Recent Progresses on quasi Parton Distribution Functions

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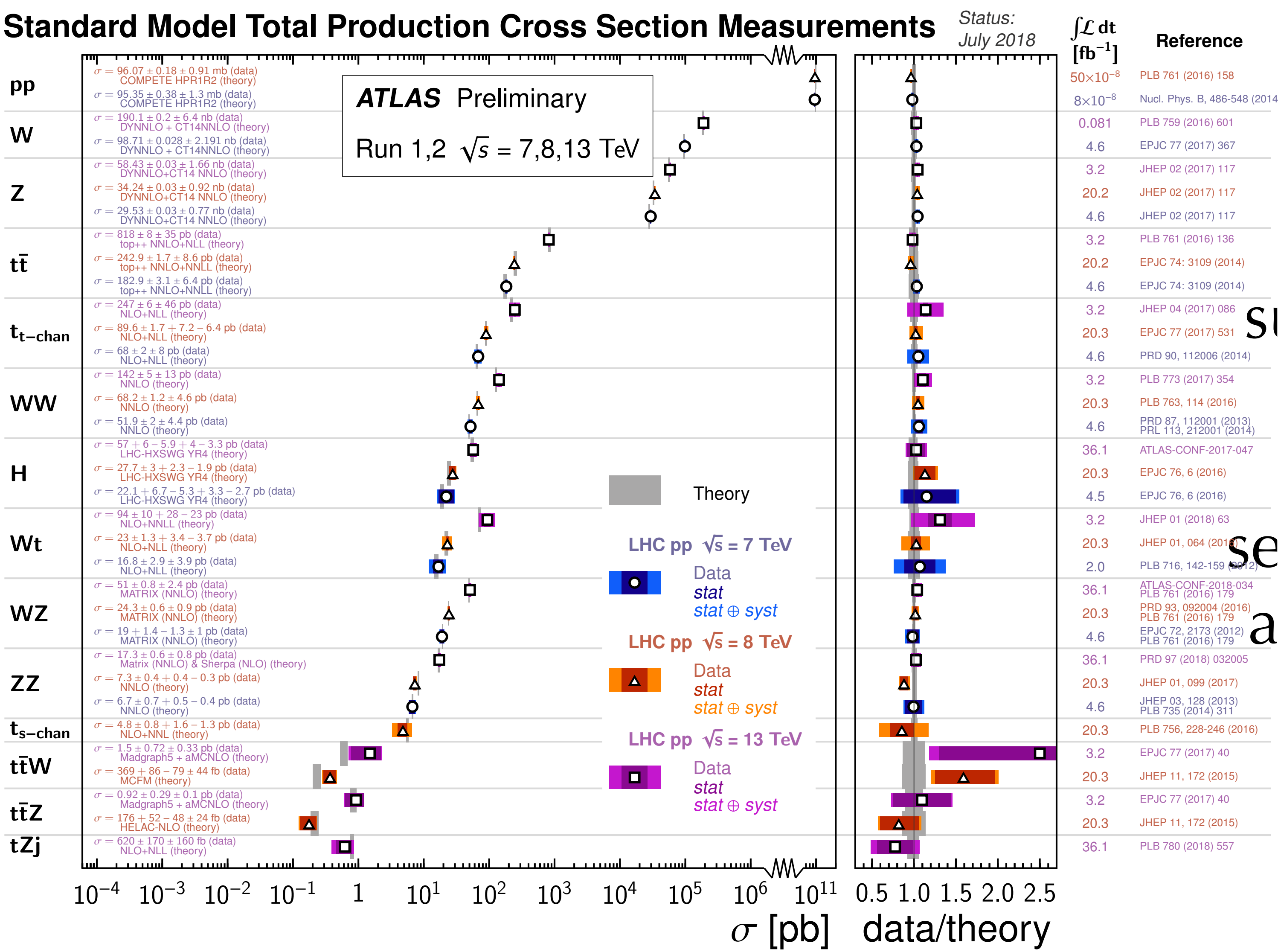
Outline

- **Parton Distribution Functions**
- **Quasi PDF and LaMET**
- **Brief Results for quark PDFs**
- **Gluon quasi PDF: renormalization**
- **Summary**

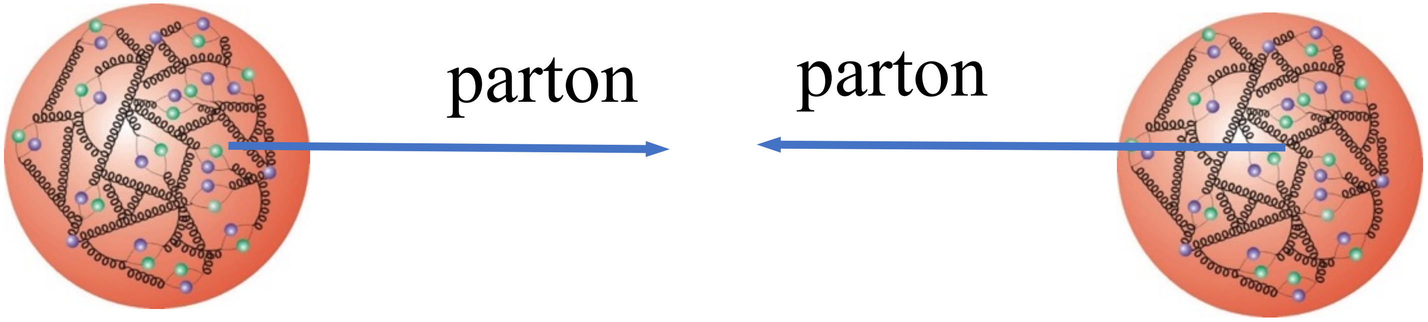
Disclaimer: There are many other proposals on PDFs, but will not covered in the talk

