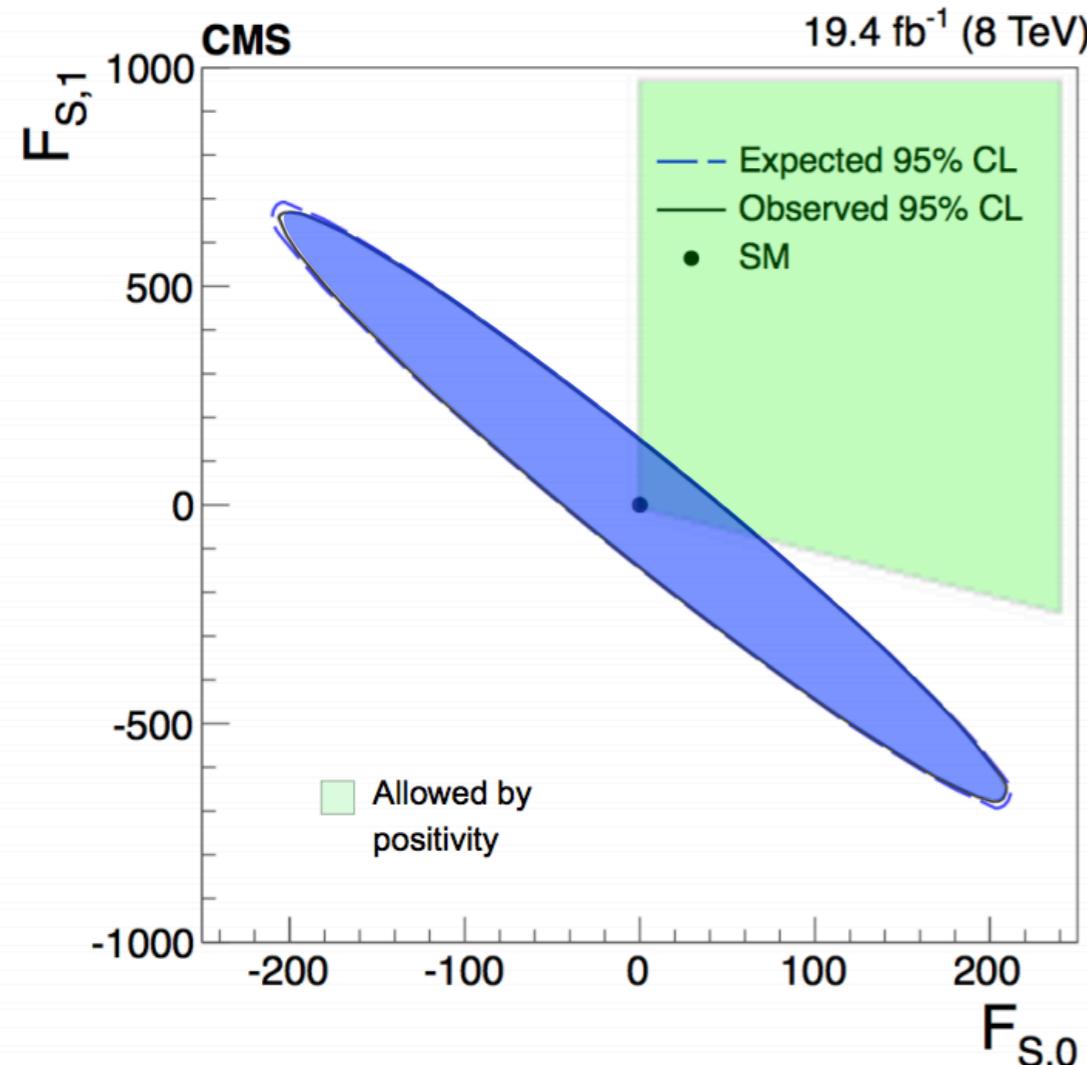
- If we believe that the BSM satisfies unitarity, dispersion relation (causality & locality), and Lorentz invariance, i.e. **positivity is true**:
 - It may enter the measurement as a prior, possibly changing the resulting limits on couplings.

Modifying the prior



[CMS, PRL 15]

Modifying the prior

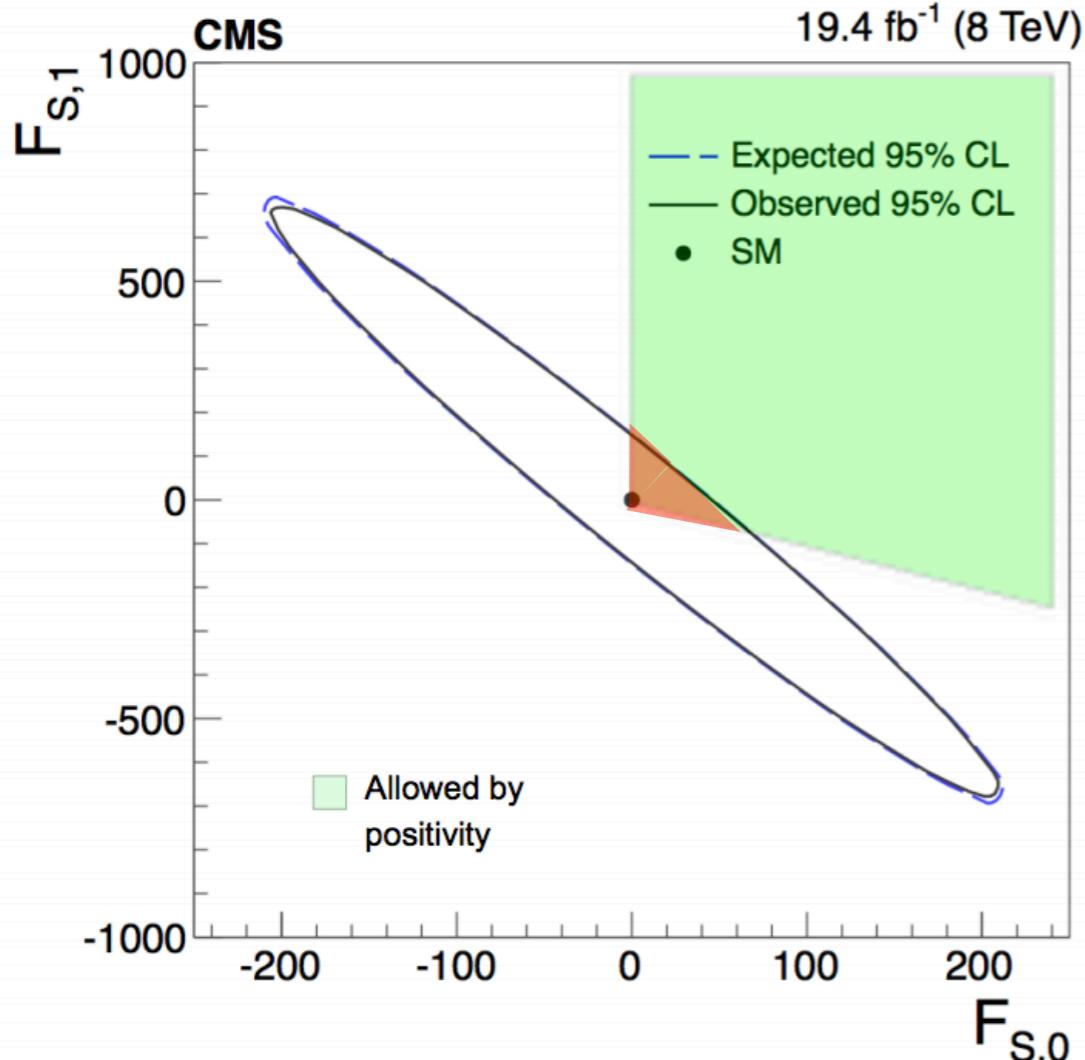


[CMS, PRL 15]

Without positivity

$$\int dF_{S,1} dF_{S,2} L = 95\%$$

Modifying the prior



[CMS, PRL 15]

