

Nambu-Goldstone Modes and the Emergence of Mass



Craig Roberts

Strong Interactions in the Standard Model

THE DILEMMA OF ATTRIBUTION

Nobel Lecture, December 8, 2004

by

H. DAVID POLITZER

California Institute of Technology (Caltech), Pasadena, USA.



- The establishment by the mid-1970's of QCD as the correct theory of the strong interactions completed what is now known prosaically as the Standard Model.
- It offers a description of all known fundamental physics except for gravity, and gravity is something that has no discernible effect when particles are studied a few at a time.
- However, the situation is a bit like the way that the Navier-Stokes equation accounts for the flow of water. The equations are at some level obviously correct, but there are only a few, limited circumstances in which their consequences can be worked out in any detail.
- Nevertheless, many leading physicists were inclined to conclude in the late 1970's that the task of basic physics was nearly complete, and we'd soon be out of jobs.
- A famous example was the inaugural lecture of Stephen Hawking as Lucasian Professor of Mathematics, a chair first held by Isaac Barrow at Cambridge University. Hawking titled his lecture, "**Is the End in Sight for Theoretical Physics?**" And he argued strongly for "**Yes**".

2013: Englert & Higgs



- The 2013 Nobel Prize in Physics was awarded to Peter Higgs and Francois Englert following discovery of the Higgs boson at the Large Hadron Collider.
- With this discovery the Standard Model of Particle Physics became complete.
- Its formulation and verification describe a remarkable story.

Where to now?



2013: Englert & Higgs



***What is
confinement?***

- “The Higgs boson is often said to give mass to everything.”
- “However, that is wrong. It only gives mass to some very simple particles, accounting for only one or two percent of the mass of more complex things ...”
- *The vast majority of mass comes from the energy needed to hold quarks together inside hadrons*

confinement



2013: Englert & Higgs

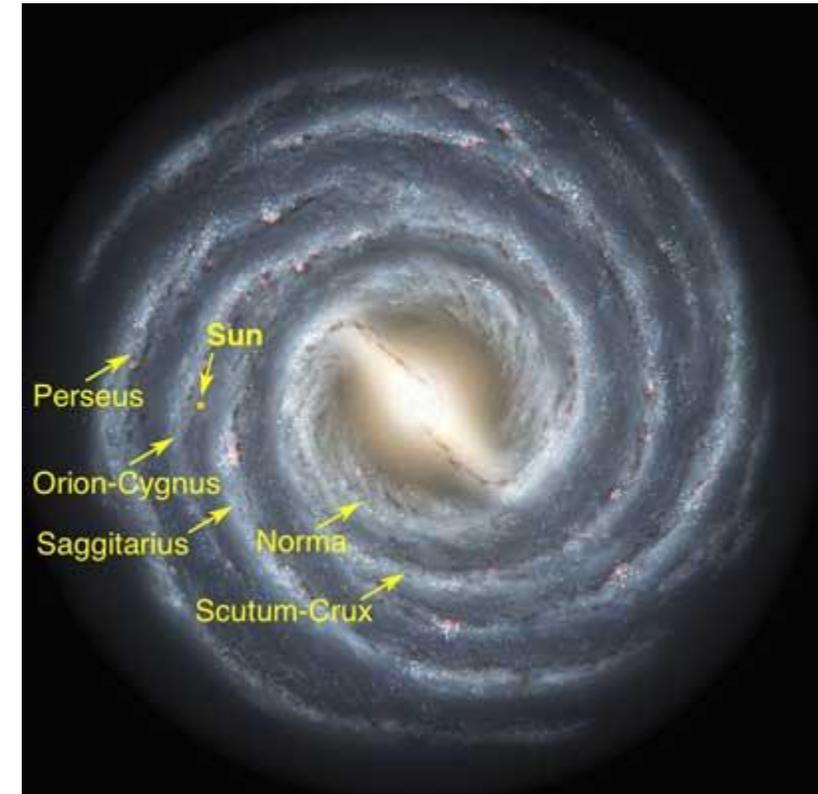


- The most important chapter of the Standard Model is the least understood.
- Quantum Chromodynamics (QCD) is that part of the Standard Model which is supposed to describe all of nuclear physics
 - Gauge bosons = gluons
 - Matter = quarks (& perhaps gluons ... hybrid bound-states)
- Yet, fifty years after the discovery of quarks, we are only just beginning to grasp how QCD moulds the basic bricks for nuclei: pions, neutrons, protons, etc.

Emergent Phenomena in the Standard Model

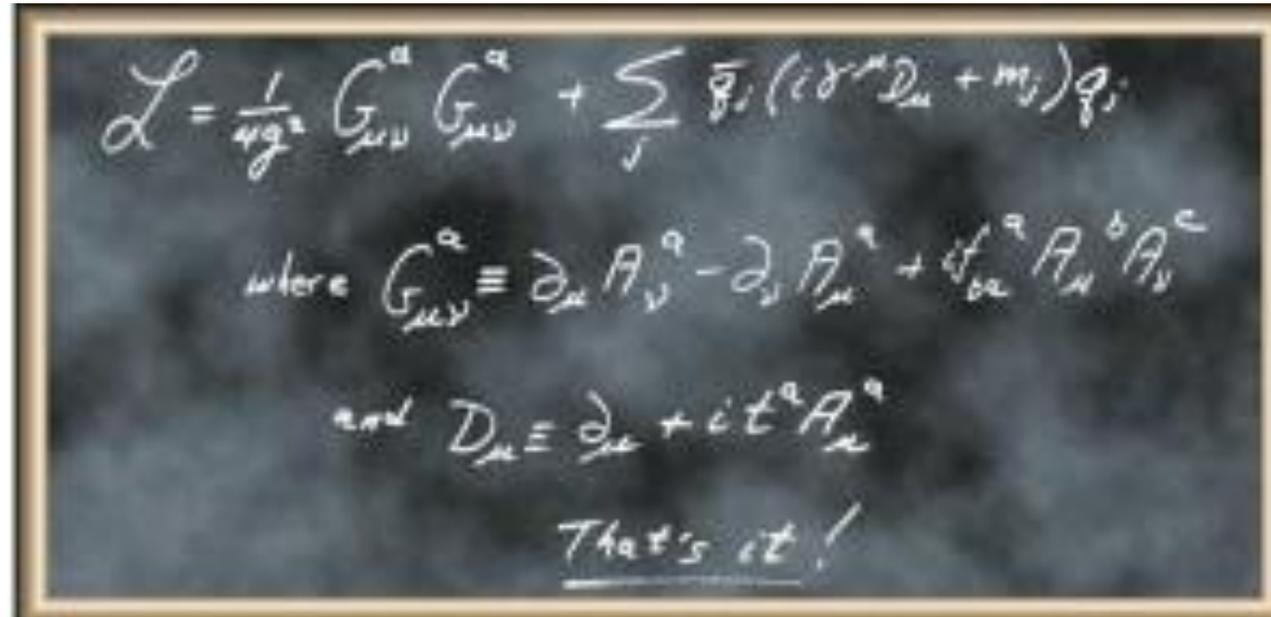
Existence of our Universe depends critically on the following empirical facts:

- Proton is massive
 - *i.e.* the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable
 - Despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (not massless, but lepton-like mass)
 - Despite being a strongly interacting composite object built from a valence-quark and valence antiquark



Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

Quantum Chromodynamics



The image shows a chalkboard with the following handwritten text:

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

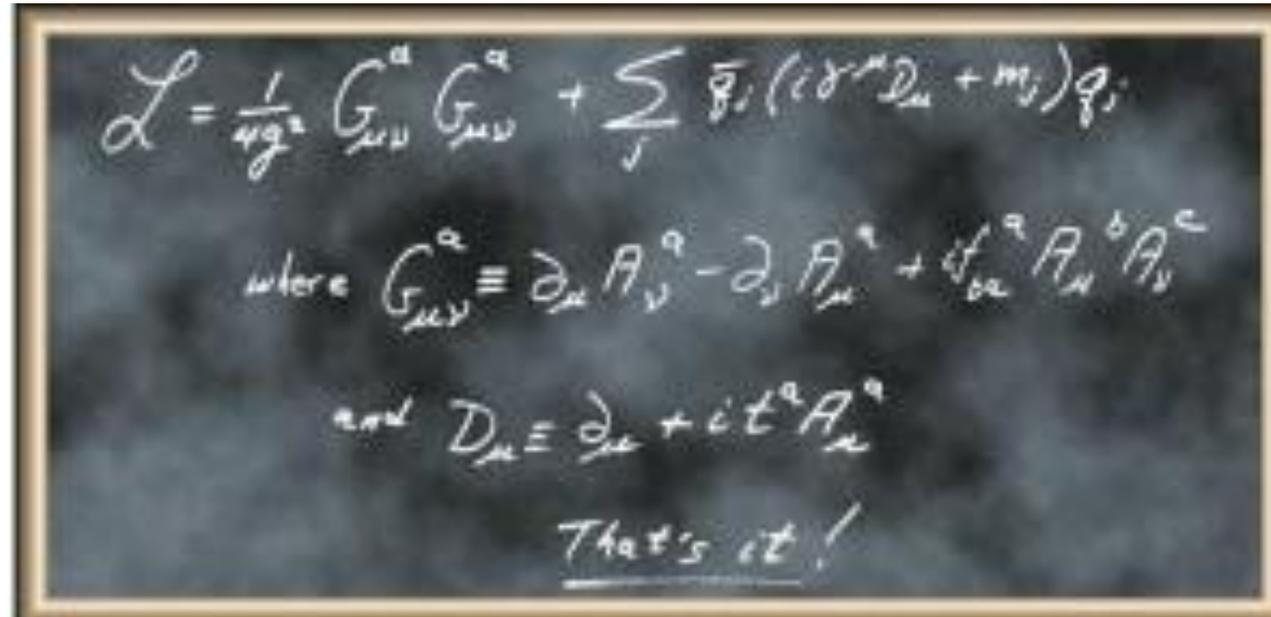
where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{abc} A_\mu^b A_\nu^c$

and $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

That's it!

- Quite possibly, the most remarkable theory we have ever invented
- One line and two definitions are responsible for the origin, mass and size of (almost) all visible matter!

Quantum Chromodynamics


$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{abc} A_\mu^b A_\nu^c$

and $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

That's it!

- Quite possibly, the most remarkable theory we have ever invented
- One line and two definitions are responsible for the origin, mass and size of (almost) all visible matter!



Strong Interactions in the Standard Model

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i(\gamma^\mu D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is said to be generated by Higgs boson.
- In connection with everyday matter, that mass is $1/250^{\text{th}}$ of the natural (empirical) scale for strong interactions,
viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated light-quark mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
 - No amount of staring at L_{QCD} can reveal that scale
- Contrast with quantum electrodynamics, *e.g.* spectrum of hydrogen levels measured in units of m_e , which appears in L_{QED}

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i(\gamma^\mu D_\mu)_{ij} - \dots) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

Whence Mass?

- Classical chromodynamics ... non-Abelian local gauge theory
- Remove the current mass ... there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics ... the theory looks the same at all length-scales ... there can be no clumps of anything ... *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem ...*
 - normal matter is constituted from light-quarks
 - the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs produces in the light-quark sector
- *Where did it all begin? ... becomes ... Where did it all come from?*

$$T_{\mu\mu} = \frac{1}{4} \beta(\alpha(\zeta)) G_{\mu\nu}^a G_{\mu\nu}^a$$

Trace Anomaly

- In a **scale invariant theory**

the *energy-momentum tensor must be traceless*: $\partial_\mu D_\mu = T_{\mu\mu} \equiv 0$

- Regularisation and renormalisation of (ultraviolet) divergences in Quantum Chromodynamics introduces a mass-scale

... *dimensional transmutation*:

Lagrangian's *constants* (couplings and masses) become dependent on a mass-scale, ζ

- $\alpha \rightarrow \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $L(m=0)$

$$\Rightarrow \partial_\mu D_\mu = T_{\mu\mu} = \delta L / \delta \sigma = \alpha \beta(\alpha) dL / d\alpha \Rightarrow \beta(\alpha) \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a = T_{\rho\rho} =: \Theta_0$$

Trace anomaly

QCD β function

Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor



Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G_{\mu\nu}^a G_{\mu\nu}^a$$

Trace Anomaly

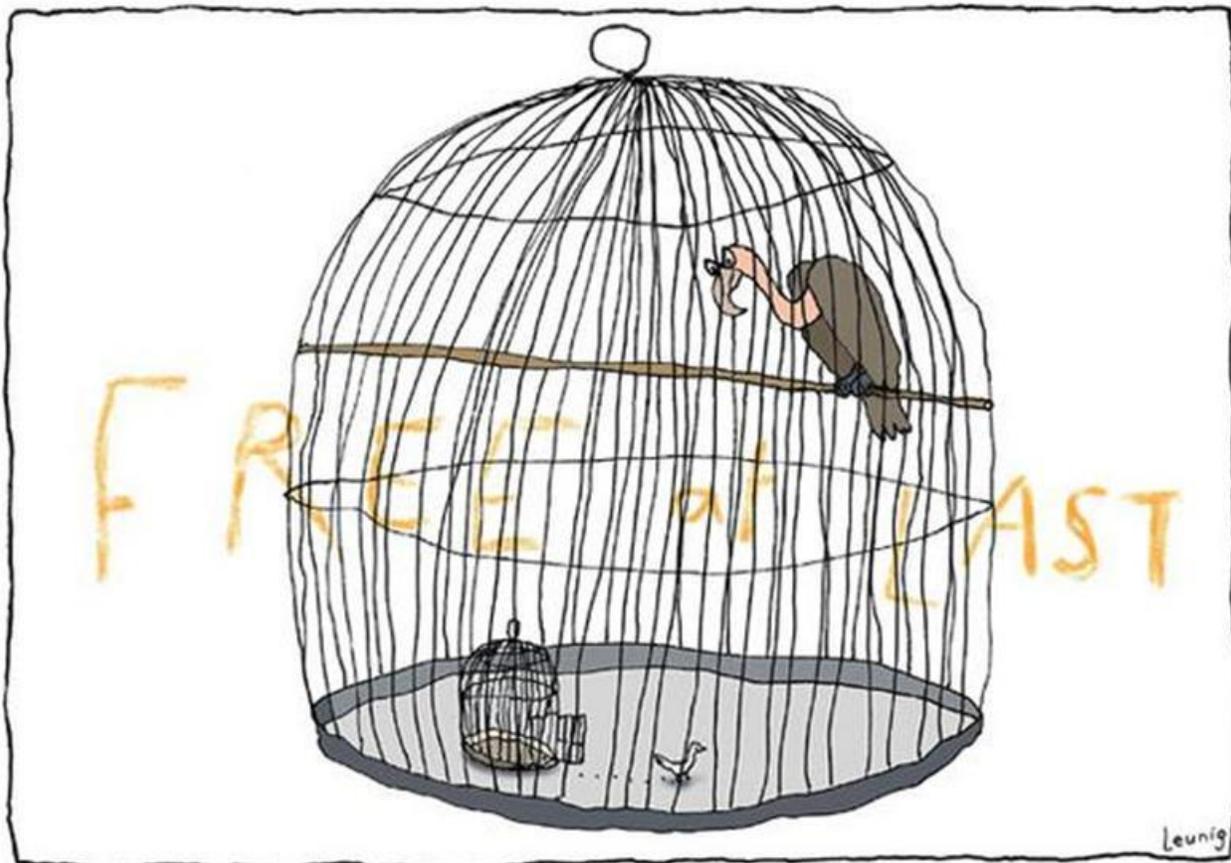
- Knowing that a trace anomaly exists does not deliver a great deal
... Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude ... consider proton:

$$\begin{aligned}\langle p(P) | T_{\mu\nu} | p(P) \rangle &= -P_\mu P_\nu \\ \langle p(P) | T_{\mu\mu} | p(P) \rangle &= -P^2 = m_p^2 \\ &= \langle p(P) | \Theta_0 | p(P) \rangle\end{aligned}$$