Success of the Standard Model(SM)



success of EW and flavor sectors but also QCD

Factorization: Parton Model; PDF

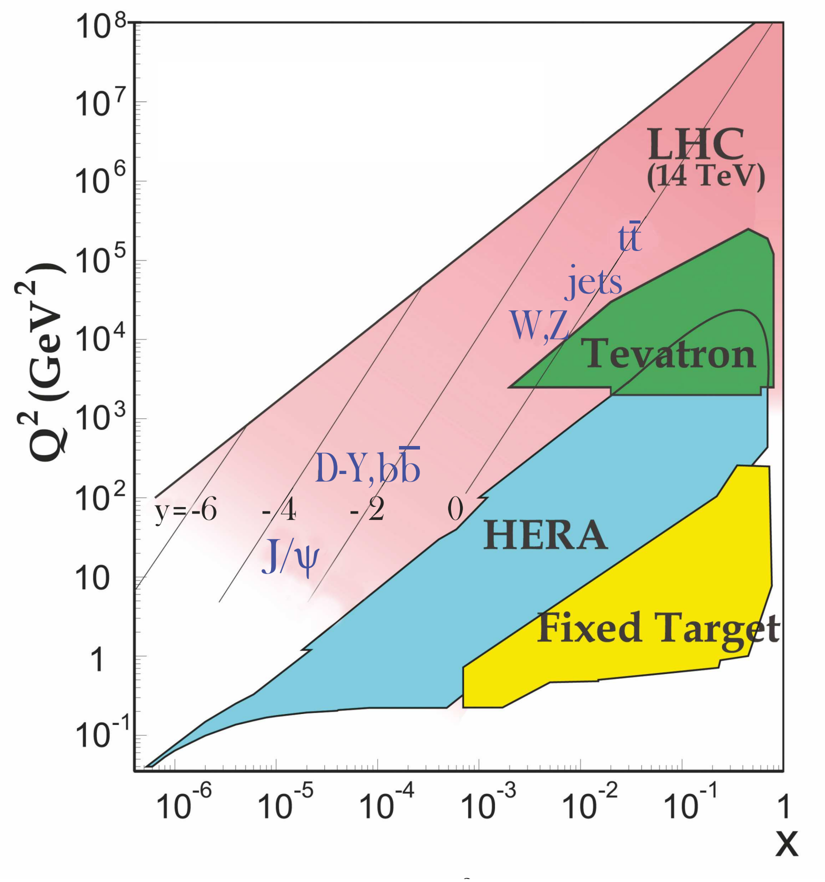


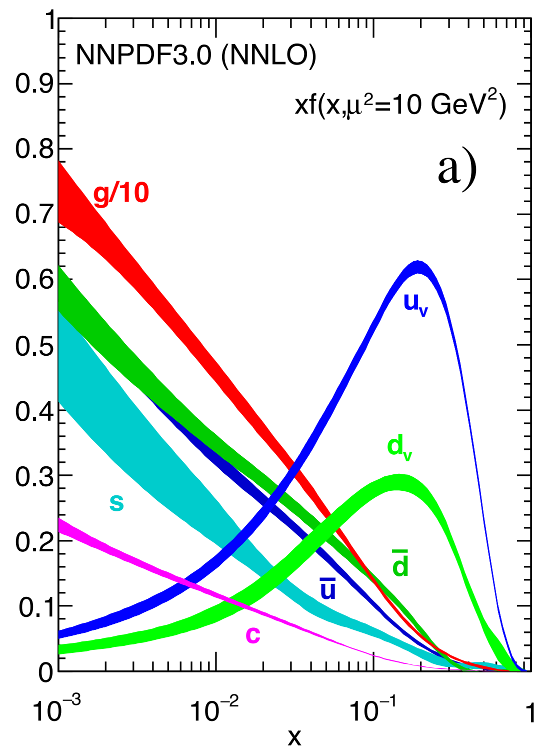
Factorization theorems:

$$d\sigma \sim \int dx_1 dx_2 * f(x_1) * f(x_2) * C(x_1, x_2, Q)$$

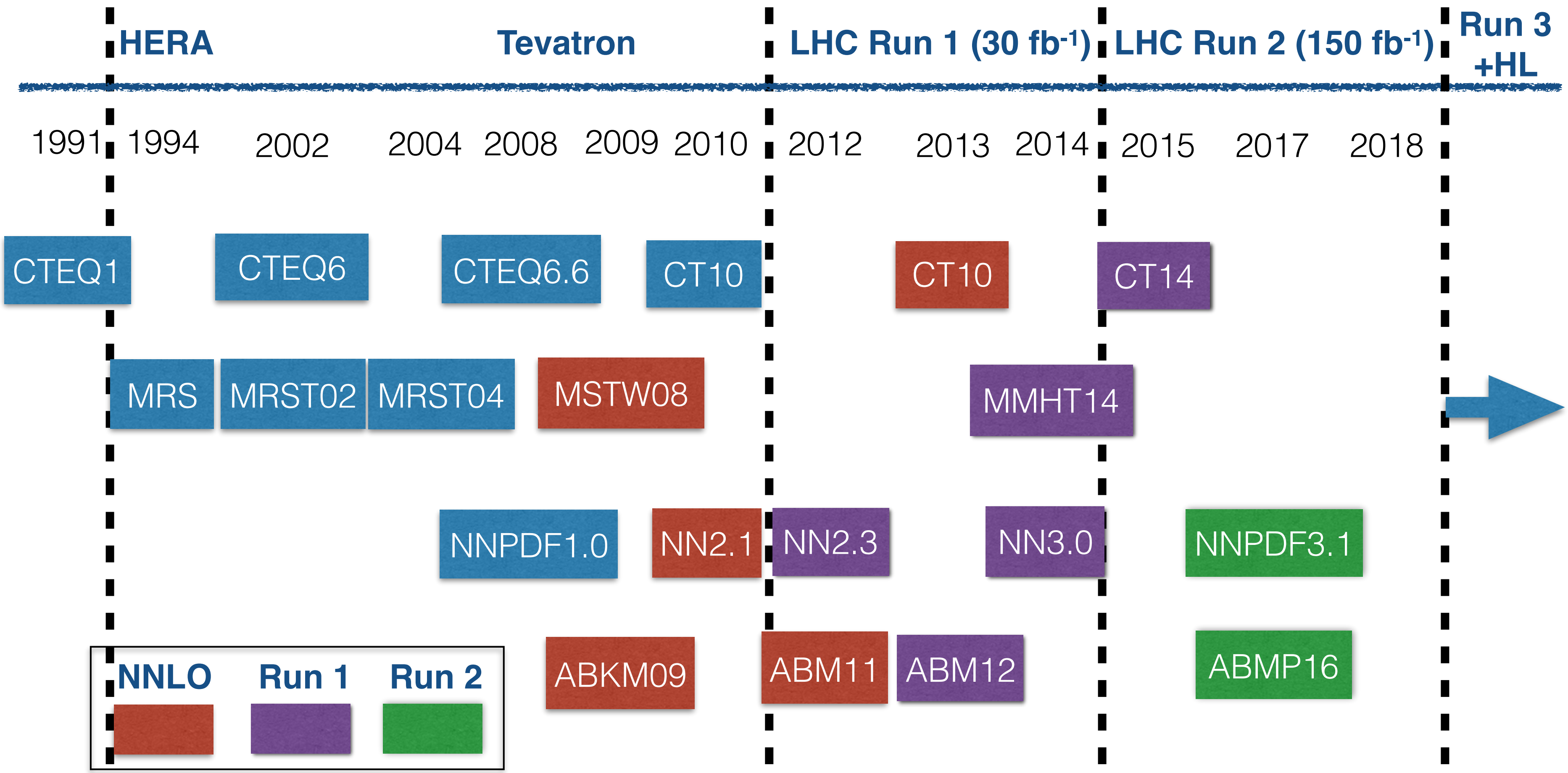
PDF: basic inputs for particle physics at hadron colliders.