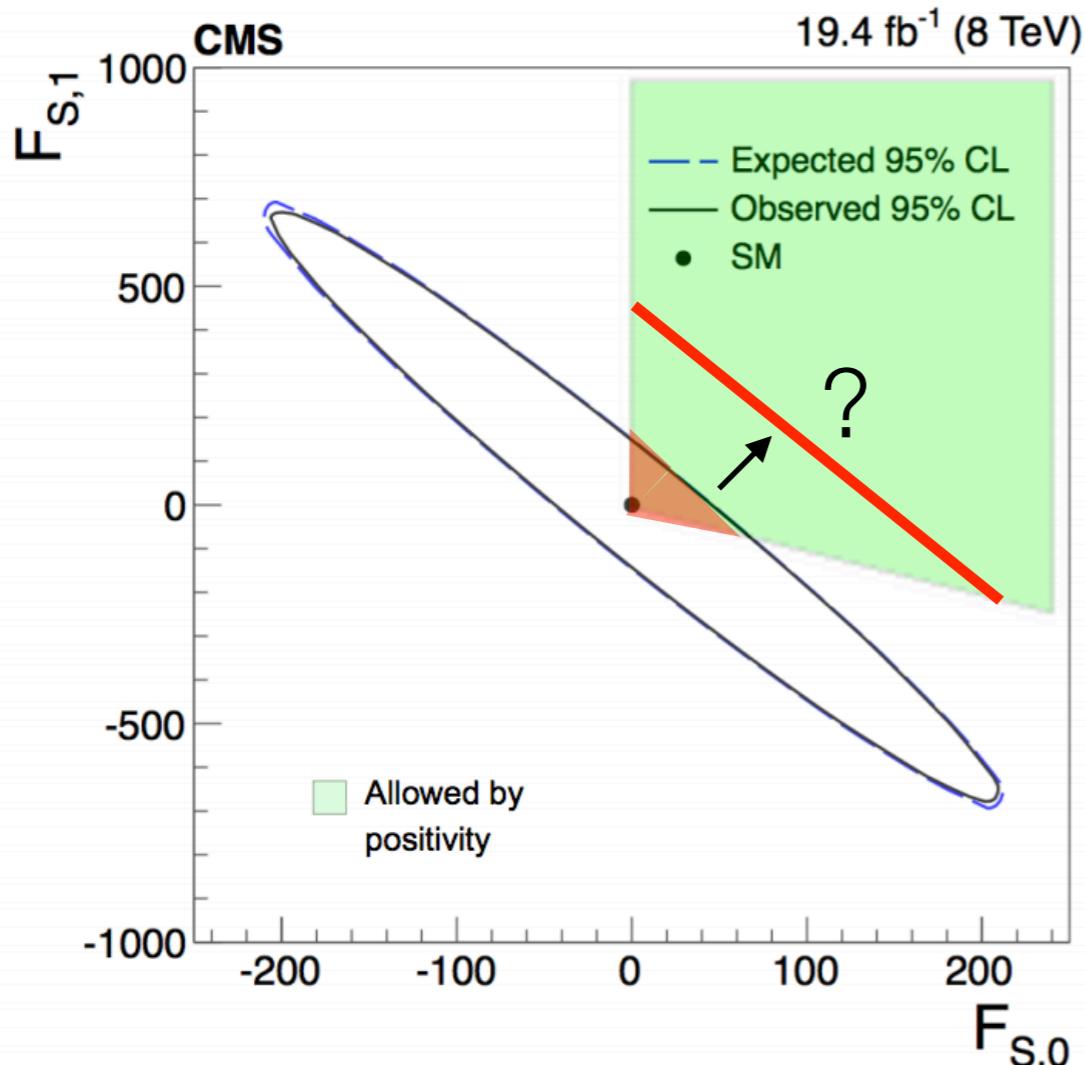
Without positivity

$$\int dF_{S,1} dF_{S,2} L = 95\%$$

With positivity

$$\frac{\int dF_{S,1} dF_{S,2} L}{\int dF_{S,1} dF_{S,2} L} = 95\%$$

Modifying the prior



[CMS, PRL 15]

Without positivity

$$\int dF_{S,1} dF_{S,2} L = 95\%$$

With positivity

$$\frac{\int dF_{S,1} dF_{S,2} L}{\int dF_{S,1} dF_{S,2} L} = 95\%$$

If you believe it

- If we believe that the BSM satisfies unitarity, dispersion relation (causality & locality), and Lorentz invariance, i.e. **positivity is true**:
 - It may enter the measurement as a prior, possibly changing the resulting limits on couplings.
 - It may help scanning the parameter space (to identify the confidence region), because there is now only 2% to scan compared with before... (for channels that are hard to unfold?)
 - It may be included in global fits either as a constraint or a prior.

Or, be more open-minded?

- Use it to **test** unitarity, dispersion relation (causality & locality), and Lorentz invariance?
- If deviation is found in the disallowed parameter space:
 - It might imply that **BSM violates one of the fundamental principles.**
 - I think this is less likely but more exciting.
 - It might imply that **LO dim-6+dim-8 SMEFT is problematic.** Either dim-6/8 loops are dominating, or dim-10 is important, or we need HEFT...
 - I think this is more likely but slightly less interesting.
 - But in either case, it is important information about BSM.

Summary

Summary

- Once we start to ask for UV completion, interesting bounds on aQGC parameters show up, carving out peculiar structures in the parameter space, reducing the parameter space by 2 orders of magnitude. This is interesting, will guide future BSM searches in VBS.
- It could either affect the measurements, e.g. by modifying the prior and changing the resulting confidence interval, or could be used to test the fundamental properties of QFT.
- These bounds are summarized by 23 analytic equations that only involve aQGC coefficients and the weak angle.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{S,0} \\ F_{S,1} \\ F_{S,2} \end{pmatrix} > 0$$

$$\begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 1 & \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} F_{M,0} \\ F_{M,1} \\ F_{M,2} \\ F_{M,3} \\ F_{M,4} \\ F_{M,5} \\ F_{M,7} \end{pmatrix} > 0$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 & \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 & \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 & \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 & \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 & \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 & \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 & \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 & \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 & \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 & \end{pmatrix} \begin{pmatrix} F_{T,0} \\ F_{T,1} \\ F_{T,2} \\ F_{T,5} \\ F_{T,6} \\ F_{T,7} \\ F_{T,8} \\ F_{T,9} \end{pmatrix} > 0$$

$$32(2F_{S,0} + F_{S,1} + F_{S,2})(2F_{T,0} + F_{T,1} + F_{T,2}) \\ - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0$$

$$8(2F_{S,0} + F_{S,1} + F_{S,2})(8F_{T,0} + 12F_{T,1} + 5F_{T,2}) \\ - \max(0, 4F_{M,0} + F_{M,1}, -4F_{M,0} + 3F_{M,1} - 2F_{M,7})^2 > 0$$

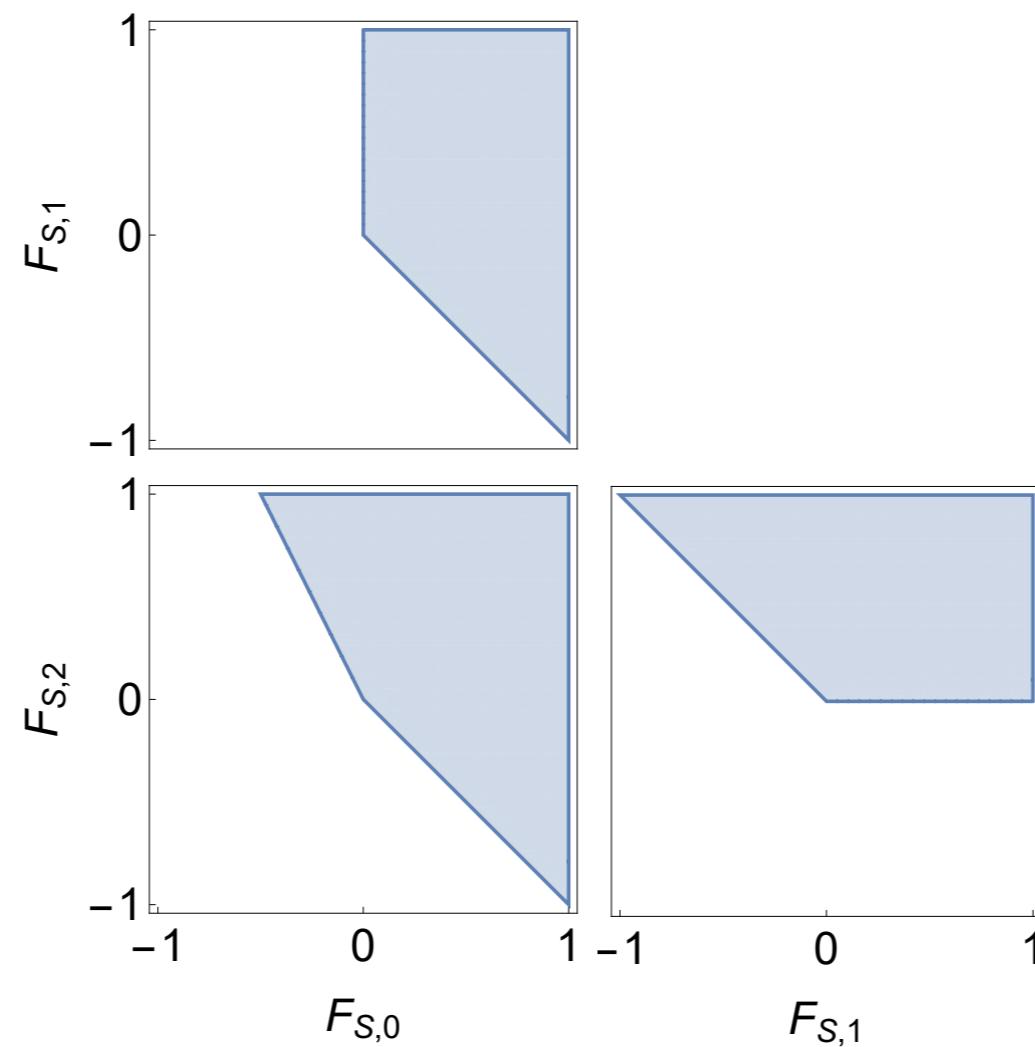
$$8(F_{S,0} + F_{S,1} + F_{S,2}) [4c_W^8(2F_{T,0} + 2F_{T,1} + F_{T,2}) + 2c_W^4 s_W^4(2F_{T,5} + 2F_{T,6} + F_{T,7}) \\ + s_W^8(2F_{T,8} + F_{T,9})] - \max[0, 2(2c_W^4 F_{M,0} + F_{M,2} s_W^4 - F_{M,4} s_W^4 + F_{M,4} s_W^2), \\ -c_W^4(4F_{M,0} - 2F_{M,1} + F_{M,7}) - 2c_W^2 F_{M,4} s_W^2 - s_W^4(2F_{M,2} - F_{M,3}) - F_{M,5}(s_W^2 - s_W^4)]^2 > 0$$

$$16(F_{S,0} + F_{S,2}) [4c_W^4(4F_{T,1} + F_{T,2}) + s_W^4(4F_{T,6} + F_{T,7})] - \max[0, +2c_W^2 F_{M,7} \\ - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5} s_W^2 + F_{M,3} s_W^4)} + 4F_{M,4} s_W^2 + F_{M,5} s_W^2, \\ - 2c_W^2 F_{M,7} - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5} s_W^2 + F_{M,3} s_W^4)} \\ - 4F_{M,4} s_W^2 - F_{M,5} s_W^2]^2 > 0$$