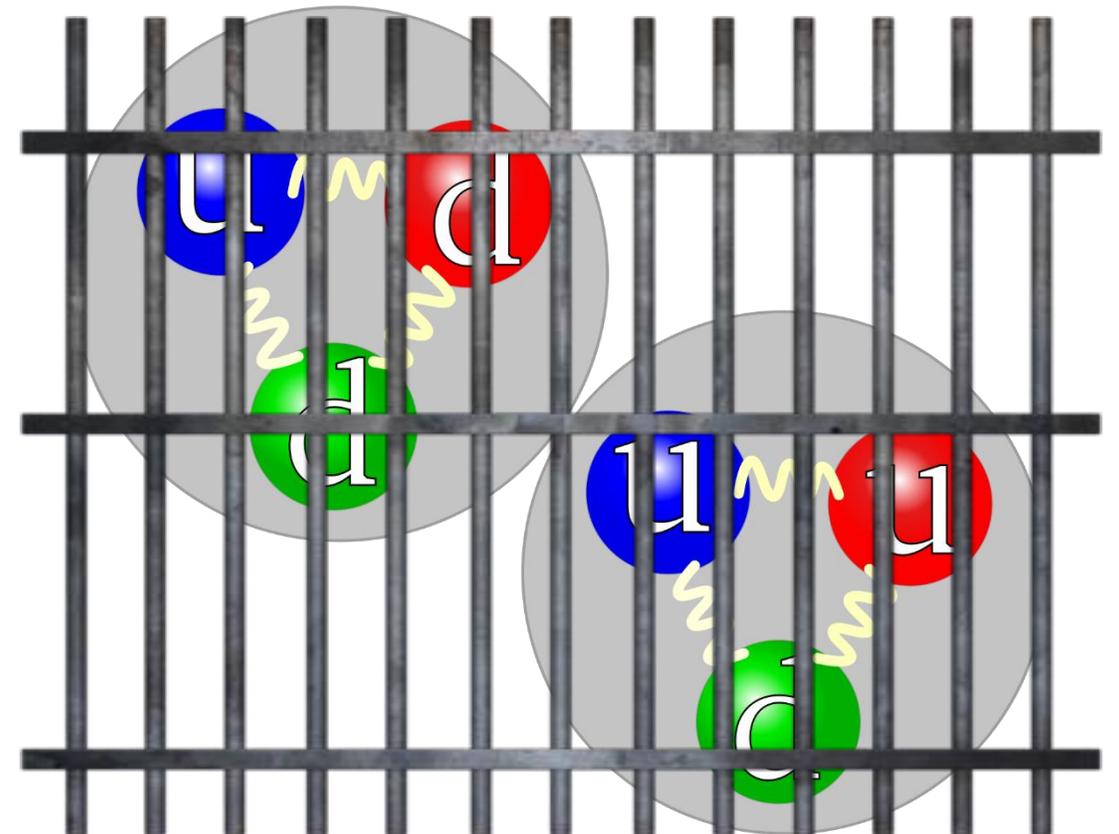
- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0
... In QCD, Θ_0 measures the strength of gluon self-interactions
... so, from one (partonic basis) perspective,
 m_p is (somehow) completely generated by glue.

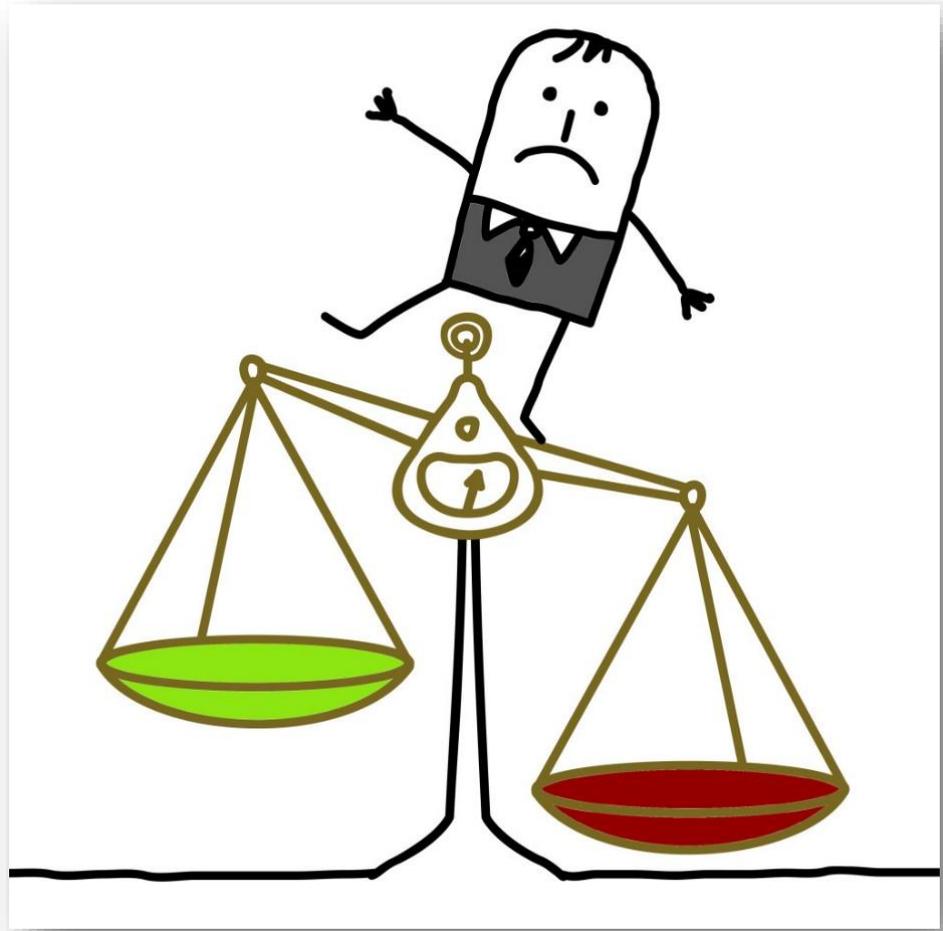
Confinement & Origin of Mass

- *The vast majority of mass comes from the energy needed to hold quarks together inside nuclei*



Craig Roberts. *The Emergence of Mass*





On the other hand ...

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G_{\mu\nu}^a G_{\mu\nu}^a$$

Trace Anomaly

- In the chiral limit

$$\langle \pi(q) | T_{\mu\nu} | \pi(q) \rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0$$

- **Does this mean** that the scale anomaly vanishes trivially in the pion state, *i.e.* **gluons contribute nothing to the pion mass?**
- Difficult way to obtain “zero”!
- Easier to imagine that “zero” owes to cancellations between different operator contributions to the expectation value of Θ_0 .
- Of course, such precise cancellation should not be an accident.
It could only arise naturally because
of some symmetry and/or symmetry-breaking pattern.

Whence “1” and yet “0” ?

$$\langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0$$

➤ *No statement of the question*

*“How does the mass of the proton arise?”
is complete without the additional clause*

*“How does the pion remain **massless**?”*

➤ Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems

- Expectation value of Θ_0 in pion is always zero,
irrespective of the size of the natural mass-scale for strong interactions = m_p

IR Behaviour of QCD

$$\Delta_{\mu\nu}^{-1}(q) = \underbrace{\dots}_{\Pi_{\mu\nu}(q)} + \dots$$

$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$
 $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

- Gluons are *supposed* to be massless
 - This is true in perturbation theory
 - **Not preserved non-perturbatively!**
- No symmetry protects four-transverse
- “ $q_\nu \Pi_{\mu\nu}(q) = 0$ ” *gluon modes*

g
 or gluon

$$I(J^P) = 0(1^-)$$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

VALUE	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
ABREU	92E	DLPH	Spin 1, not 0
ALEXANDER	91H	OPAL	Spin 1, not 0
BEHREND	82D	CELL	Spin 1, not 0
BERGER	80D	PLUT	Spin 1, not 0
BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu et al.	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander et al.	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend et al.	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger et al.	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik et al.	(TASSO Collab.)

IR Behaviour of QCD

$$\Delta_{\mu\nu}^{-1}(q) = \dots + \underbrace{\dots}_{\Pi_{\mu\nu}(q)} + \dots$$

$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$
 $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$

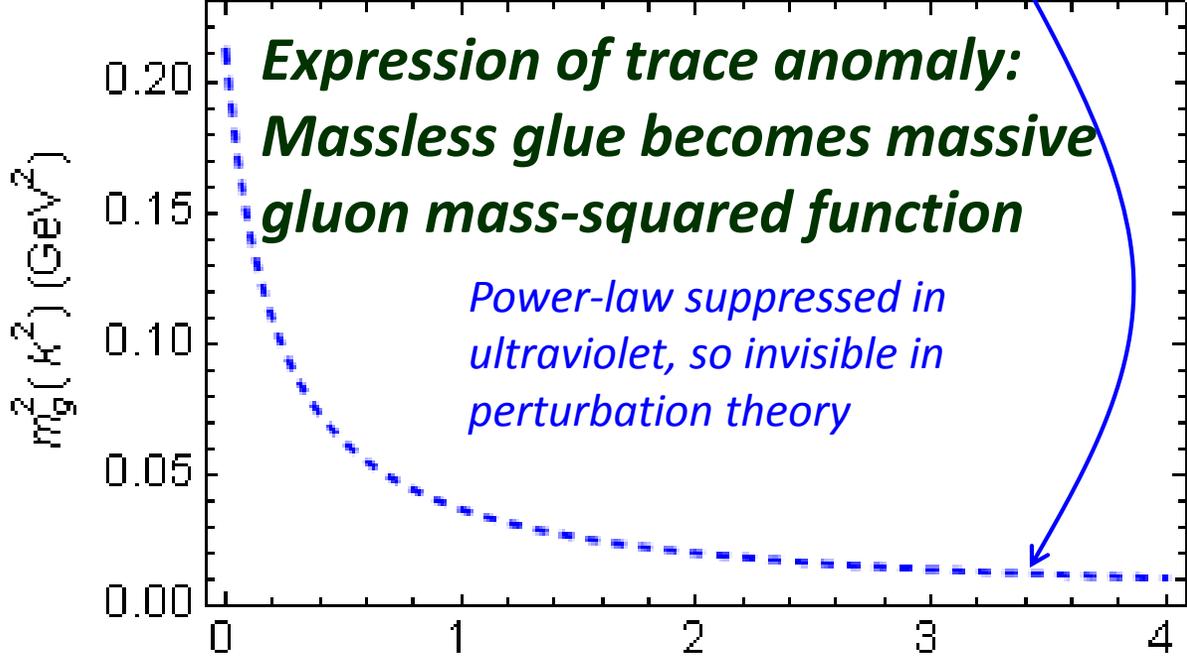
➤ Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2; \zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi, \quad m_g^2(0) = (0.46 \text{ GeV})^2$$

➤ Gluons are **cannibals** – a particle species whose members become massive by eating each other!

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2} \quad \mu_g \approx \frac{1}{2} m_p$$



Combining DSE, IQCD and pQCD analyses of QCD's gauge sector

Dynamical mass generation in continuum quantum chromodynamics
 J.M. Cornwall, *Phys. Rev. D* **26** (1981) 1453

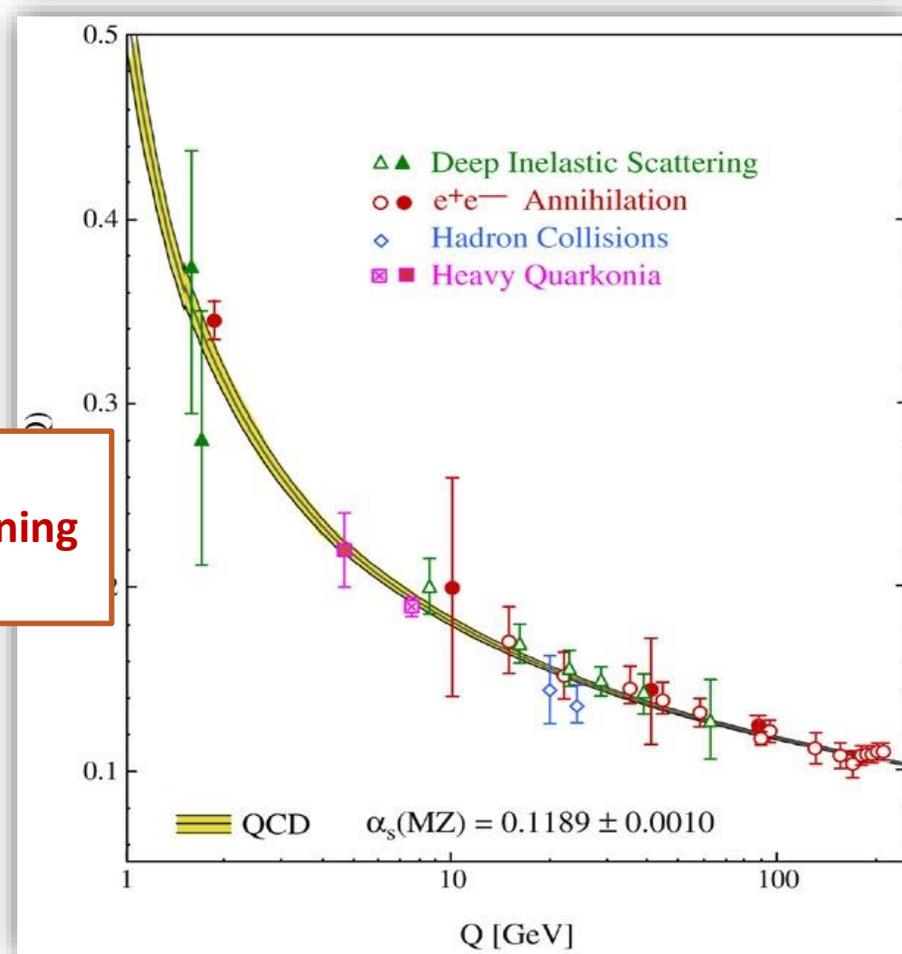
The Gluon Mass Generation Mechanism: A Concise Primer
 A.C. Aguilar, D. Binosi, J. Papavassiliou, *Front. Phys.* **11** (2016) 111203