Global Fit of Data





Global Fit of Data



From Jun Gao

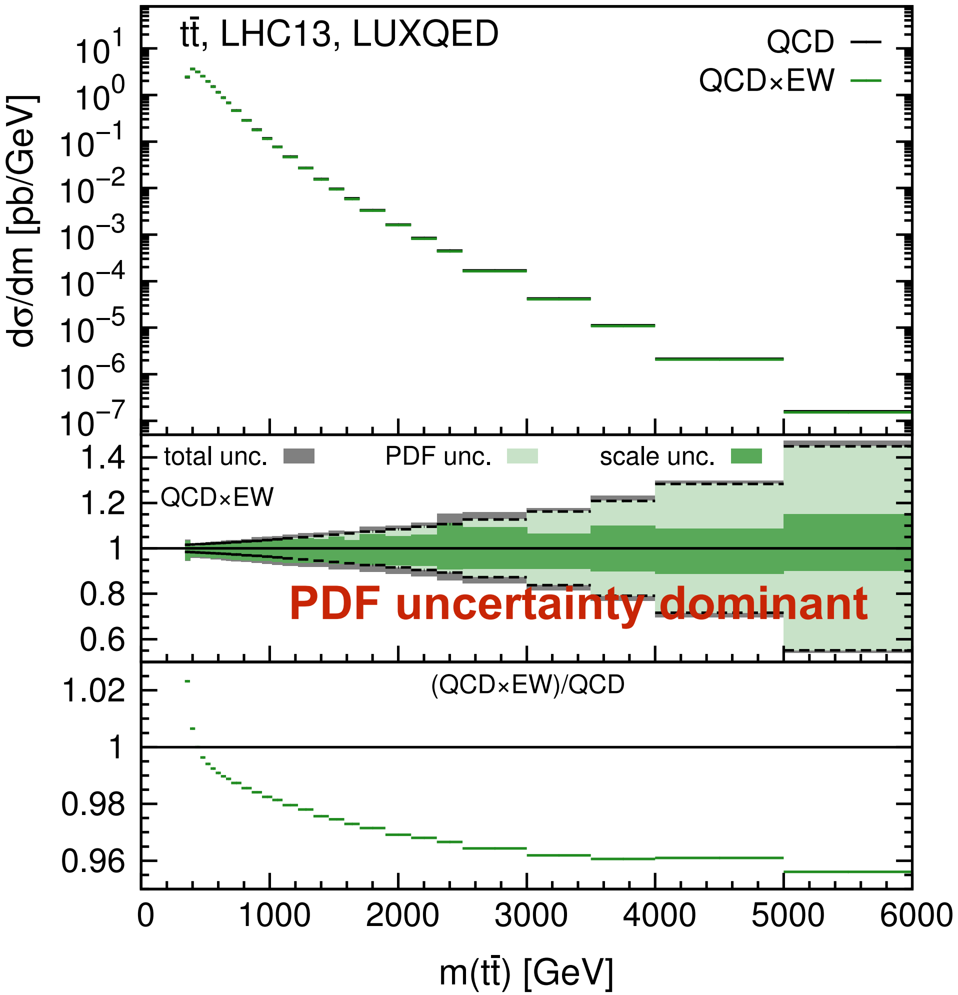
PDF From First Principle?

- Fitting Results rely on data

- First-principle calculation can cover regions where experiments cannot constrain so well

- The cost of improving calculations could be much lower than building large experiments.

Gluon PDF



1705.04105v2

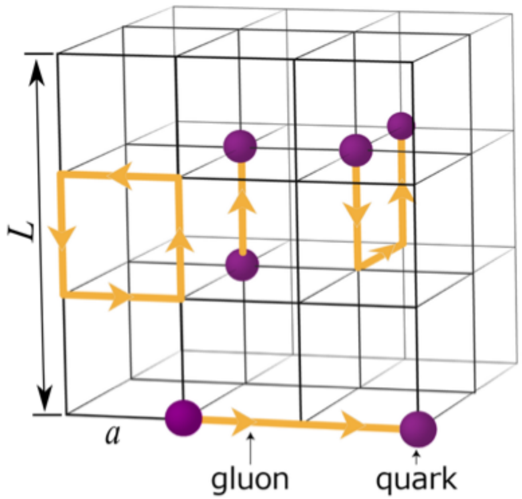
PDF at large x gives dominant errors: important to study heavy particles.

Lattice QCD(K.G.Wilson,1974)

- Numerical simulation in discretized Euclidean space-time

- Finite volume (L should be large)

- Finite lattice spacing (a should be small)



Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.

Lattice QCD: PDF?

PDF (or more general parton physics):
Minkowski space, real time
infinite momentum frame, on the light-cone

Lattice QCD:

Euclidean space, imaginary time (t=i*tau)
Difficulty in time

$$x_E^{\mu}\, x_E^{\mu} = 0,\ x_E^{\mu} = (0,0,0,0)$$

Unable to distinguish local operator and light-cone operator
Sign problem in simulating real-time dynamics.

Lattice QCD: PDF?

One can form local moments to get rid of the time-dependence

- $\langle x^n \rangle = \int f(x) x^n dx$: matrix elements of local operators
- However, one can only calculate lowest few moments in practice.
- Higher moments quickly become noisy.

$$\int_0^1 dx\, x^n q(x,\mu) dx = a_n(\mu) \propto \left\langle P \middle| \bar{\psi}(0)\gamma^+ \overbrace{i\overleftrightarrow{D}^+ \cdots i\overleftrightarrow{D}^+}^{n} \psi(0) \middle| P \right\rangle$$

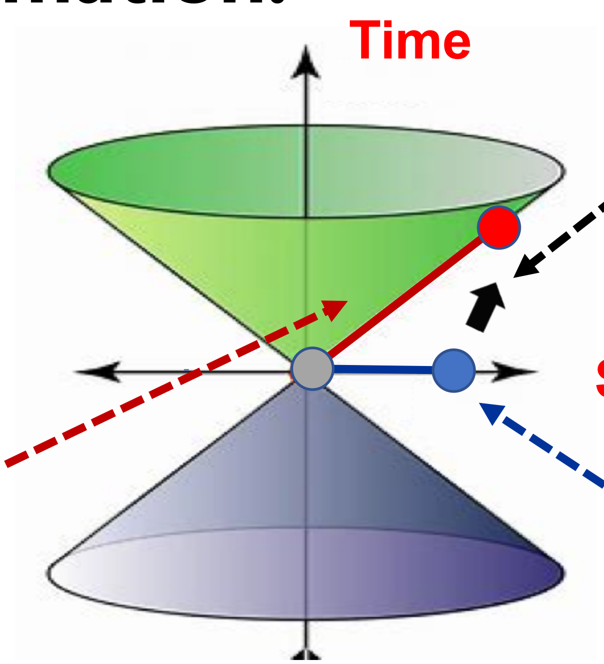
Quasi Parton Distribution Functions
and
Large Momentum Effective Theory
(LaMET)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002
X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412