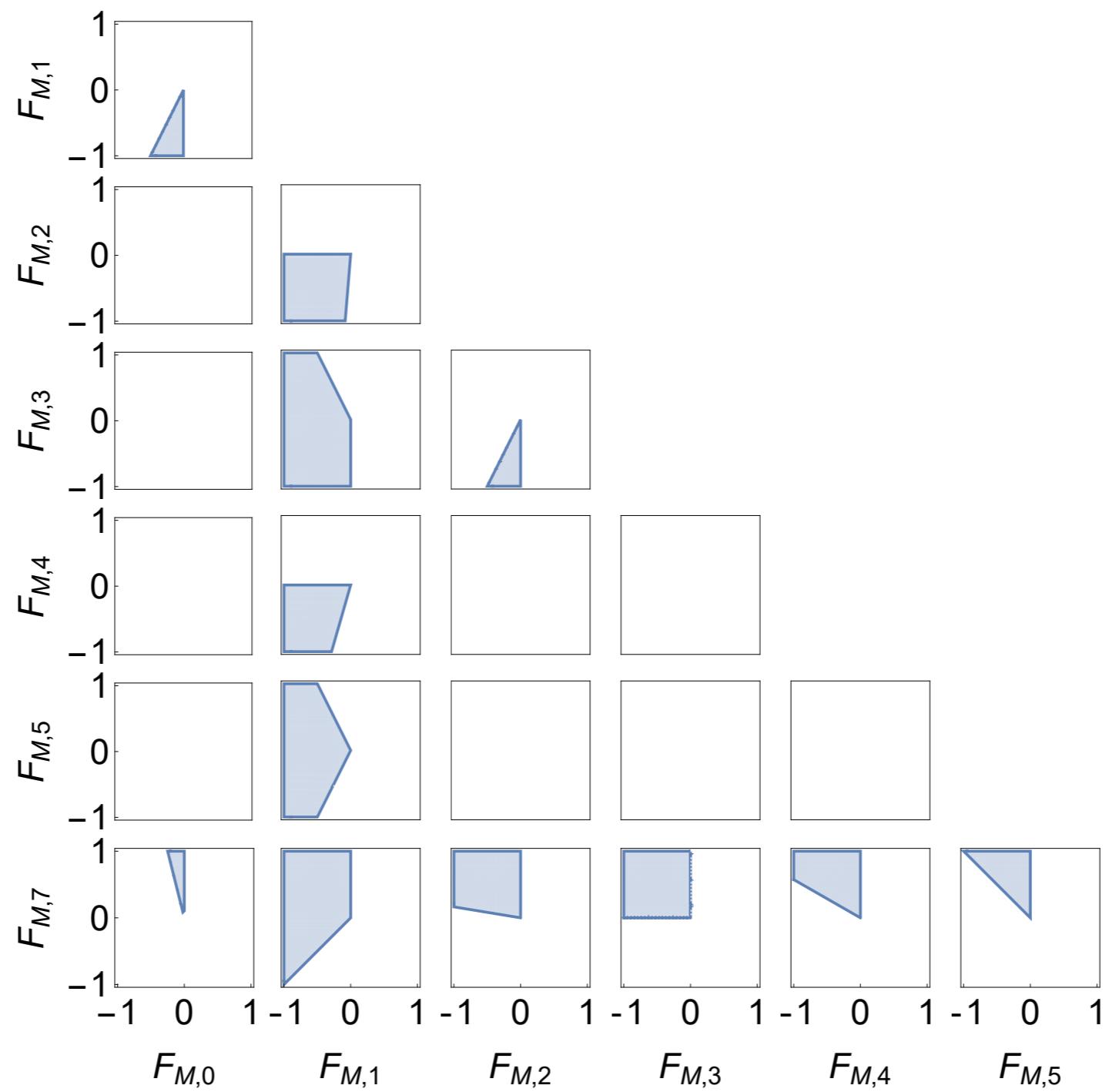
Thank you

All 2D subspace

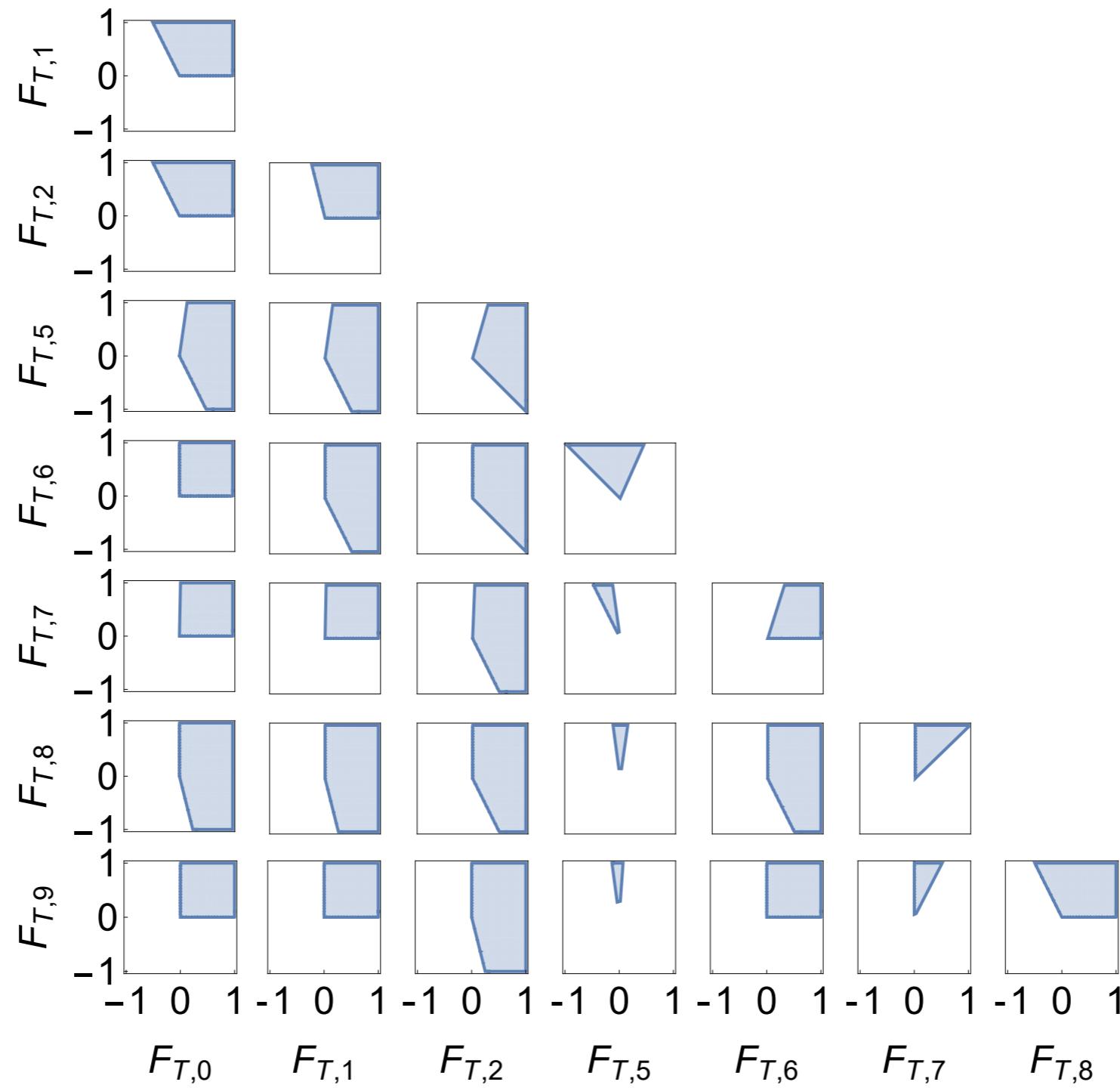
- For quick comparison with EXP results.
- There is no correlation across different S/M/T categories.
(Quadratic ones show up only in 3D)



All 2D subspace



All 2D subspace



18D case: volume

- Linear ones: completely decoupled in S, M, T subspaces

$$V_S = 0.38, \quad V_M = 0.34, \quad V_T = 0.16$$

$$V_S V_M V_T = 0.022$$

- Quadratic:

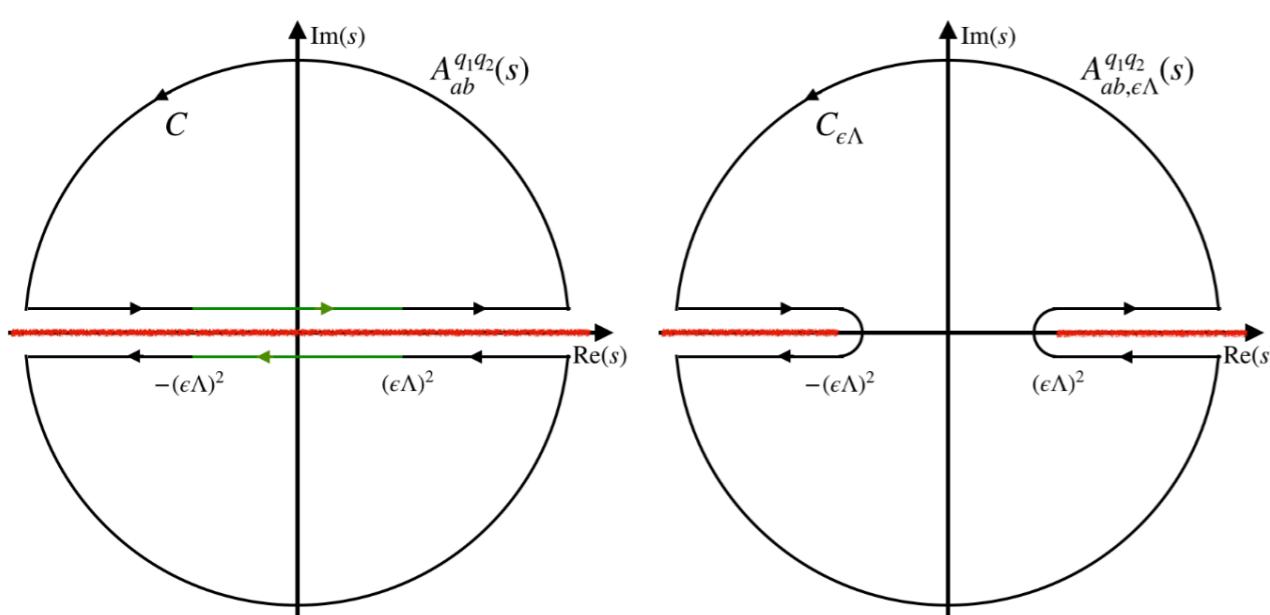
	WW 1st	WW 2nd	ZZ	WZ	All
Volume	0.20	0.22	0.20	0.12	0.05

- All together: 2.1%

Most of the constraining power
is in the linear ones

Positivity bounds (for SMEFT)

- SM loops give branch cuts covering the entire real axis
- Define new amplitude with low energy discontinuity subtracted below $\epsilon\Lambda$, $\epsilon \lesssim 1$



$$\begin{aligned} B_{ab,\epsilon\Lambda}^{q_1 q_2}(s) &\equiv A_{ab}^{q_1 q_2}(s) - \frac{1}{2\pi i} \int_{-(\epsilon\Lambda)^2}^{+(\epsilon\Lambda)^2} ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{s' - s} \\ &= \frac{1}{2\pi i} \oint_C ds' \frac{A_{ab}^{q_1 q_2}(s')}{s' - s} - \frac{1}{2\pi i} \int_{-(\epsilon\Lambda)^2}^{+(\epsilon\Lambda)^2} ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{s' - s} \end{aligned}$$

$$\begin{aligned} f_{ab,\epsilon\Lambda}^{q_1 q_2}(s) &\equiv \frac{1}{2} \frac{d^2 B_{ab,\epsilon\Lambda}^{q_1 q_2}(s)}{ds^2} \\ &= \frac{1}{2\pi i} \left(\int_{-\infty}^{-(\epsilon\Lambda)^2} + \int_{+(\epsilon\Lambda)^2}^{\infty} \right) ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{(s' - s)^3} \end{aligned}$$