Interaction model for the gap equation, S.-x.Qin et al.,
[arXiv:1108.0603 \[nucl-th\]](https://arxiv.org/abs/1108.0603), *Phys. Rev. C* **84** (2011) 042202(R) [5 pages]



This is where we live

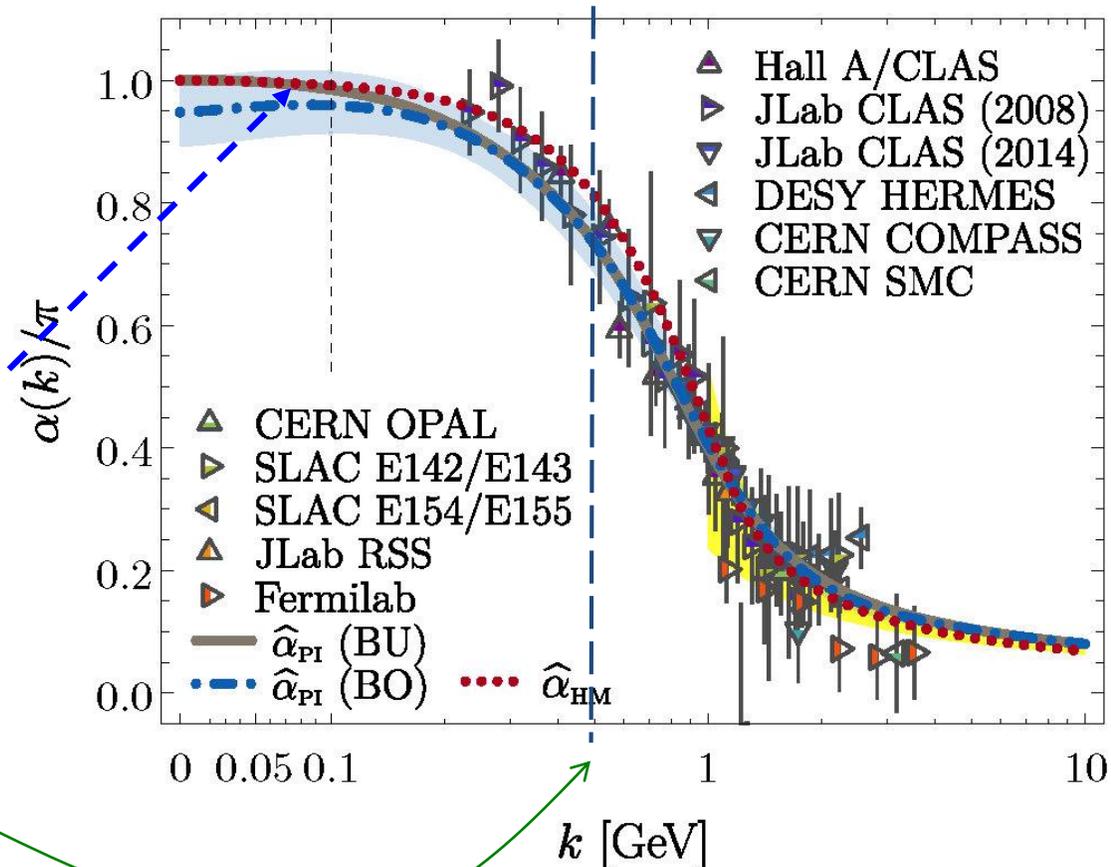
←
What's happening
out here?!



QCD's Running Coupling

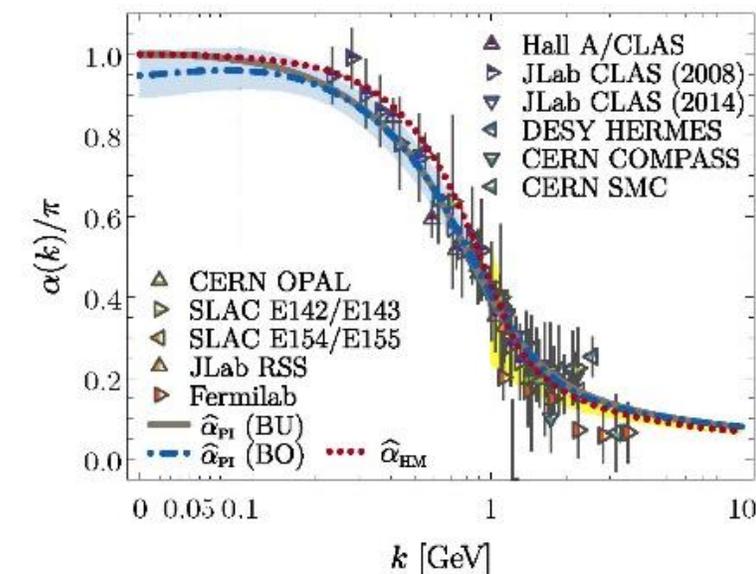
Process-independent effective-charge in QCD

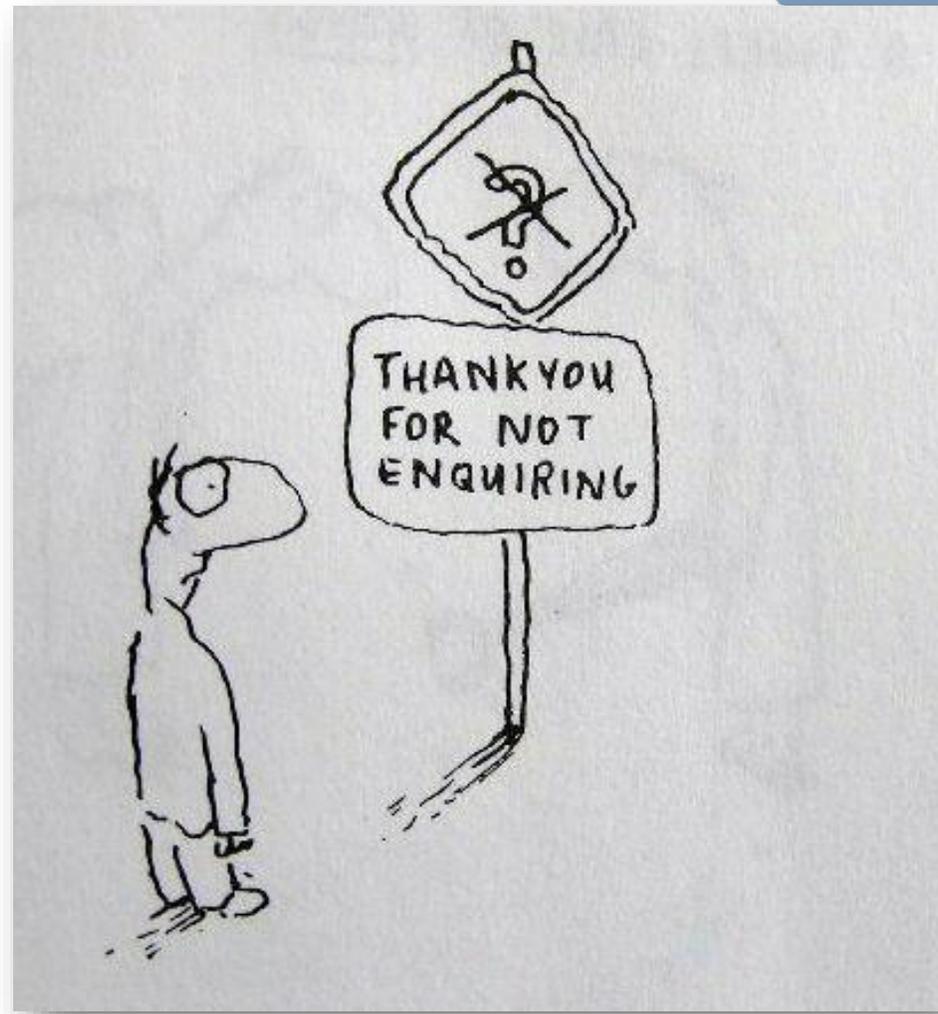
- Modern continuum & lattice methods for analysing gauge sector enable “Gell-Mann – Low” running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD’s gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx \frac{1}{2} m_p$



Process-independent effective-charge in QCD

- $\hat{\alpha}_{PI}$ is a new type of effective charge
 - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
 - process-independent
 - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
 - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale, which also serves to eliminate the Gribov ambiguity
- Asymptotic freedom \Rightarrow QCD is well-defined at UV momenta
- **QCD is therefore unique amongst known 4D quantum field theories**
 - **Potentially, defined & internally consistent at all momenta**





Enigma of Mass

Pion's Goldberger-Treiman relation

- Pion's Bethe-Salpeter amplitude
 Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) + \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right]$$

- Dressed-quark propagator
- Axial-vector Ward-Takahashi identity entails

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$f_{\pi} E_{\pi}(k; P = 0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known

*Rudimentary version of this relation is
apparent in Nambu's Nobel Prize work*

Model independent
Gauge independent
Scheme independent

$$f_{\pi} E_{\pi}(p^2) = B(p^2)$$

The most fundamental
expression of Goldstone's
Theorem and PCAC

Model independent
Gauge independent
Scheme independent

$$f_{\pi} E_{\pi}(p^2) = B(p^2)$$

**Pion exists if, and only if,
mass is dynamically
generated**

$$\langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0$$

Whence “0” ?

$$\langle p(P) | \Theta_0 | p(P) \rangle = m_p^2, \quad \langle \pi(q) | \Theta_0 | \pi(q) \rangle = 0$$

Whence “0” ?

The answer is algebraic

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations
 - Bethe-Salpeter Kernel:
 - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
 - Interactions of arbitrary but enumerable complexity involving these “basis vectors”
 - Chiral limit:
 - Algebraic proof
 - at any & each finite order in symmetry-preserving construction of kernels for
 - » the gap (quark dressing)
 - » and Bethe-Salpeter (bound-state) equations,
 - there is a precise cancellation between
 - » mass-generating effect of dressing the valence-quarks
 - » and attraction introduced by the scattering events
 - Cancellation guarantees that
 - simple system, which began massless,
 - becomes a complex system, with
 - » a nontrivial bound-state wave function
 - » attached to a pole in the scattering matrix, which remains at $P^2=0$...
 - Interacting, bound system remains massless!

Pion masslessness

➤ Obtain a coupled set of gap- and Bethe-Salpeter equations

– Bethe

- va
- th
- In

– Chira

- A

Quantum field theory statement:
In the pseudoscalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

ch have given

» mass
 » and

$$f_{\pi} E_{\pi}(p^2) = B(p^2)$$

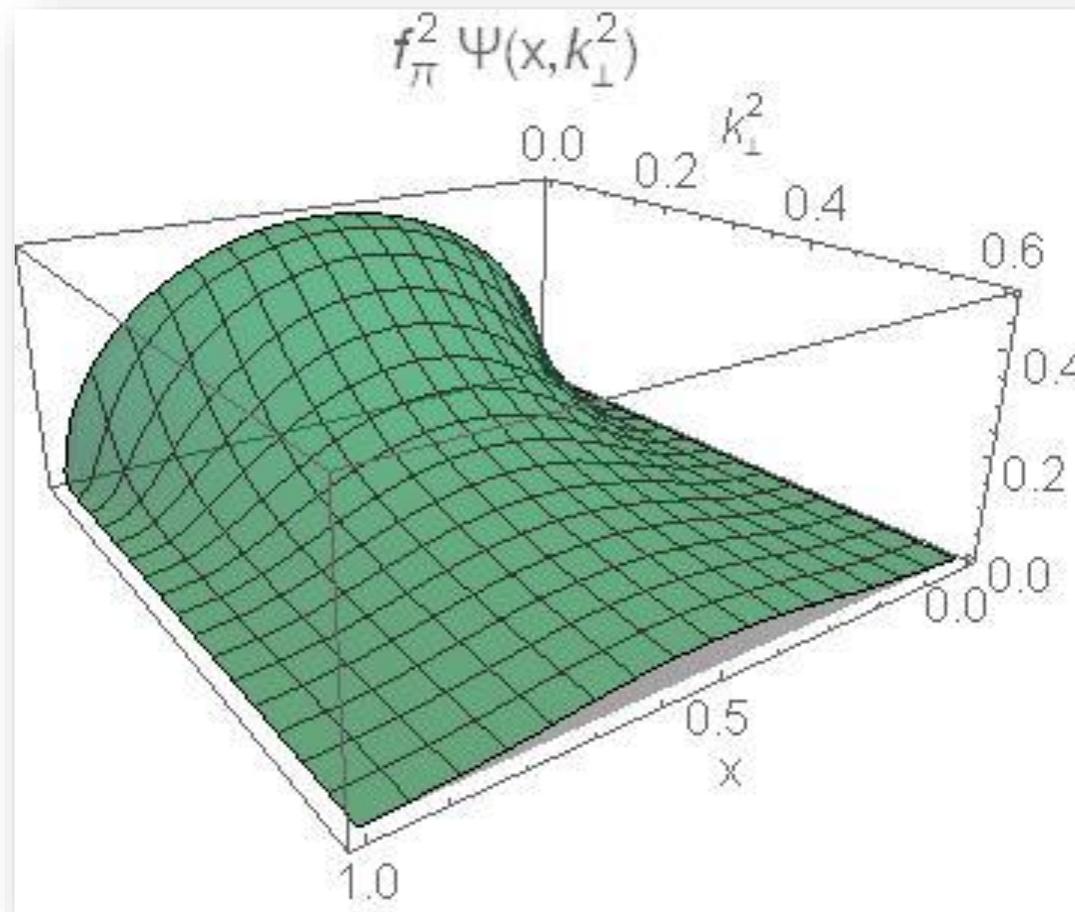
• Cancellation guarantees that

- simple system, which began massless,
- becomes a complex system, with
 - » a nontrivial bound-state wave function
 - » attached to a pole in the scattering matrix, which remains at $P^2=0$...

• Interacting, bound system remains massless!



