Results for Pomeron and Odderon parameter using IR regulators

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23 - 29 June, Quy Nhon, Vietnam

Outline

- Introduction
- BFKL kernel
- IR regulators and Running Coupling constants
- Numerical results
- Summary and Outlook

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PRD 95 (2017) 014013
arXiv 1908.0 to appear

Introduction

- Scattering process: description in terms of QCD (parton distributions) in the Regge Limit involving Strong Interaction

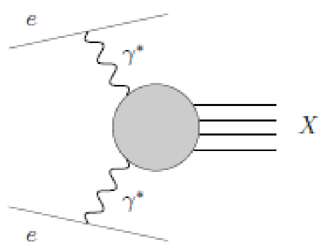
- In the Regge Limit becomes an effective 2+1 dimensional : transversal space and rapidity (Lipatov effective action)

- At very small transverse distances pQCD and BFKL Pomeron (1958)

- At very large transverse distances before QCD era, there was the Reggeon Field Theory description Gribov

- The Pomeron state of two gluon and C=1

- 1973 Odderon Lukaszuk and Nicoleskus the partner of the Pomeron (C =-1) 3 gluons



There are another states with 3, 4 gluons….

Open Talk Peter Bussey Result from HERA: evidence for the Pomeron

Reggeon Field Theory before QCD

P.D.B. Collins, *An introduction to Regge theory and high energy physics*, Cambridge University Press, Cambridge, 1977.

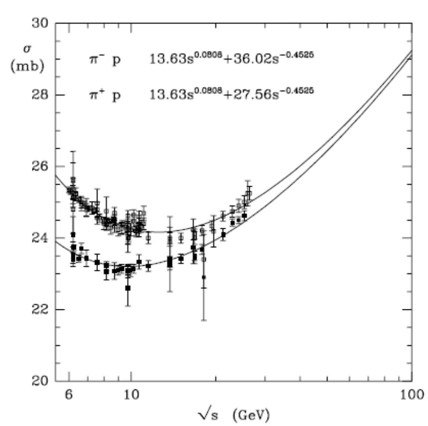
- V. N. Gribov introduce in the 60´s
- Scattering amplitude at high energies for hadrons is according Regge Theory
- The exchange are "quasi particles" characterized by its Regge trajectories : $\alpha_i(t)$
- Leading Pole: is Called Pomeron with vacuum quantum numbers

$$\alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t$$

- $\mu = \alpha_0 - 1$ is the Pomeron intercept and α' is the slope
- According to the Regge theory the contribution to the total Cross section, is given by:

$\sigma_T = A_i s^{\alpha_i(0)-1}$ A. Donnachie and Landshoff : arXiv 1309.1292

- $\mu = \alpha_0 - 1 = 0.08$ and $\alpha' = 0.25$ GeV−2



$$\alpha_P(t) = 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$$

Soft Pomeron

$\alpha_P(0) = 1.07 \pm 0.02\ (\mathrm{stat.}) \pm 0.06$(syst.)

$\alpha_P{}' = 0.25 \pm 0.25\ (\mathrm{stat.})$GeV−2

Leszek Adamczyk Talk

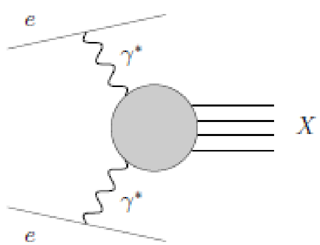
Hard Pomeron pQCD

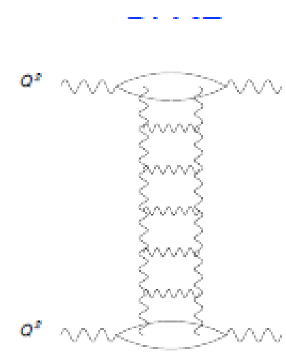
$p + p \rightarrow p + p$

- For Hard processes short Transversal distances

we consider pQCD → Hard Pomeron dominate scattering

$$\alpha_P(0) \approx 1 + 4\,\frac{\alpha_s N_c}{\pi}\ln 2 \qquad Hard\ BFKL/QCD$$





$\gamma^*\gamma^*$-scattering, Mueller-Navelet,..

Soft Pomeron vs Hard Pomeron

$$\alpha_P(0) \approx 1 + 4\,\frac{\alpha_s N_c}{\pi}\ln 2 \qquad Hard\ BFKL/QCD$$

For Hard processes UV region
short Transversal distances
Large Momenta, large but finite energies

$$\alpha_P(0) \approx 1.08 \qquad soft$$

For soft processes
largest Transversal distances
small Momenta, large but finite energies

$\alpha_{P,k}(0)$ *can be considered as a variable which depends on the sizes of the projectiles.*

How we can connect regions of different sizes and different sorts of Pomerons

Use: *Functional Renormalization Group**

Results

- We reproduce the values of the critical exponets universality class of Percolation
- The convergence is under control with the increasing the local truncation

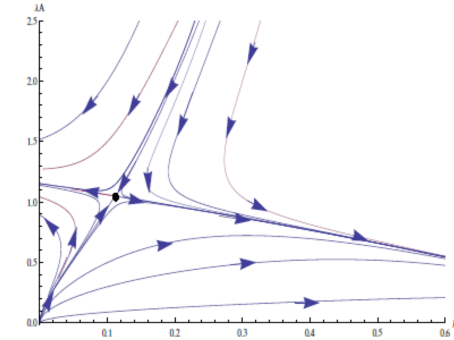
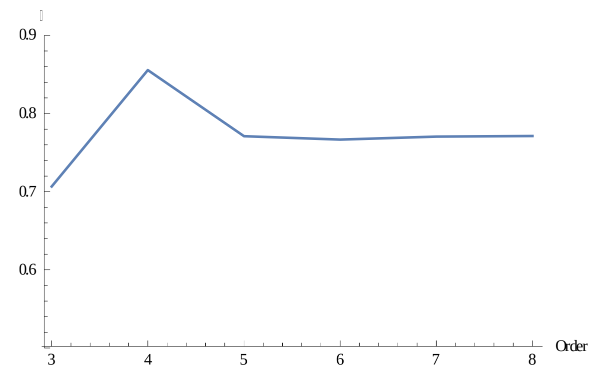
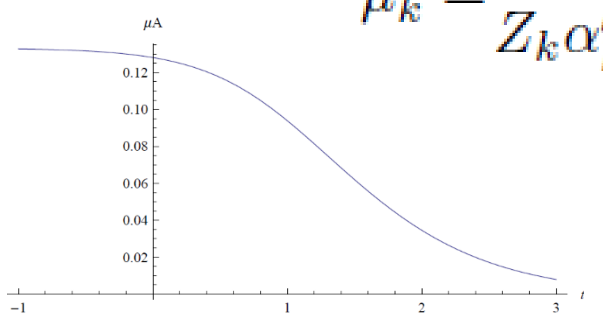