Positivity bounds (for SMEFT)

- EFT contribution: tree level approximation, gives

$$f \approx \frac{d^2 A(\mu^2)}{ds^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

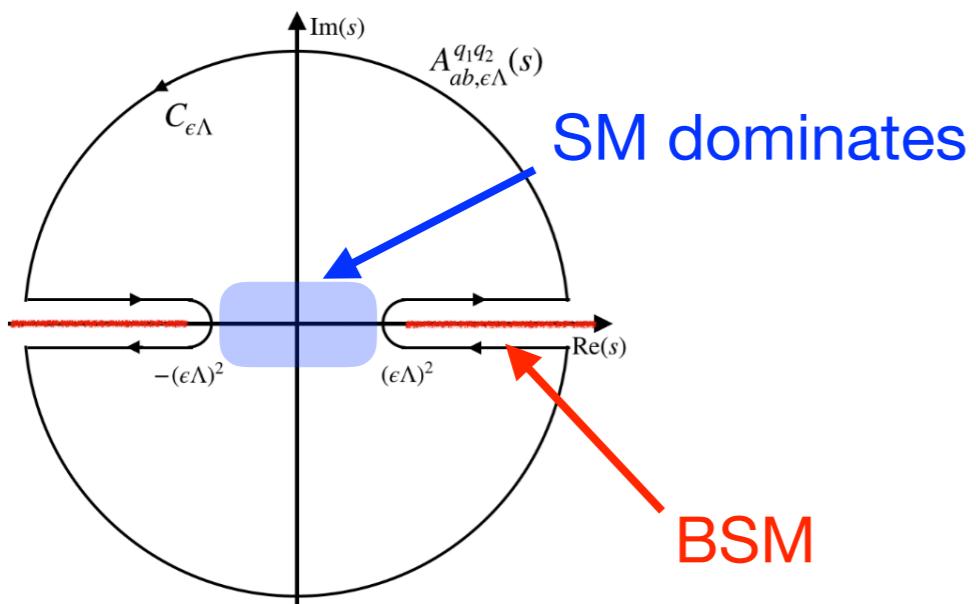
- SM loops: subtracted in $-(\epsilon\Lambda)^2 < s < (\epsilon\Lambda)^2$

Positivity bounds (for SMEFT)

- EFT contribution: tree level approximation, gives

$$f \approx \frac{d^2 A(\mu^2)}{ds^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

- SM loops: subtracted in $-(\epsilon\Lambda)^2 < s < (\epsilon\Lambda)^2$



- Remaining loop can be computed with

$$\frac{1}{2\pi i} \left(\int_{-\infty}^{-(\epsilon\Lambda)^2} + \int_{+(\epsilon\Lambda)^2}^{\infty} \right) ds' \frac{\text{Disc} A_{ab}^{q_1 q_2}(s')}{(s' - s)^3}$$

E.g. $f_{\epsilon\Lambda}^{00,WW} = 0.038 \text{ TeV}^{-4}$

compared with

$\mathcal{O}(1) \text{ TeV}^{-4}$ with current bounds

- SM can be completely removed in weakly coupled UV.

t-channel pole?

- Tree level diagram: s/t , vanishes with 2 subtractions.
 - Can use finite t as a regulator.
 - For indefinite polarization, the bounds will have an additional term that goes like $t^{1/2}$.
- Loop level pole:
 - Xsec of all channels except $WW>WW$
 - $\text{Im} (A(s) - A^{WW}(s)) = [(s - M_-^2)(s - M_+^2)]^{1/2} \sigma_{WW}(s) > 0,$
 - Subtract diagrams with WW cuts
(up to additional higher orders)
- Or simply run the positivity argument for the leading BSM order.

18D case

- In general, **a, b and be complex, i.e. 12 DOFs.** But we can **remove 6 of them**, by parameter redefinition

$$|a_3|^2 = T_a^2, \quad |b_3|^2 = T_b^2$$

$$a_3 b_3^* = T_a T_b e^{i\delta}, \quad a_3 b_3 = T_a T_b e^{i\Delta}$$

$$a_1 b_1^* + a_2 b_2^* = \cos \phi e^{i(\alpha+\delta)}, \quad a_1 b_1 + a_2 b_2 = \cos \psi e^{i(\beta+\Delta)}$$

$$|a_1|^2 + |a_2|^2 = 1, \quad |b_1|^2 + |b_2|^2 = 1$$

- Positivity conditions depend on 6 parameters:

$$T_a, T_b \in [0, +\infty)$$

$$\cos \phi, \cos \psi \in [0, 1]$$

$$\cos \alpha, \cos \beta \in [-1, 1]$$

18D case: WW

Before

$$\begin{aligned} & 2A_1(8F_{T,0} + 12F_{T,1} + 5F_{T,2}) + 6A_2F_{T,2} + (A_3 + A'_3)(-2F_{M,1} + F_{M,7}) \\ & + 2A_4(8F_{T,1} + F_{T,2}) + 2A'_4(8F_{T,0} + 4F_{T,1} + F_{T,2}) + A'_5(4F_{M,0} - F_{M,1} + F_{M,7}) \\ & + 4A_6(2F_{S,0} + F_{S,1} + F_{S,2}) > 0 \end{aligned}$$

$$\begin{aligned} A_1 &\equiv |a_1|^2|b_1|^2 + |a_2|^2|b_2|^2, & A_4 &\equiv a_1a_2^*b_1b_2^* + c.c., \\ A_2 &\equiv |a_1|^2|b_2|^2 + |a_2|^2|b_1|^2, & A'_4 &\equiv a_1a_2^*b_1^*b_2 + c.c., \\ A_3 &\equiv (|b_1|^2 + |b_2|^2)|a_3|^2, & A_5 &\equiv (a_1b_1 + a_2b_2)a_3^*b_3^* + c.c., \\ A'_3 &\equiv (|a_1|^2 + |a_2|^2)|b_3|^2, & A'_5 &\equiv -(a_1b_1^* + a_2b_2^*)a_3^*b_3 + c.c. \\ A''_3 &\equiv |b_1|^2|a_3|^2 & A_6 &\equiv |a_3|^2|b_3|^2, \end{aligned}$$

After

$$f_{WW}(T_a, T_b, R, W, V) = f_1(T_a^2 + T_b^2) + f_2T_a^2T_b^2 + f_3W^2 + f_4T_aT_bRW + f_5V^2 + f_6 > 0$$

$$\begin{aligned} \cos\phi &\rightarrow W \\ \cos\psi &\rightarrow V \\ \cos\alpha &\rightarrow R \end{aligned}$$

$$\begin{aligned} f_1 &= -2F_{M,1} + F_{M,7} \\ f_2 &= 8F_{S,0} + 4F_{S,1} + 4F_{S,2} \\ f_3 &= 16F_{T,0} + 8F_{T,1} + 2F_{T,2} \\ f_4 &= -8F_{M,0} + 2F_{M,1} - 2F_{M,7} \\ f_5 &= 16F_{T,1} + 2F_{T,2} \\ f_6 &= 6F_{T,2} \end{aligned}$$

$$T_a, T_b \in [0, +\infty),$$

$$W \in [0, 1], \quad R \in [-1, 1],$$

$$V \in [0, 1]$$

18D case: WW

$$f_{WW}(T_a, T_b, R, W, V) = f_1(T_a^2 + T_b^2) + f_2 T_a^2 T_b^2 + f_3 W^2 + f_4 T_a T_b R W + f_5 V^2 + f_6 > 0$$

$$T_a, T_b \in [0, +\infty), \quad W \in [0, 1], \quad R \in [-1, 1], \quad V \in [0, 1]$$

Coefs.

- Minimum value of f_{WW} has to exist:

$$f_1 > 0, \quad f_2 > 0$$

- Minimum value of f_{WW} is

$$f_6 + \min(f_5, 0) + \min\left(0, f_3 - \frac{\max(0, -2f_1 + |f_4|)^2}{4f_2}\right)$$

$$\begin{aligned} f_1 &= -2F_{M,1} + F_{M,7} \\ f_2 &= 8F_{S,0} + 4F_{S,1} + 4F_{S,2} \\ f_3 &= 16F_{T,0} + 8F_{T,1} + 2F_{T,2} \\ f_4 &= -8F_{M,0} + 2F_{M,1} - 2F_{M,7} \\ f_5 &= 16F_{T,1} + 2F_{T,2} \\ f_6 &= 6F_{T,2} \end{aligned}$$

and has to be positive:

$$f_6 > 0, \quad f_3 + f_6 > 0, \quad f_5 + f_6 > 0, \quad f_3 + f_5 + f_6 > 0$$

$$4f_2(f_3 + f_6) > \max(0, |f_4| - 2f_1)^2,$$

$$4f_2(f_3 + f_5 + f_6) > \max(0, |f_4| - 2f_1)^2,$$

18D case: WW

$$f_{WW}(T_a, T_b, R, W, V) = f_1(T_a^2 + T_b^2) + f_2 T_a^2 T_b^2 + f_3 W^2 + f_4 T_a T_b$$

$$T_a, T_b \in [0, +\infty), \quad W \in [0, 1], \quad R \in [-1, 1], \quad V \in$$