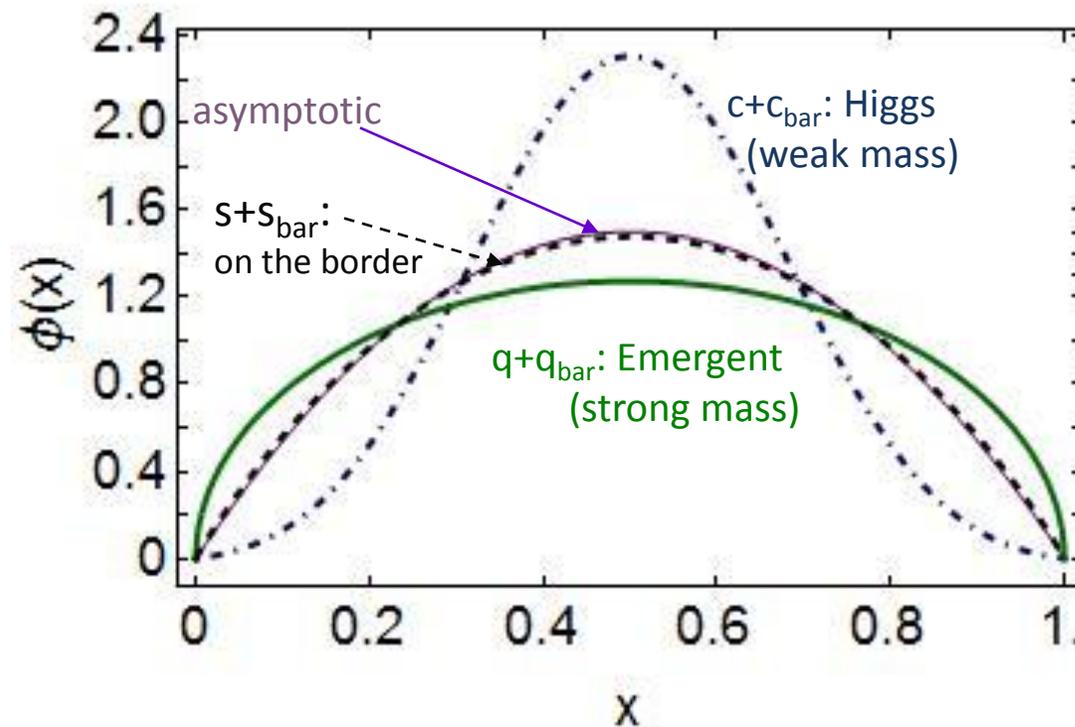
Observing Mass

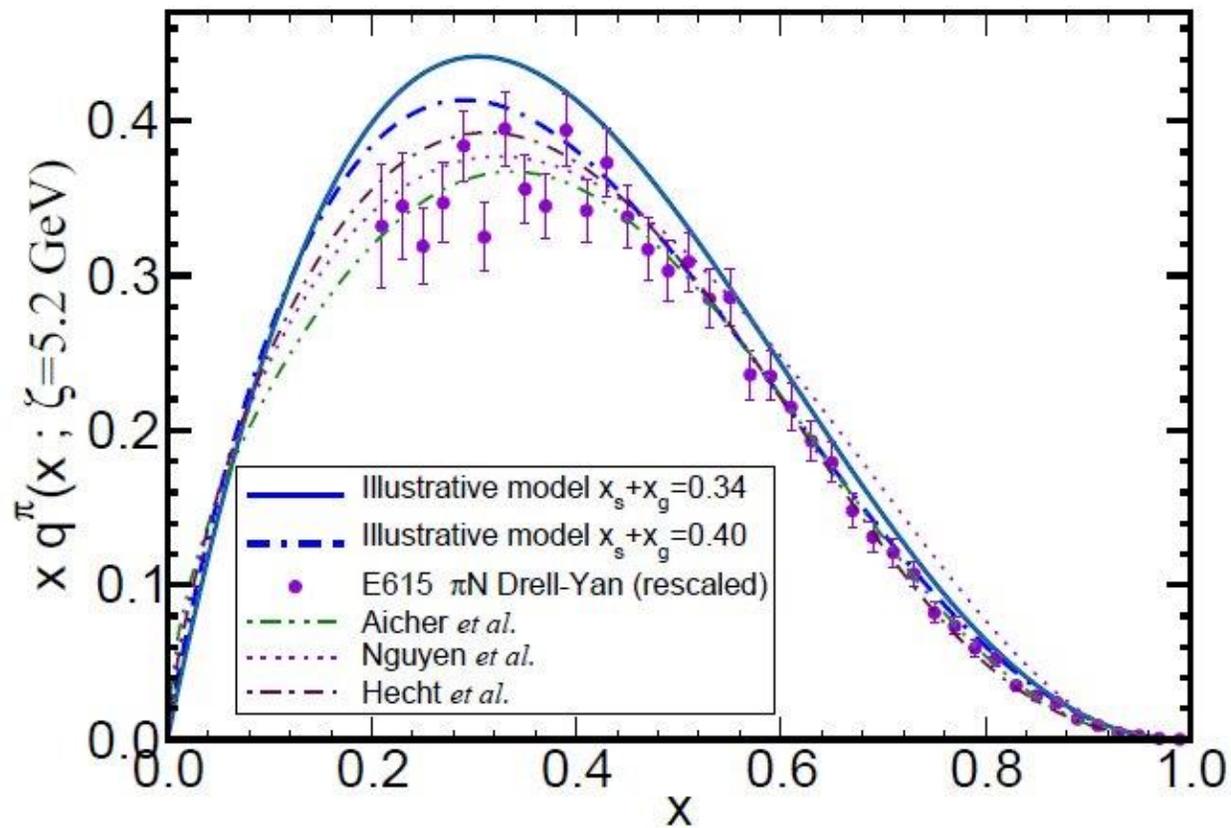


Meson Wave Functions

Emergent Mass vs. Higgs Mechanism

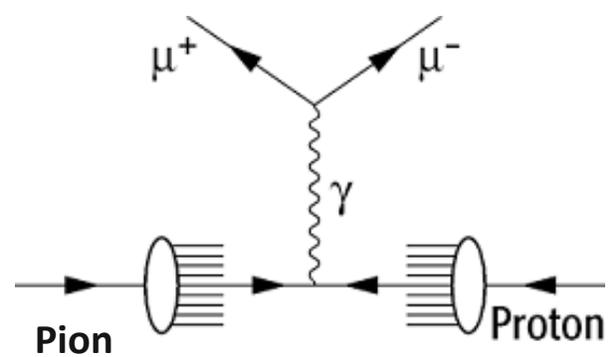
- When does Higgs mechanism begin to influence mass generation?
- limit $m_{\text{quark}} \rightarrow \infty$
 $\varphi(x) \rightarrow \delta(x-\frac{1}{2})$
- limit $m_{\text{quark}} \rightarrow 0$
 $\varphi(x) \sim (8/\pi) [x(1-x)]^{\frac{1}{2}}$
- Transition boundary lies just above m_{strange}
- *Comparison between distributions of light-quarks and those involving strange-quarks is good place to seek signals for strong-mass generation*





π & K Valence-quark Distribution Functions

Continuum QCD prediction of π valence-quark distributions



- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

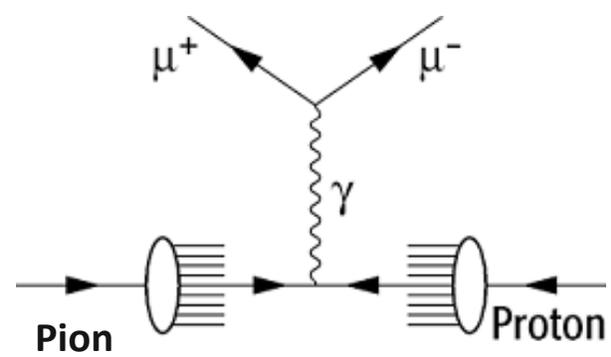
$$\pi p \rightarrow \mu^+ \mu^- X$$

- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 \gg m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a hadronic resolving scale, $\zeta_H \dots u_\pi(x; \zeta_H) \sim (1-x)^{2\beta}$
namely, the large- x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.
- In QCD, $\beta=1$ and hence

$${}^{QCD} u_\pi(x; \zeta_H) \sim (1-x)^2$$

$${}^{QCD}: Q > \zeta_H \Rightarrow 2 \rightarrow 2+\gamma, \gamma > 0$$

Empirical status of the Pion's valence-quark distributions



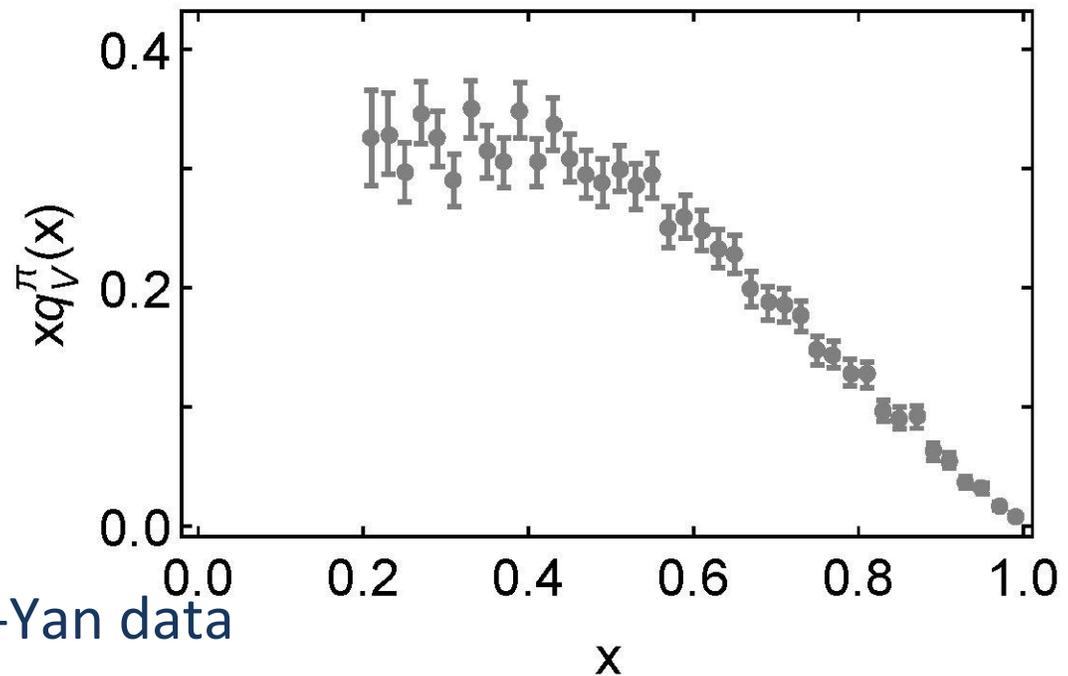
- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

$$\pi p \rightarrow \mu^+ \mu^- X$$

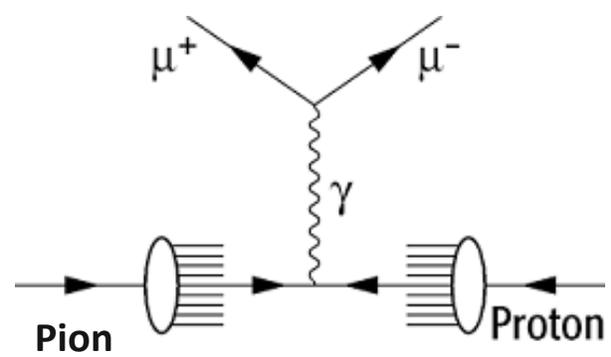
- Three experiments:
 - CERN (1983 & 1985)
 - FNAL (1989).
- None more recent
- Conway *et al.*

[Phys. Rev. D **39**, 92 \(1989\)](#)

- Leading-order analysis of the Drell-Yan data
- ~ 400 citations



Empirical status of the Pion's valence-quark distributions



- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

$$\pi p \rightarrow \mu^+ \mu^- X$$

- Three experiments:
 - CERN (1983 & 1985)
 - FNAL (1989).
- None more recent
- Conway *et al.*

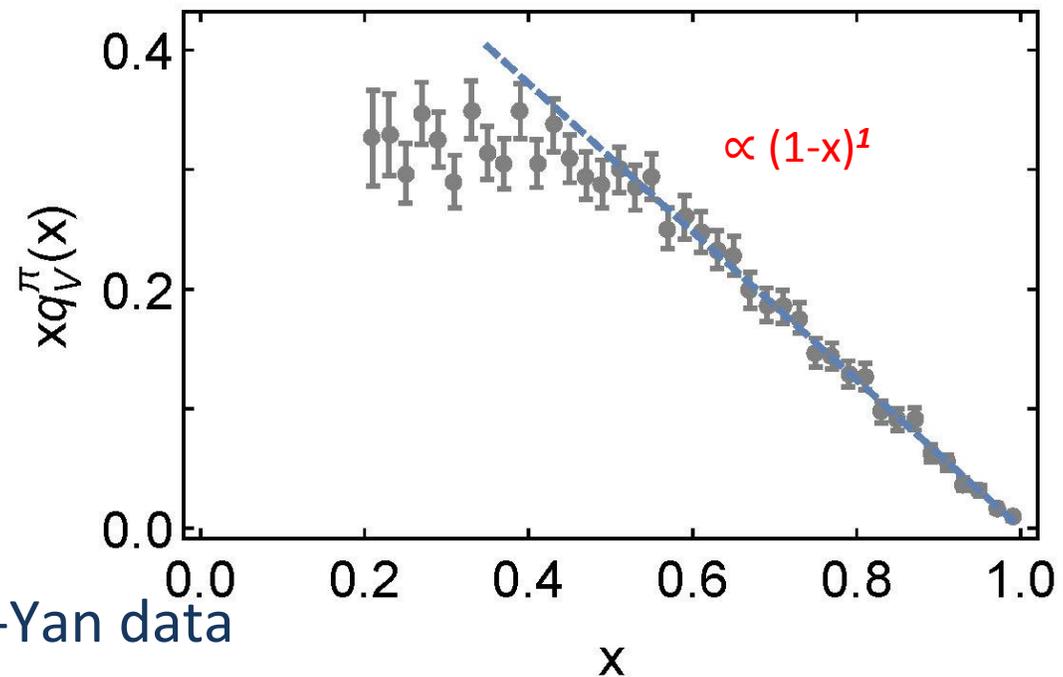
[Phys. Rev. D 39, 92 \(1989\)](#)