Quasi-PDFs

$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

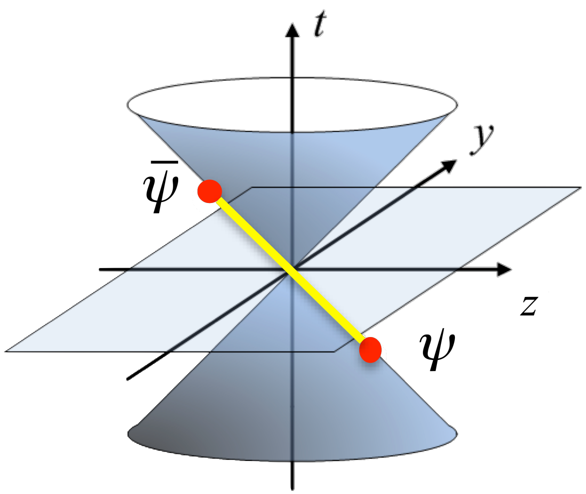
Frame transformation:



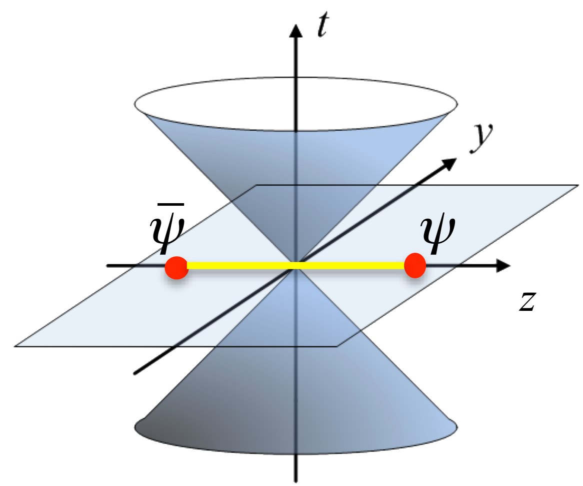
PDF:
light-cone correlation

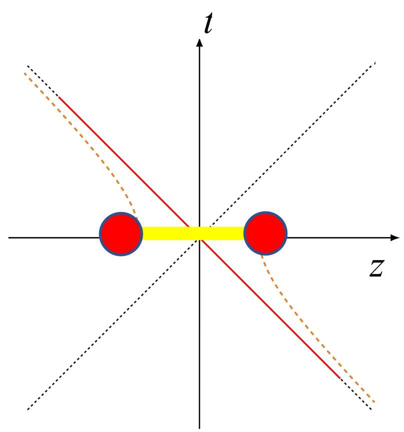
equal time correlation

Quasi-PDFs



Lorentz boost





PDF:
light-cone separation;
Cannot be calculated on the lattice

Quasi-PDF :
Equal-time correlation;
Directly calculable on the lattice

$$\tilde{q}(x, \mu^2, P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

Quasi-PDFs: **Finite but large Pz**

- The distribution at a finite but large Pz shall be calculable in lattice QCD.

- Since it differs from the standard PDF by simply an infinite Pz limit, it shall have the same infrared (collinear) physics.

- It shall be related to the standard PDF by a matching factor $Z(\frac{\mu}{Pz})$ which is perturbatively calculable.

$$Z(x,\mu/P^z)=\delta(x-1)+\frac{\alpha_s}{2\pi}Z^{(1)}(x,\mu/P^z)+\ldots$$

Progress on quasi PDF

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

Many Progress have been made on quasi PDFs, see Reviews:
Zhao, Int.J.Mod.Phys. A33 (2019);
Alexandrou et al. (ETMC), 1902.00587.

- ✓ Formalism: factorization, renormalization, power corrections
- ✓ Matching: perturbative corrections to Z
- ✓ Lattice QCD calculations

Progress on quasi PDF

Lattice Collaboration working on quasi-PDFs:

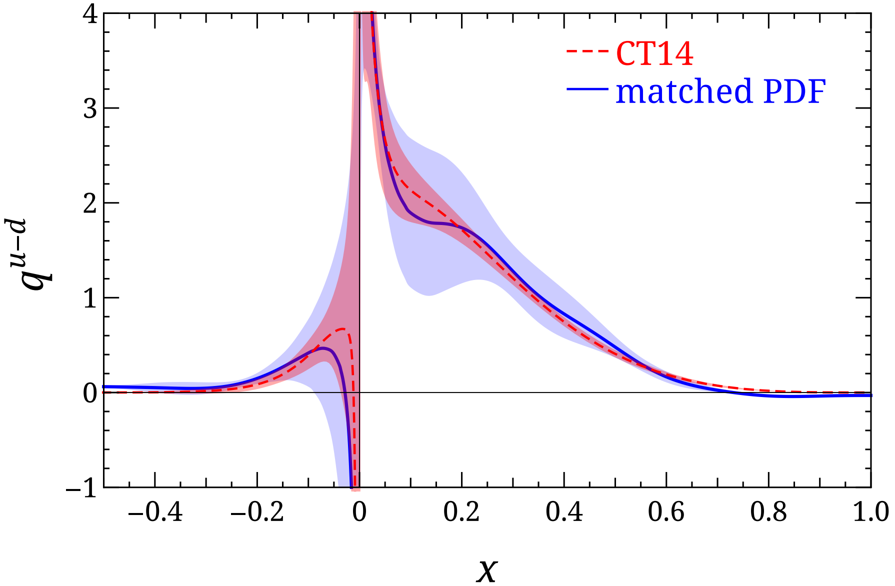
- **Lattice Parton Physics Project (LP3) Collaboration**
  J.W. Chen (National Taiwan U.), T. Ishikawa (T.-D. Lee Institute), L. Jin (U. Connecticut and BNL), R.-Z. Li (Michigan State U.), H.-W. Lin (Michigan State U.), Y.-S. Liu (TDLI), A. Schaefer (U. Regensburg), Y.-B. Yang (Michigan State U.), J.-H. Zhang (U. Regensburg), R. Zhang (Beijing Inst. Theory), and Y. Zhao (MIT), et al. (to be updated)
- **European Twisted Mass Collaboration (ETMC)**
  C. Alexandrou (U. Cyprus) , M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.
- **DESY, Zeuthen** J. Green, et al.
- **Brookhaven group**
  T. Izubuchi, L. Jin, K. Kallidonis, N. Karthik, S. Mukherje, P. Petreczky, C. Schugert, S. Syritsyn.

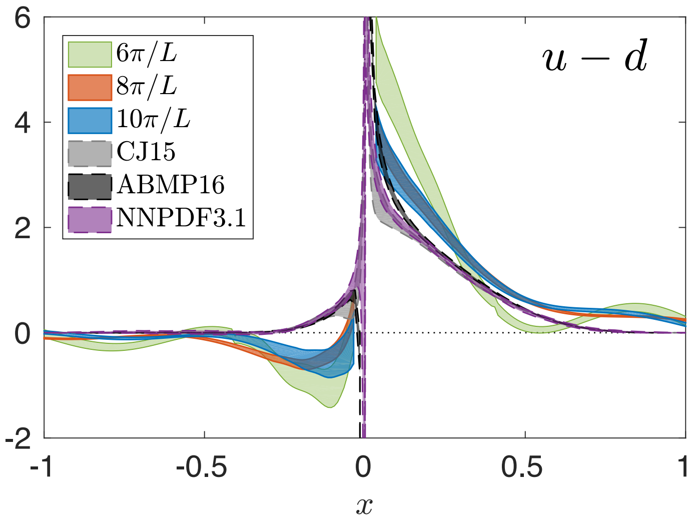
Progresses on quasi PDF:quark

$$u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x)$$

LP3: 1803.04393

ETMC:1803.02685



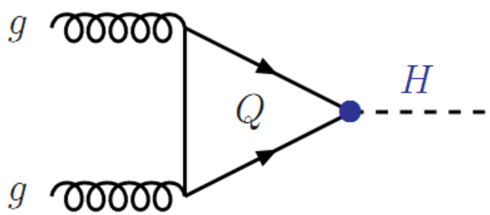


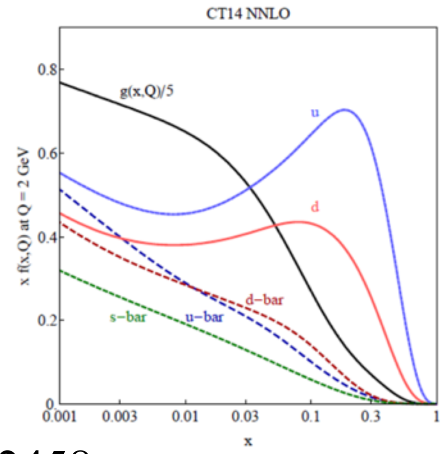
Progresses on quasi PDF

- ✓ Gluons: Renormalizability; Perturbative Matching, Lattice Simulations

- ✓ Generalized PDF, measurable in hard exclusive processes such as deeply virtual Compton scattering:
  - Perturbative Matching in RI/MOM Scheme: Liu, W.W. Xu, Zhang[2], Zhao[2],arXiv:1902.00307.
  - First lattice calculation of pion GPD, Chen, Lin and Zhang, arXiv: 1904.12376.
  - Preliminary results for quasi-GPDs (ETMC), M. Constantinou's talk at QCD Evolution 2019.
- ✓ Transverse Momentum Dependent PDF: definition,
  - Ji, Sun, Xiong and Yuan, PRD91 (2015);
  - Ji, Jin, Yuan, Zhang and Zhao, PRD99 (2019);
  - M. Ebert, I. Stewart, Zhao PRD99 (2019);
  - M. Ebert, I. Stewart, Zhao, arXiv:1901.03685.
  - ….