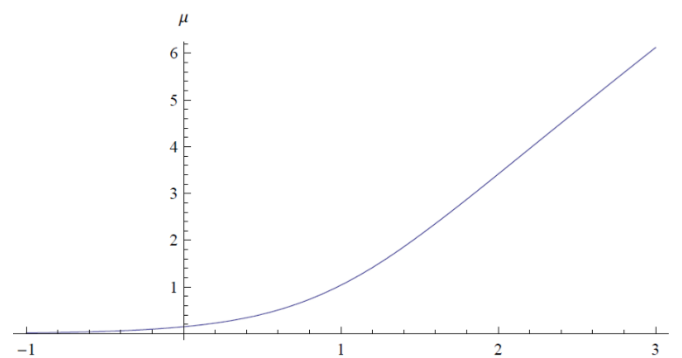
Fig.2: Some trajectories of the flow equation.

truncation	3	4	5	6	7	8
exponent ν	0.74	0.75	0.73	0.73	0.73	0.73
mass $\tilde{\mu}_{eff}$	0.33	0.362	0.384	0.383	0.397	0.397
$-i\psi_{0,diag}$	0.058	0.072	0.074	0.074	0.0.074	0.074
$-iu_0$	0.173	0.213	0.218	0.218	0.218	0.218



$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$





JHEP 1603 (2016) 201

Next Application: Physical Odderon

- We study the effect of the Odderon in the Pomeron

Bartels, Contreras, Vacca 2017

- fixed point analysis indicates:
- ✓ Odderon with intercept one should exist
- ✓ there are no transitions from pure Pomeron states to states containing Odderons (number of Odd. conservation)

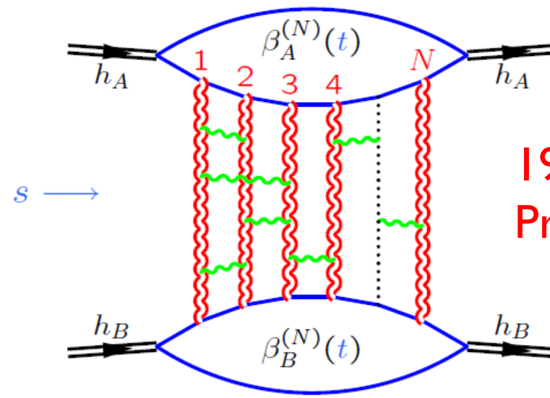
We need more non perturbative information about:

- Intercept
- Slop
- Pomeron Vertices

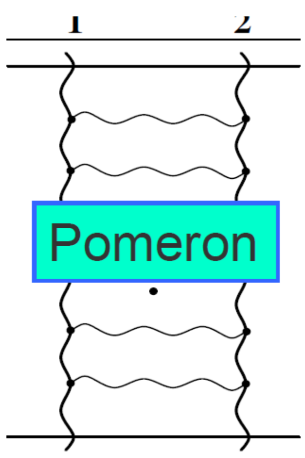
How we can study the Intercept and the Slope?

There are another states with 3, 4 gluons

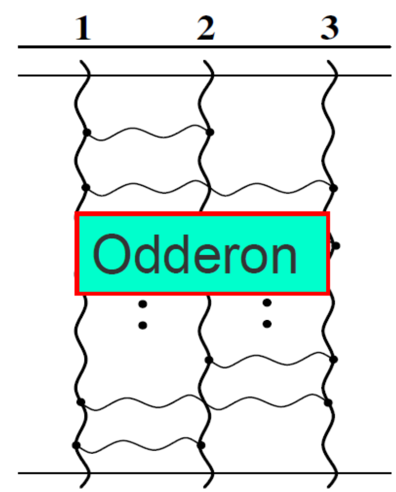
Multi-Reggeons equation BKP



1980 J. Bartels; J. Kwiecinski and M. Praszalowicz



$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

What is new 2018!!

TOTEM total elastic and diffractive cross section Measurement

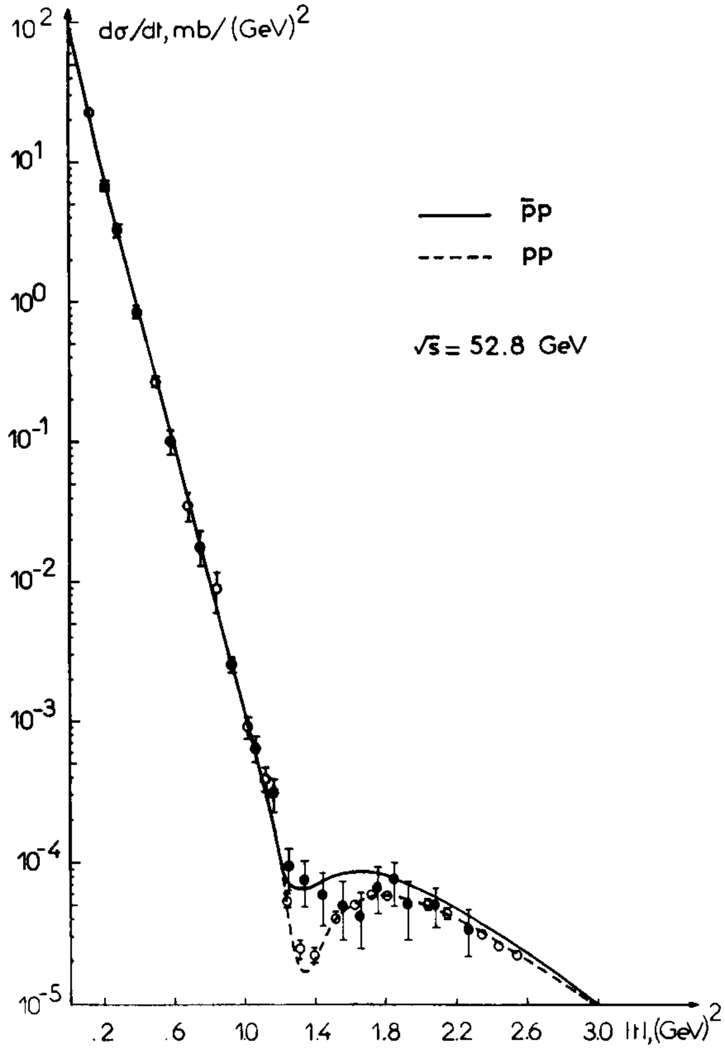
ScienceDaily® (/)