- Leading-order analysis of the Drell-Yan data

➤ **Controversial!**

Craig Roberts. The Emergence of Mass

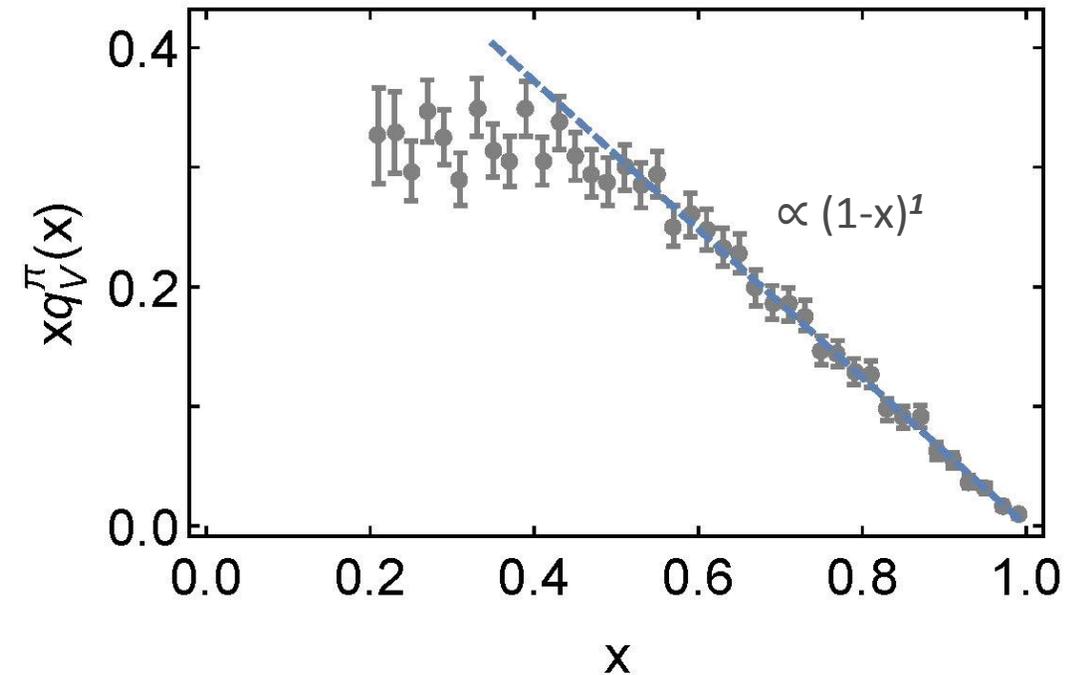
$${}^{QCD} u_{\pi}(x; \zeta > \zeta_H) \sim (1-x)^{2+\gamma}$$



π valence-quark distributions

20 Years of Evolution

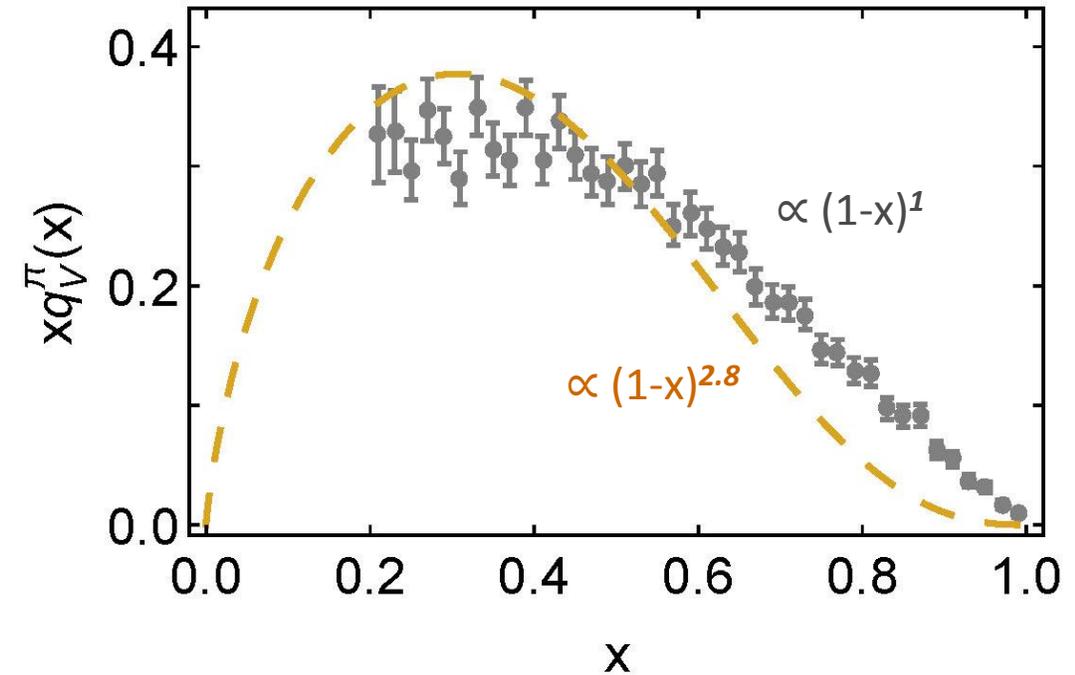
- 1989 ... Conway *et al.* [Phys. Rev. D 39, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data



π valence-quark distributions

20 Years of Evolution

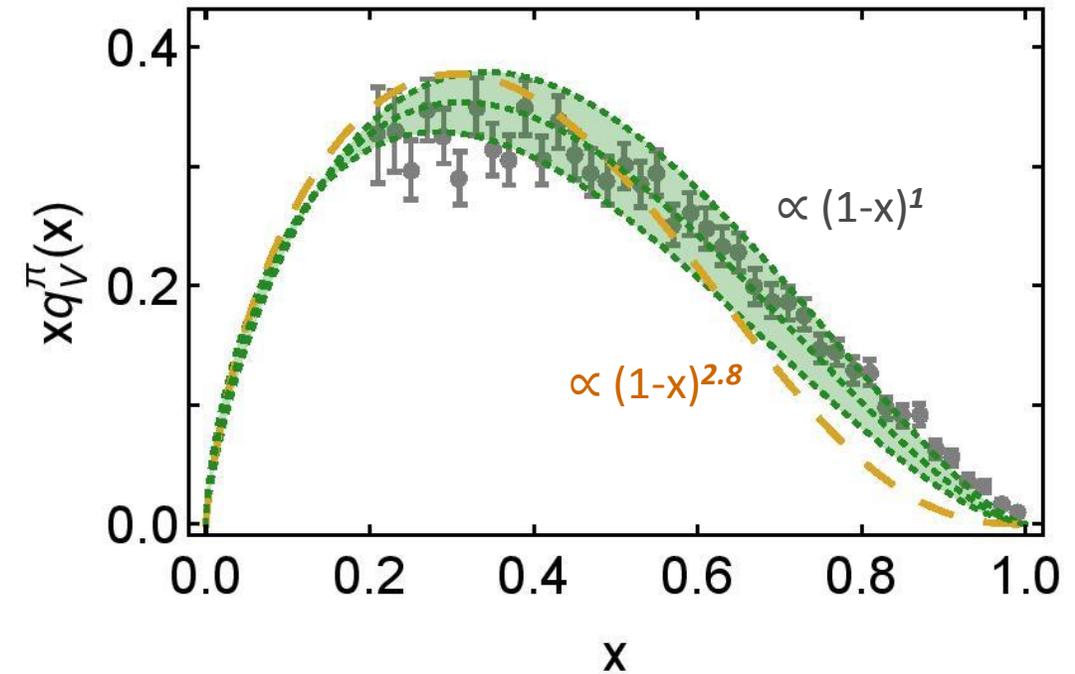
- 1989 ... Conway *et al.* [Phys. Rev. D **39**, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data
- 2000 ... Hecht *et al.* [Phys.Rev. C **63** \(2001\) 025213](#)
 - QCD-connected model prediction



π valence-quark distributions

20 Years of Evolution

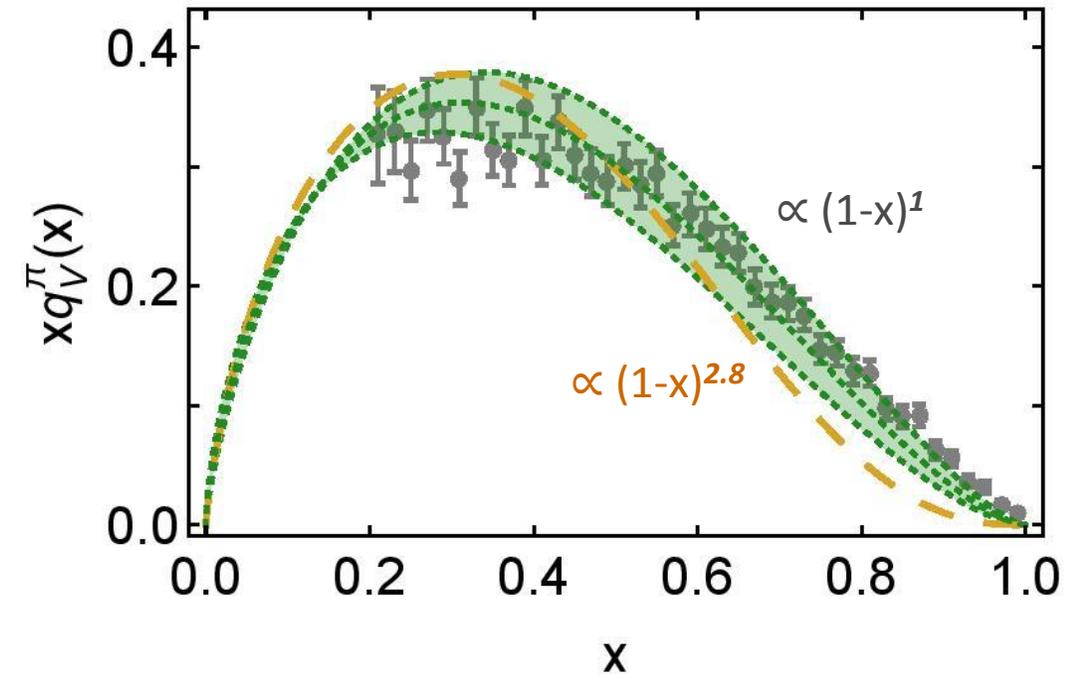
- 1989 ... Conway *et al.* [Phys. Rev. D **39**, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data
- 2000 ... Hecht *et al.* [Phys.Rev. C **63** \(2001\) 025213](#)
 - QCD-connected model prediction
- 2005 ... Wijesooriya, Reimer, Holt, [Phys. Rev. C **72** \(2005\) 065203](#)
 - Partial NLO analysis of E615 data
 - Large-x power-law $\rightarrow 1.54 \pm 0.08$



π valence-quark distributions

20 Years of Evolution

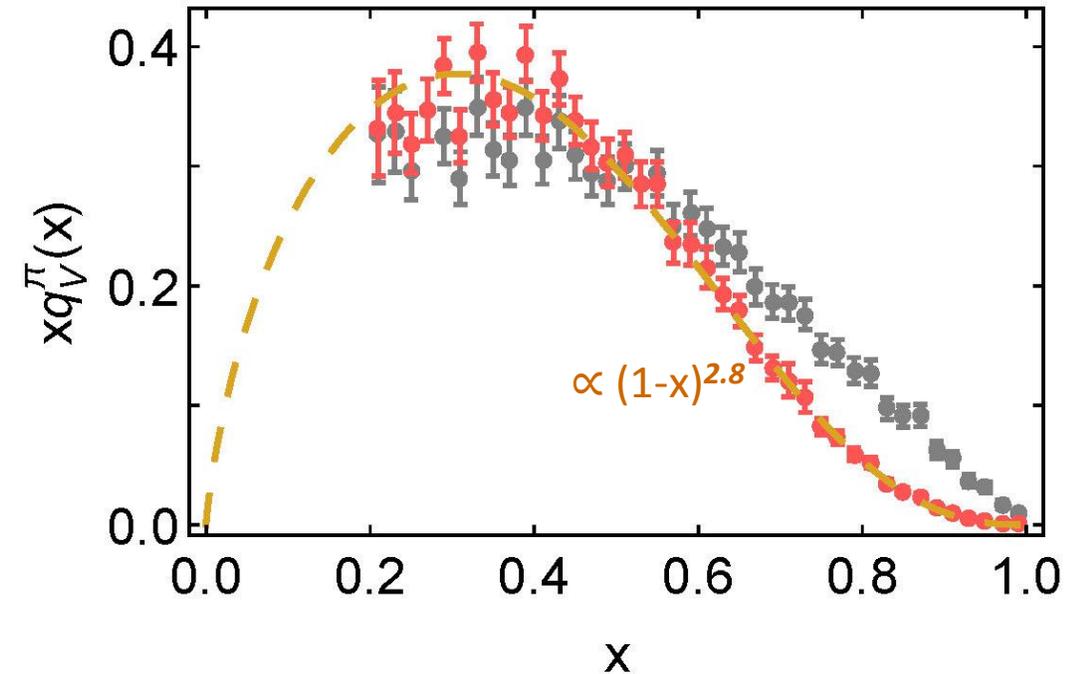
- 1989 ... Conway *et al.* [Phys. Rev. D **39**, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data
- 2000 ... Hecht *et al.* [Phys.Rev. C **63** \(2001\) 025213](#)
 - QCD-connected model prediction
- 2005 ... Wijesooriya, Reimer, Holt [Phys. Rev. C **72** \(2005\) 065203](#)
 - Partial NLO analysis of E615 data
 - Large-x power-law $\rightarrow 1.54 \pm 0.08$
- 2010/02 ... Controversy highlighted: Holt & Roberts, [Rev. Mod. Phys. **82** \(2010\) 2991-3044](#)



π valence-quark distributions

20 Years of Evolution

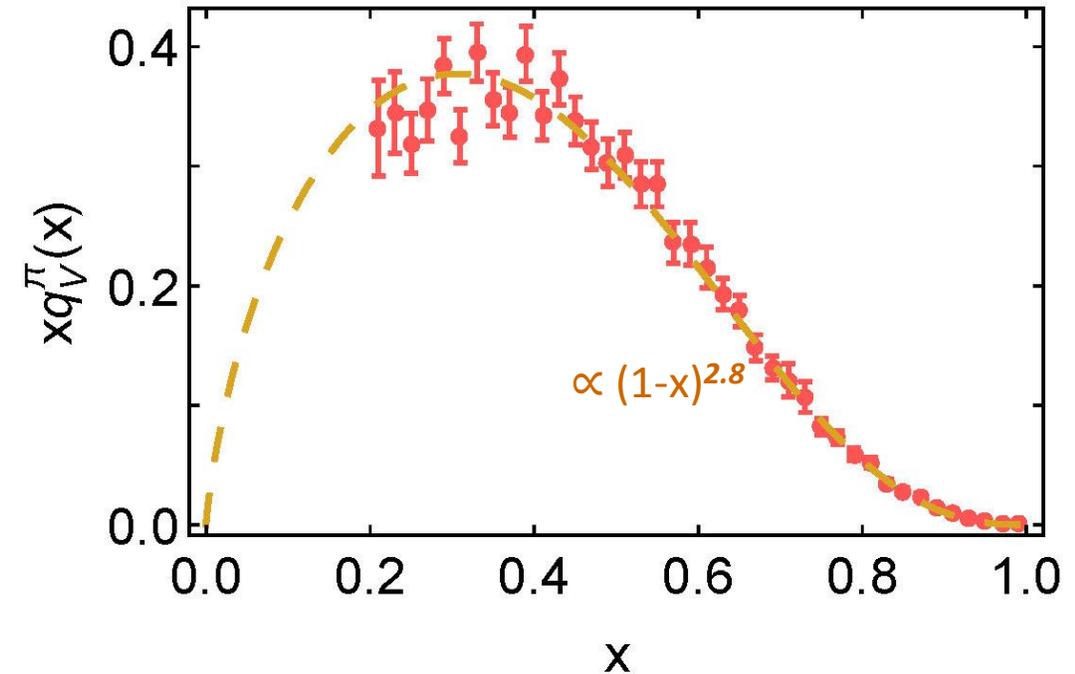
- 1989 ... Conway *et al.* [Phys. Rev. D **39**, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data
- 2000 ... Hecht *et al.* [Phys.Rev. C **63** \(2001\) 025213](#)
 - QCD-connected model prediction
- 2010/02 ... Controversy highlighted: Holt & Roberts, [Rev. Mod. Phys. **82** \(2010\) 2991-3044](#)
- 2010/09 ... Reconsideration of data: Aicher *et al.*, [Phys. Rev. Lett. **105** \(2010\) 252003](#)
 - Consistent next-to-leading-order analysis
 - Large-x power-law $\rightarrow 2.6(1)$