- Minimum value of f_{WW} has to exist:

$$f_1 > 0, \quad f_2 > 0$$

- Minimum value of f_{WW} is

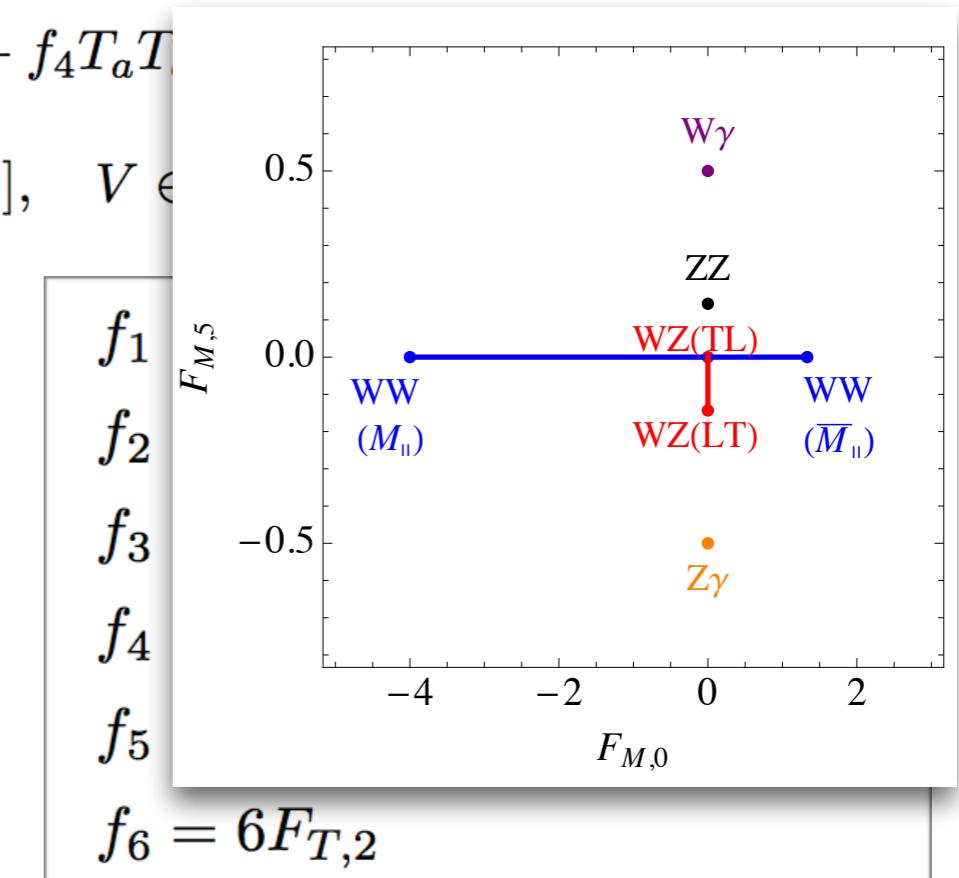
$$f_6 + \min(f_5, 0) + \min\left(0, f_3 - \frac{\max(0, -2f_1 + |f_4|)^2}{4f_2}\right)$$

and has to be positive:

$$f_6 > 0, \quad f_3 + f_6 > 0, \quad f_5 + f_6 > 0, \quad f_3 + f_5 + f_6 > 0$$

$$4f_2(f_3 + f_6) > \max(0, |f_4| - 2f_1)^2,$$

$$4f_2(f_3 + f_5 + f_6) > \max(0, |f_4| - 2f_1)^2,$$



They are like the endpoints in the pyramid case

18D case: WW

$$f_{WW}(T_a, T_b, R, W, V) = f_1(T_a^2 + T_b^2) + f_2 T_a^2 T_b^2 + f_3 W^2 + f_4 T_a T_b R W + f_5 V^2 + f_6 > 0$$

$$T_a, T_b \in [0, +\infty), \quad W \in [0, 1], \quad R \in [-1, 1], \quad V \in [0, 1]$$

Coefs.

- Minimum value of f_{WW} has to exist:

$$f_1 > 0, \quad f_2 > 0$$

- Minimum value of f_{WW} is

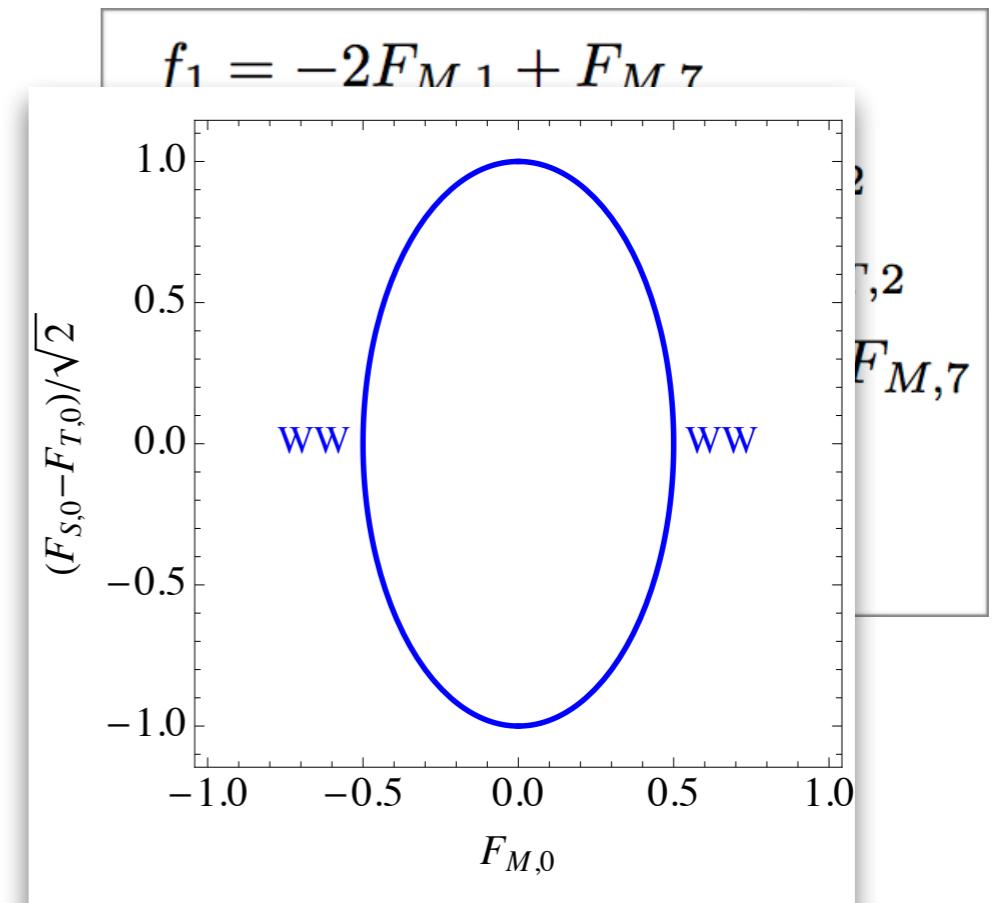
$$f_6 + \min(f_5, 0) + \min\left(0, f_3 - \frac{\max(0, -2f_1 + |f_4|)^2}{4f_2}\right)$$

and has to be positive:

$$f_6 > 0, \quad f_3 + f_6 > 0, \quad f_5 + f_6 > 0, \quad f_3 + f_5 + f_6 > 0$$

$$4f_2(f_3 + f_6) > \max(0, |f_4| - 2f_1)^2,$$

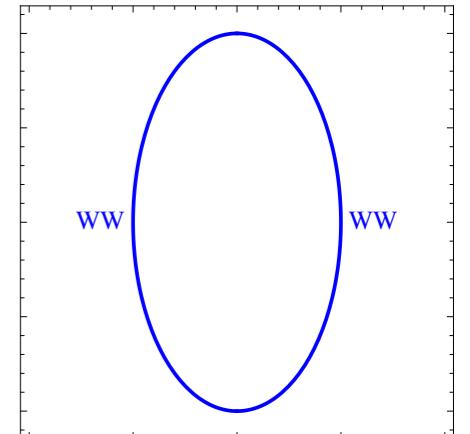
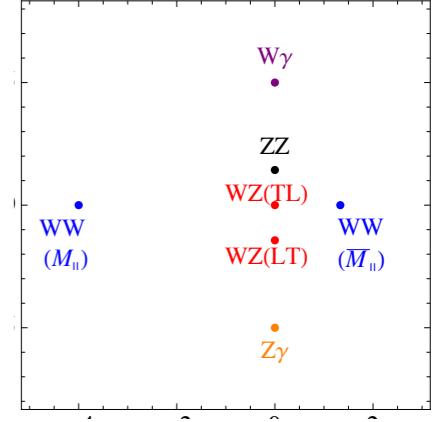
$$4f_2(f_3 + f_5 + f_6) > \max(0, |f_4| - 2f_1)^2,$$