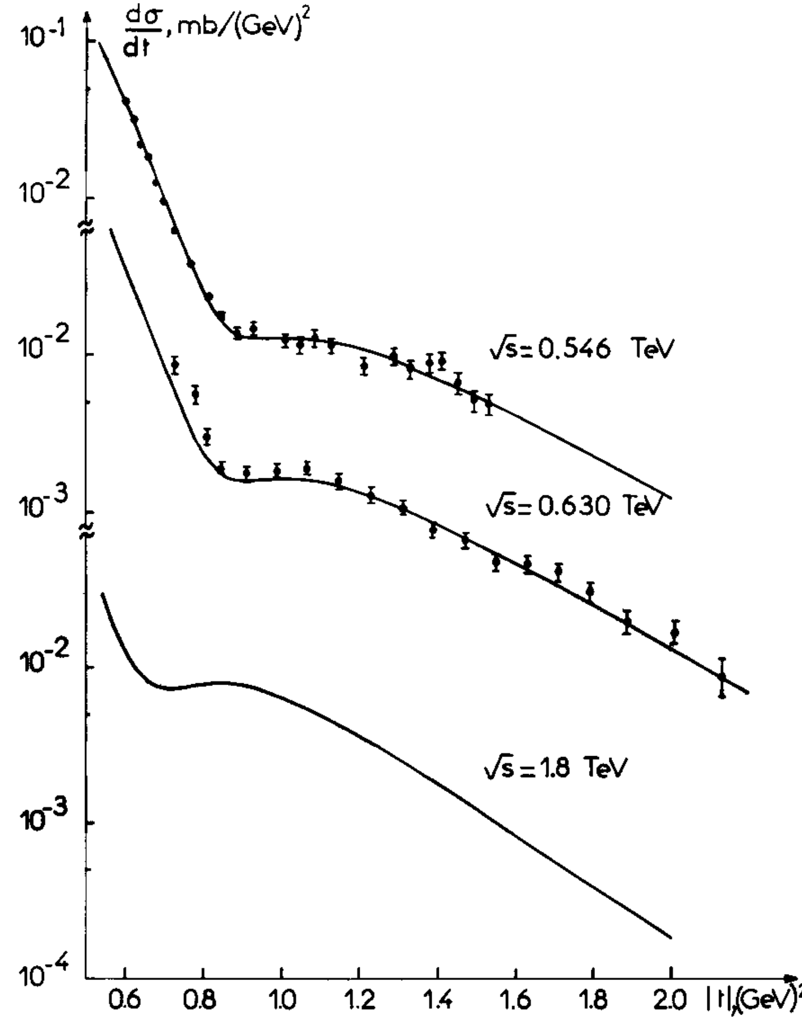
Your source for the latest research news

Meet the 'odderon': Large Hadron Collider experiment shows potential evidence of quasiparticle sought for decades

Date:
February 1, 2018
Source:
University of Kansas
Summary:
A team of high-energy experimental particle physicists has uncovered possible evidence of a subatomic quasiparticle dubbed an 'odderon' that -- until now -- had only been theorized to exist.

FULL STORY





Talk Tuesday: Roman Pasecknik

- This is evidence for the non-perturbative Odderon

21 Jan 2018

Did TOTEM experiment discover the Odderon?

Evgenij Martynov[a], Basarab Nicolescu[b]

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Abstract

The present study shows that the new TOTEM datum $\rho^{pp} = 0.098 \pm 0.01$ can be considered as the first experimental discovery of the Odderon, namely in its maximal form.

Keywords: Froissaron, Maximal Odderon, total cross sections, the phase of the forward amplitude.

1. Introduction

Very recently, the TOTEM experiment released the following values at $\sqrt{s} = 13$ TeV of pp total cross section σ^{pp} and ρ^{pp} parameter [1]

The Odderon is defined as a singularity in the complex j-plane, located at $j = 1$ when $t = 0$ and which contributes to the odd-under-crossing amplitude F_-. It was first introduced in 1973 on the theoretical basis of

Convergence properties of Lévy expansions: implications for Odderon and proton structure

T. Csörgö, R. Pasechnik and A. Ster arXiv:1903.08235

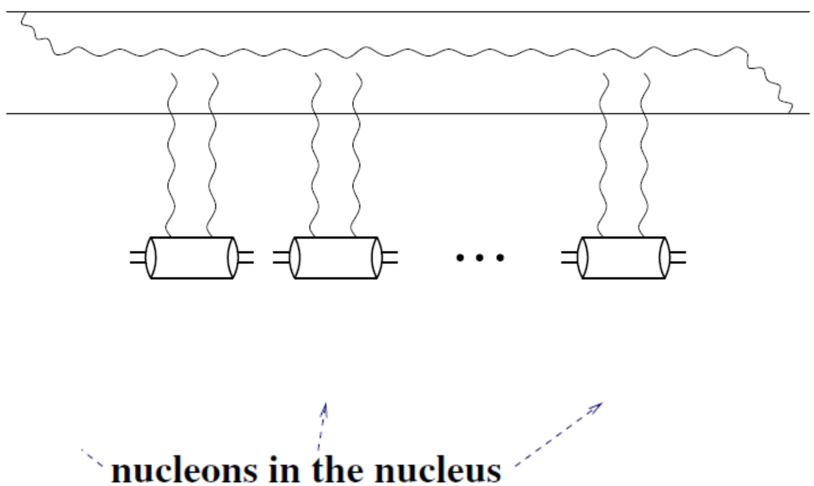
Odderon, HEGS model and LHC data

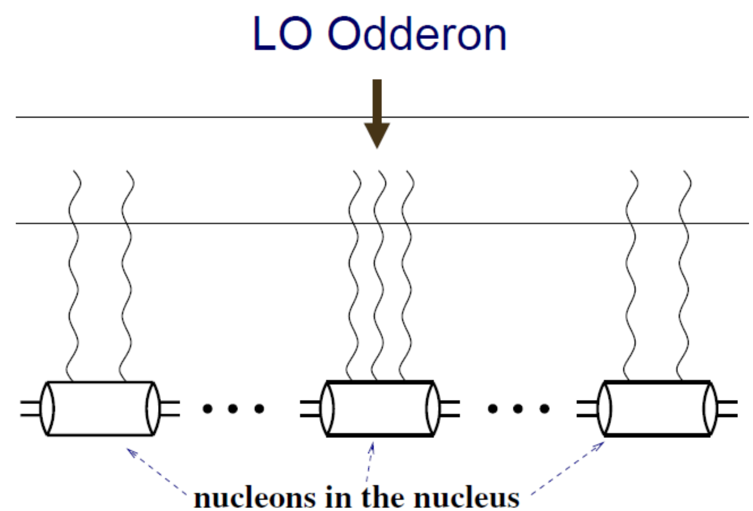
O.V. Selyugin(a) and J.R. Cudell(b) arXiv:1810.11538

Leading Pomeron Contributions and the TOTEM Data at 13 TeV

M. Broilo, E.G.S. Luna, M.J. Menon arXiv:1803.06560

How we can see the Odderon!!