Gluon quasi PDF: Renormalization



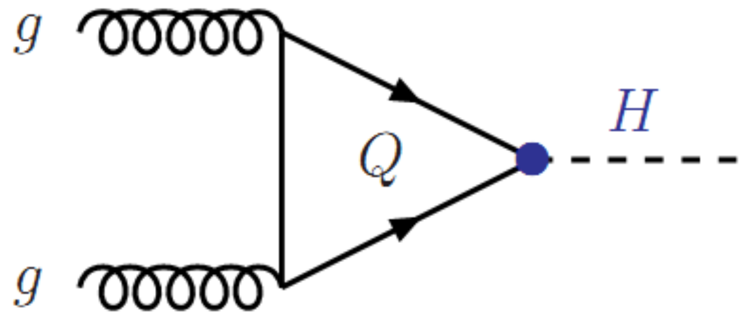


WW,Zhao,Zhu,1708.02458
WW, Zhao, 1712.03830
Zhang, Ji, Schafer, WW, Zhao,1808.10824
WW, Zhang, Zhao, Zhu, 1904.00978

See also Li, Ma, Qiu, 1809.01836

Gluon PDF

Higgs Production:
gluon-gluon fusion



Cross sections are calculated by Zürich group at N^3LO QCD and NLO EW accuracies [Anastasiou:2016cez]

mH=125.09 GeV, $\sqrt{s}$=13 TeV

σ=48.52pb

Total Uncertainty: 3.9% (Gaussian)
PDF: 1.9%
α_s: 2.6%

quasi PDF for gluon: definition?

Definition of quasi and light-cone gluon distribution

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ix\xi^- P^+} \langle P|F^+{}_i(\xi^-) W(\xi^-,0,L_{n^+}) F^{i+}(0)|P\rangle$$

$$\tilde{f}_{g/H}(x,\mu) = \int \frac{dz}{2\pi x P^z} e^{-ixzP^z} \langle P|F^z{}_i(z) W(z,0,L_{n^z}) F^{iz}(0)|P\rangle$$

- Field Strength Tensor: F
- i sums over transverse directions (i=1,2) or full directions
- W(z_1,z_2, C) is a Wilson line along contour C.

Renormalization of gluon PDF: Auxiliary Field

Gervais and Neveu, 1980

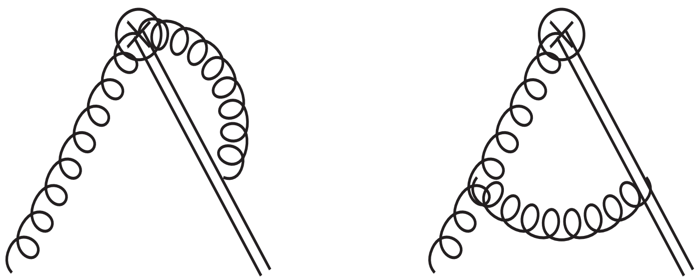
Wilson line $W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1)\bar{\mathcal{Z}}(\lambda_2)\rangle_z$

Gauge invariant non-local operators
pairs of gauge invariant composite local operators

$$F^a_{\mu\nu}(z_1)W_{ab}(z_1, z_2; C)F^b_{\rho\sigma}(z_2) = \left\langle (F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1)) \middle| \overline{(\mathcal{Z}_b}(\lambda_2)F^b_{\rho\sigma}(z_2)) \right\rangle$$
$$= \Omega^{(1)}_{\mu\nu}(z_1)\overline{\Omega^{(1)}_{\rho\sigma}}(z_2)$$

$$\Omega^{(1)}_{\mu\nu}(z_1) = F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1))$$

Renormalization of gluon PDF: One Loop diagrams



$$I_1 = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{4-d}(A_a^\nu n^\mu - A_a^\mu n^\nu) n\cdot\partial \mathcal{Z}_a/n^2 - \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Z}_a + reg.\right\},$$

$$I_2 = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{4-d}\left[\frac{1}{4}F_a^{\mu\nu}\mathcal{Z}_a + \frac{1}{2}\left(F_a^{\mu\rho}n_\nu n_\rho - F_a^{\nu\rho}n_\mu n_\rho\right)/n^2 + \frac{1}{2}(A_a^\mu n^\nu - A_a^\nu n^\mu) n\cdot\partial\mathcal{Z}_a/n^2\right]\right.$$
$$\left. + \frac{\pi\mu}{3-d}(n^\mu A_a^\nu - n^\nu A_a^\mu)\mathcal{Z}_a + reg.\right\},$$

No power divergence!

