Recent Progresses on quasi Parton Distribution Functions

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Outline

- Parton Distribution Functions
- Quasi PDF and LaMET
- Brief Results for quark PDFs
- Gluon quasi PDF: renormalization
- Summary

Disclaimer: There are many other proposals on PDFs, but will not covered in the talk
Factorization theorems:

\[ d\sigma \sim \int dx_1 dx_2 \cdot f(x_1) \cdot f(x_2) \cdot C(x_1, x_2, Q) \]

PDF: basic inputs for particle physics at hadron colliders.
Figure 18.5: The bands are x times the unpolarized (a,b) parton distributions \( f(x) \) where \( f = u, v, d, s, \bar{u}, c, \bar{c}, b, g \) obtained in N N L O N N P D F 3.0 global analysis [56] at scales \( \mu^2 = 10 \text{ GeV}^2 \) (left) and \( \mu^2 = 10^4 \text{ GeV}^2 \) (right), with \( \alpha_s(M_Z^2) = 0.118 \). The analogous results obtained in the NNLO MMHT analysis can be found in Fig. 1 of Ref [55]. The corresponding polarized parton distributions are shown (c,d), obtained in NLO with NNPDFpol1.1 [15].
PDF updates

✦ Percent-level precision of PDFs requires careful evaluations of various systematic uncertainty sources from data and theory at the LHC. Main challenging are inclusion of theoretical uncertainties, possible tensions between different data sets and correlations of experimental systematics.

From Jun Gao
PDF From First Principle?

• Fitting Results rely on data

• First-principle calculation can cover regions where experiments cannot constrain so well

• The cost of improving calculations could be much lower than building large experiments.
PDF at large $x$ gives dominant errors: important to study heavy particles.
Lattice QCD (K.G. Wilson, 1974)

- Numerical simulation in discretized Euclidean space-time
- Finite volume (L should be large)
- Finite lattice spacing (a should be small)

Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.
Lattice QCD: PDF?

PDF (or more general parton physics):
Minkowski space, real time
infinite momentum frame, on the light-cone

Lattice QCD:

Euclidean space, imaginary time (t=i*tau)
Difficulty in time
\[ x_\mu^E x_\mu^E = 0, \quad x_\mu^E = (0,0,0,0) \]

Unable to distinguish local operator and light-cone operator
Sign problem in simulating real-time dynamics.
Lattice QCD: PDF?

One can form local moments to get rid of the time-dependence

- $\langle x^n \rangle = \int f(x)x^n \, dx$ : matrix elements of local operators
- However, one can only calculate lowest few moments in practice.
- Higher moments quickly become noisy.

\[
\int_0^1 dx \, x^n q(x, \mu) dx = a_n(\mu) \propto \langle P | \psi(0) \gamma^+ i\bar{D}^+ \cdots i\bar{D}^+ \psi(0) | P \rangle
\]
Quasi Parton Distribution Functions
and
Large Momentum Effective Theory
(LaMET)

Quasi-PDFs

\[ \tilde{q}(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^z \times \exp \left( -ig \int_0^z dz' A^z(z') \right) | \psi(0) \rangle P \]

Frame transformation:

PDF: light-cone correlation

equal time correlation
Quasi-PDFs

PDF:
light-cone separation;
Cannot be calculated on the lattice

Lorentz boost

Quasi-PDF :
Equal-time correlation;
Directly calculable on the lattice

\[
\tilde{q}(x, \mu^2, P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z \left( \frac{x}{y} \right) \frac{\mu}{P^z} q(y, \mu^2) + O \left( \Lambda^2 / (P^z)^2, M^2 / (P^z)^2 \right) ,
\]
Quasi-PDFs: Finite but large $P_z$

- The distribution at a finite but large $P_z$ shall be calculable in lattice QCD.

- Since it differs from the standard PDF by simply an infinite $P_z$ limit, it shall have the same infrared (collinear) physics.

- It shall be related to the standard PDF by a matching factor $Z\left(\frac{\mu}{P_z}\right)$ which is perturbatively calculable.

$$Z(x, \mu/P_z) = \delta(x - 1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P_z) + ...$$
Many Progress have been made on quasi PDFs, see Reviews:
Zhao, Int.J.Mod.Phys. A33 (2019);
Alexandrou et al. (ETMC), 1902.00587.