For a nuclear target, two-gluon multiple rescatterings bring in powers of $\alpha_S^2 A^{1/3} \sim 1$, so that single odderon exchange makes the amplitude to be of the order $\alpha_S^3 A^{1/3} \sim \alpha_S << 1$

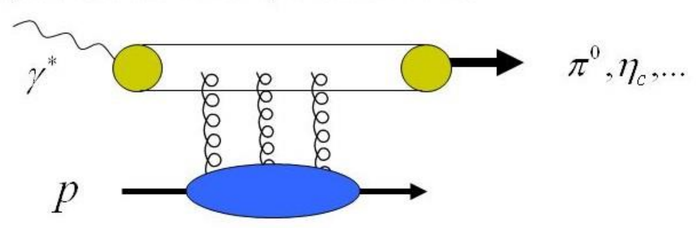
experiment vs. phenomenology

- only few experimental evidences for the non-perturbative Odderon
- We are looking now only in channels where the Odderon is hidden by the huge Pomeron contribution
- We need experimental and theoretical evidence for the perturbative and non perturbative

Searching for the Odderon in Ultraperipheral Proton-Ion Collisions at the LHC non perturbative Odderon.

L.A. Harland-Lang, V.A. Khoze, A.D. Martinb and M.G. Ryskin arXiv:1811.12705

Diffractive pseudoscalar meson production in DIS



Talk Tomorrow: Tomasz Stebel
J/psi and eta_c
Chung I Tan
Pomeron/Odderon Ads/CFT

Talk Today: Antoni Szczurek
Searching Odderon Pion meson

Odderon is a crucial test of QCD

- pQCD the Odderon was studied and is bound state of three reggeized gluons

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

Solutions for the BKP equation:

Hard Odderon

- Janik-Wosiek 1999 with an intercept $\alpha_O = 0.96$
- Bartels, Lipatov, Vacca 2000 with an intercept exactly equal to one
- Variational calculations $\alpha_O > 1$ and $\alpha_O < 1$
- Phenomenology: The slope of the Odderon trajectory is very small : near 0

  B. Nicolescu

- Lattice and Spectroscopy several calculations, all indicating a low intercept.

However, the way in which this intercept is identified in lattice calculations is questionable.

H. B. Meyer and M. J. Teper, Phys. Lett. B605 (2005) 344

H. B. Meyer, PhD thesis at Oxford, hep-lat/0508002

Another way to introduce no perturbative aspects: QCD Description and BFKL kernel

Balitsky Fadin, Kuraev Lipatov 1977

- QCD in the high energy limit behaves in such a way that scattering amplitudes can describe by Pomeron (RFT).
- The BFKL Pomeron which has been studied up to NLO in perturbation theory is a composite states of Reggeized gluons.
  runge Resumation of the Ladder at leading log approximation (multi Regge Kinemmatics MRK) Lipatov, Bartels
- QCD Odderon: bound state of 3 reggeized gluons
- This interaction: local in rapidity and transversal space, can be described by an effective action: Lipatov´s Action
  1995 L. N. Lipatov, Gauge invariant effective action for high-energy processes in QCD
- $S[A, a, ..]$ A reggiezed Gluon and a normal gluons
- Scattering process can be described using BFKL Green Function : G(t ,t´, Y)

$$A(x,Q^2) = \int dt\, dt'\, \Phi_\gamma(Q^2, t)\, G(t, t', Y)\, \Phi_P(t')$$

where Y = ln(1/x) Rapidity

$\Phi_\gamma(Q^2, t)$ describe the coupling of the gluon (perturbatively calculable) with transverse momentum k to a photon of virtuality Q^2 and $\Phi_P(t)$ describes the coupling of a gluon of transverse momentum k´ to the target proton

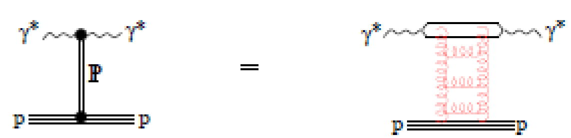
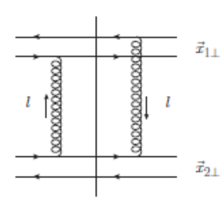
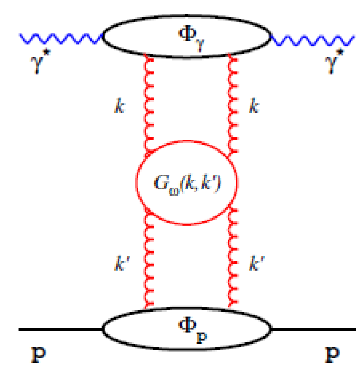


FIG. 4: The virtual photon interacts via its hadronic fluctuations which are $\bar{q}q$ dipoles and more complicated Fock states. The Pomeron exchange is illustrated as a perturbative ladder.





BFKL equation

Balitsky Fadin Kuraev Lipatov

*A(s,***k***,***k***)* **is** the amplitude for the scattering of a gluon with transverse momentum **k** off another gluon with transverse momentum **k**´ at center of mass energy √ *s* and it is found to obey an evolution equation

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') = \delta(k^2 - k'^2) + \int dq^2\, \mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}'),$$

$\widetilde{K}(k,k')$ **is the BFKL kernel**

The kernel is obtained by summing all graphs which contribute an effective "gluon ladder".

Using a Mellin Transformation the Green function evolution can be solved in terms of the $\phi_\omega(k)$ eigenfunctions of the Kernel

$$\omega\, \phi_\omega(k) = \bar{\alpha} \int \frac{d^2k'}{2\pi} \widetilde{K}(k,k')\phi_\omega(k')$$

$$\tilde{K}(\mathbf{q},\mathbf{k},\mathbf{k}') = K(\mathbf{q},\mathbf{k},\mathbf{k}') + \delta^{(2)}(\mathbf{k}-\mathbf{k}')\left(\omega_g(\mathbf{q}_1^2) + \omega_g(\mathbf{q}_2^2)\right)$$

- In more general form we define the BFKL Greens functions G(q´,q - q´; q´´, q´´- q |w) as an infinite sum of ladder diagram and satisfies the integral equation: (Bethe Salpeter resummation)

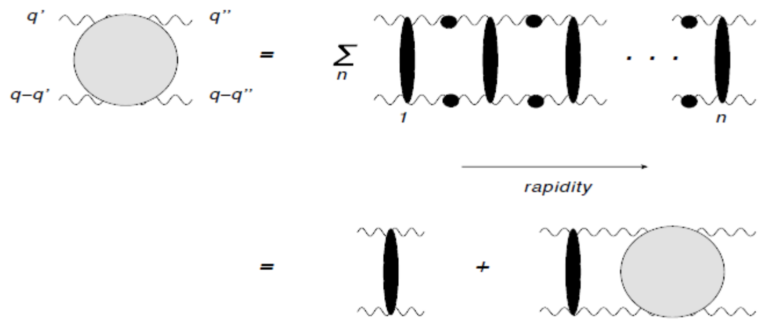


Figure 1: the BFKL ladder

$$G(q', q-q'; q'', q-q''|\omega) = K_{\text{BFKL}}(q, q-q'; q'', q-q'') \qquad (2.1)$$
$$+ \int d^2k K_{\text{BFKL}}(q, q-q'; k, q-k) \frac{1}{k^2(q-k)^2} \frac{1}{\omega - \omega_g(k) - \omega_g(q-k)} G(k, q-k; q'', q-q''|\omega)$$