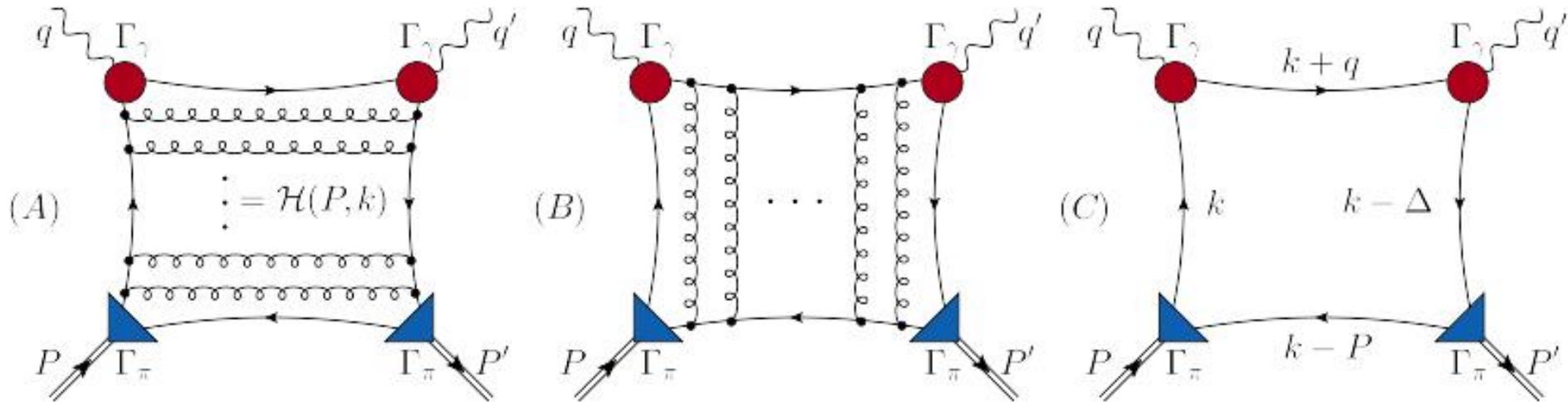


π valence-quark distributions

20 Years of Evolution

- 1989 ... Conway *et al.* [Phys. Rev. D **39**, 92 \(1989\)](#)
 - Leading-order analysis of Drell-Yan data
- 2000 ... Hecht *et al.* [Phys.Rev. C **63** \(2001\) 025213](#)
 - QCD-connected model prediction
- 2010/02 ... Controversy highlighted: Holt & Roberts, [Rev. Mod. Phys. **82** \(2010\) 2991-3044](#)
- 2010/09 ... Reconsideration of data: Aicher *et al.*, [Phys. Rev. Lett. **105** \(2010\) 252003](#)
 - Consistent next-to-leading-order analysis
 - Large-x power-law $\rightarrow 2.6(1)$



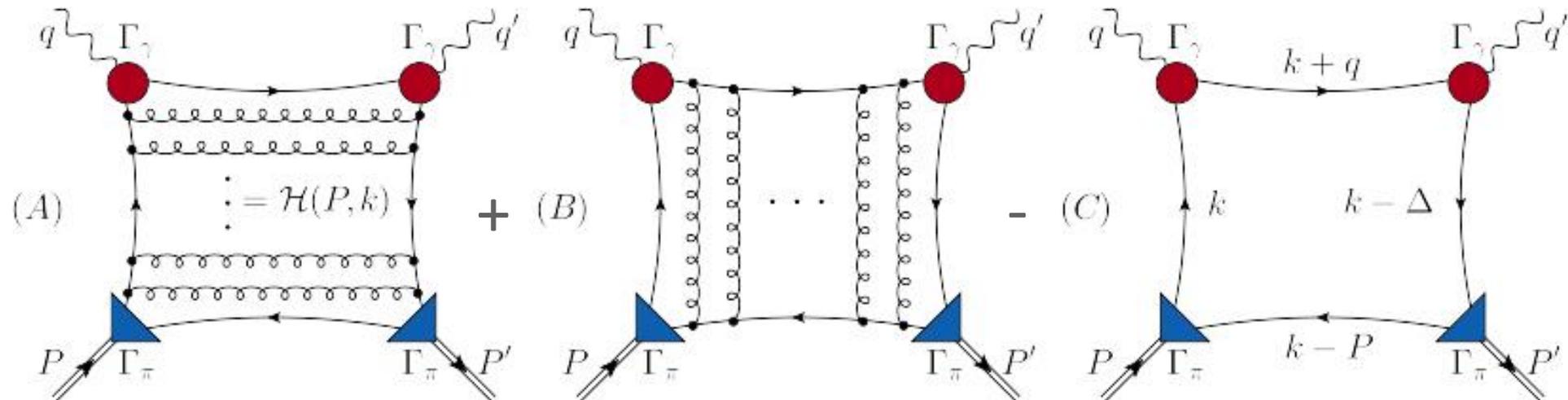


Symmetry, Symmetry Breaking, & Pion Structure

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Pion structure function is given by imaginary part of the virtual-photon – pion forward Compton scattering amplitude:

$$\gamma^*(q) + \pi(P) \rightarrow \gamma^*(q) + \pi(P)$$
- Any nonperturbative analysis will compute the structure function at an hadronic scale, ζ_H
- Using the leading-order truncation of the continuum bound-state problem, *this* collection of diagrams is necessary and sufficient to preserve all Ward-Green-Takahashi identities:



Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Continuum calculations should be renormalised at hadronic scale, where dressed quasiparticles are the correct degrees-of-freedom.
 - Recognises that a given meson's Poincaré covariant wave function and correlated vertices, too, must evolve with ζ ... [Lepage:1979zb, Efremov:1979qk, Lepage:1980fj]
- Such evolution enables dressed-quark and -antiquark degrees-of-freedom, in terms of which the wave function is expressed at a given scale $\zeta^2 = Q^2$, to *undress* ...
 - split into less-well dressed partons via emission of gluons and sea quarks as prescribed by QCD dynamics.
- These effects are automatically incorporated in bound-state problems when complete quark-antiquark scattering kernel is used
 - aspects are lost when kernel is truncated, *e.g.* RL truncation.

- ✓ m_α = essentially nonperturbative scale whose existence ensures that modes with $k^2 \leq m_\alpha^2$ are screened from interactions.
- ✓ m_α therefore serves to define the natural boundary between soft and hard physics

- What is the natural value for the hadronic scale, ζ_H ?
- Recall QCD's process-independent effective charge
- This running-coupling saturates in the infrared:

$$\alpha_{PI}(k^2=0) \approx \pi$$

owing to dynamical generation of gluon mass-scale

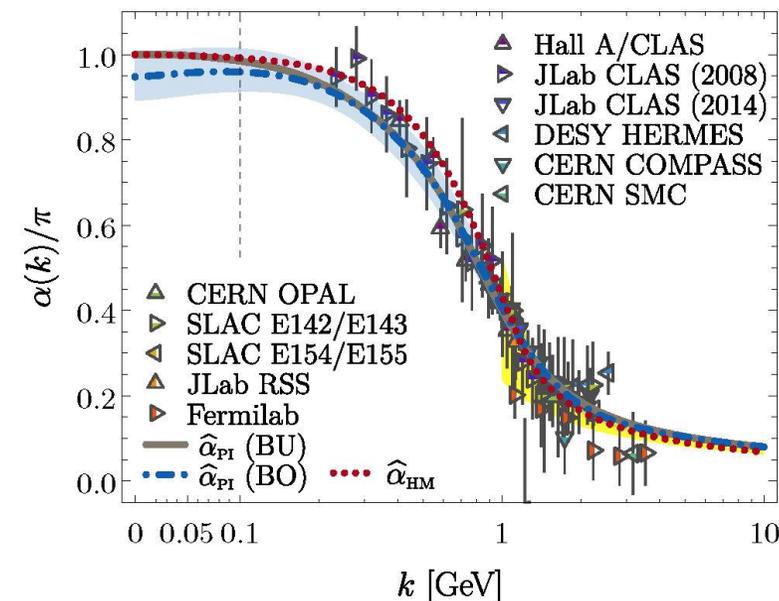
- These features and a smooth connection with pQCD are expressed in the following algebraic expression

$$\beta_0 = 11 - (2/3) n_f$$

$$m_\alpha = 0.3 \text{ GeV} \sim \Lambda_{\text{QCD}}$$

$$\alpha_{PI}(k^2) = \frac{4\pi}{\beta_0 \ln[(m_\alpha^2 + k^2)/\Lambda_{\text{QCD}}^2]}$$

Hadronic Scale = ζ_H

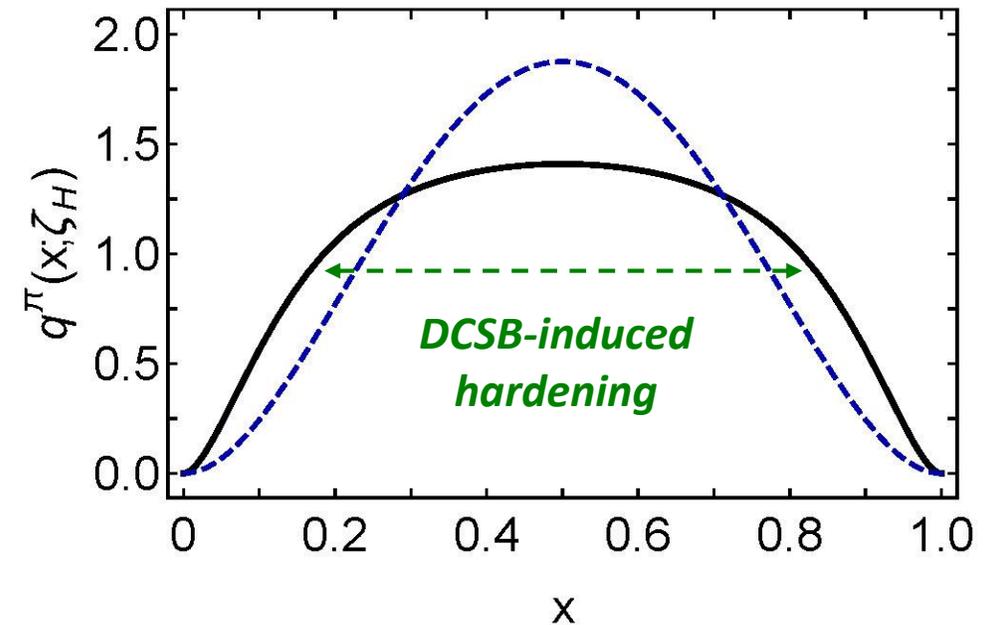


Identify $\zeta_H = m_\alpha$

$q^\pi(x, \zeta_H)$

- Reconstruct PDF from Mellin moments using novel numerical and algebraic techniques
- Dashed Blue = scale-free parton-model-like result $30 x^2 (1-x)^2$
- Solid Black = continuum-QCD prediction
- ✓ Distribution is a broad concave function.
- ✓ Similar effect observed in the pion's leading-twist valence-quark distribution amplitude [Chang:2013pq] & those of other mesons
- ✓ Cause is identical: $q^\pi(x, \zeta_H)$ is hardened owing to DCSB
 - = realisation of the mechanism responsible for the emergence of mass in the Standard Model
- ✓ DCSB is expressed in momentum-dependence of all QCD Schwinger functions.
- ✓ Therefore manifest in pointwise behaviour of wave functions, elastic and transition form factors, *etc.*; and as now seen, also in parton distributions.
- ✓ Expected, given the connection between light-front wave functions and parton distributions

$$q^\pi(x; \zeta_H) = 213.32 x^2 (1-x)^2 \times [1 - 2.9342 \sqrt{x(1-x)} + 2.2911 x(1-x)]$$



- $q^\pi(x, \zeta_H)$ computed at $\zeta_H = m_\alpha$... but ...
 - existing IQCD calculations of low-order moments & phenomenological fits to pion parton distributions are typically quoted at $\zeta_2 = 2$ GeV
 - and the scale relevant to the E615 data is $\zeta_5 = 5.2$ GeV
- Therefore employ QCD evolution
 - $q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_2) \rightarrow q^\pi(x, \zeta_5)$
using the process-independent running coupling
- Given that $\zeta_H = m_\alpha$ is fixed by the analysis, all results are predictions
- $\alpha_{p_l}(\zeta_H)/(2\pi) = 0.20$ & $[\alpha_{p_l}(\zeta_H)/(2\pi)]^2 = 0.04$
... so LO evolution should serve as a good approximation
- Results reported with $\zeta_H \rightarrow (1 \pm 0.1) \zeta_H$

$q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_2)$

- ✓ Nonsinglet evolution for valence-quark

$$\langle 2x \rangle_q^\pi = 0.48(3)$$
- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry only 1/2 pion's light-front momentum
- ✓ At ζ_H , pion is solely bound-state of dressed-quark and dressed-antiquark
- ✓ Glue and sea distributions are zero at ζ_H
- ✓ g & S distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle x \rangle_g^\pi = 0.41(2), \quad \langle x \rangle_{\text{sea}}^\pi = 0.11(2)$$