- Formalism: factorization, renormalization, power corrections
- Matching: perturbative corrections to $Z$
- Lattice QCD calculations
Progress on quasi PDF

Lattice Collaboration working on quasi-PDFs:

- **Lattice Parton Physics Project (LP3) Collaboration**

- **European Twisted Mass Collaboration (ETMC)**
  C. Alexandrou (U. Cyprus), M. Constantinou (Temple U.), K. Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.

- **DESY, Zeuthen** J. Green, et al.

- **Brookhaven group**
Progresses on quasi PDF: quark

\[ u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x) \]

LP3: 1803.04393

ETMC: 1803.02685
Progresses on quasi PDF

✓ Gluons: Renormalizability; Perturbative Matching, Lattice Simulations

✓ Generalized PDF, measurable in hard exclusive processes such as deeply virtual Compton scattering:
  - Preliminary results for quasi-GPDs (ETMC), M. Constantinou’s talk at QCD Evolution 2019.

✓ Transverse Momentum Dependent PDF: definition,
  - Ji, Sun, Xiong and Yuan, PRD91 (2015);
  - Ji, Jin, Yuan, Zhang and Zhao, PRD99 (2019);
  - M. Ebert, I. Stewart, Zhao PRD99 (2019);
  - ....
Gluon quasi PDF: Renormalization

WW, Zhao, Zhu, 1708.02458
WW, Zhao, 1712.03830
Zhang, Ji, Schafer, WW, Zhao, 1808.10824
WW, Zhang, Zhao, Zhu, 1904.00978

See also Li, Ma, Qiu, 1809.01836
Gluon PDF

Higgs Production:
gluon-gluon fusion

Cross sections are calculated by Zürich group at N³LO QCD and NLO EW accuracies [Anastasiou:2016cez]
mH=125.09 GeV, √s=13 TeV

σ=48.52pb

Total Uncertainty: 3.9% (Gaussian)
PDF: 1.9%
αs: 2.6%
Definition of quasi and light-cone gluon distribution

\[ f_{g/H}(x, \mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ix\xi^-p^+} \langle P|F^+_i(\xi^-)W(\xi^-, 0, L_{n^+})F^{i^+}(0)|P \rangle \]

\[ f_{g/H}(x, \mu) = \int \frac{dz}{2\pi x P^z} e^{-ixzp^z} \langle P|F^z_i(z)W(z, 0, L_{n^z})F^{i^z}(0)|P \rangle \]

- Field Strength Tensor: F
- \(i\) sums over transverse directions (i=1,2) or full directions
- \(W(z_1, z_2, C)\) is a Wilson line along contour C.
Wilson line

\[ W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1) \bar{\mathcal{Z}}(\lambda_2) \rangle_z. \]

Gauge invariant non-local operators
pairs of gauge invariant composite local operators

\[ F^a_{\mu\nu}(z_1) W_{ab}(z_1, z_2; C) F^b_{\rho\sigma}(z_2) = \langle (F^a_{\mu\nu}(z_1) \mathcal{Z}_a(\lambda_1)) | (\mathcal{Z}_b(\lambda_2) F^b_{\rho\sigma}(z_2)) \rangle \]
\[ = \Omega^{(1)}_{\mu\nu}(z_1) \Omega^{(1)}_{\rho\sigma}(z_2) \]
\[ \Omega^{(1)}_{\mu\nu}(z_1) = F^a_{\mu\nu}(z_1) \mathcal{Z}_a(\lambda_1) \]

Renormalization of gluon PDF:
Auxiliary Field

Gervais and Neveu, 1980
Renormalization of gluon PDF:
One Loop diagrams

\[ I_1 = \frac{\alpha_s C_A}{\pi} \left\{ \frac{1}{4-d} \left( A_a^{\nu} n^\mu - A_a^{\mu} n^\nu \right) n \cdot \partial Z_a / n^2 - \frac{\pi \mu}{3-d} (n^\mu A_a^{\nu} - n^\nu A_a^{\mu}) Z_a + \text{reg.} \right\}, \]

\[ I_2 = \frac{\alpha_s C_A}{\pi} \left\{ \frac{1}{4-d} \left[ \frac{1}{4} F_a^{\mu \nu} Z_a + \frac{1}{2} (F_a^{\mu \rho} n_\nu n_\rho - F_a^{\nu \rho} n_\mu n_\rho) / n^2 + \frac{1}{2} (A_a^{\mu} n^\nu - A_a^{\nu} n^\mu) n \cdot \partial Z_a / n^2 \right] + \frac{\pi \mu}{3-d} (n^\mu A_a^{\nu} - n^\nu A_a^{\mu}) Z_a + \text{reg.} \right\}, \]