- Where the wavy denote the Reggeized Gluons: $\frac{1}{k^2}\frac{1}{\omega - \omega_g(k^2)}$.
- And the real part $K_{\text{BFKL}}(q, q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi}\left(-q^2 + \frac{q''^2(q-q')^2}{(q'-q'')^2} + \frac{q'^2(q-q'')^2}{(q'-q'')^2}\right),$
- Virtual part (Gluon trajectory) $\omega_g(q^2) = -q^2\frac{\bar{\alpha}_s}{4\pi}\int d^2k \frac{1}{k^2(q-k)^2}$
- One can reorganize this expression and the final BFKL kernel is given by

$$\tilde{K}_{\text{BFKL}}(q, q-q'', q-q'') = K_{\text{BFKL}}(q, q-q'; q'', q-q'') + \delta^{(2)}(q'-q'')\left(\omega_g(q') + \omega_g(q-q')\right)$$
(2.9)

Regularized equation for BFKL

We need IR regulator in the propagator in order study the ladder diagram in the NLO.

- ✓ **BFKL with IR regulator it not new:**
- ➢ **Lipatov 1986** — **mass regulator**
- ➢ **Braun and Vacca 1999** — **bootstrap conditions**
- ➢ **Levin, Lipatov and Siddikov 2014** — **mass regulator**
- ➢ **Kowalski, Lipatov, Ross 2014** — **boundary conditions**
- ➢ **Bartels, Contreras and Vacca 2018/19** — **Wilsonian or ERG regulator**

- We introduce the following momentum regulator for the propagator for the Gluon

$$\frac{1}{q^2} \to \frac{1}{[q^2 + R_k(q^2)]}$$

$$R_k(q^2) = (k^2 - q^2)\,\theta(k^2 - q^2) \quad \textbf{Wilsonian Regulator}$$

$$R_k(q^2) = M^2 \qquad \textbf{Mass Regulator}$$

Trajectory Gluons

$$\omega_g(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2k \frac{1}{k^2 (q-k)^2}$$

$$\omega_{gk}(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2l \frac{1}{[l^2 + R_k(l^2)]\,[(q-l)^2 + R_k((q-l)^2]}$$

- BFKL Kernel

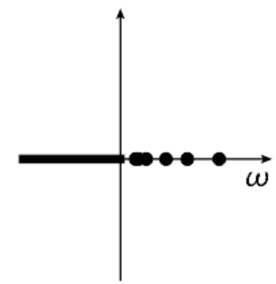
$$K_{\mathrm{BFKL}}(q', q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi} \left(-q^2 \frac{(q'-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2} + \frac{q''^2 (q-q')^2 + q'^2 (q-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2)} \right)$$

What is important in this result

We study the 4 point BFKL kernel in the color singlet state of the t-chanel

- ✓ **the spectral descomposition with a Wilsonian or Mass IR regulated BFKL kernel,**
- ✓ **with running coupling constant**

$$\mathcal{G}_k = \frac{1}{\omega - \tilde{K}} = \sum_n \frac{\psi_{n,k}(q', q-q')\psi^*_{n,k}(q'', q-q'')}{\omega - \omega_{n.k}} + \text{continuous part},$$



$$\omega\, \phi_\omega(k) = \bar{\alpha} \int \frac{d^2 k'}{2\pi} \widetilde{K}(k, k')\phi_\omega(k')$$

- IR Regulator
- Numerical Solution: lattice formulation with N = 600
- Eigenvalues ------ intercept
- Eigenfunctions: bound states
- Running coupling

$$\alpha_s(\mathrm{q}^2) = \frac{3.41}{\beta_0\left[\ln(\mathrm{q}^2 + R_0^2) + \ln\frac{m_h^2}{\Lambda^2_{QCD}}\right]}$$

$$\bar{\alpha}_s(\mathrm{q}^2) = \alpha_s(\mathrm{q}^2)\frac{N_c}{\pi}$$

- ✓ Slope: we leave the forward direction and consider the q^2 dependence of the eigenvalues and the slope is

$$\alpha\,(q^2) = \omega(q^2) \sim \omega^0 + q^2\alpha'$$

$$\alpha' = \frac{d\omega}{dq^2} \quad when \quad q^2 \to 0$$

$$\omega_n(q^2) = \omega_n^{(0)} + q^2 \frac{\int d^2k \int d^2k' f_n(k'^2)\left[K^{(1)}(\mathbf{k},\mathbf{k}') + 2\delta^{(2)}(\mathbf{k}-\mathbf{k}')\omega_g^{(1)}(k^2)\right] f_n(k^2)}{\int d^2k |f_n(k^2)|^2}$$

Mass Regulator

$$\omega_g((\frac{\mathbf{q}}{2}+\mathbf{k}')^2) = -\frac{\bar{\alpha}_s}{2\pi}\int d^2k'' \frac{(\frac{\mathbf{q}}{2}+\mathbf{k}')^2}{D(\mathbf{k}'')\Big[D(\frac{\mathbf{q}}{2}+\mathbf{k}'-\mathbf{k}'')+D(\mathbf{k}'')\Big]}$$

$$\omega_g(\mathrm{k}^2) = -\frac{\bar{\alpha}_s}{4\pi}\int d^2k' \frac{\mathrm{k}^2+m^2}{(\mathrm{k}'^2+m^2)((\mathrm{k}-\mathrm{k}')^2+m^2)}$$
$$= -\frac{\bar{\alpha}_s}{2\pi}\int d^2k' \frac{\mathrm{k}^2+m^2}{(\mathrm{k}'^2+m^2)(\mathrm{k}'^2+(\mathrm{k}-\mathrm{k}')^2+2m^2)}\,.$$

$$\frac{2\pi}{\bar{\alpha}_s}K(\mathrm{q},\mathrm{k},\mathrm{k}') = \sqrt{\frac{\mathrm{q}_1^2+m^2}{\mathrm{q}_2^2+m^2}}\frac{1}{(\mathrm{k}-\mathrm{k}')^2+m^2}\sqrt{\frac{\mathrm{q}_2'^2+m^2}{\mathrm{q}_1'^2+m^2}}$$
$$+\sqrt{\frac{\mathrm{q}_2^2+m^2}{\mathrm{q}_1^2+m^2}}\frac{1}{(\mathrm{k}-\mathrm{k}')^2+m^2}\sqrt{\frac{\mathrm{q}_1'^2+m^2}{\mathrm{q}_2'^2+m^2}}$$
$$-\frac{\mathrm{q}^2+\frac{N_c^2+1}{N_c^2}m^2}{\sqrt{(\mathrm{q}_1^2+m^2)(\mathrm{q}_2^2+m^2)(\mathrm{q}_1'^2+m^2)(\mathrm{q}_2'^2+m^2)}}$$