They are like the circle
in the cone case

18D case: key polarizations

	Notation	Example of a_i, b_i values	General form
Linear	L	$\vec{a} = (0, 0, 1), \vec{b} = (0, 0, 1)$	$\vec{a} = (0, 0, m), \vec{b} = (0, 0, n)$
	LT	$\vec{a} = (0, 0, 1), \vec{b} = (1, 0, 0)$	$\vec{a} = (0, 0, m_3), \vec{b} = (n_1, n_2, 0)$
	TL	$\vec{a} = (1, 0, 0), \vec{b} = (0, 0, 1)$	$\vec{a} = (m_1, m_2, 0), \vec{b} = (0, 0, n_3)$
	T_{\perp}	$\vec{a} = (1, 0, 0), \vec{b} = (0, 1, 0)$	$\vec{a} = e^{i\rho}(k_1, k_2, 0), \vec{b} = e^{i\sigma}(k_2, -k_1, 0)$
	T_{\parallel}	$\vec{a} = (1, 0, 0), \vec{b} = (1, 0, 0)$	$\vec{a} = e^{i\rho}(k_1, k_2, 0), \vec{b} = e^{i\sigma}(k_1, k_2, 0)$
	T_{++}	$\vec{a} = (1, i, 0), \vec{b} = (1, -i, 0)$	$\vec{a} = e^{i\rho}(k_1, \pm ik_1, 0), \vec{b} = e^{i\sigma}(k_1, \mp ik_1, 0)$
	T_{+-}	$\vec{a} = (1, i, 0), \vec{b} = (1, i, 0)$	$\vec{a} = e^{i\rho}(k_1, \pm ik_1, 0), \vec{b} = e^{i\sigma}(k_1, \pm ik_1, 0)$
	M_{+-}	$\vec{a} = (x, ix, 1), \vec{b} = (x, ix, 1)$	$\vec{a} = e^{i\rho}(k_1, \pm ik_1, e^{i\gamma}k_3), \vec{b} = e^{i\sigma}(k_1, \pm ik_1, e^{i\gamma}k_3)$
Quadratic and quartic	M_{\parallel}	$\vec{a} = (x, 0, 1), \vec{b} = (x, 0, 1)$	$\vec{a} = e^{i\rho}(k_1, k_2, e^{i\gamma}k_3), \vec{b} = e^{i\sigma}(k_1, k_2, e^{i\gamma}k_3)$
	M_I	$\vec{a} = (ix, 0, 1), \vec{b} = (ix, 0, 1)$	$\vec{a} = e^{i\rho}(ik_1, ik_2, k_3), \vec{b} = e^{i\sigma}(ik_1, ik_2, k_3)$
	\bar{M}_{+-}	$\vec{a} = (x, ix, 1), \vec{b} = (x, ix, -1)$	$\vec{a} = e^{i\rho}(k_1, \pm ik_1, e^{i\gamma}k_3), \vec{b} = e^{i\sigma}(k_1, \pm ik_1, -e^{i\gamma}k_3)$
	\bar{M}_{\parallel}	$\vec{a} = (x, 0, 1), \vec{b} = (x, 0, -1)$	$\vec{a} = e^{i\rho}(k_1, k_2, e^{i\gamma}k_3), \vec{b} = e^{i\sigma}(k_1, k_2, -e^{i\gamma}k_3)$
	\bar{M}_I	$\vec{a} = (ix, 0, 1), \vec{b} = (ix, 0, -1)$	$\vec{a} = e^{i\rho}(ik_1, ik_2, k_3), \vec{b} = e^{i\sigma}(ik_1, ik_2, -k_3)$
	M'_I	$\vec{a} = (ix, 0, \sqrt{h''_1}),$ $\vec{b} = (ix, 0, \sqrt{h'_1})$	$\vec{a} = e^{i\rho}(ik_1, ik_2, k_3), \vec{b} = e^{i\sigma}(ik_1, ik_2, k'_3),$ $h'_1 k_3^2 = h''_1 k'^2_3, k_3 k'_3 > 0$
	\bar{M}'_I	$\vec{a} = (ix, 0, \sqrt{h''_1}),$ $\vec{b} = (ix, 0, -\sqrt{h'_1})$	$\vec{a} = e^{i\rho}(ik_1, ik_2, k_3), \vec{b} = e^{i\sigma}(ik_1, ik_2, k'_3),$ $h'_1 k_3^2 = h''_1 k'^2_3, k_3 k'_3 < 0$
	$m_i, n_i \in \mathbb{C}, x, k_i, k'_i \in \mathbb{R}, \rho, \sigma, \gamma \in [0, 2\pi)$		



18D case: linear bounds

$$M_{S,ij} F_{S,j} > 0$$

$$F_{S,i} = (F_{S,0}, F_{S,1}, F_{S,2})^T$$

$$M_{M,ij} F_{M,j} > 0$$

$$F_{M,i} = (F_{M,0}, F_{M,1}, F_{M,2}, F_{M,3}, F_{M,4}, F_{M,5}, F_{M,7})^T$$

$$M_{T,ij} F_{T,j} > 0$$

$$F_{T,i} = (F_{T,0}, F_{T,1}, F_{T,2}, F_{T,5}, F_{T,6}, F_{T,7}, F_{T,8}, F_{T,9})^T$$

$$M_S = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{array}{l} WW, L \\ ZZ, L \\ WZ, L \end{array}$$

$$M_M = \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix} \begin{array}{l} ZZ, LT \& TL \\ WZ, LT \\ WW, LT \& TL; WZ, TL \\ W\gamma, LT \\ Z\gamma, LT \end{array}$$

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{array}{l} WW, T_\perp \\ WW, T_{++} \\ WW, T_{+-} \\ WW, T_\parallel \\ ZZ, T_\parallel \\ *ZZ, T_\perp \\ *WZ, T_\perp \\ WZ, T_\parallel \\ W\gamma, T_\perp \\ W\gamma, T_\parallel \\ *Z\gamma, T_\perp \\ Z\gamma, T_\parallel \\ \gamma\gamma, T_\perp \\ \gamma\gamma, T_\parallel \end{array}$$

18D case: linear bounds

$$M_{S,ij} F_{S,j} > 0$$

$$F_{S,i} = (F_{S,0}, F_{S,1}, F_{S,2})^T$$

$$M_{M,ij} F_{M,j} > 0$$

$$F_{M,i} = (F_{M,0}, F_{M,1}, F_{M,2}, F_{M,3}, F_{M,4}, F_{M,5}, F_{M,7})^T$$

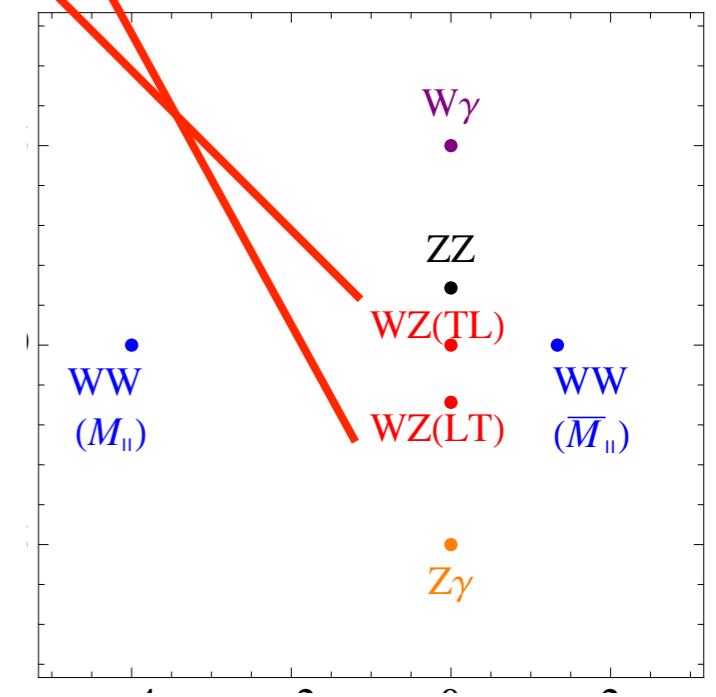
$$M_{T,ij} F_{T,j} > 0$$

$$F_{T,i} = (F_{T,0}, F_{T,1}, F_{T,2}, F_{T,5}, F_{T,6}, F_{T,7}, F_{T,8}, F_{T,9})^T$$

$$M_S = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{array}{l} WW, L \\ ZZ, L \\ WZ, L \end{array}$$

$$M_M = \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix} \begin{array}{l} ZZ, LT \& TL \\ WZ, LT \\ WW, LT \& TL; WZ, TL \\ W\gamma, LT \\ Z\gamma, LT \end{array}$$

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{array}{l} WW, T_\perp \\ WW, T_{++} \\ WW, T_{+-} \\ WW, T_\parallel \\ ZZ, T_\parallel \\ *ZZ, T_\perp \\ *WZ, T_\perp \\ WZ, T_\parallel \\ W\gamma, T_\perp \\ W\gamma, T_\parallel \\ *Z\gamma, T_\perp \\ Z\gamma, T_\parallel \\ \gamma\gamma, T_\perp \\ \gamma\gamma, T_\parallel \end{array}$$

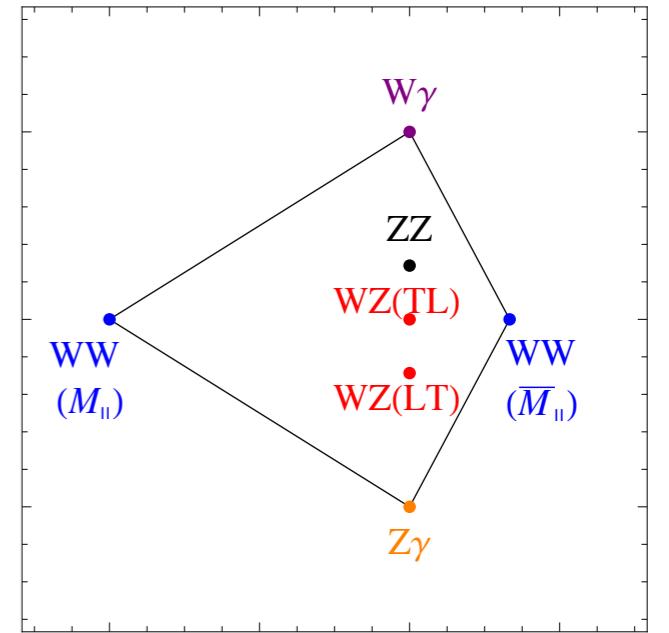


18D case: T subspace

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{array}{l} WW, T_{\perp} \\ WW, T_{++} \\ WW, T_{+-} \\ WW, T_{\parallel} \\ ZZ, T_{\parallel} \\ *ZZ, T_{\perp} \\ *WZ, T_{\perp} \\ WZ, T_{\parallel} \\ W\gamma, T_{\perp} \\ W\gamma, T_{\parallel} \\ *Z\gamma, T_{\perp} \\ Z\gamma, T_{\parallel} \\ \gamma\gamma, T_{\perp} \\ \gamma\gamma, T_{\parallel} \end{array}$$

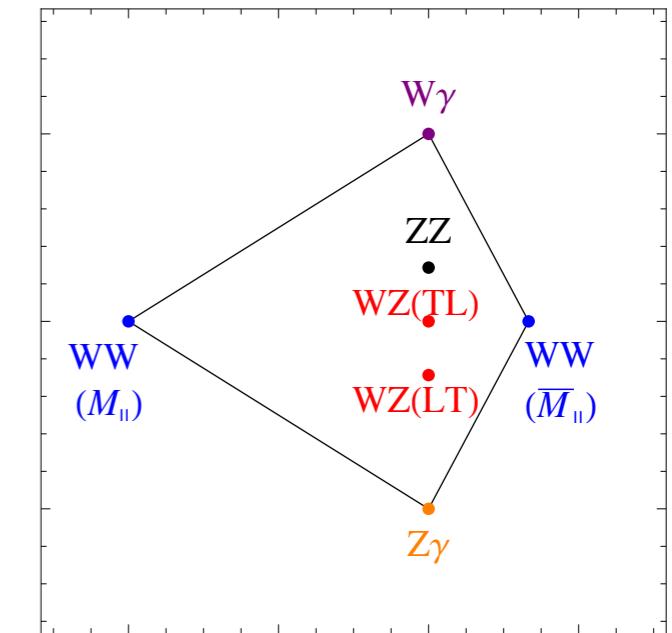
18D case: T subspace

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{array}{l} WW, T_{\perp} \\ WW, T_{++} \\ WW, T_{+-} \\ WW, T_{\parallel} \\ ZZ, T_{\parallel} \\ *ZZ, T_{\perp} \\ *WZ, T_{\perp} \\ WZ, T_{\parallel} \\ W\gamma, T_{\perp} \\ W\gamma, T_{\parallel} \\ *Z\gamma, T_{\perp} \\ Z\gamma, T_{\parallel} \\ \gamma\gamma, T_{\perp} \\ \gamma\gamma, T_{\parallel} \end{array}$$