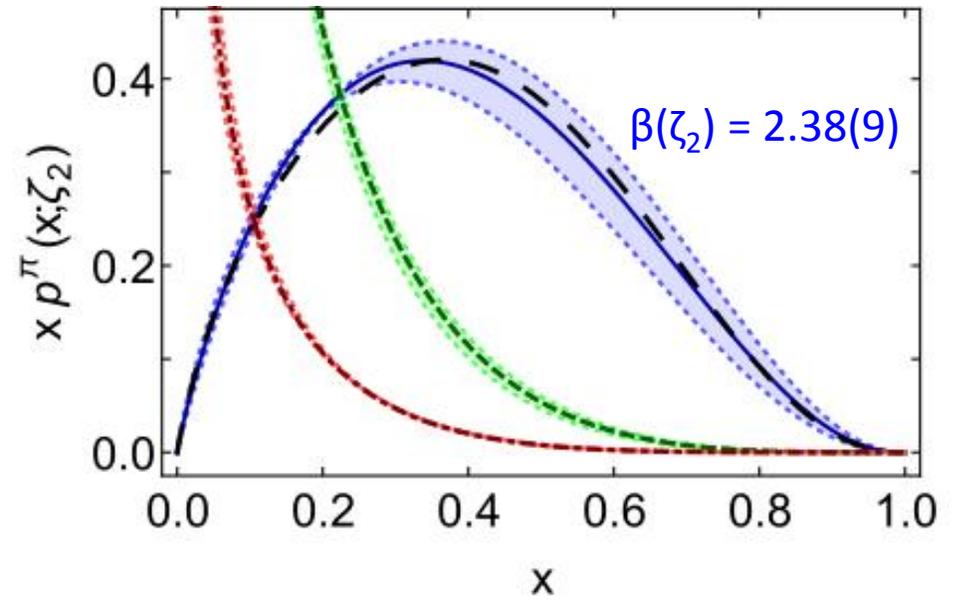


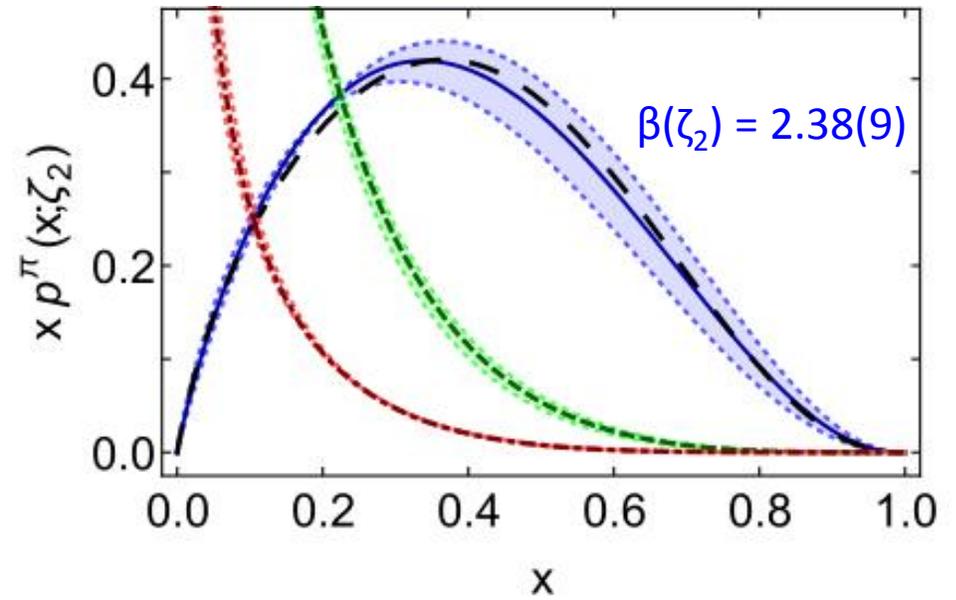
FIG. 6. Pion valence-quark momentum distribution function, $xq^\pi(x; \zeta)$, evolved $\zeta_H \rightarrow \zeta_2 = 2 \text{ GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_2 result from Ref. [12]. Eqs. (39), (40): gluon momentum distribution in pion, $xg^\pi(x; \zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^\pi(x; \zeta_2)$ – dot-dashed (red) curve within shaded band. In all cases, the shaded band indicates the effect of $\zeta_H \rightarrow \zeta_H(1 \pm 0.1)$.

$q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_2)$

- ✓ Nonsinglet evolution for valence-quark

$$\langle 2x \rangle_q^\pi = 0.48(3)$$
- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry only 1/2 pion's light-front momentum
- ✓ Pion is solely bound-state of dressed-quark and dressed-antiquark at ζ_H
- ✓ Glue and sea distributions are zero at ζ_H
- ✓ g & S distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle x \rangle_g^\pi = 0.41(2), \quad \langle x \rangle_{\text{sea}}^\pi = 0.11(2)$$



$$x p^\pi(x; \zeta) = \mathcal{A} x^\alpha (1 - x)^\beta, \quad (39)$$

with the coefficient and powers listed here ($p = g = \text{glue}$, $p = S = \text{sea}$):

	p	\mathcal{A}	α	β	
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13	(40)
	sea	0.13 ∓ 0.01	-0.53 ∓ 0.05	4.51 ± 0.03	
ζ_5	g	0.34 ∓ 0.04	-0.62 ∓ 0.04	3.75 ± 0.12	
	sea	0.12 ± 0.02	-0.61 ∓ 0.07	4.77 ± 0.03	

$q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark
- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry less-than 1/2 pion's light-front momentum
- ✓ g & S distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle 2x \rangle_q^\pi = 0.42(3)$$

$$\langle x \rangle_g^\pi = 0.45(1), \quad \langle x \rangle_{\text{sea}}^\pi = 0.14(2)$$

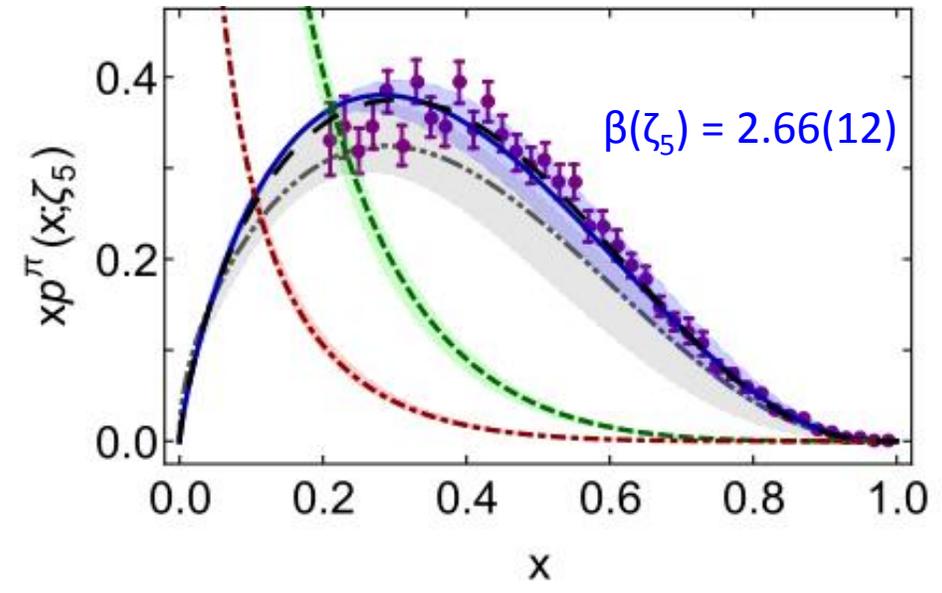


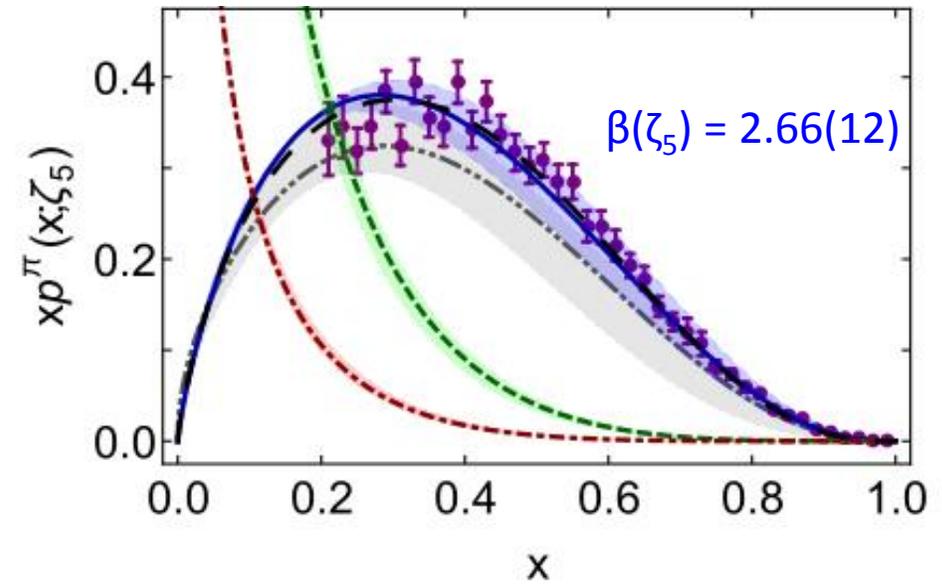
FIG. 7. Pion valence-quark momentum distribution function, $xq^\pi(x; \zeta)$, evolved $\zeta_H \rightarrow \zeta_5 = 5.2 \text{ GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_5 result from Ref. [12]. Gluon momentum distribution in pion, $xg^\pi(x; \zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^\pi(x; \zeta_2)$ – dot-dashed (red) curve within shaded band. See Eqs. (39), (40). In all the above cases, the shaded band indicates the effect of $\zeta_H \rightarrow \zeta_H(1 \pm 0.1)$. Dot-dot-dashed (grey) curve within shaded band – lQCD result [31]. Data (purple) from Ref. [9], rescaled according to the analysis in Ref. [14].

$q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark
- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry less-than 1/2 pion's light-front momentum
- ✓ g & S distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle 2x \rangle_q^\pi = 0.42 (3)$$

$$\langle x \rangle_g^\pi = 0.45(1), \quad \langle x \rangle_{\text{sea}}^\pi = 0.14(2)$$



$$xp^\pi(x; \zeta) = \mathcal{A} x^\alpha (1 - x)^\beta, \quad (39)$$

with the coefficient and powers listed here ($p = g = \text{glue}$, $p = S = \text{sea}$):

	p	\mathcal{A}	α	β	
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13	. (40)
	sea	0.13 ∓ 0.01	-0.53 ∓ 0.05	4.51 ± 0.03	
ζ_5	g	0.34 ∓ 0.04	-0.62 ∓ 0.04	3.75 ± 0.12	
	sea	0.12 ± 0.02	-0.61 ∓ 0.07	4.77 ± 0.03	

$q^\pi(x, \zeta_H) \rightarrow q^\pi(x, \zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark

$$\langle 2x \rangle_q^\pi = 0.42 (3)$$

- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry less-than 1/2 pion's light-front momentum
- ✓ dot-dot-dashed (grey) = IQCD result for the pion valence-quark distribution function [Sufian:2019bol]
- ✓ Pointwise form of the IQCD prediction agrees with continuum result (within errors)
- ✓ **Significant**: two disparate treatments of pion structure have arrived at the same prediction for $q^\pi(x, \zeta_5)$

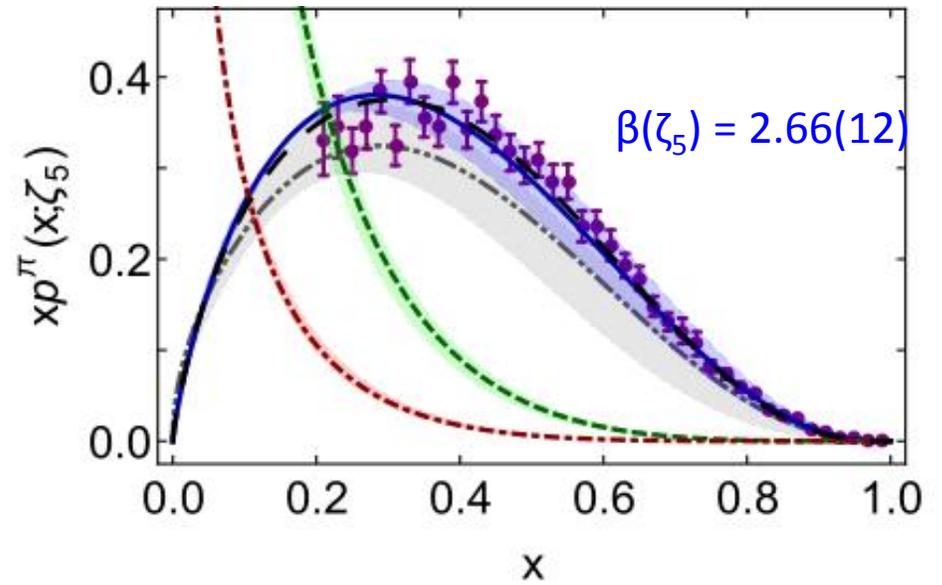


FIG. 7. Pion valence-quark momentum distribution function, $xq^\pi(x; \zeta)$, evolved $\zeta_H \rightarrow \zeta_5 = 5.2 \text{ GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_5 result from Ref. [12]. Gluon momentum distribution in pion, $xg^\pi(x; \zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^\pi(x; \zeta_2)$ – dot-dashed (red) curve within shaded band. See Eqs. (39), (40). In all the above cases, the shaded band indicates the effect of $\zeta_H \rightarrow \zeta_H(1 \pm 0.1)$. Dot-dot-dashed (grey) curve within shaded band – IQCD result [31]. Data (purple) from Ref. [9], rescaled according to the analysis in Ref. [14].

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Continuum and Lattice results agree on valence-quark distributions
 - No parameters varied to achieve this outcome
 - Nor any other
- Remarkable, modern confluence,
 - Suggests that real strides are being made toward understanding pion structure.
- Realistic predictions for glue & sea content of pion
 - results match phenomenological expectations
- ***After 30 years, experimental facilities are available/planned that can validate these crucial aspects of Strong QCD***

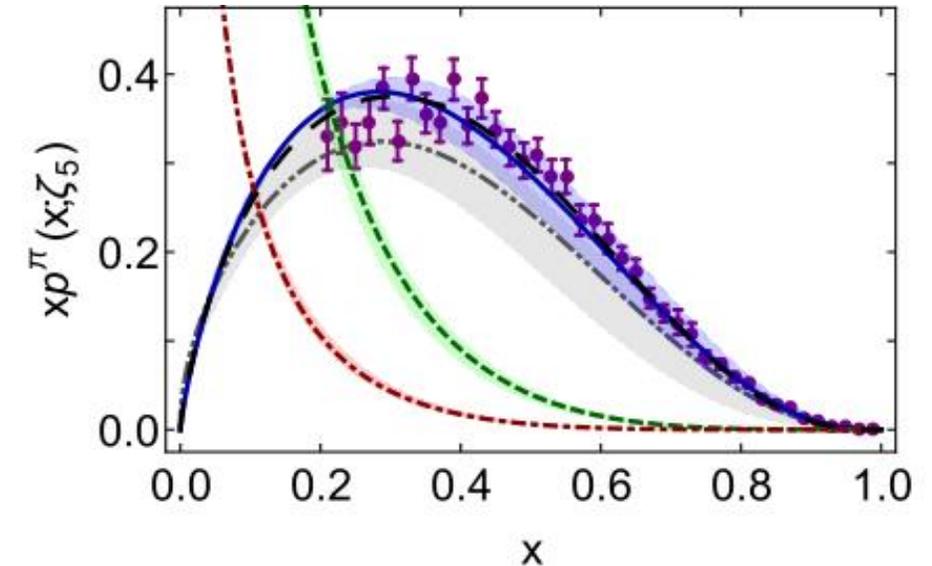


FIG. 7. Pion valence-quark momentum distribution function, $xq^\pi(x; \zeta)$, evolved $\zeta_H \rightarrow \zeta_5 = 5.2 \text{ GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_5 result from Ref. [12]. Gluon momentum distribution in pion, $xg^\pi(x; \zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^\pi(x; \zeta_2)$ – dot-dashed (red) curve within shaded band. See Eqs. (39), (40). In all the above cases, the shaded band indicates the effect of $\zeta_H \rightarrow \zeta_H(1 \pm 0.1)$. Dot-dot-dashed (grey) curve within shaded band – lQCD result [31]. Data (purple) from Ref. [9], rescaled according to the analysis in Ref. [14].

π & K PDFs

- Urgent need for Newer Data
 - Persistent controversy regarding the Bjorken- $x \simeq 1$ behaviour of the pion's valence-quark PDF \Rightarrow phenomenological analyses should include soft-gluon resummation
 - Confluence of continuum and lattice results \Rightarrow Prediction that *must* be checked
 - Single modest-quality measurement of $u^K(x)/u^\pi(x)$ (1980) cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question
- Similar technique might also serve for the kaon ... TDIS experiment approved at JLab
- Future:
 - New mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs
 - “Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER)” [<http://arxiv.org/abs/arXiv:1808.00848>]
 - EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.

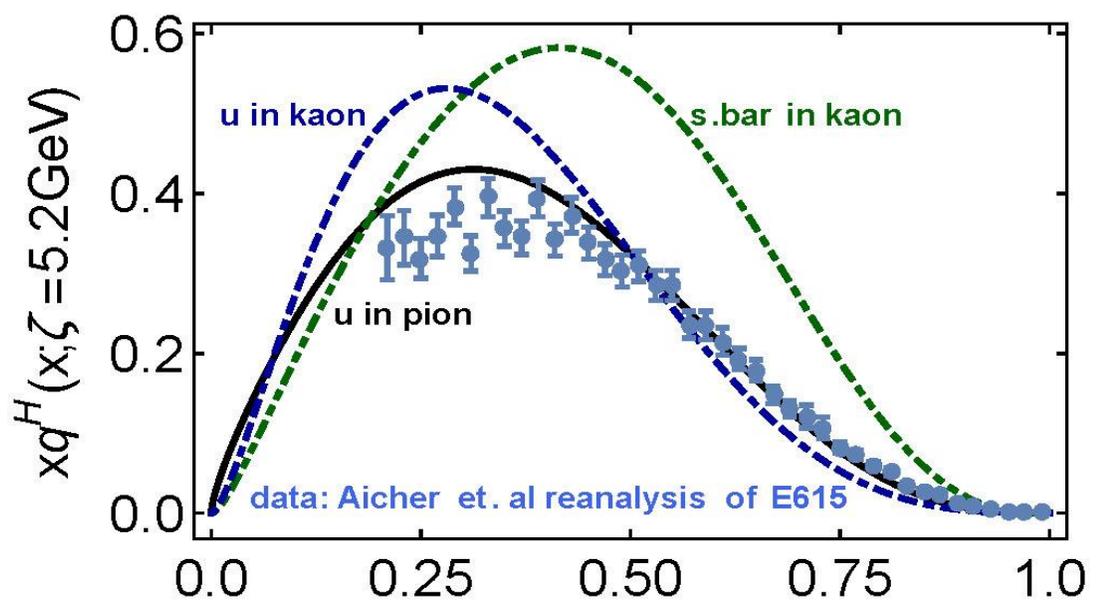
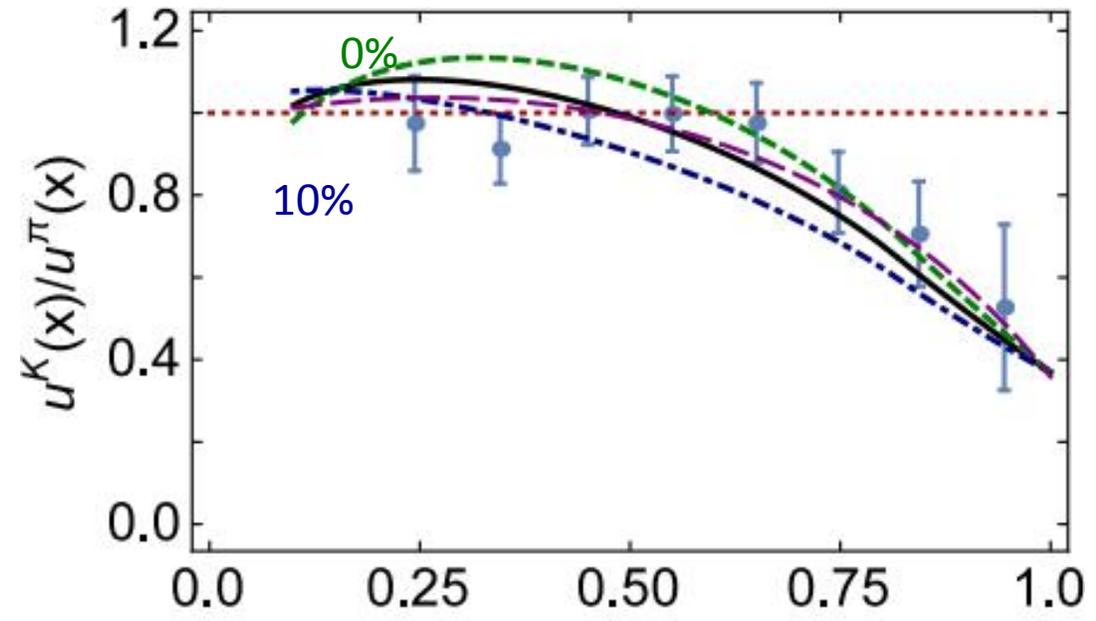
Kaon's gluon content

- $\langle x \rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
 \Rightarrow Valence quarks carry 95% of kaon's momentum at ζ_H
- DGLAP-evolved to ζ_2

q	$\langle x \rangle_q^K$	$\langle x^2 \rangle_q^K$	$\langle x^3 \rangle_q^K$
u	0.28	0.11	0.048
\bar{s}	0.36	0.17	0.092

Valence-quarks carry $\frac{2}{3}$ of kaon's light-front momentum

Cf. Only $\frac{1}{2}$ for the pion



π & K PDFs

- Marked differences between π & K gluon content
 - ζ_H :
 - Whilst $\frac{1}{3} \sim \frac{1}{5}$ of pion's light-front momentum carried by glue
 - **Only $\frac{1}{20}$ of the kaon's light-front momentum lies with glue**
 - $\zeta_2^2 = 4 \text{ GeV}^2$
 - Glue carries $\frac{1}{2}$ of pion's momentum but only $\frac{1}{3}$ of kaon's momentum
 - Evident in differences between large- x behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of π
 - Nearly complete cancellation between one-particle dressing and binding attraction in this almost-massless pseudoscalar system

$$2 \text{ Mass}_Q + U_g \approx 0$$



π & K PDFs

- Understanding the emergence and structure of Nambu-Goldstone modes in the Standard Model is critical to solving the Standard Model ...
 - Nambu-Goldstone modes are nonpointlike!
 - Intimately connected with origin of mass!
 - Possibly/Probably(?) inseparable from expression of confinement!
- Difference between gluon content of π & K is measurable
 - mesonic Drell-Yan measurements at modern facilities
 - using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model



Pion and Kaon Structure at an Electron-Ion Collider

... White Paper & Review ...

- Provides analysis of the mass budget of the pion and proton in QCD
- Discusses the special role of the kaon, which lies near the boundary between dominance of Higgs- and strong-mass generation mechanisms
- Explains the need for a coherent effort in phenomenology & continuum calculations, in exascale computing, and in experiments ... to make progress in understanding the origins of hadron masses and the distribution of that mass within them.
- Compares the unique capabilities foreseen at an electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA)
- Describes five key experimental measurements, enabled by the EIC and aimed at delivering fundamental insights and generating concrete answers to the questions of
 - How does mass and structure arise in the pion and kaon, the Standard Model's NG modes?

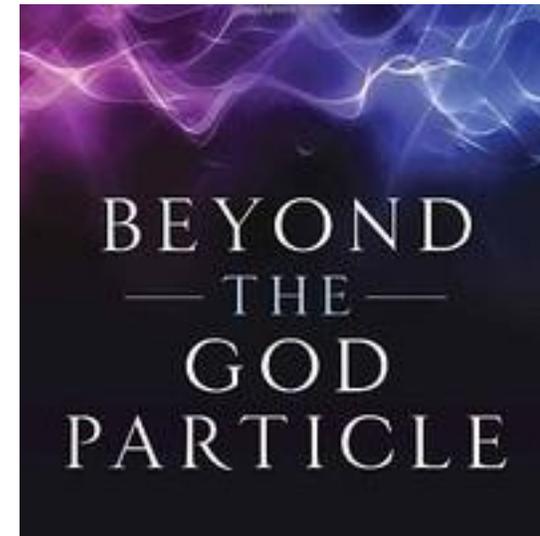
N.B. Their surprisingly low mass is critical to the evolution of our Universe.



Epilogue



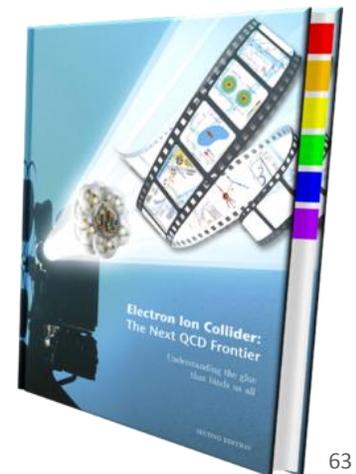
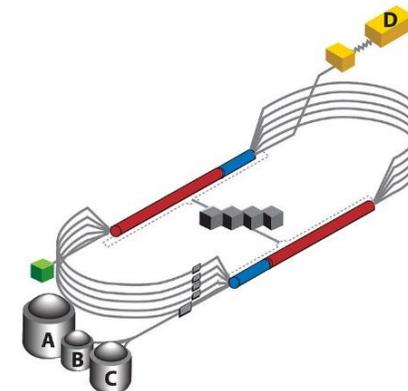
- LHC has NOT found the “God Particle” because the Higgs boson is NOT the origin of mass
 - Higgs-boson only produces a little bit of mass
 - Higgs-generated mass-scales explain neither the proton’s mass nor the pion’s (*near-*) masslessness
 - Hence LHC has, as yet, taught us very little about the origin, structure and nature of the nuclei whose existence support the Cosmos
- Strong interaction sector of the Standard Model, *i.e.* QCD, is the key to understanding the origin, existence and properties of (almost) all known matter



- Challenge: Explain & Understand the Origin & Distribution of the Bulk of Visible Mass
- Current Paradigm: Quantum Chromodynamics
- QCD is plausibly a mathematically well-defined quantum field theory,
The only one we've ever produced
 - Consequently, it is a worthwhile paradigm for developing Beyond-SM theories
- Challenge is to reveal the content of strong-QCD
- **Tough Problem**
- *Progress* and *Insights*
being delivered by amalgam of
 - Experiment ... Phenomenology ... Theory
- Must continue into the modern era of
new opportunities at existing and planned facilities

A New QCD facility at the M2 beam line of the CERN SPS*

COMPASS++[†]/AMBER[‡]



Collaborators: 2017 - Now

Thankyou for listening And my collaborators for contributing

Students, Postdocs, Profs.

1. Zhao-Qian YAO (Nanjing U.)
2. Yin-Zhen XU (Nanjing U.)
3. Marco BEDOLLA (Genova, U Michoacán)
4. Chen CHEN (Giessen, UNESP - São Paulo, USTC & IIT);
5. Muyang CHEN (NKU, PKU)
6. Zhu-Fang CUI (Nanjing U.) ;
7. Minghui DING (ECT*, ANL, Nankai U.) ;
8. Fei GAO (Heidelberg, Valencia, Peking U.) ;
9. Bo-Lin LI (Nanjing U.)
10. Ya LU (Nanjing U.)
11. Cédric MEZRAG (INFN-Roma, ANL, IRFU-Saclay) ;
12. Khépani RAYA (Nankai U., U Michoacán);
13. Adnan Bashir (U Michoacán);
14. Daniele Binosi (ECT*)
15. Volker Burkert (JLab)
16. Lei Chang (Nankai U.) ;
17. Xiao-Yun Chen (Jinling Inst. Tech., Nanjing)
18. Feliciano C. De Soto Borrero (UPO);
19. Tanja Horn (Catholic U. America)
20. Gastão Krein (UNESP – São Paulo)
21. Yu-Xin Liu (PKU);
22. Joannis Papavassiliou (U.Valencia)
23. Jia-Lun Ping (Nanjing Normal U.)
24. Si-xue Qin (Chongqing U.);
25. Jose Rodriguez Quintero (U. Huelva) ;
26. Elena Santopinto (Genova)
27. Jorge Segovia (U. Pablo de Olavide = UPO);
28. Sebastian Schmidt (IAS-FZJ & JARA);
29. Shaolong Wan (USTC) ;
30. Qing-Wu Wang (Sichuan U)
31. Shu-Sheng XU (NJUPT, Nanjing U.)
32. Pei-Lin Yin (NJUPT)
33. Hong-Shi Zong (Nanjing U)



Pion and Kaon Structure at an Electron-Ion Collider

... Five Key Measurements ...

- i. Measurement of pion and kaon structure functions and their GPDs will render insights into quark and gluon energy contributions to hadron masses.
- ii. Measurements of open charm production will settle the question of whether gluons persist or disappear within pions in the chiral limit – if they persist it proves the cancellation of terms that must occur such that the pion mass is driven by Higgs-generated current quark masses, albeit with a huge emergent magnification factor.
- iii. Measurement of pion form factor up to $Q^2 \approx 35 \text{ GeV}^2$, which can be quantitatively related to emergent-mass acquisition from dynamical chiral symmetry breaking.
- iv. Measurement of the behavior of (valence) u -quarks in the pion and kaon, which gives a quantitative measure of the contributions of gluons to NG boson masses and differences between the impacts of emergent- and Higgs-mass generating mechanisms.
- v. Measurement of the fragmentation of quarks into pions and kaons, a timelike analogue of mass acquisition, which can potentially reveal relationships between dynamical chiral symmetry breaking and the confinement mechanism.