Numerical Results NLO Pomeron

$$r_n = <\ln \mathbf{q}^2> = \frac{\int d\mathbf{k}^2 |\psi_n(\mathbf{k})|^2 \ln \mathbf{k}^2}{\int d\mathbf{k}^2 |\psi_(\mathbf{k})|^2}$$

	energy			slope			radius		
n	k=0.54	k=1	k=5	k=0.54	k=1	k=5	k=0.54	k=1	k=5
1	-0.53	-0.43	-0.29	0.17	0.034	0.00065	2.2	4.7	3.4×10^{1}
2	-0.25	-0.22	-0.17	0.040	0.0091	0.00022	5.2×10^{1}	1.2×10^{2}	1.0×10^{3}
3	-0.16	-0.15	-0.12	0.017	0.0042	0.00012	1.4×10^{3}	3.3×10^{3}	2.8×10^{4}
4	-0.12	-0.11	-0.097	0.0097	0.0024	0.000070	3.7×10^{4}	8.9×10^{4}	7.8×10^{5}
5	-0.094	-0.089	-0.079	0.0061	0.0016	0.000048	1.0×10^{6}	2.5×10^{6}	2.2×10^{7}
6	-0.077	-0.074	-0.067	0.0042	0.0011	0.000035	2.8×10^{7}	6.7×10^{7}	6.0×10^{8}
7	-0.066	-0.064	-0.058	0.0040	0.00083	0.000026	7.8×10^{8}	1.9×10^{9}	1.7×10^{10}
8	-0.058	-0.056	-0.052	0.0023	0.00064	0.000021	2.2×10^{10}	5.2×10^{10}	4.6×10^{11}

Table 4: Numerical values for eigenvalues, slopes and radii, with running coupling and for different values of the IR cutoff k

Results independent of the regulator

see J. Bartels, C. Contreras G. P Vacca JHEP 1901(2019) 004

Wave Function Pomeron

- The support of the Wave function with n > 2, is defined in the UV region

- N=1 is concentrated in the soft region

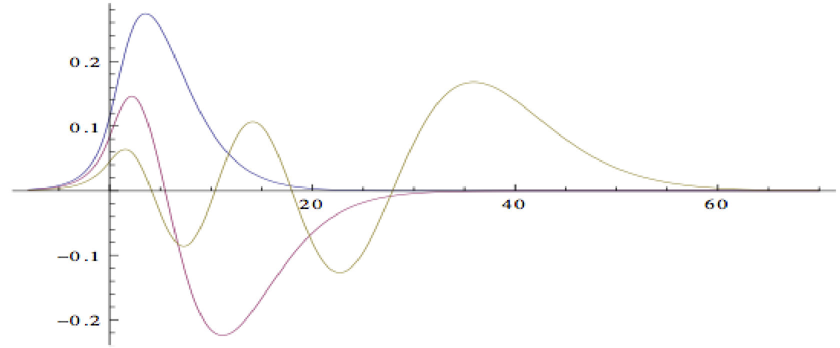


Figure 22: three leading wavefunctions (No 1,2,5) as a function of $\ln q^2$.

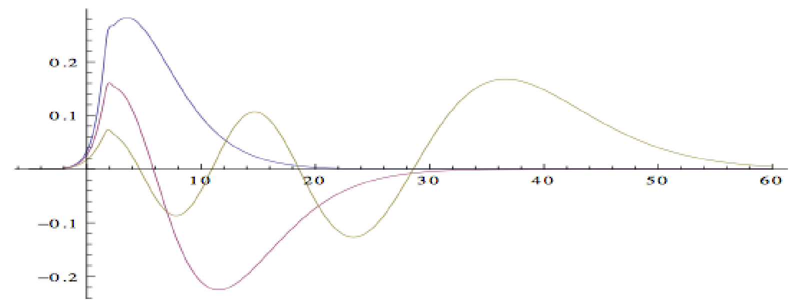


Figure 24: three leading wavefunctions (No 1,2,5) for the scale $k = 1$GeV, as a function of $\ln q^2$

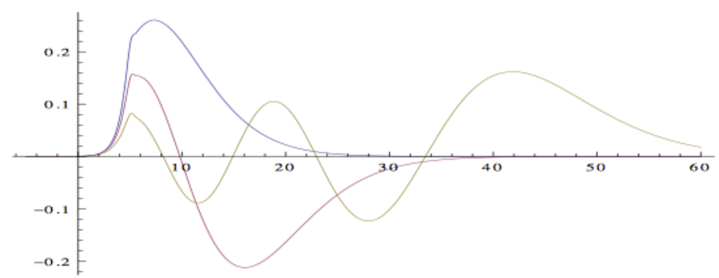


Figure 25: three leading wavefunctions (No 1,2,5) for the scale $k = 5$GeV, as a function of $\ln q^2$

Odderon

a remarkable feature of this solution of the three gluons bound state is that it coincides with the two gluon BFKL solution with conformal spin n=1

$$\omega f(\mathbf{k}) = \frac{\bar{\alpha}_s}{2\pi}\int d^2k'\left[\frac{2f(\mathbf{k}')(\mathbf{k}'^2+m^2)-2f(\mathbf{k})(\mathbf{k}^2+m^2)}{(\mathbf{k}'^2+m^2)((\mathbf{k}-\mathbf{k}')^2+m^2)} - \frac{\frac{N_c^2+1}{N_c^2}m^2}{(\mathbf{k}^2+m^2)(\mathbf{k}'^2+m^2)}f(\mathbf{k}')\right]$$
$$+\frac{\bar{\alpha}_s}{2\pi}\int d^2k'\frac{2f(\mathbf{k})(\mathbf{k}^2+m^2)}{(\mathbf{k}'^2+m^2)(\mathbf{k}'^2+(\mathbf{k}-\mathbf{k}')^2+2m^2)}. \tag{2.10}$$

$$f(\mathbf{k}) = e^{i\varphi}\tilde{f}(|\mathbf{k}|)$$

$$\omega\tilde{f}(|\mathbf{k}|) = \frac{\bar{\alpha}_s}{2\pi}\int d^2k'\left[\frac{2\tilde{f}(|\mathbf{k}'|)e^{i(\varphi'-\varphi)}(\mathbf{k}'^2+m^2)-2\tilde{f}(|\mathbf{k}|)|(\mathbf{k}^2+m^2)}{(\mathbf{k}'^2+m^2)((\mathbf{k}-\mathbf{k}')^2+m^2)}\right]$$
$$+\frac{\bar{\alpha}_s}{2\pi}\int d^2k'\frac{2\tilde{f}(|\mathbf{k}|)(\mathbf{k}^2+m^2)}{(\mathbf{k}'^2+m^2)(\mathbf{k}'^2+(\mathbf{k}-\mathbf{k}')^2+2m^2)}\Big].$$