Renormalization of gluon quasi-PDF

Three operators with the same quantum number

$$\Omega^{(1)}_{\mu\nu} = F^a_{\mu\nu}\mathcal{Z}_a,$$
$$\Omega^{(2)}_{\mu\nu} = \Omega^{(1)}_{\mu\alpha}\frac{\dot{x}_\alpha \dot{x}_\nu}{\dot{x}^2} - \Omega^{(1)}_{\nu\alpha}\frac{\dot{x}_\alpha \dot{x}_\mu}{\dot{x}^2},$$
$$\Omega^{(3)}_{\mu\nu} = |\dot{x}|^{-2}(\dot{x}_\mu A^a_\nu - \dot{x}_\nu A^a_\mu)(D\mathcal{Z})_a,$$

$$\begin{pmatrix}\Omega^{\mu\nu}_{1,R}\\ \Omega^{\mu\nu}_{2,R}\\ \Omega^{\mu\nu}_{3,R}\end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{22}-Z_{11} & Z_{13}\\ 0 & Z_{22} & Z_{13}\\ 0 & 0 & Z_{33}\end{pmatrix}\begin{pmatrix}\Omega^{\mu\nu}_{1}\\ \Omega^{\mu\nu}_{2}\\ \Omega^{\mu\nu}_{3}\end{pmatrix}.$$

Different components are renormalized differently!

$$\begin{pmatrix}\Omega^{z\mu}_{1,R}\\ \Omega^{z\mu}_{3,R}\end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13}\\ 0 & Z_{33}\end{pmatrix}\begin{pmatrix}\Omega^{z\mu}_{1}\\ \Omega^{z\mu}_{3}\end{pmatrix};$$
$$\Omega^{ti}_{1,R} = Z_{11}\Omega^{ti}_{1}$$

Renormalization of gluon PDF:

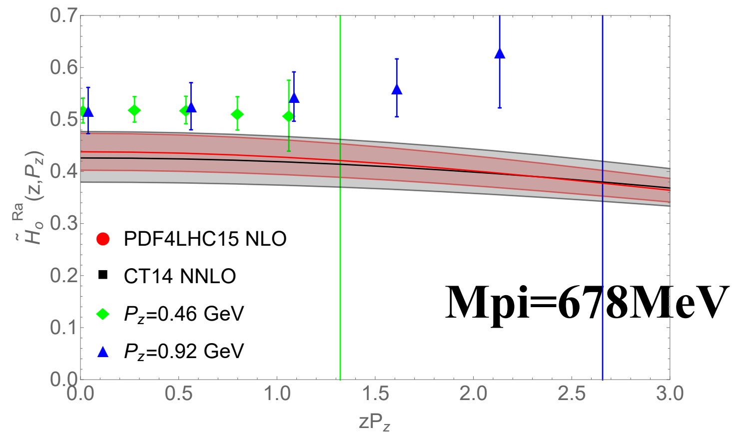
Multiplicatively Renormalizable Operators

$$O^{(1)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ t}(z_2),$$
$$O^{(2)}(z_1, z_2) \equiv F^{zi}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$
$$O^{(3)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$
$$O^{(4)}(z_1, z_2) \equiv F^{z\mu}(z_1)L(z_1, z_2)F_\mu^{\ z}(z_2),$$

Four multiplicative Renormalizable operators can be used to define gluon quasi-PDFs

Zhang, Ji, Schafer, WW, Zhao,1808.10824 (PRL 2019)

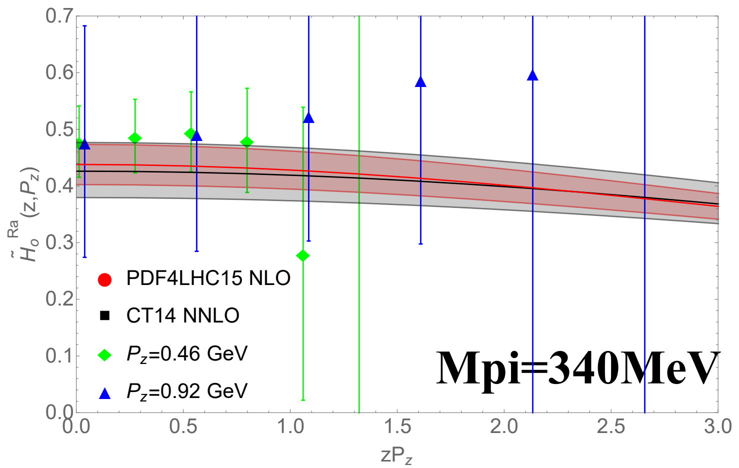
First Lattice Simulation



Fan, Yang, Anthony, Lin, Liu,1808.02077

$$\tilde{H}_0(z,P_z) = \langle P|\mathcal{O}_0(z)|P\rangle,$$

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t_\mu, F^{\mu t}; z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu_\nu, F^\nu_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2}$$



In future:

- More precise
- Physical Pion
- Large Momentum
- Quark-gluon Mixing

Summary

LaMET: Parton physics demands new ideas to solve non-perturbative QCD. Many Progresses are achieved.

Gluon Quasi PDF:
Renormalizability; RI/MOM subtraction; Factorization; One-loop matching; polarized PDF;
Mixing on the lattice; BRST/ghost on lattice (p^2/epsilon);

In near future, we expect:

- ✓ Lattice calculation of quark PDFs: 10%
- ✓ Better constraints x~1
- ✓ Distributions: gluon, TMD, GPD

Thank you very much!