No power divergence!
Renormalization of gluon quasi-PDF

Three operators with the same quantum number

\[
\begin{align*}
\Omega^{(1)}_{\mu\nu} &= F_{\mu\nu} \mathcal{Z}_a, \\
\Omega^{(2)}_{\mu\nu} &= \Omega^{(1)}_{\mu\alpha} \frac{\dot{x}_\alpha \dot{x}_\nu}{\dot{x}^2} - \Omega^{(1)}_{\nu\alpha} \frac{\dot{x}_\alpha \dot{x}_\mu}{\dot{x}^2}, \\
\Omega^{(3)}_{\mu\nu} &= |\dot{x}|^{-2} (\dot{x}_\mu A^a_\mu - \dot{x}_\nu A^a_\nu) (D \mathcal{Z})_a,
\end{align*}
\]

\[
\begin{pmatrix}
\Omega^\mu_{1,R} \\
\Omega^\mu_{2,R} \\
\Omega^\mu_{3,R}
\end{pmatrix} =
\begin{pmatrix}
Z_{11} & Z_{22} - Z_{11} & Z_{13} \\
0 & Z_{22} & Z_{13} \\
0 & 0 & Z_{33}
\end{pmatrix}
\begin{pmatrix}
\Omega^\mu_{1,R} \\
\Omega^\mu_{2,R} \\
\Omega^\mu_{3,R}
\end{pmatrix}.
\]

Different components are renormalized differently!

\[
\begin{pmatrix}
\Omega^{z\mu}_{1,R} \\
\Omega^{z\mu}_{3,R}
\end{pmatrix} =
\begin{pmatrix}
Z_{22} & Z_{13} \\
0 & Z_{33}
\end{pmatrix}
\begin{pmatrix}
\Omega^{z\mu}_{1} \\
\Omega^{z\mu}_{3}
\end{pmatrix};
\]

\[
\Omega^{t_i}_{1,R} = Z_{11} \Omega^{t_i}_{1}.
\]
Renormalization of gluon PDF: 
Multiplicatively Renormalizable Operators

\[ O^{(1)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F^t_i(z_2), \]
\[ O^{(2)}(z_1, z_2) \equiv F^{zi}(z_1)L(z_1, z_2)F_i^z(z_2), \]
\[ O^{(3)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^z(z_2), \]
\[ O^{(4)}(z_1, z_2) \equiv F^{z\mu}(z_1)L(z_1, z_2)F_\mu^z(z_2), \]

Four multiplicative Renormalizable operators can be used to define gluon quasi-PDfs

Zhang, Ji, Schafer, WW, Zhao,1808.10824 (PRL 2019)
Fan, Yang, Anthony, Lin, Liu, 1808.02077

$\tilde{H}_0(z, P_z) = \langle P | O_0(z) | P \rangle,$

$$O_0 \equiv \frac{P_0 \left( O \left( F_\mu^t, F_\mu^\nu; z \right) - \frac{1}{4} g^{tt} O \left( F_\nu^\mu, F_\mu^\nu; z \right) \right)}{\frac{3}{4} P_0^2 + \frac{1}{4} P_z^2}.$$ 

In future:
- More precise
- Physical Pion
- Large Momentum
- Quark-gluon Mixing
Summary

LaMET: Parton physics demands new ideas to solve non-perturbative QCD. Many Progresses are achieved.

Gluon Quasi PDF:
Renormalizability; RI/MOM subtraction; Factorization; One-loop matching; polarized PDF;
Mixing on the lattice; BRST/ghost on lattice ($p^2/\epsilon$);

In near future, we expect:
✓ Lattice calculation of quark PDFs: 10%
✓ Better constraints $x \sim 1$
✓ Distributions: gluon, TMD, GPD

Thank you very much!