Why this kind of solutions

$$f(\mathbf{k}) = e^{i\varphi} \tilde{f}(|\mathbf{k}|)$$

- ✓ Lipatov found using conformal algebra that the Eigen function of the BFKL Kernel for fixed coupling without regulator

$$E^{n,\nu}(\rho_0,\rho_1) = \left(\frac{\rho_{01}}{\rho_0\,\rho_1}\right)^{\frac{1+n}{2}+i\nu} \left(\frac{\rho^*{}_{01}}{\rho^*{}_0\,\rho^*{}_1}\right)^{\frac{1-n}{2}+i\nu}$$

- ✓ And the Eigenvalues

$$\chi(n,\nu) = \psi(1) - \frac{1}{2}\psi\left(\frac{1+|n|}{2}+i\nu\right) - \frac{1}{2}\psi\left(\frac{1+|n|}{2}-i\nu\right)$$

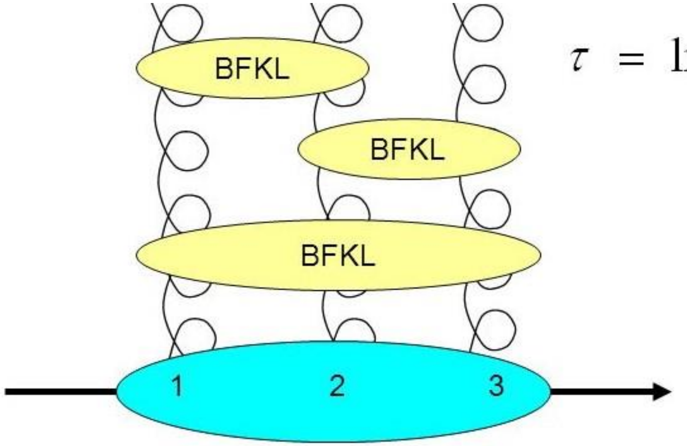
- ✓ The Odderon solution look like

$$\int_{-\infty}^{\infty} d\nu \sum_{odd\,n} \exp\left[\frac{2\alpha_S N_C}{\pi}\chi(n,\nu)Y\right] E^{n,\nu}(\rho_0,\rho_1)$$

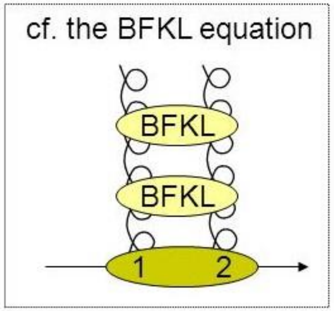
- ✓ Using saddle point in the integration at large rapidity Y, the dominant contribution correspond to *n =1*

The BKP Equation for the Odderon:

the exchange of the BFKL kernel between all possible pairs in one step of evolution



$\tau = \ln s$: rapidity



Numerical Results Intercept

$E_1 = 0.000032, \quad E_2 = 0.000289, \quad E_3 = 0.000802$

Espectrum

- E_n =- ω_n , start in a very small Leading eigenvalues
- Finite number of eigenvalues (lattice size) and end at maximal positive values

We know that

- ω_{Odderon} = 0.00003 for α_s= 0.2
- Odderon´s accumulate at zero and then the cut stars at ω_O=0

➢ Lattice size effect

➢ Discrete eigenvalues are correct: spacing between E_n is larger to the lattice size

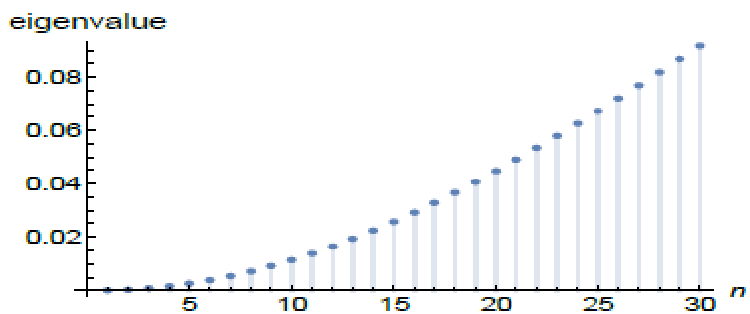


Figure 1: The first 30 eigenvalues of the Odderon with fixed coupling

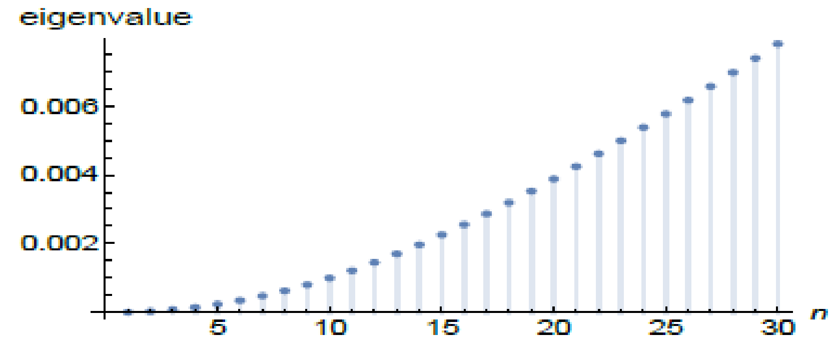


Figure 5: The first 30 eigenvalues

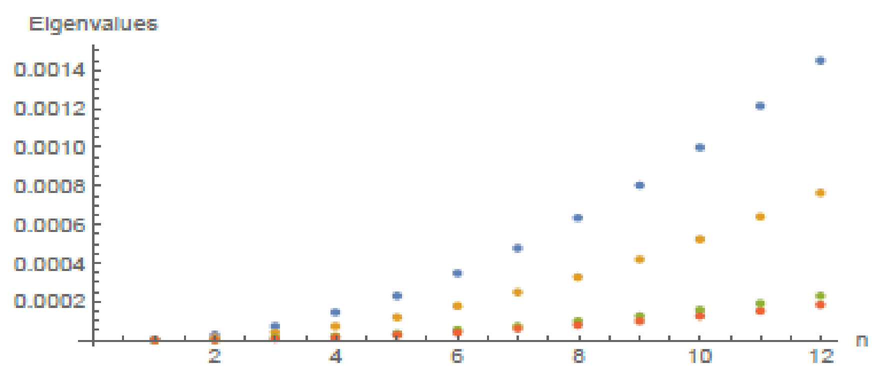
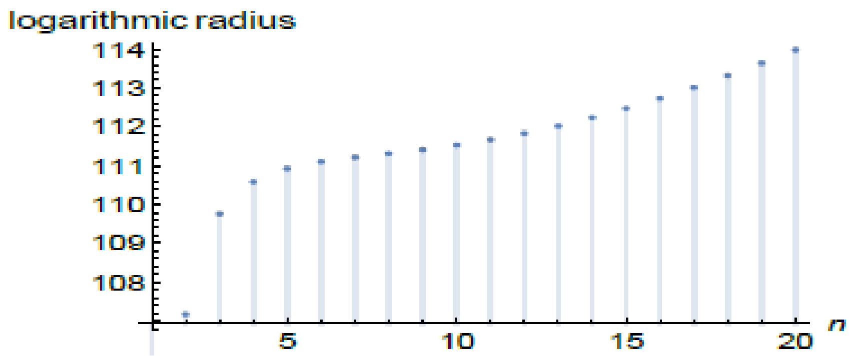