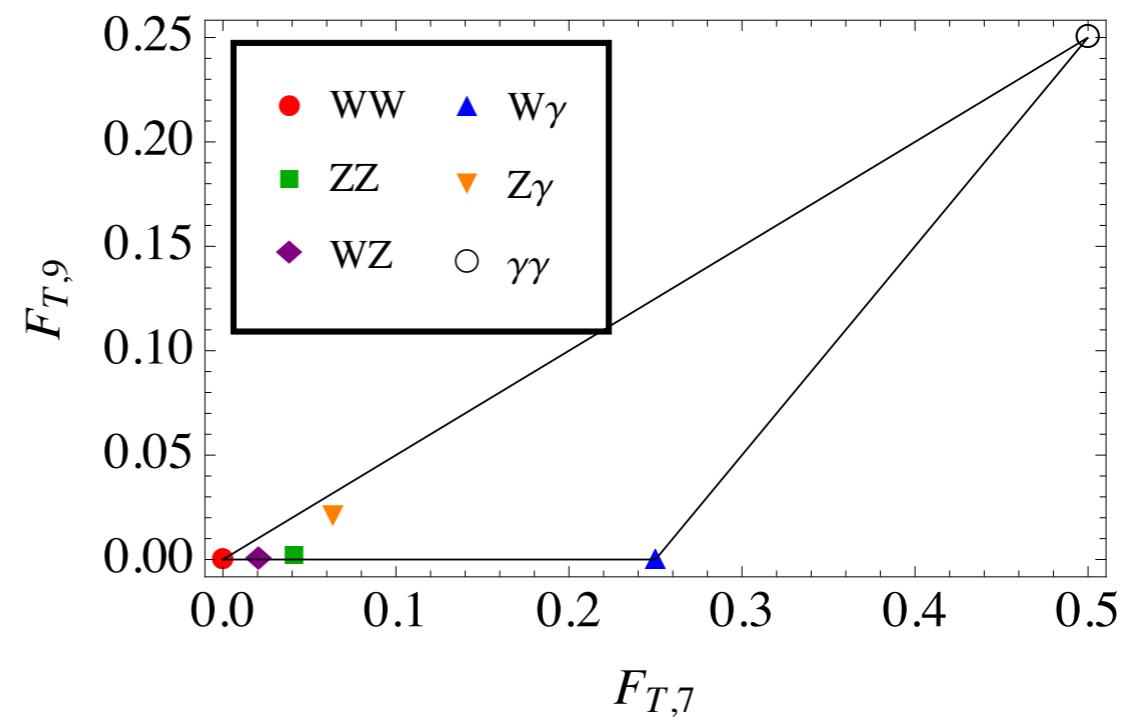


18D case: T subspace

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & WW, T_{\perp} \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & WW, T_{++} \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & WW, T_{+-} \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 & WW, T_{\parallel} \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 & ZZ, T_{\parallel} \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 & *ZZ, T_{\perp} \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 & *WZ, T_{\perp} \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 & WZ, T_{\parallel} \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 & W\gamma, T_{\perp} \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 & W\gamma, T_{\parallel} \\ 0 & 0 & 8c_W^4 & 0 & 0 & (c_W^2 - s_W^2)^2 & 0 & 2s_W^4 & *Z\gamma, T_{\perp} \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4(c_W^2 - s_W^2)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 & Z\gamma, T_{\parallel} \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 & \gamma\gamma, T_{\perp} \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 & \gamma\gamma, T_{\parallel} \end{pmatrix}$$



- Focusing on the T perpendicular cases, we can easily see that ZZ, WZ, and Z γ are redundant.
- So M_T describes a 11-edge 8-D pyramid.

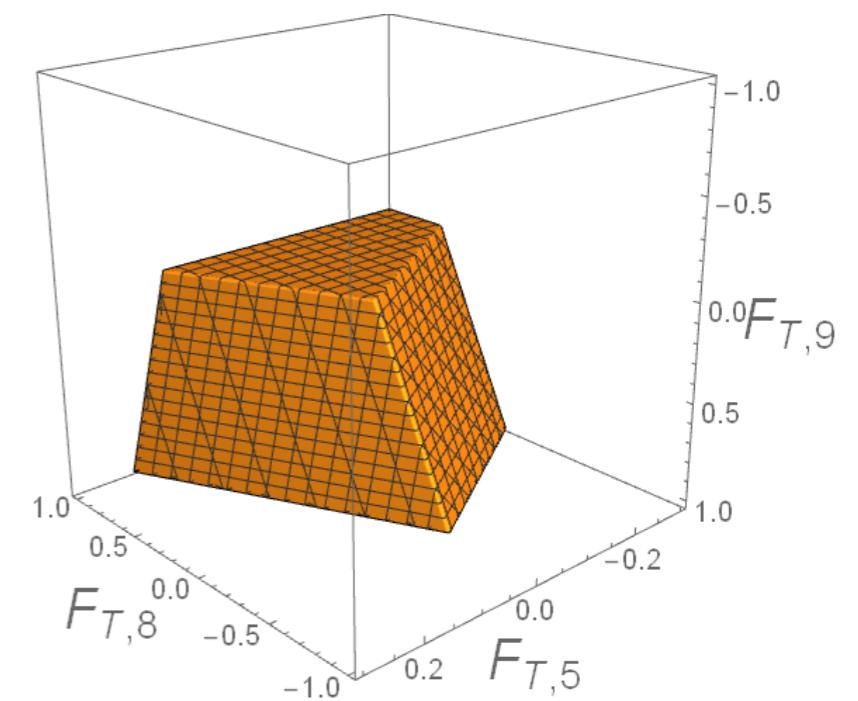
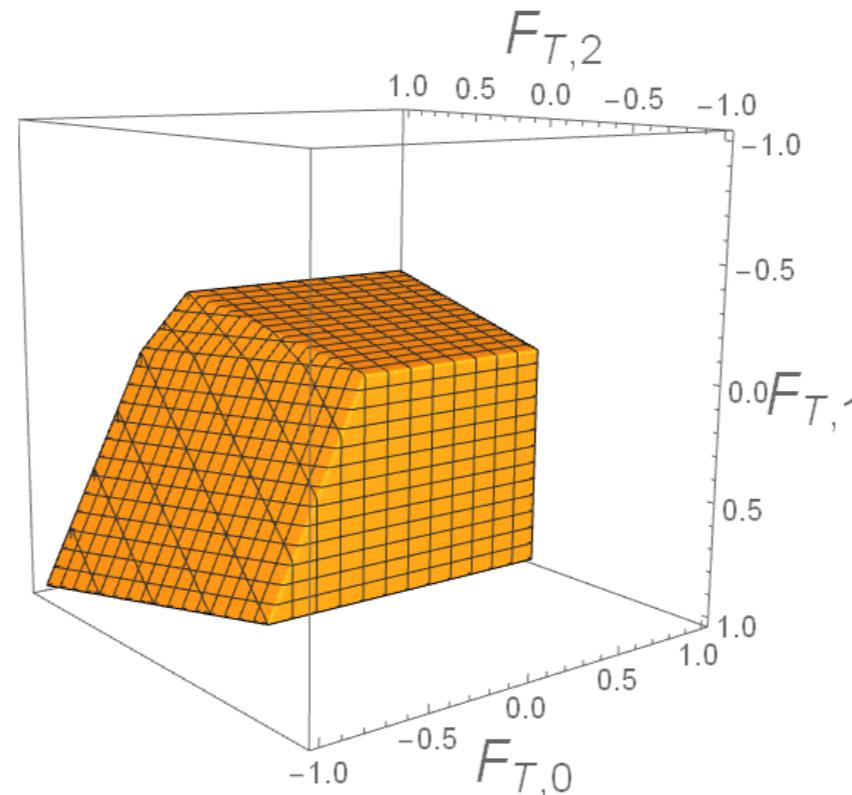
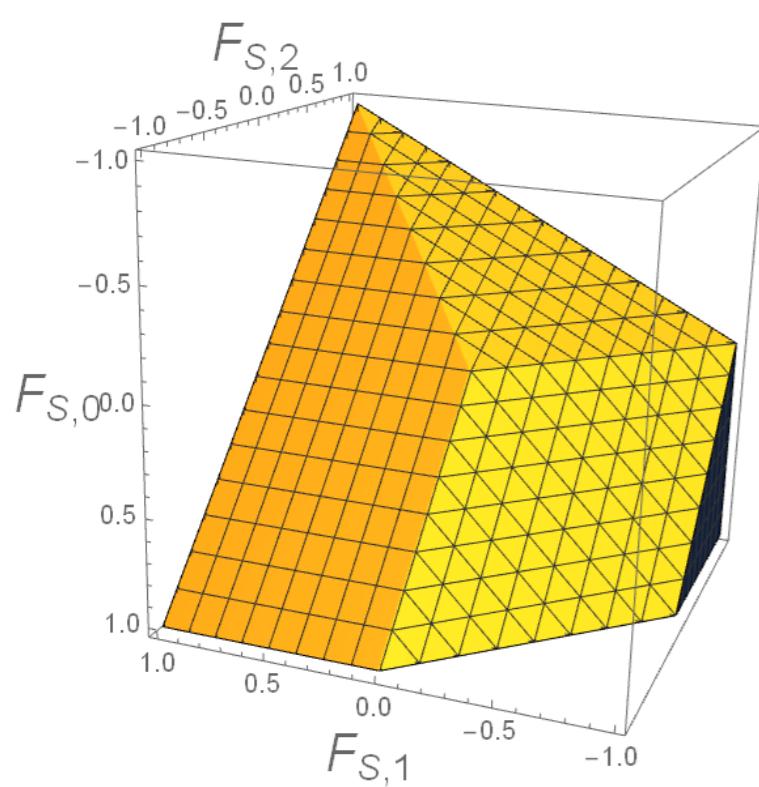