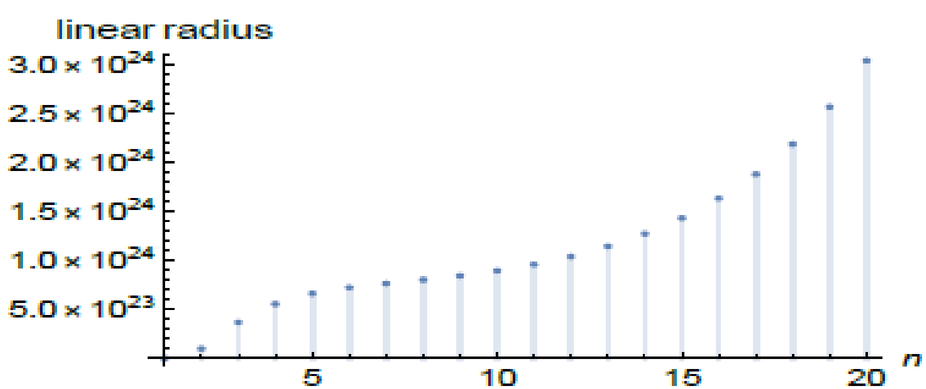
Figure 7: Behavior of the eigenvalues when we increase the larger of the lattice, $t_{max} = 80, 100, 150$ and 170.

Numerical Results

$$r_n = <\ln \mathbf{q}^2> = \frac{\int d\mathbf{k}^2 |\psi_n(\mathbf{k})|^2 \ln \mathbf{k}^2}{\int d\mathbf{k}^2 |\psi_(\mathbf{k})|^2}$$



$$< ln\, k^2 >_1 = 87.97$$
$$< ln\, k^2 >_2 = 107.26$$
$$< ln\, k^2 >_3 = 109.86\,,$$



$$r_1 = 6.83 \times 10^{18}\, GeV$$
$$r_2 = 1.06 \times 10^{23}\, GeV$$
$$r_3 = 3.89 \times 10^{23}\, GeV.$$

Wave Function

- The support of the
 Wave function with n > 2,
is defined in the UV region

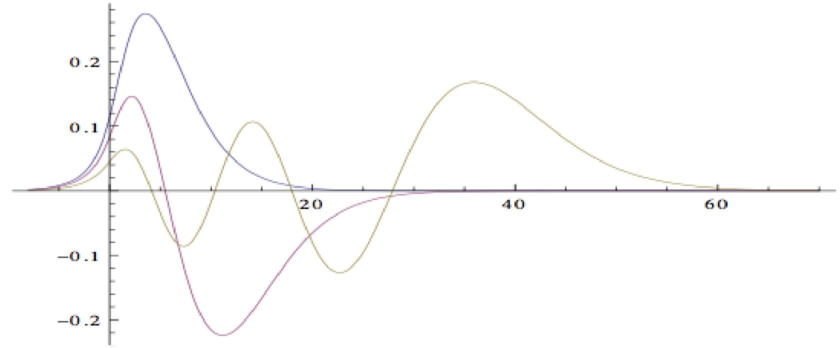


Figure 22: three leading wavefunctions (No 1,2,5) as a function of $\ln q^2$.

- The support of the
 Wave function with n, is
defined in the all region

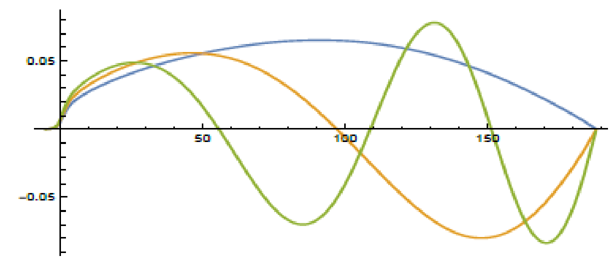


Figure 2: The first three absolute value of the wavefunction as a function of lnq^2

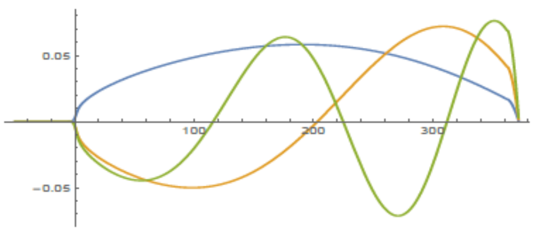


Figure 8: Behavior of the wave function for larger of the lattice $t_{max} = 170$.

Numerical Results Slope

Very small value

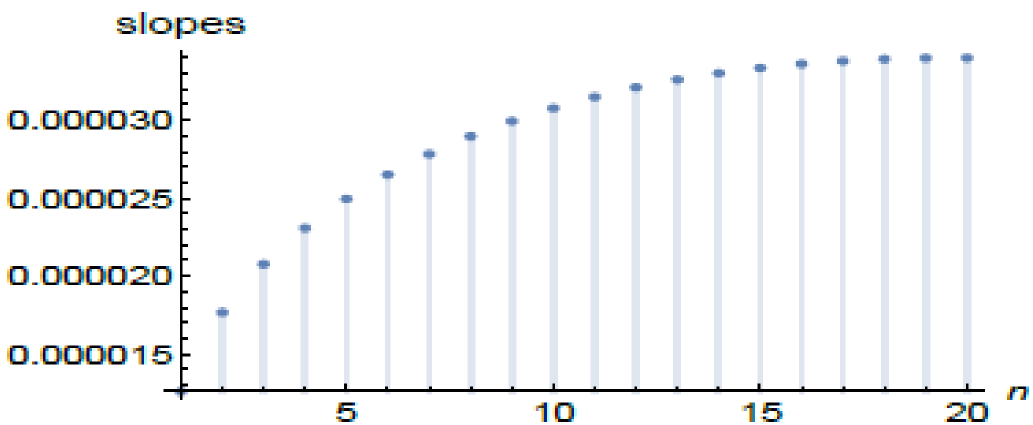


Figure 6: q^2-slopes

Summary* and outlook

- Using Numerical analysis
- Massive infrared regulator
- Running coupling constant

Eigenvalues are consistent with the BLV solution

Slopes are relative small and different from Pomeron

WF: all the leading eigenstate are in UV region

The slope of the Odderon trajectory is very small: near 0

*M. Braun and G. P. Vacca similar result using bootstrap (private communication)

In the future:

These Pomeron and Odderon solutions can help to study the N-Green function

We can study Triple Pomeron - Odderon Vertex.

Thank you

$$\mathcal{G}(t, t', Y) \; = \; \sum_{n=1}^{\infty} x^{-\omega_n} f_{\omega_n}((t) f^*_{\omega_n}(t')$$