Some 3D subspace

- SSS & TTT: only linear ones matter

$$\left(\sum_i a_i F_{S,i} \right) \left(\sum_i b_i F_{T,i} \right) > \max \left(0, \sum_i c_i F_{M,i}, \sum_i d_i F_{M,i} \right)^2$$

Trivial if M operators turned off



Some 3D subspace

- MMM: may have linear and quadratic shapes.
- i.e. there may be pyramids or cones.

$$\left(\sum_i a_i F_{S,i} \right) \left(\sum_i b_i F_{T,i} \right) > \max \left(0, \sum_i c_i F_{M,i}, \sum_i d_i F_{M,i} \right)^2$$

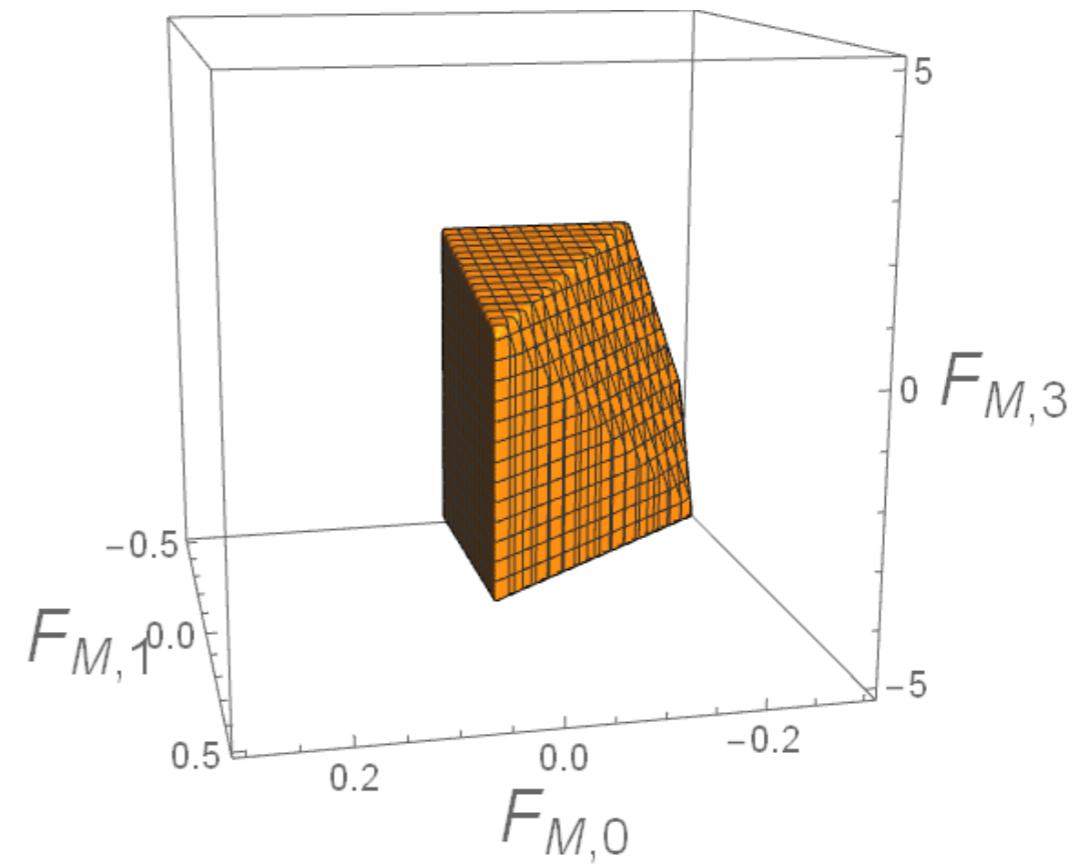
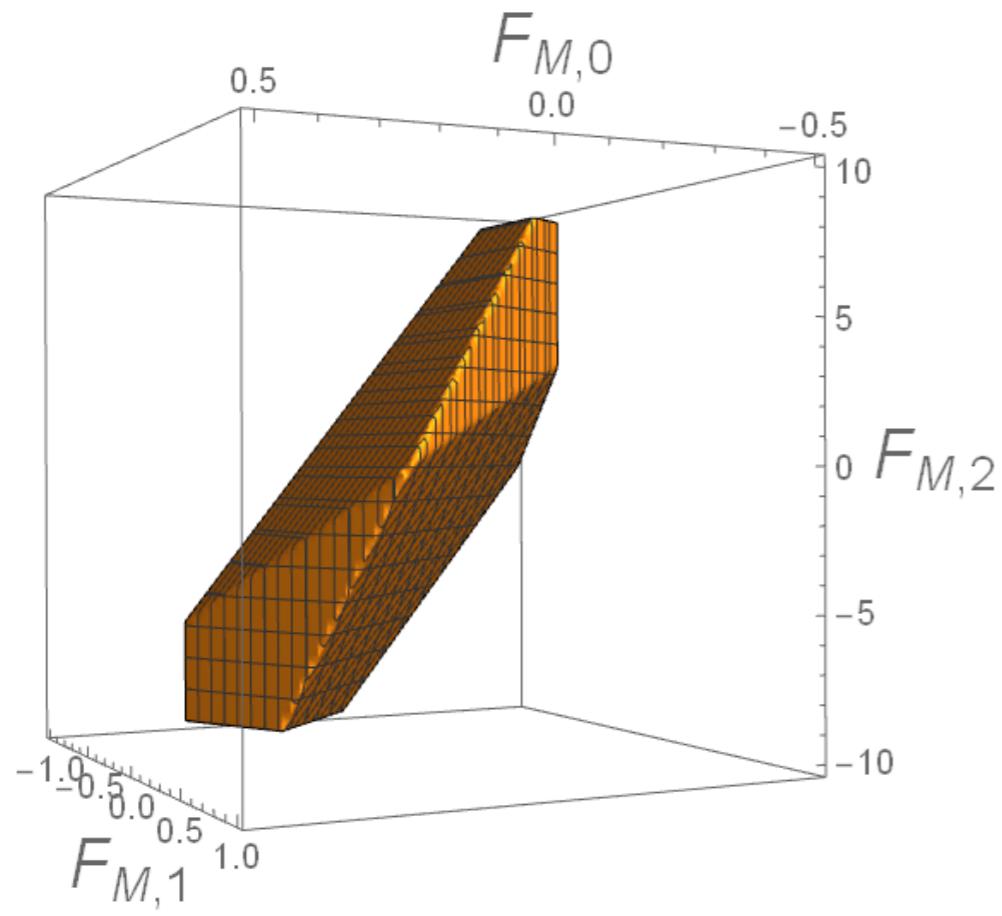
Quadratic becomes linear if S/T = 0

$$\begin{aligned} & 16(F_{S,0} + F_{S,2}) [4c_W^4(4F_{T,1} + F_{T,2}) + s_W^4(4F_{T,6} + F_{T,7})] - \max [0, +2c_W^2 F_{M,7} \\ & - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5} s_W^2 + F_{M,3} s_W^4)} + 4F_{M,4} s_W^2 + F_{M,5} s_W^2, \\ & - 2c_W^2 F_{M,7} - 2\sqrt{(2F_{M,1} - F_{M,7})(c_W^4(2F_{M,1} - F_{M,7}) + c_W^2 F_{M,5} s_W^2 + F_{M,3} s_W^4)} \\ & - 4F_{M,4} s_W^2 - F_{M,5} s_W^2]^2 > 0 \end{aligned} \quad (3.)$$

Quartic becomes quadratic if S/T = 0

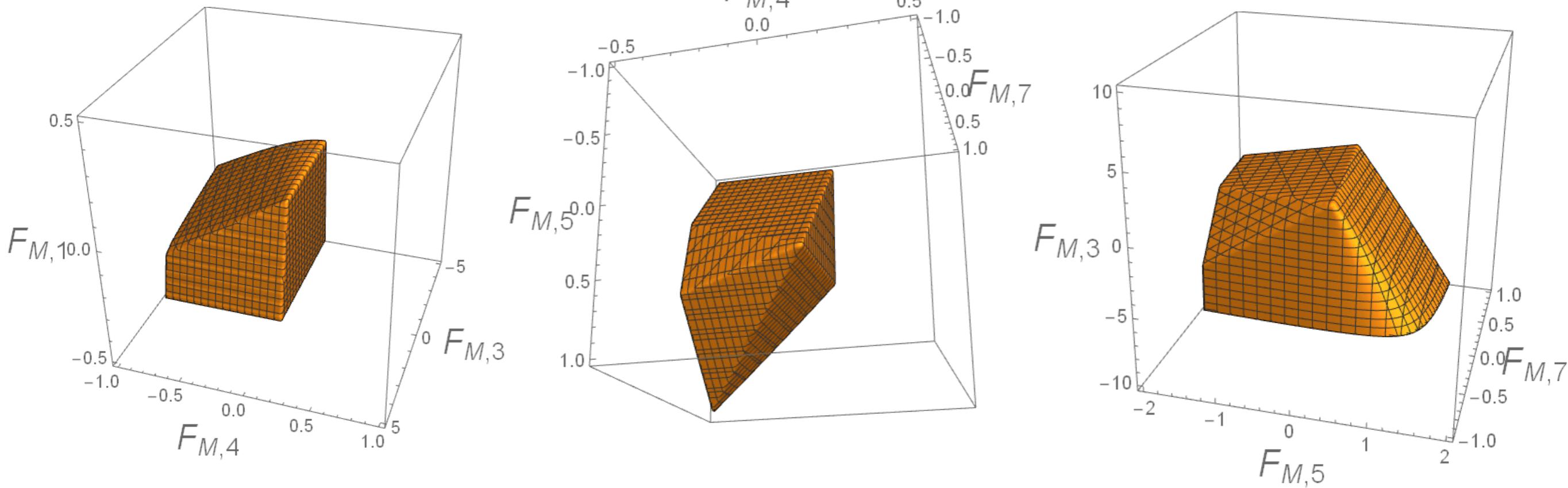
Some 3D subspace

- MMM: may have linear and quadratic
- Pyramids:



Some 3D subspace

- MMM: may have linear and quadratic
- Combinations of pyramids and cones:



Some 3D subspace

- SMT:

