TMD evolution from SIDIS data

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Outlook

I will discuss some of the (TMD) physics in COMPASS measurements

Collins, Sivers, Unpolarized functions (**crucial**)

Challenges remain. I review some.

TMD evolution hard to see in data
(Sivers and Collins effects).
Accurate determination of unpolarized functions
Issues with normalization
Matching between small and large qT

Kinematics of applicability of factorization theorems

How to extract TMDs?

Ingredients:

Data

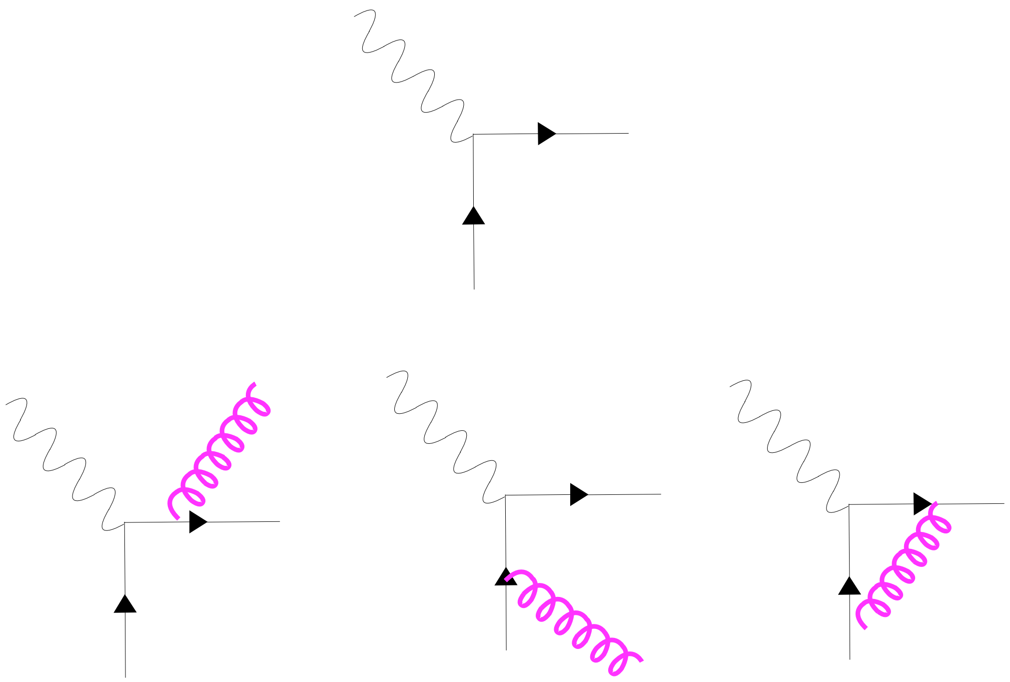
+

Theoretical framework

Recipe is tricky though, several challenges.

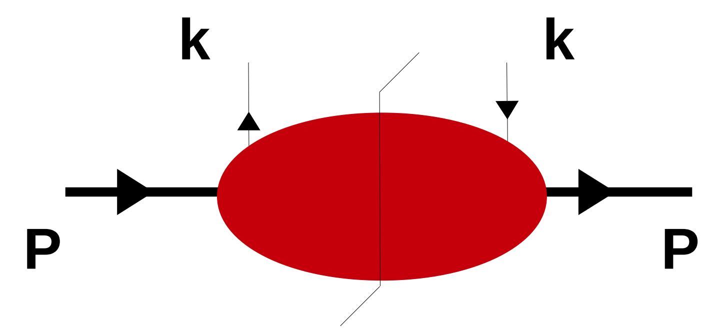
Theoretical Framework: Factorization theorems

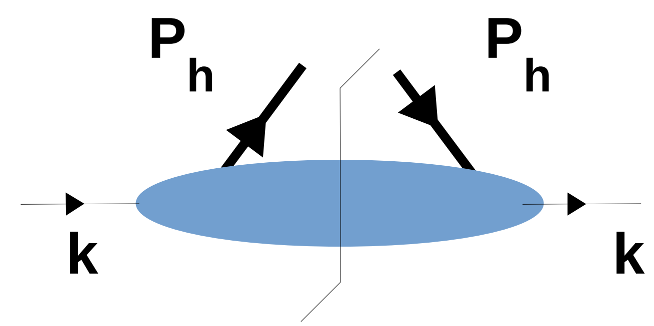
Short distance effects.



pQCD

Long distance physics





Non-perturbative content

Theoretical Framework: Factorization theorems

W **(TMD region)**

$$\sum_q \mathcal{H}_q \text{ F.T.} \left\{ \tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\; \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

Fourier Transform of:

$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}^{in}_{ji}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$
$$\times \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left(\gamma_F(\mu; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu}\right) \gamma_K(\mu) \right) \right\}$$
$$\times \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\},$$

pQCD

Input (extraction from collinear cross section)

Non-perturbative functions to extract from data.

General Strategy: (Possible recipe)

(1) Map kinematical dependencies of different experiments. Simple models, "snap shots" of TMDs.

(2) Study how much information can be inferred on certain effects, e.g. TMD evolution, strictly from data.

(3) Use information from (1), (2) to build full TMD picture (CSS, SCET, other TMD factorization schemes)

- i) Test importance on input information (collinear PDFs & FFs, TMD models).
- ii) Errors of factorization (optimal kinematical regime?).
- iii) Balance between constraints from theory and information obtained from statistical analyses, model comparison.

General Strategy: (Possible recipe)

(1) Map kinematical dependencies of different experiments. Simple models, "snap shots" of TMDs.

(2) Study how much information can be inferred on certain effects, e.g. TMD evolution, strictly from data.

Gaussian model for TMDs in momentum space

(1) & (2)

Transversity and Collins function.

Transversity

$$\Delta_T q(x, k_\perp) = \Delta_T q(x) \, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

$$\mathcal{N}_q^T(x) = N_q^T \, x^\alpha (1-x)^\beta \, \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}$$

$$(q = u_v, d_v)$$

$$\Delta_T q(x, Q_0^2) = \mathcal{N}_q^T(x, Q_0^2) \, \frac{1}{2} \, [f_{q/p}(x, Q_0^2) + \Delta q(x, Q_0^2)]$$

Collins function

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \tilde{\Delta}^N D_{h/q^\uparrow}(z) \, h(p_\perp) \, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\tilde{\Delta}^N D_{h/q^\uparrow}(z, Q_0^2) = 2 \, \mathcal{N}_q^C(z, Q_0^2) \, D_{h/q}(z, Q_0^2)$$

$$h(p_\perp) = \sqrt{2e} \, \frac{p_\perp}{M_C} \, e^{-p_\perp^2/M_C^2}$$

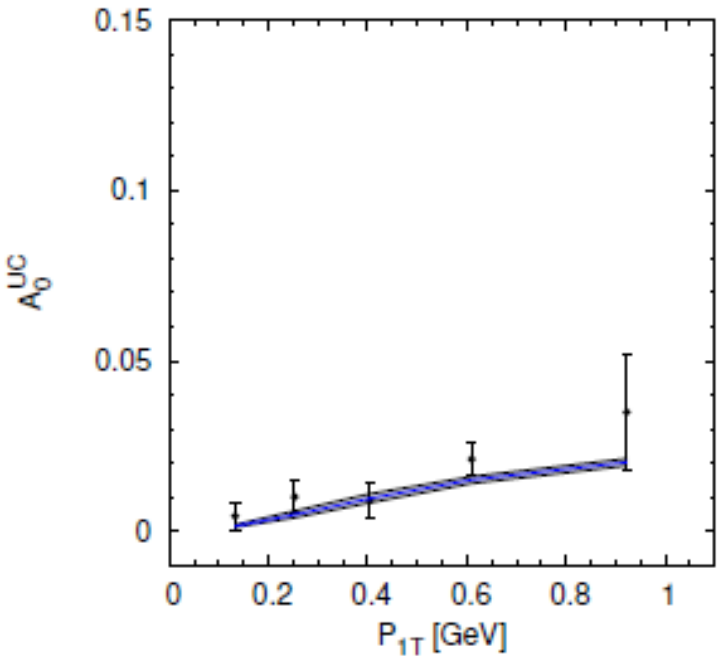
$$\mathcal{N}_{\rm fav}^C(z) = N_{\rm fav}^C \, z^\gamma (1-z)^\delta \, \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma \delta^\delta}$$

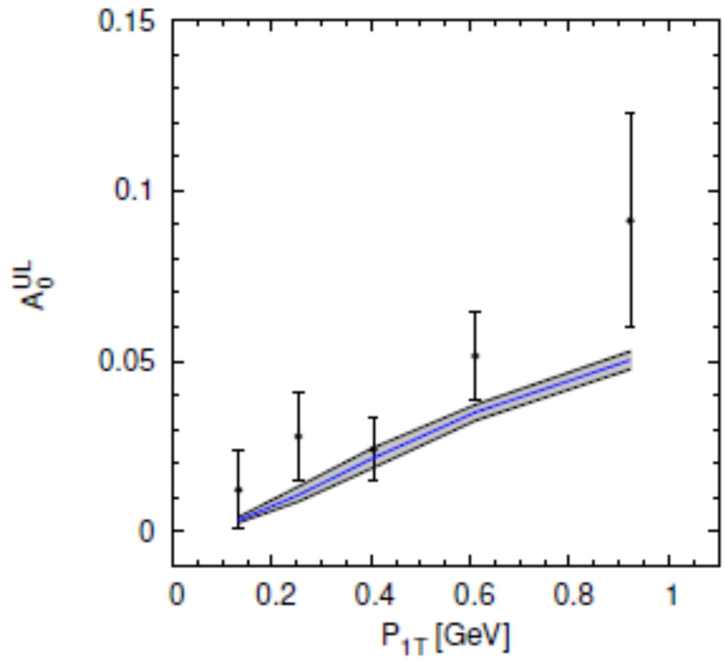
$$\mathcal{N}_{\rm dis}^C(z) = N_{\rm dis}^C$$

(1) & (2)

Scale dependence?

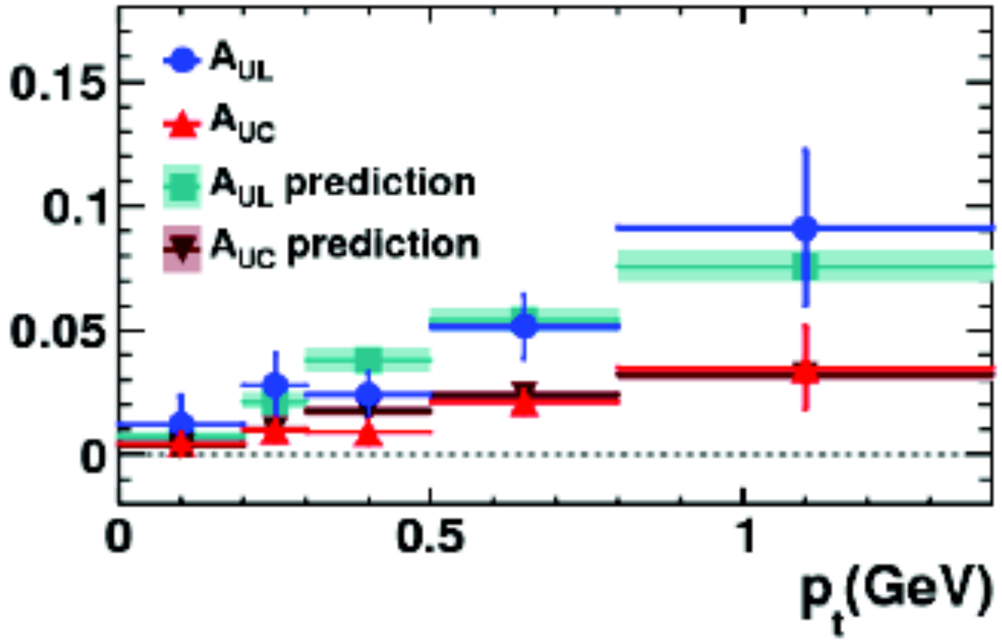
$Q^2 = 13\ GeV^2$





Predictions for BES III

Simple gaussian picture



Picture within QCD-factorization

(3)

Kang, Prokudin, Sun, Yuan

Phys.Rev. D93 (2016) no.1, 014009
arXiv:1505.05589 [hep-ph] JLAB-THY-15-2044

(1) & (2)

Sivers asymmetry in SIDIS

$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S d\phi_h \left[d\sigma^\uparrow - d\sigma^\downarrow\right]\sin(\phi_h-\phi_S)}{\int d\phi_S d\phi_h \left[d\sigma^\uparrow + d\sigma^\downarrow\right]} = \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU}}$$

Generalized Parton Picture (no evolution)

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 4N_q x^{\alpha_q}(1-x)^{\beta_q}\,\frac{M_p}{\langle k_\perp^2\rangle_S}\,k_\perp\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle_S}$$

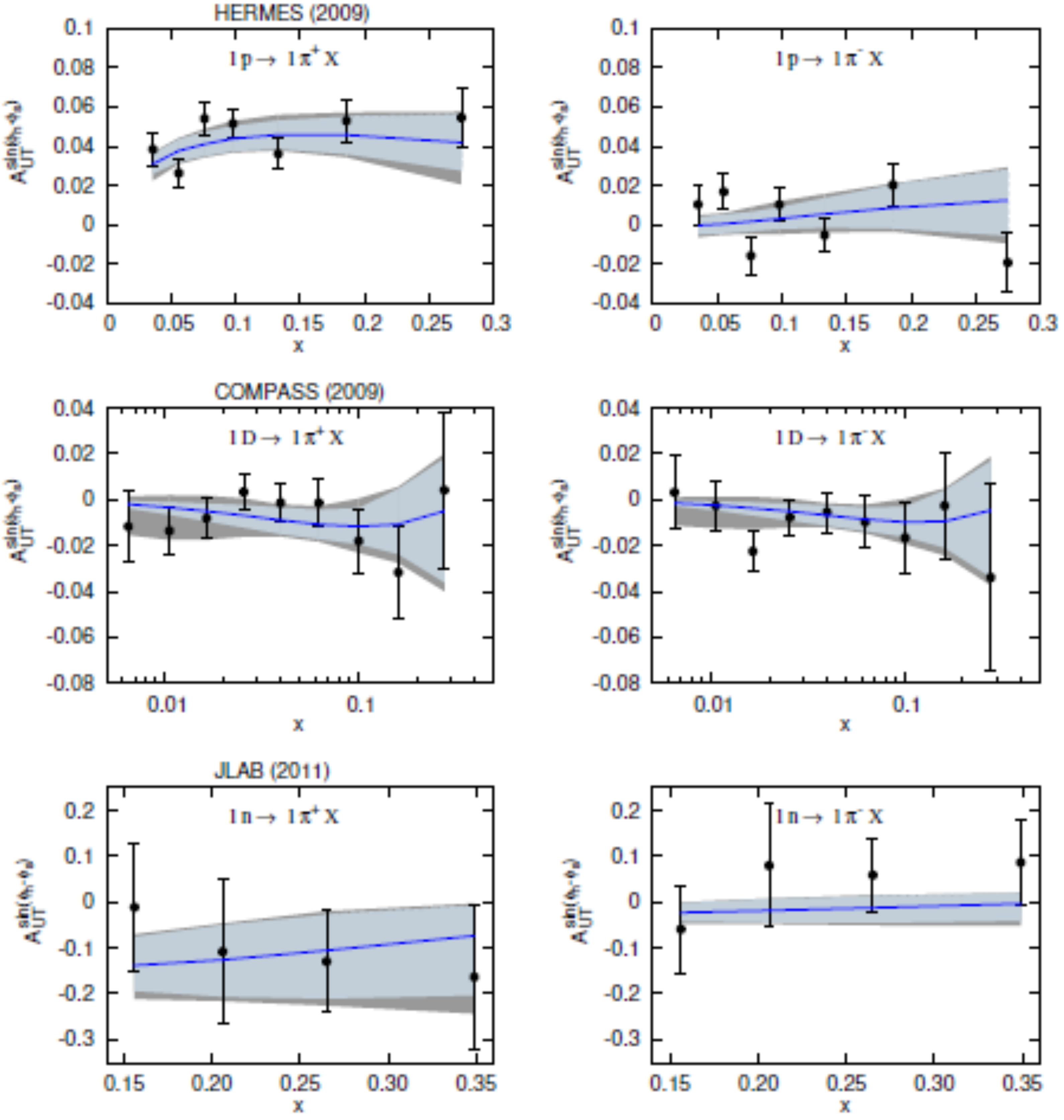
No constraint from collinear PDF

Gaussian ansatz

- **M. Boglione, U. D'Alesio, C. Flore, JOGH , JHEP 1807 (2018) 148**

(1) & (2)

Sivers asymmetry in SIDIS



(1) & (2)

Sivers asymmetry in SIDIS

n. of data points = 220		
One flavour fits (3 parameters)		
	$\chi^2_{\rm tot}$	$\chi^2_{\rm dof}$
u	408	1.88
d	914	4.21
Two flavour fits (5 parameters)		
	$\chi^2_{\rm tot}$	$\chi^2_{\rm dof}$
$u, \bar{u}$	266	1.24
$u, \bar{d}$	228	1.06
$\boldsymbol{u, d}$	**213**	0.99

Is it Reasonable to increase number of parameters?

(1) & (2)

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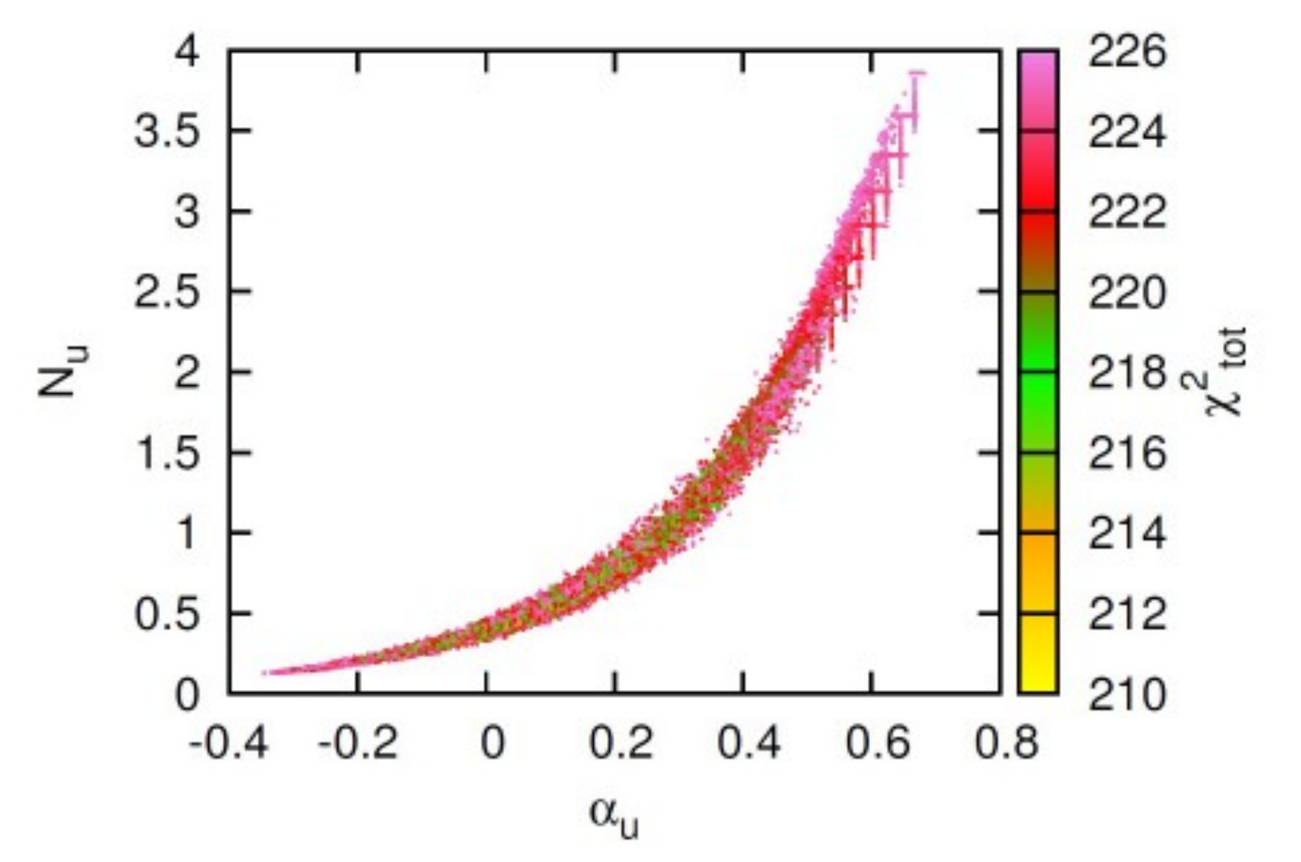
One more parameter (per flavor)



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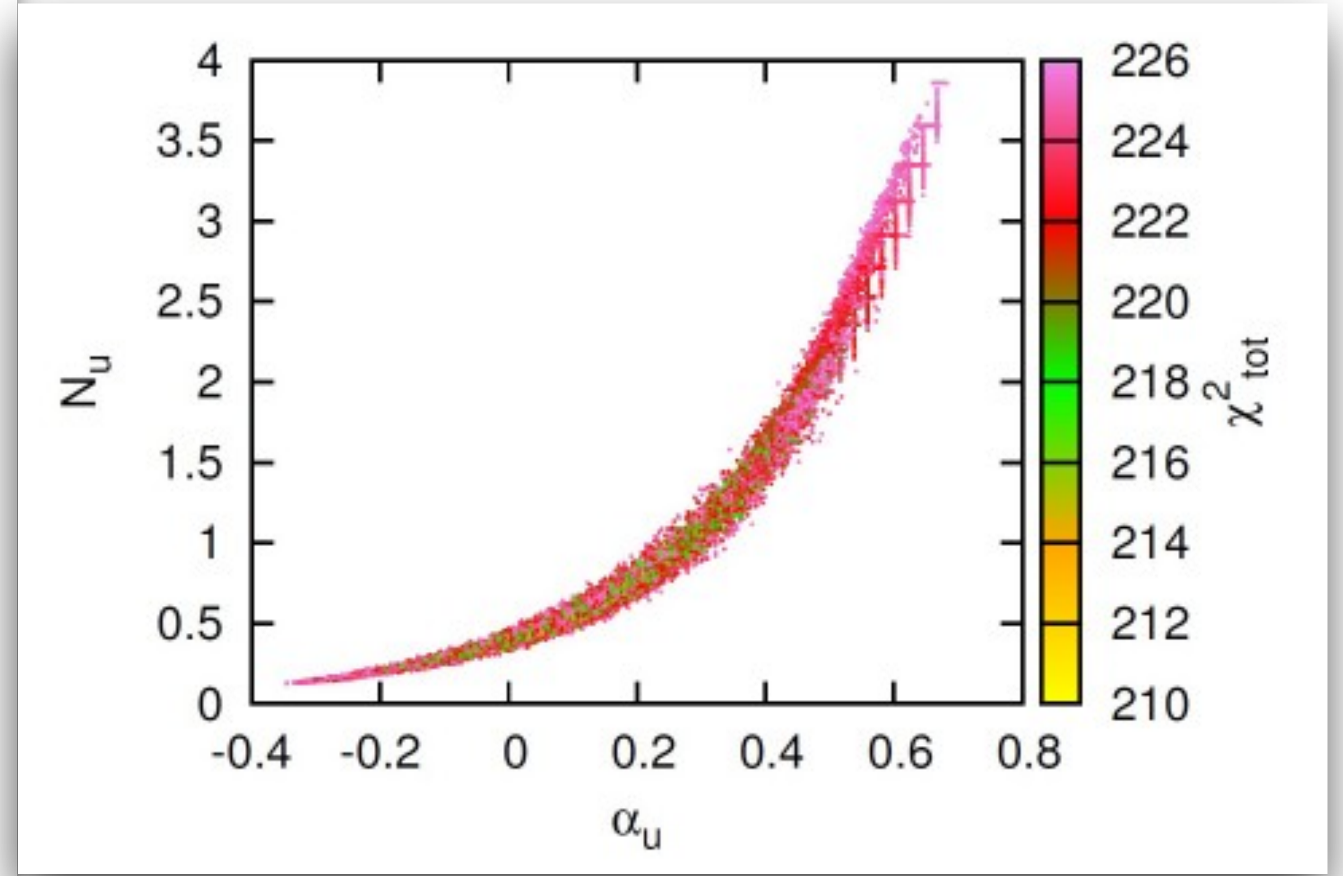
$\boldsymbol{u, d}$	**213**	0.99

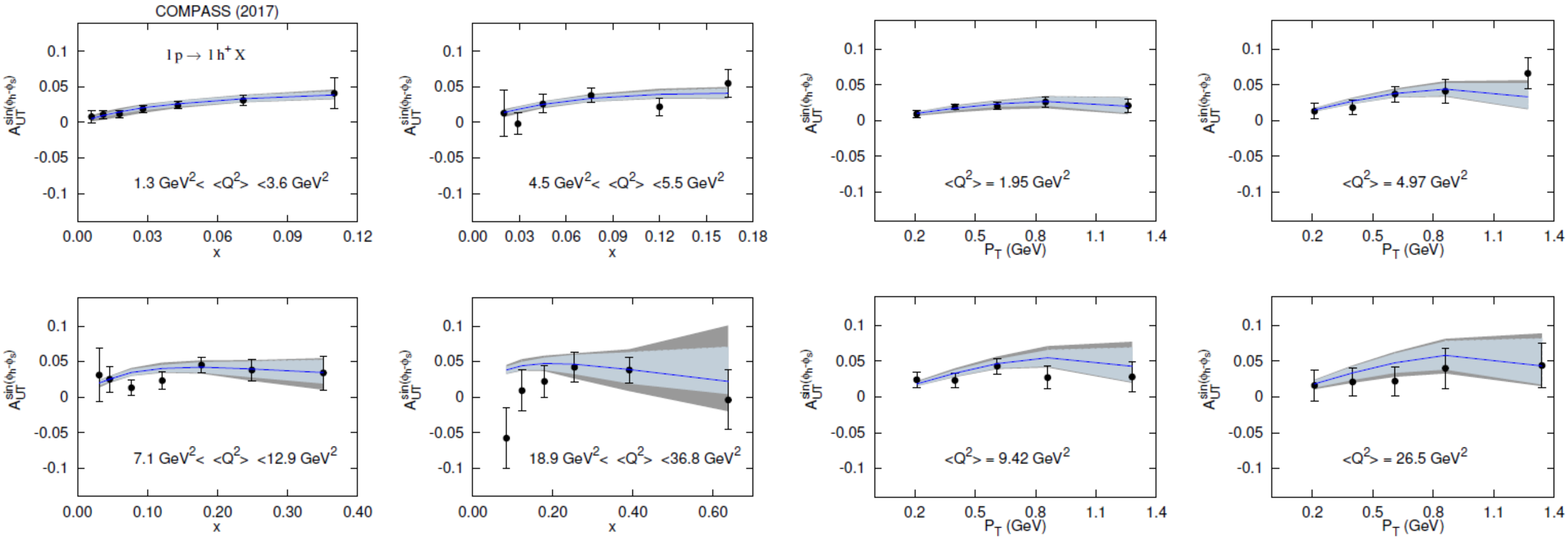
Rough limit on number of parameters (benchmark)

One more parameter (per flavor)



(1) & (2)

Sivers asymmetry in SIDIS



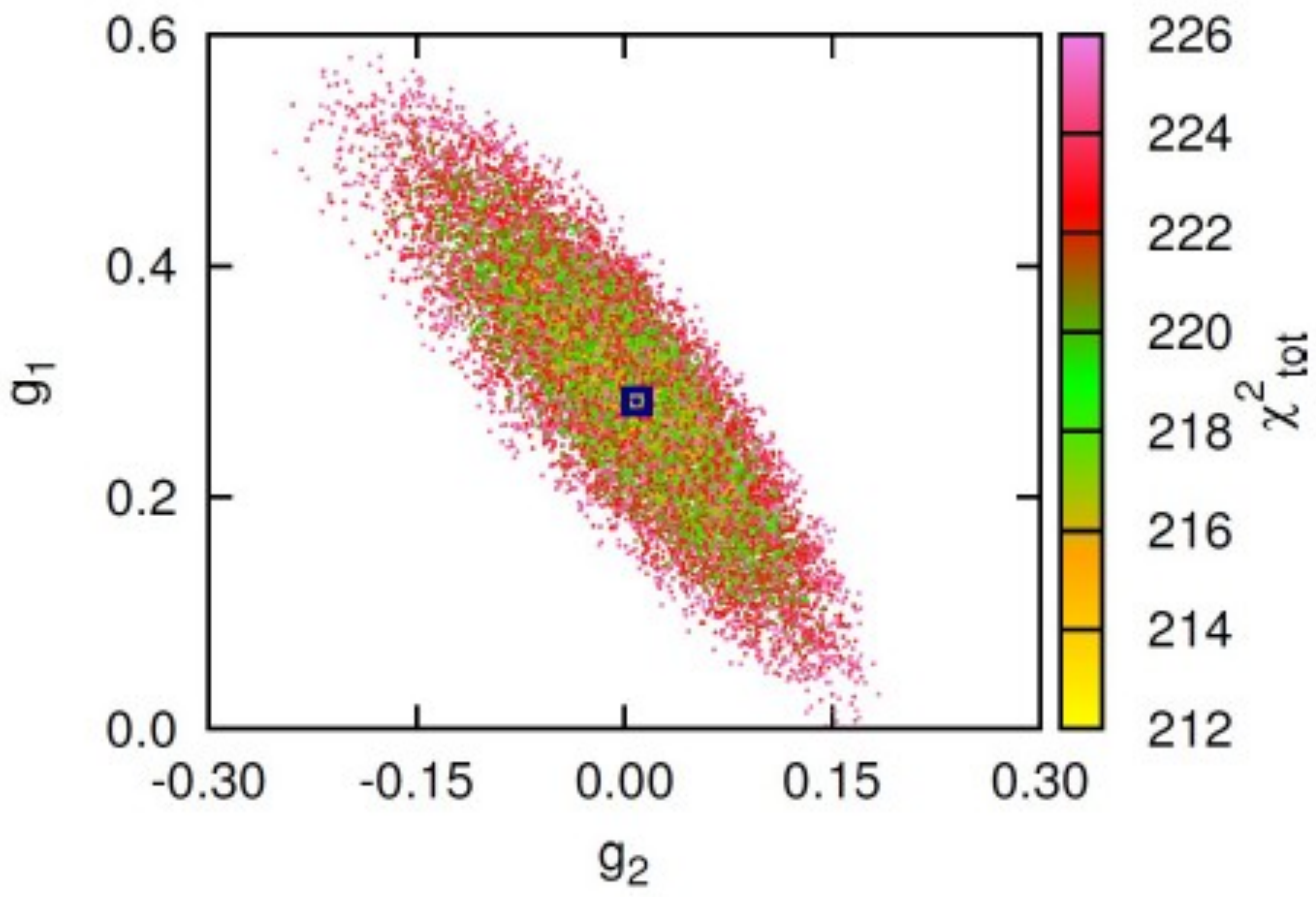
(1) & (2)

Sivers asymmetry in SIDIS

Signals of scale dependence

$$\langle k_\perp^2 \rangle_S = g_1 + g_2 \ln \frac{Q^2}{Q_0^2}$$

g2 here to “mimic” TMD evolution



No “visible” sign TMD evolution, expected:
It washes out in the ratio of the asymmetry

Recapitulating so far:

Signals of TMD evolution are not so “visible” in asymmetries (ratios)

Important to look at correlations of parameters.

One may get a rough idea of a reasonable number of parameters appropriate for an analysis by comparing to some ‘benchmark’ (simple model)

Note that parameter number may increase if adding more constraints (whether correct or incorrect).

TMD, QCD definition (CSS2 scheme)
To many moving parts …

W **(TMD region)**

$$\sum_q \mathcal{H}_q \ \text{F.T.} \left\{ \tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q) \ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

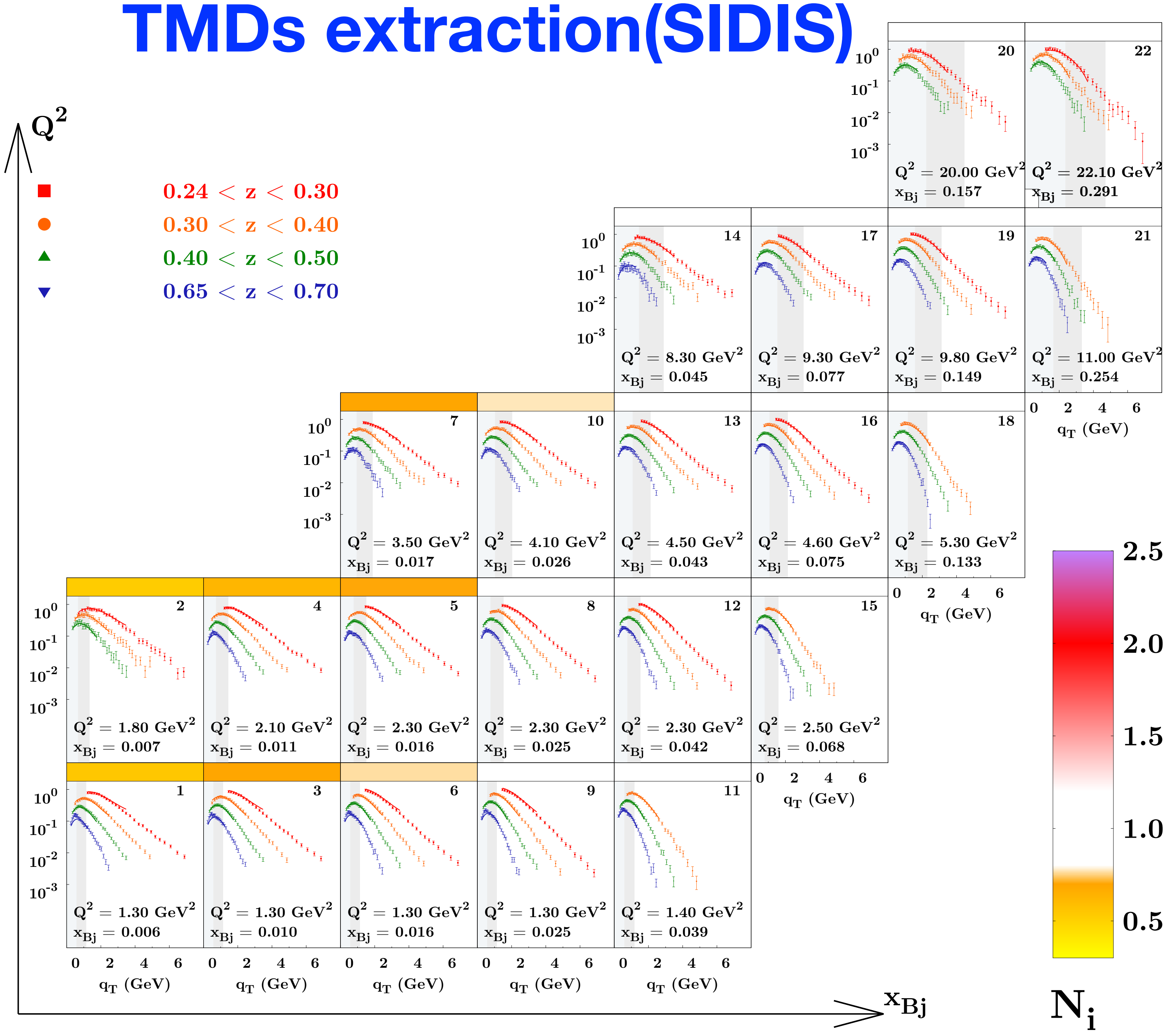
Fourier Transform of:

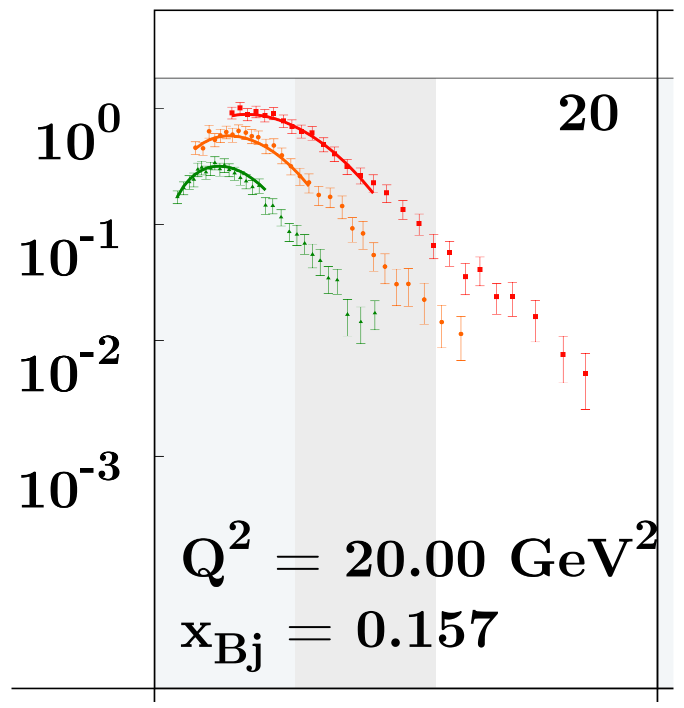
$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$
$$\times \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left(\gamma_F(\mu; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\}$$
$$\times \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\},$$

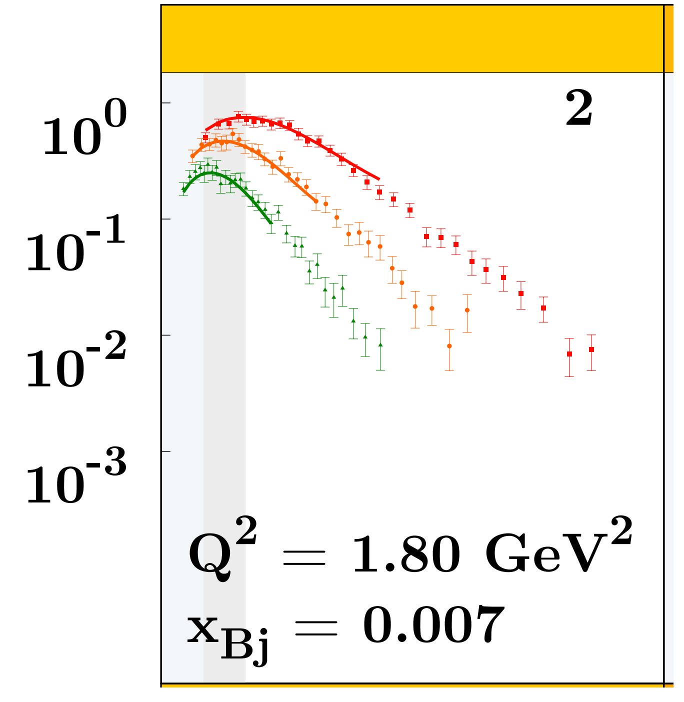
- (blue line) pQCD
- (green line) Input (extraction from collinear cross section)
- (red line) Non-perturbative functions to extract from data.

Some issues with unpolarized TMDs extraction(SIDIS)

$\mathcal{O}(\alpha_s^0)$



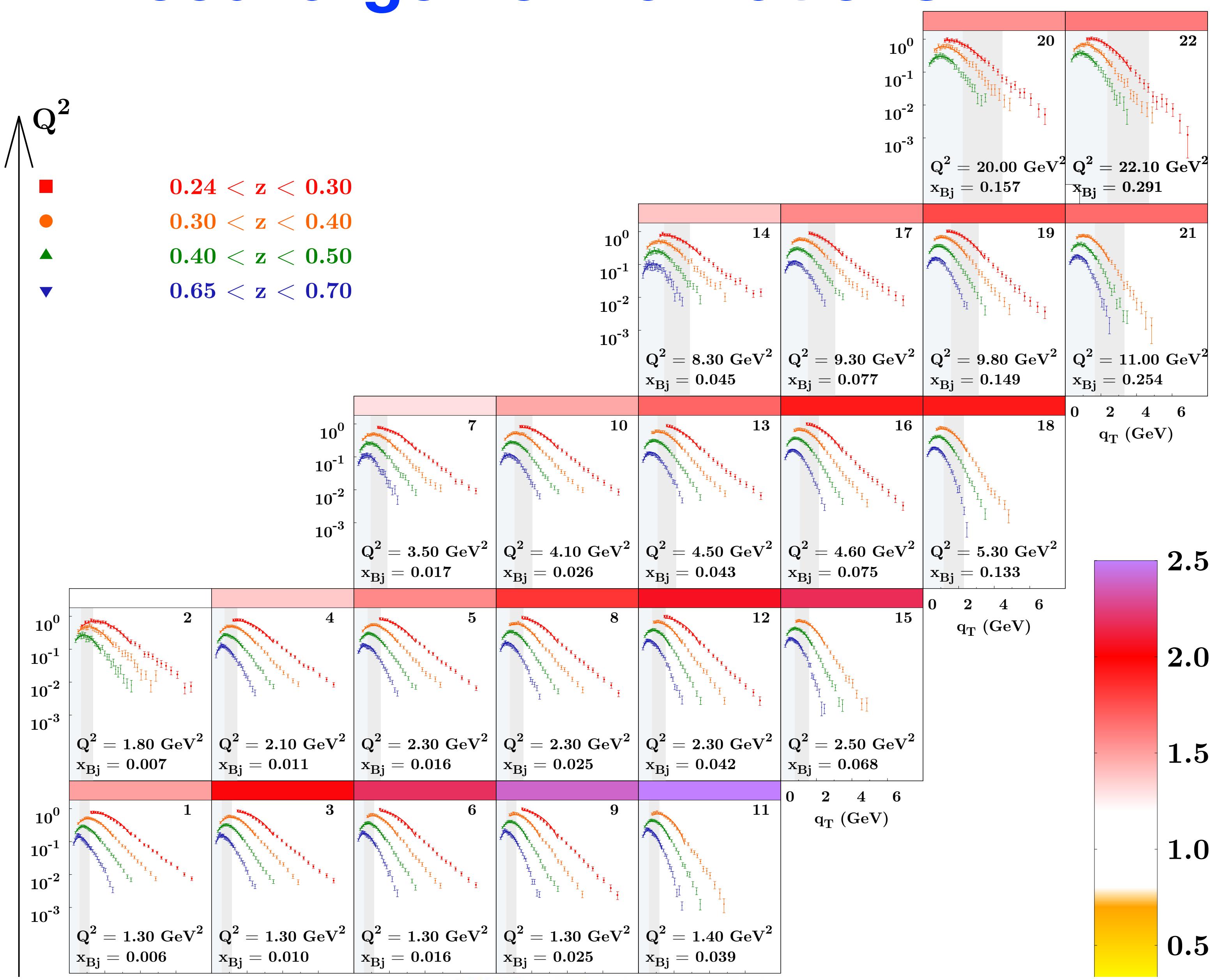


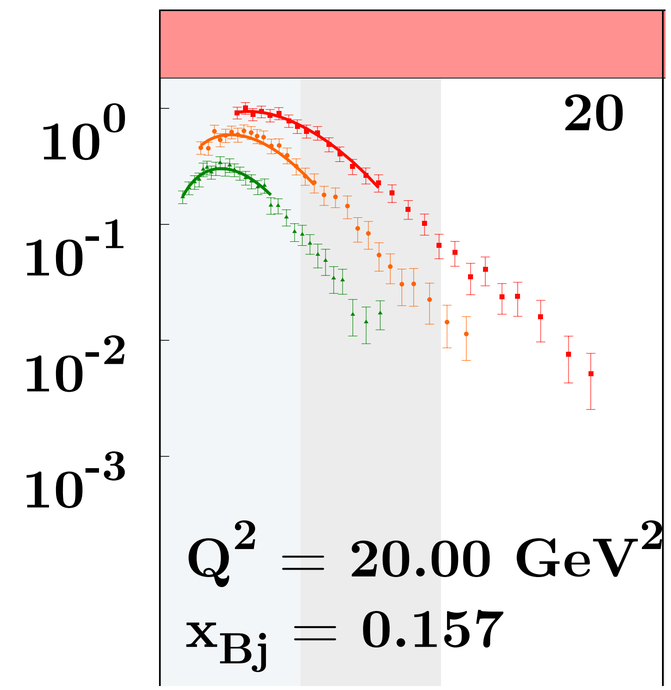


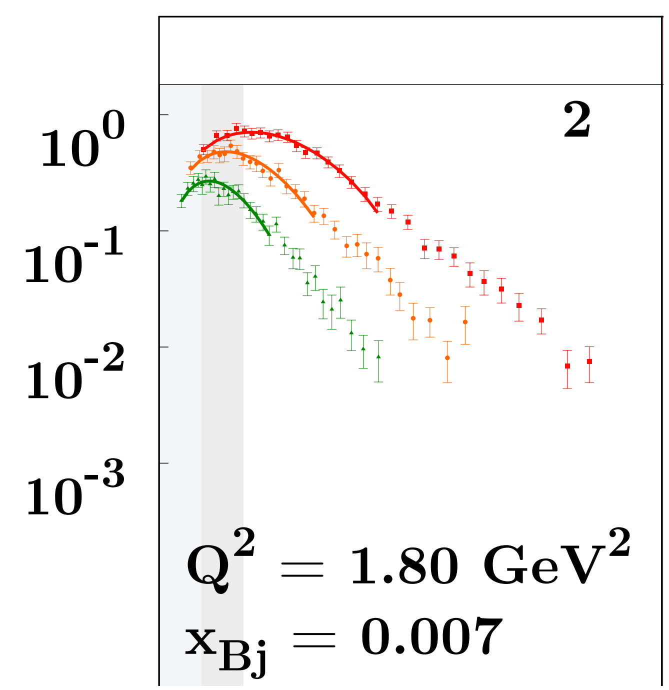
$$\tilde{F}_j = f(x, \mu_b) \exp\left\{ g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Need large normalizations

$\mathcal{O}(\alpha_s)$







$$\tilde{F}_j(x, b_T, Q, \zeta_F) = \left(\frac{\sqrt{\zeta_F}}{\mu_b}\right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{in}(x/\hat{x}, b_*, \mu_b, \mu_b^2) f_i(\hat{x}, \mu_b)$$
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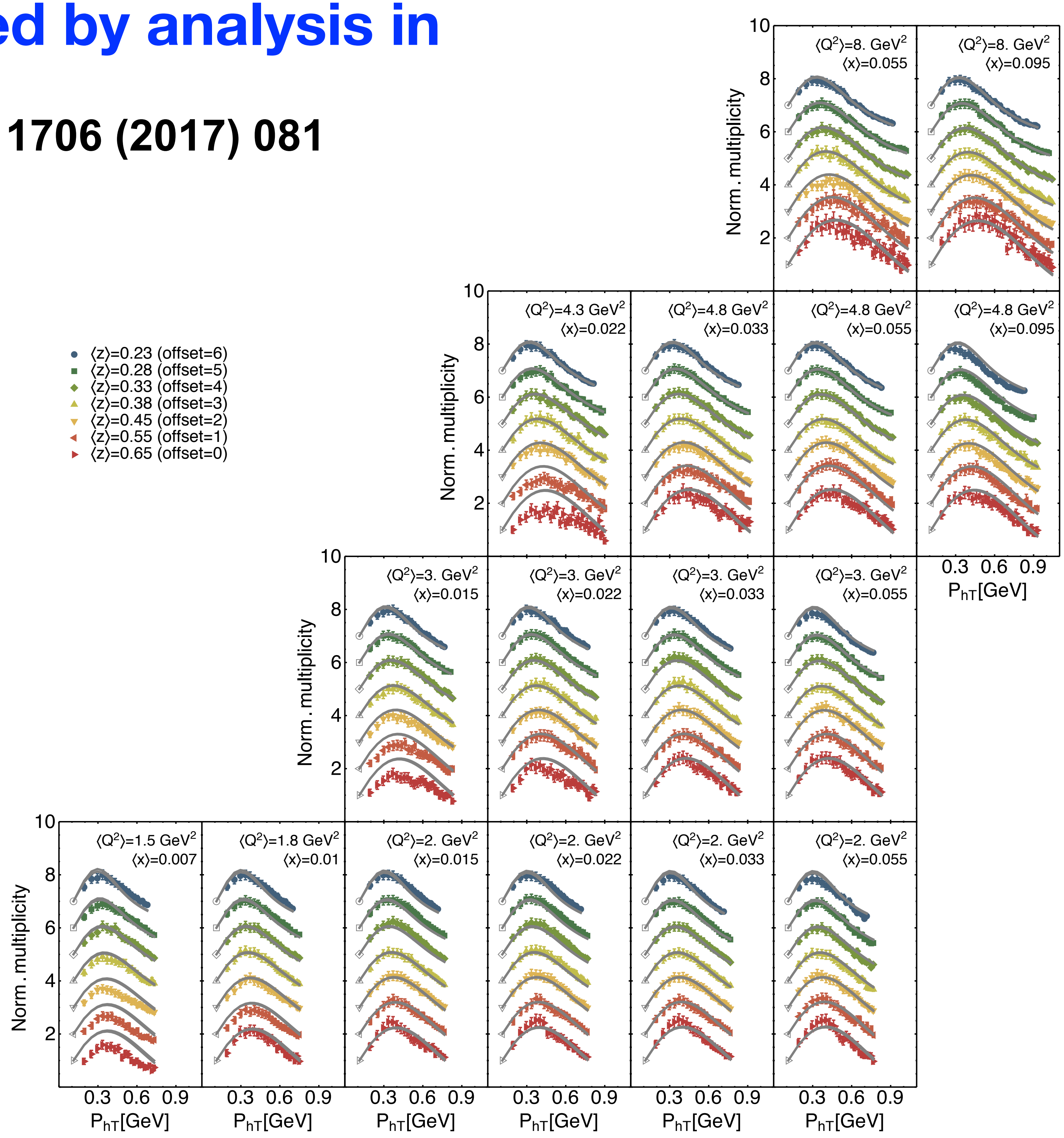
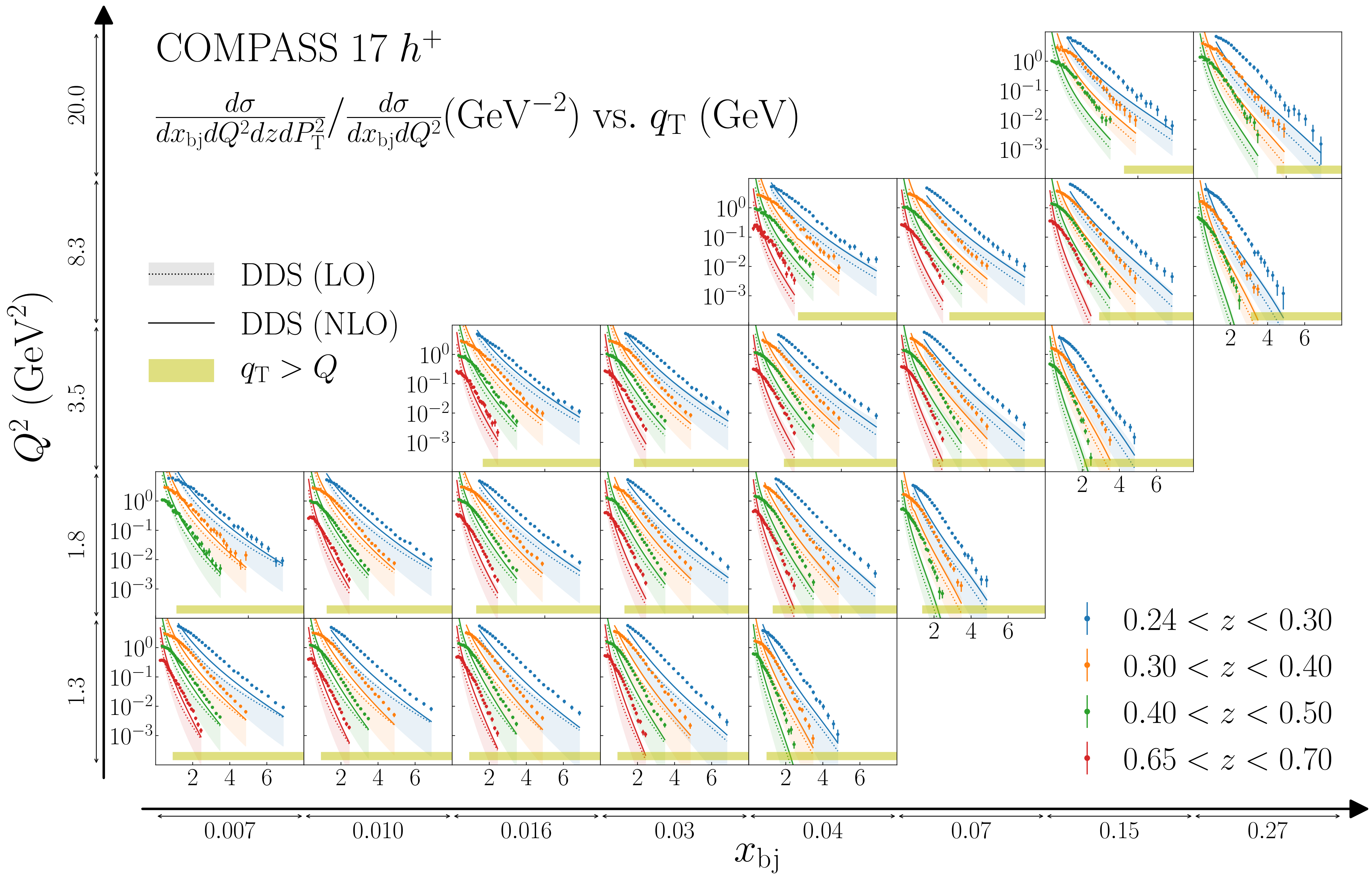
$$\times \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\},$$

Similar normalization adjustment needed by analysis in

JHEP 1706 (2017) 081



Also issues in non-TMD region (large qT cross section) using DDS code



JOGH, Rogers, Sato, Wang
Phys.Rev. D98 (2018) no.11, 114005

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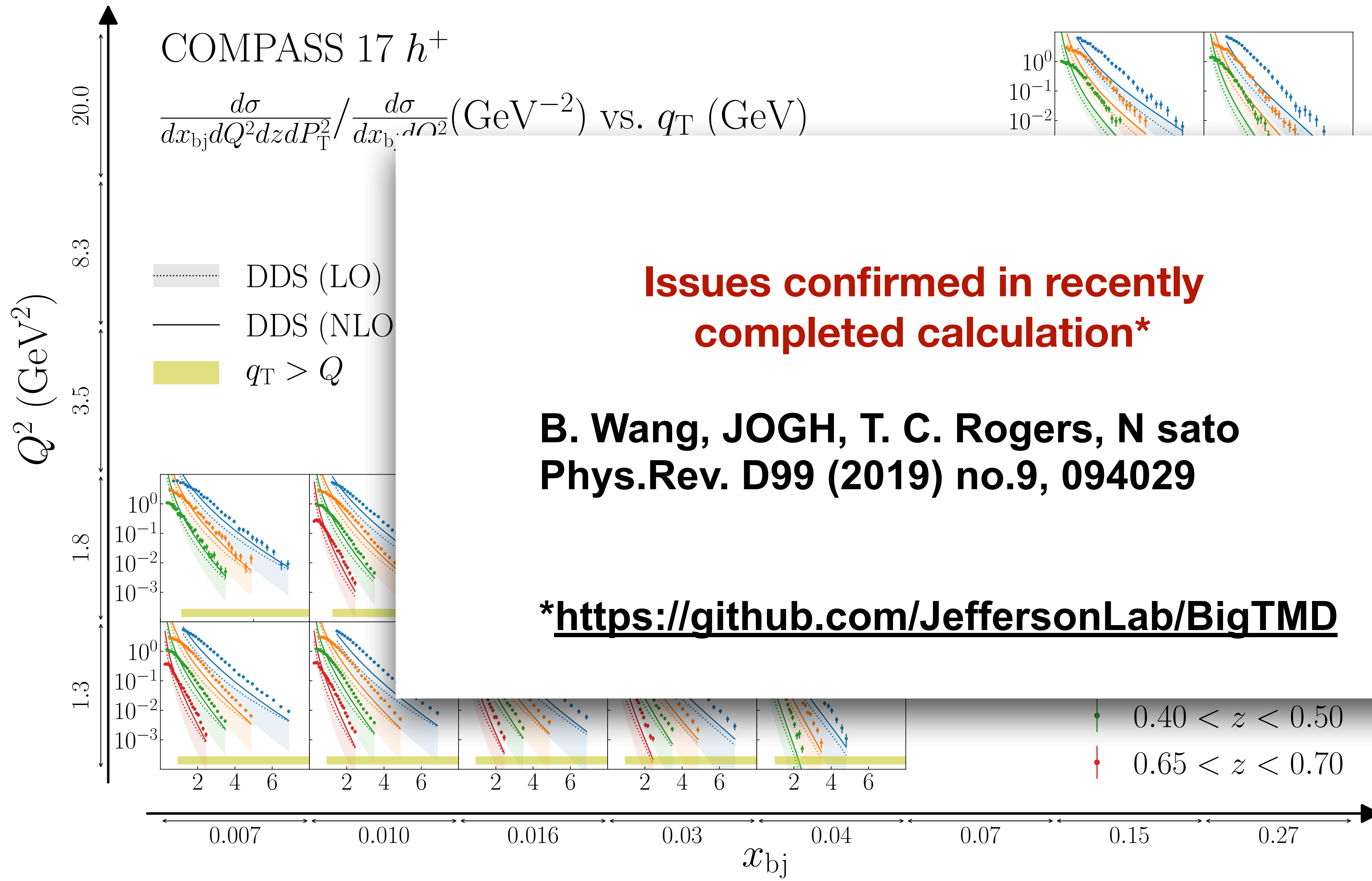

Issues confirmed in recently completed calculation*

B. Wang, JOGH, T. C. Rogers, N sato
Phys.Rev. D99 (2019) no.9, 094029

***https://github.com/JeffersonLab/BigTMD**

JOGH, Rogers, Sato, Wang
Phys.Rev. D98 (2018) no.11, 114005

Some challenges:

One can still infer information about the evolution

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$$\sum_q \mathcal{H}_q \text{ F.T.} \left\{ \tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

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$$\times \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\},$$

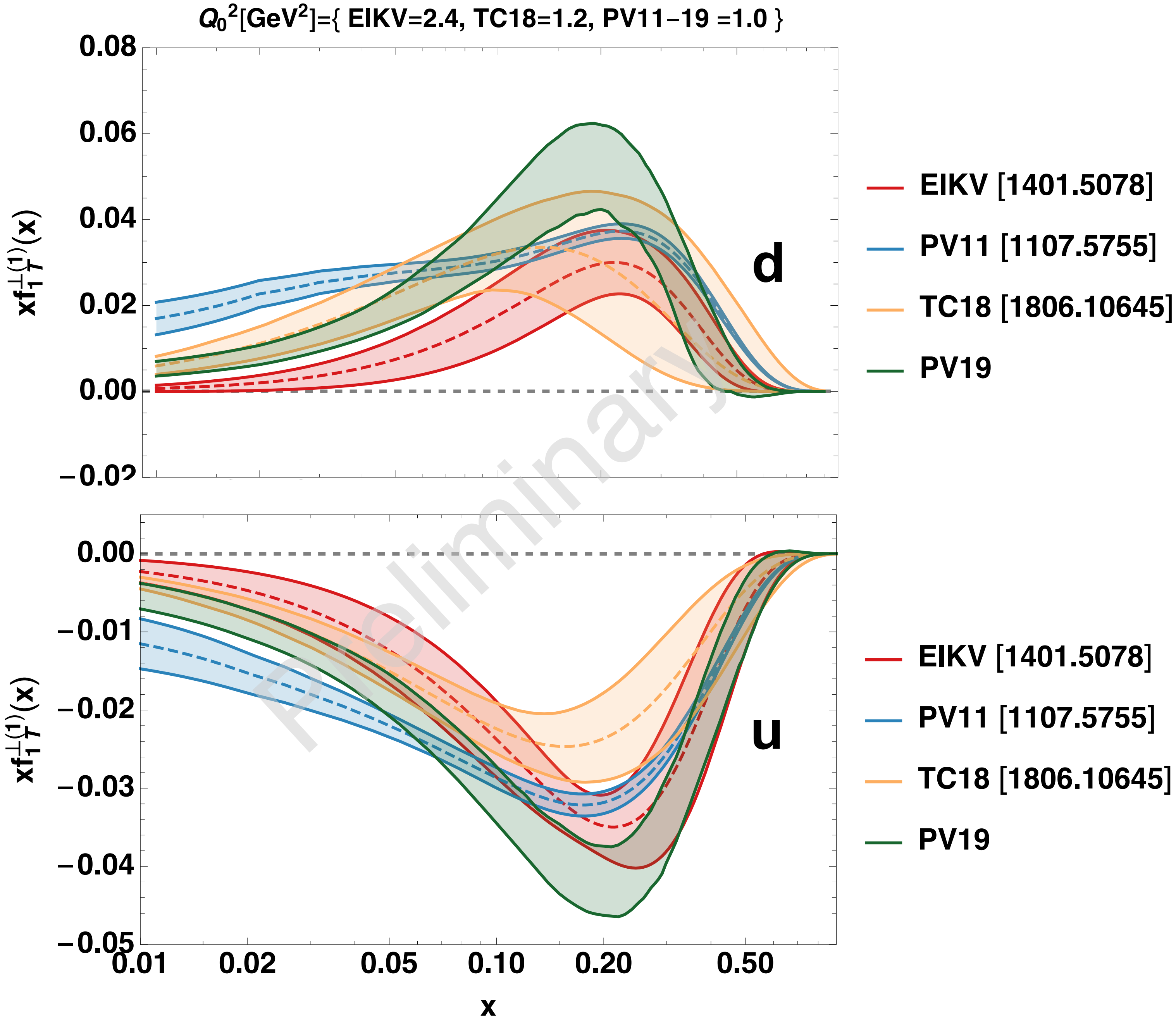
(blue) pQCD

(green) Input (extraction from collinear cross section)

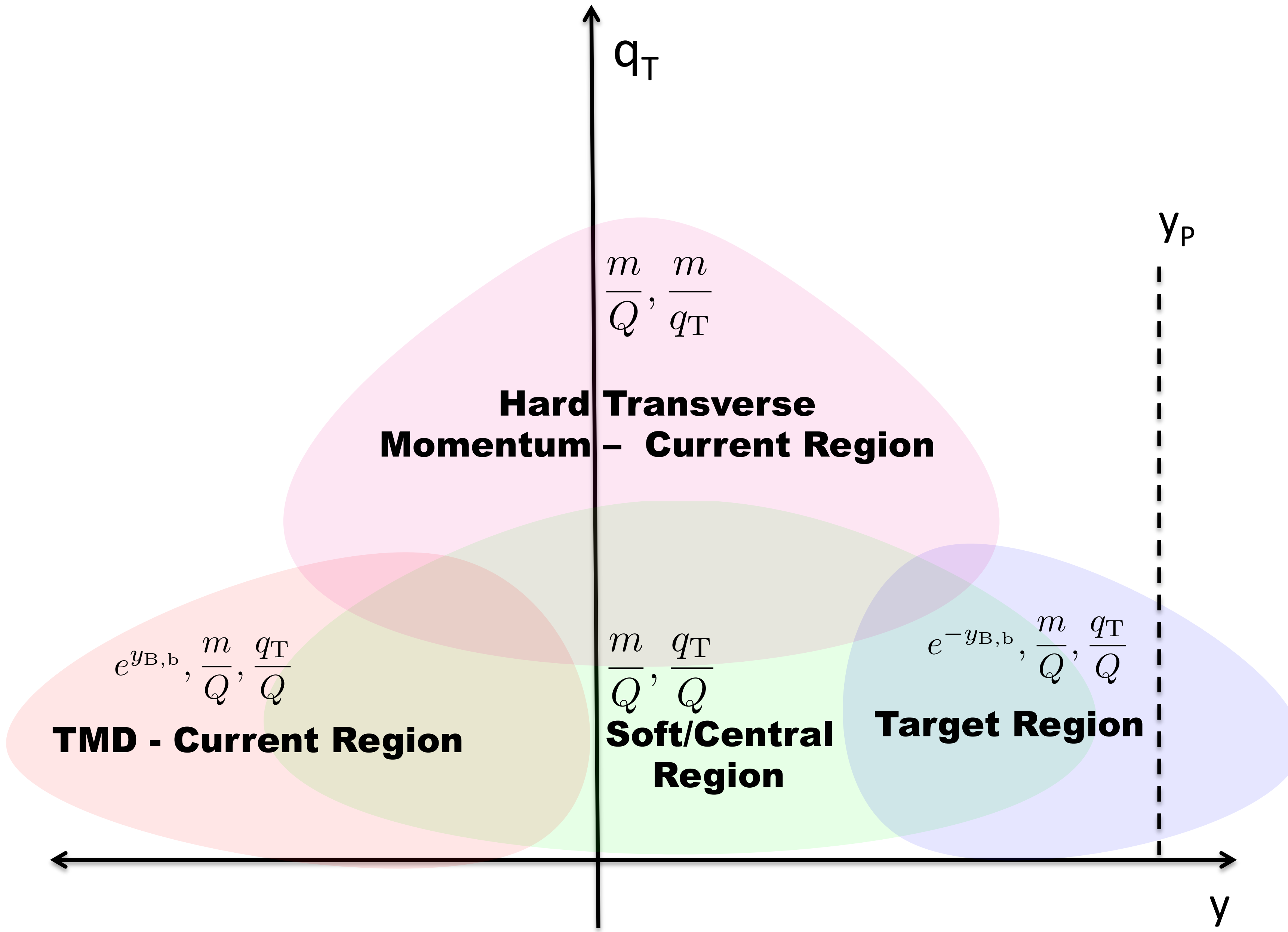
(red) Non-perturbative functions to extract from data.

Issues on normalization likely propagate here

Results comparison

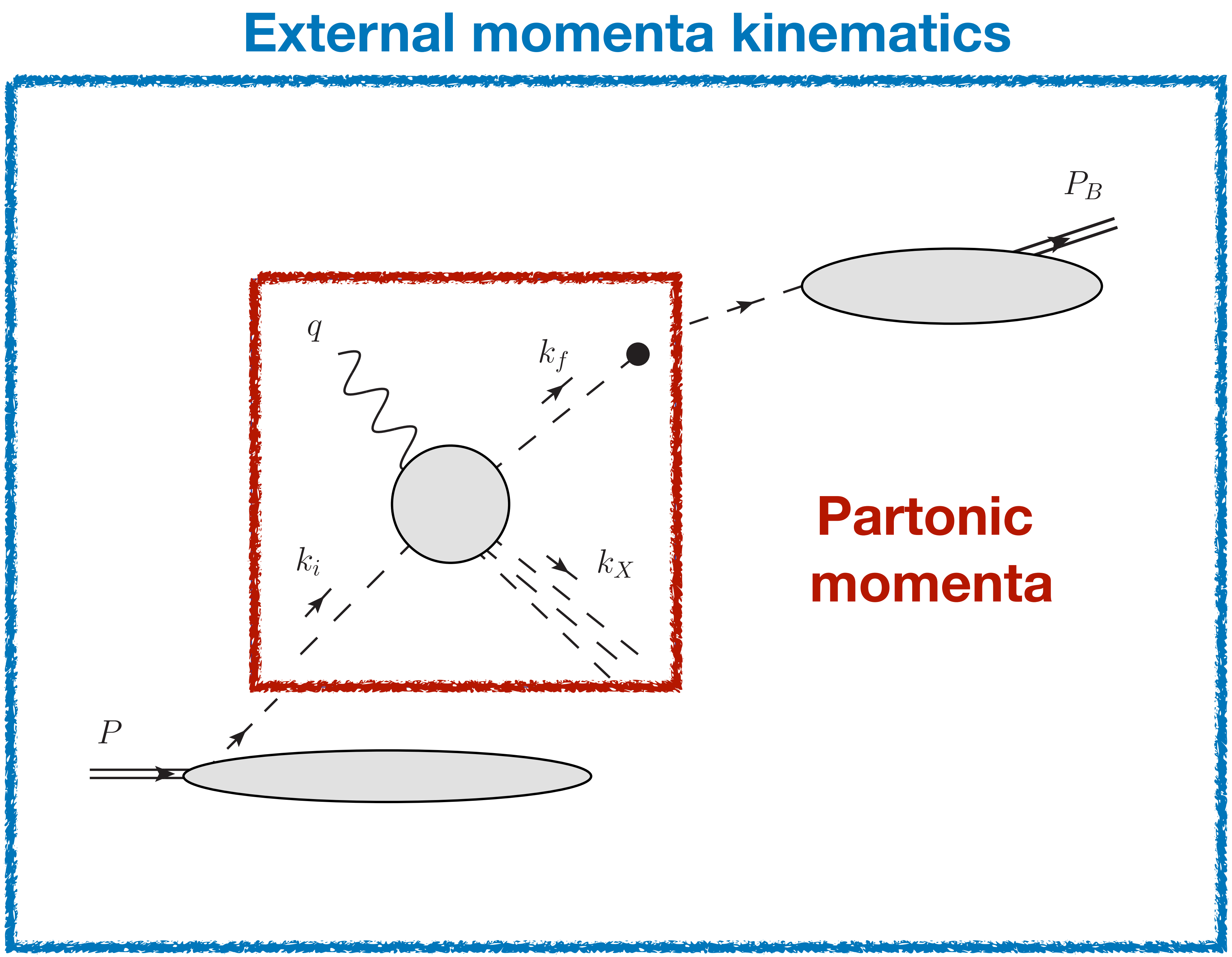


Slide by Filippo Delcarro (JLAB)
-QCD evolution 2019-

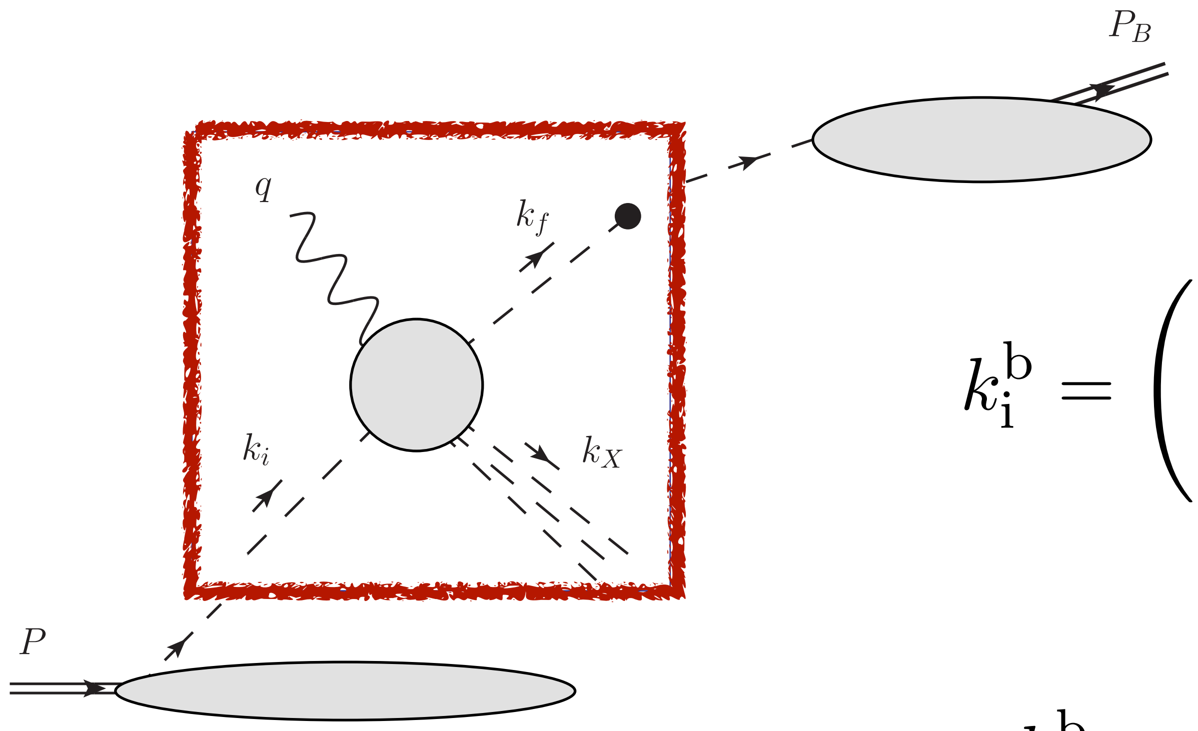


Based on:
M.Boglione, A. Dotson, L. Gamberg, S. Gordon, JOGH, A. Prokudin, T. C. Rogers, N. Sato
Submitted to J.High Energy Phys.
JLAB-THY-19-2920
e-Print: arXiv:1904.12882

Different types of approximations



Partonic kinematics



$$k_{\rm i}^{\rm b} = \left(\frac{Q}{\hat{x}_{\rm N}\sqrt{2}}, \frac{\hat{x}_{\rm N}(k_{\rm i}^2 + \mathbf{k}_{\rm i,b,T}^2)}{\sqrt{2}Q}, \mathbf{k}_{\rm i,b,T} \right)$$

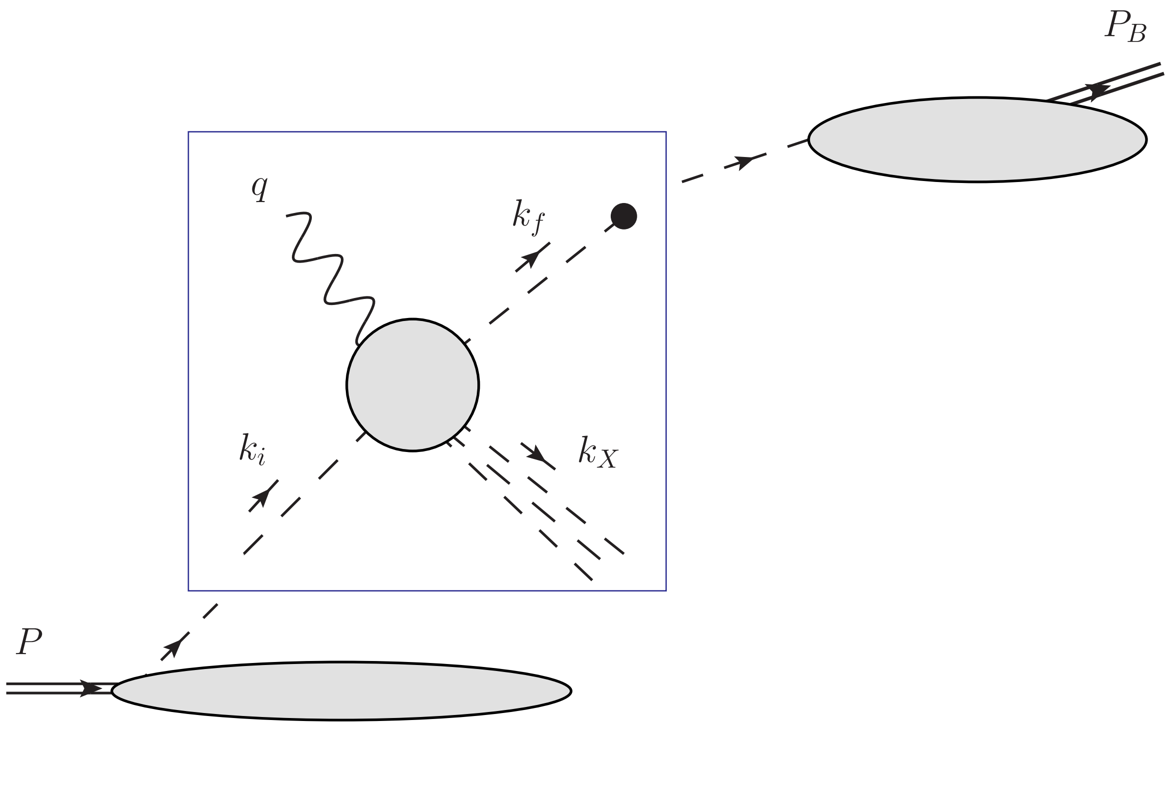
$$k_{\rm f}^{\rm b} = \left(\frac{\mathbf{k}_{\rm f,b,T}^2 + k_{\rm f}^2}{\sqrt{2}\hat{z}_{\rm N}Q}, \frac{\hat{z}_{\rm N}Q}{\sqrt{2}}, \mathbf{k}_{\rm f,b,T} \right)$$

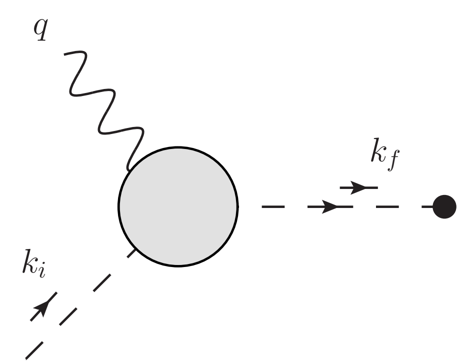
$$k_{\rm i}^+ \equiv \xi P_{\rm b}^+$$

$$\hat{x}_{\rm N} \equiv -\frac{q_{\rm b}^+}{k_{\rm i,b}^+} = \frac{x_{\rm N}}{\xi}$$

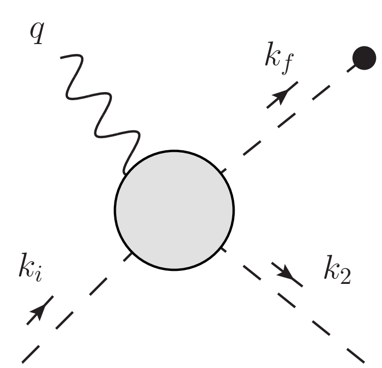
$$P_{\rm B,b}^- \equiv \zeta k_{\rm f}^-$$

$$\hat{z}_{\rm N} \equiv \frac{k_{\rm f,b}^-}{q_{\rm b}^-} = \frac{z_{\rm N}}{\zeta}$$

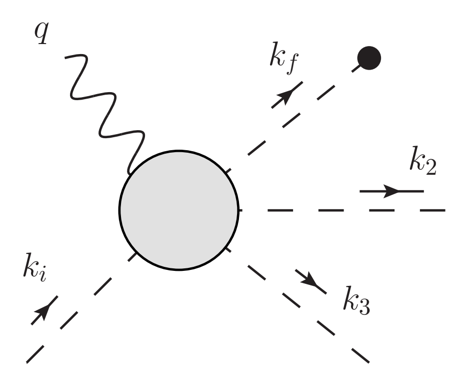




(a)

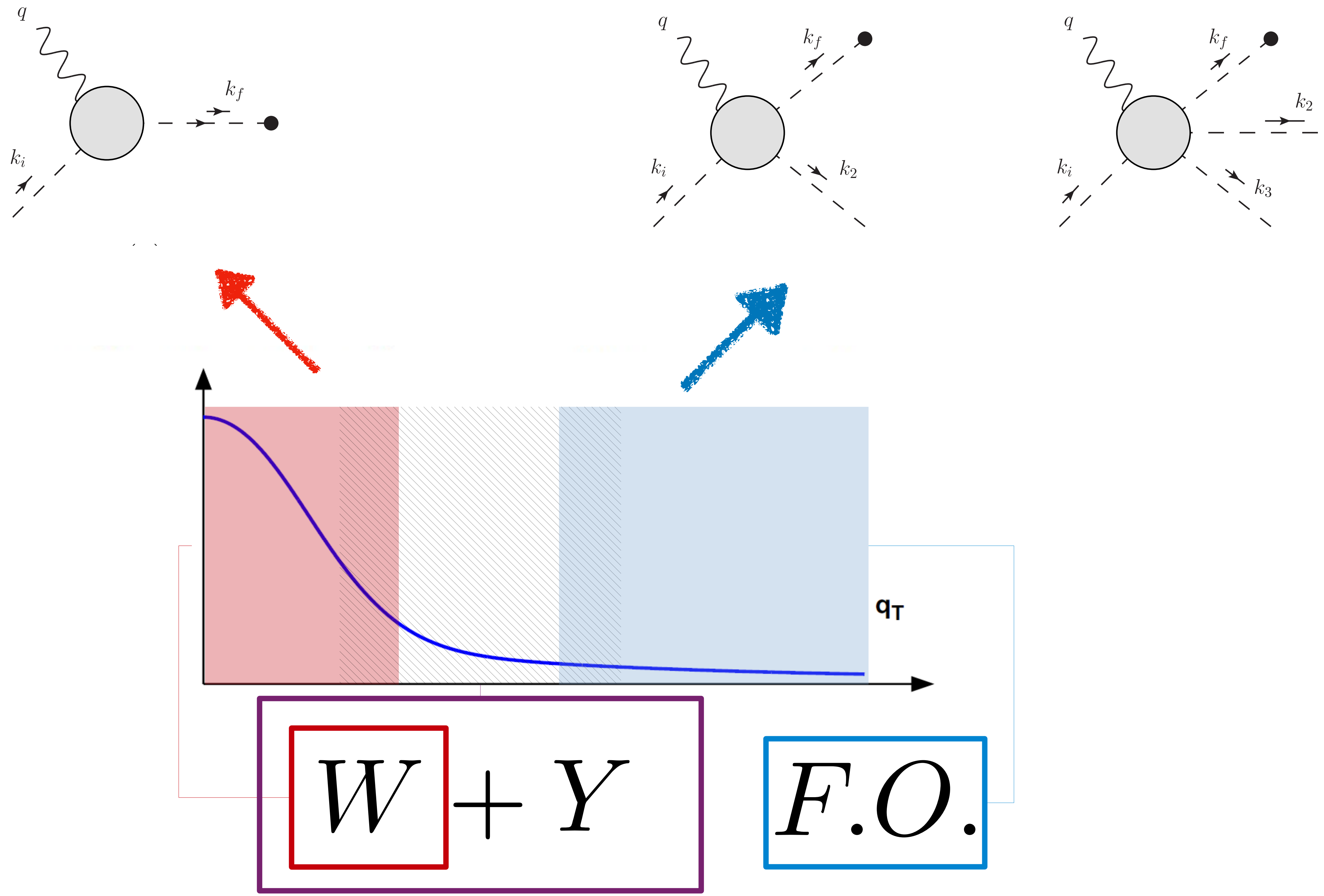


(b)

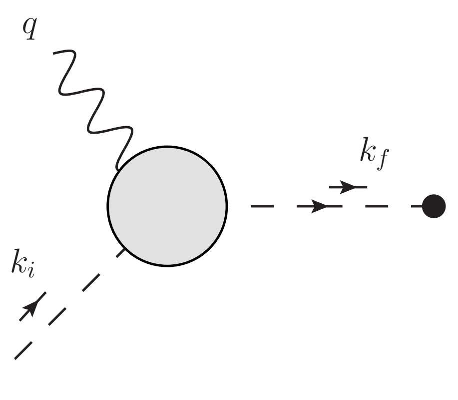


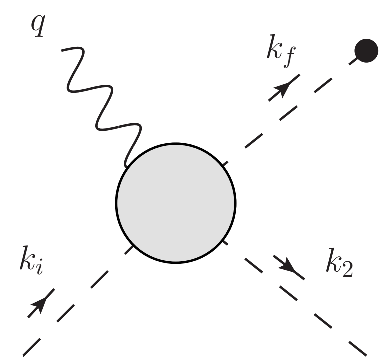
(c)

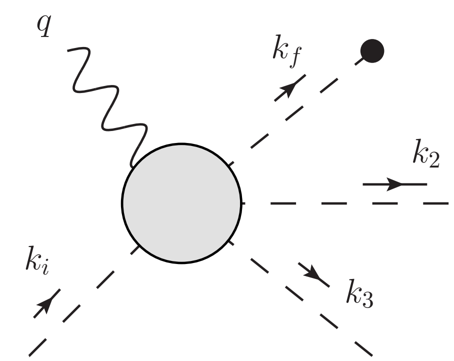
$$k \equiv k_{\mathrm{f}} - q$$



$$k^2 = (k_f - q)^2$$





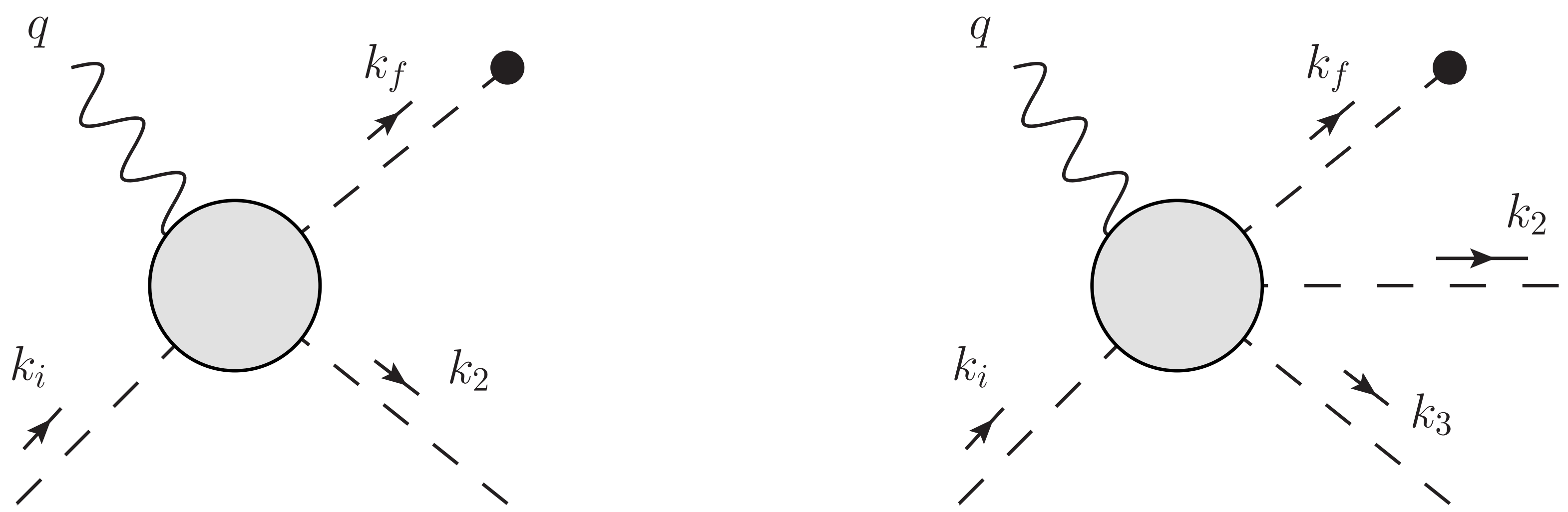


$$k^2 = 0$$

$$k^2 \neq 0$$

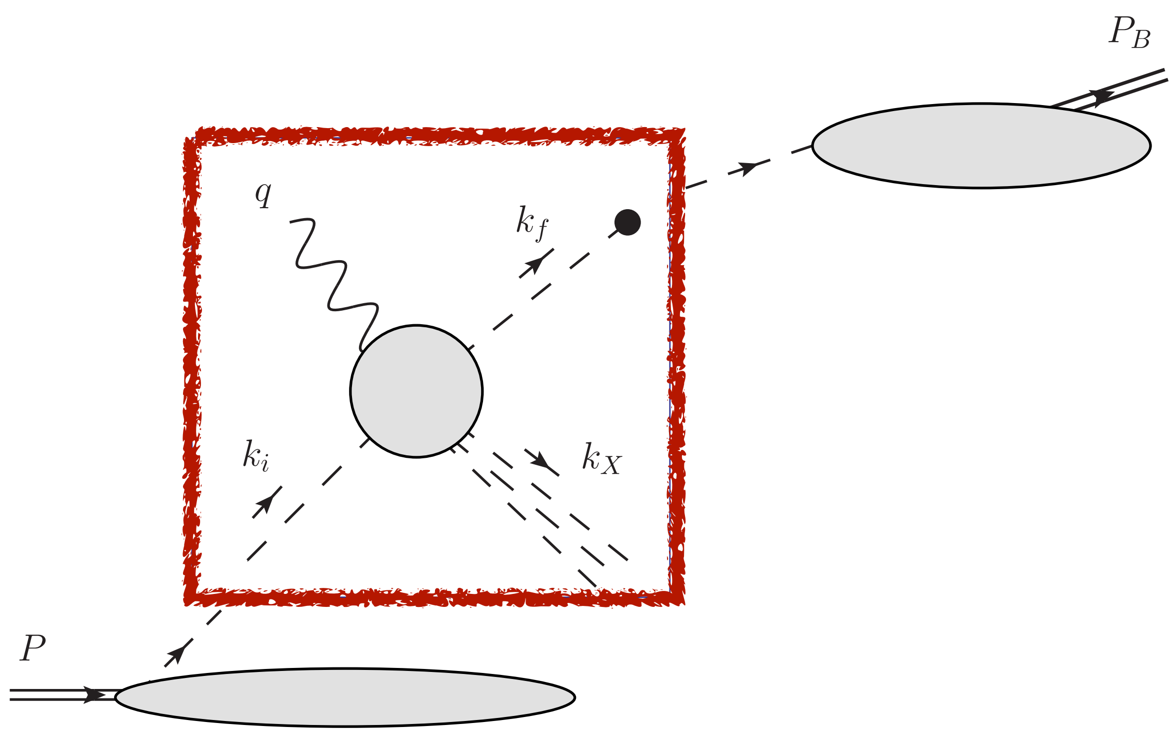
Allows to distinguish handbag from real emission kinematics

$$k_{\mathrm{X}}^2 = (k_{\mathrm{i}} + q - k_{\mathrm{f}})^2$$



$k_X^2 = 0$ $k_X^2 \neq 0$

Higher order pQCD corrections in large qT cross section associated to larger values of virtuality spectator



$$k_{\mathrm{X}}^2 = (k_{\mathrm{i}} + q - k_{\mathrm{f}})^2$$

$$k \equiv k_{\mathrm{f}} - q$$

$$\text{Collinearity} = R_1 \equiv \frac{P_{\mathrm{B}} \cdot k_{\mathrm{f}}}{P_{\mathrm{B}} \cdot k_{\mathrm{i}}}$$

$$\text{Transverse Hardness Ratio} = R_2 \equiv \frac{|k^2|}{Q^2}$$

$$\text{Spectator Virtuality Ratio} = R_3 \equiv \frac{|k_{\mathrm{X}}^2|}{Q^2}$$

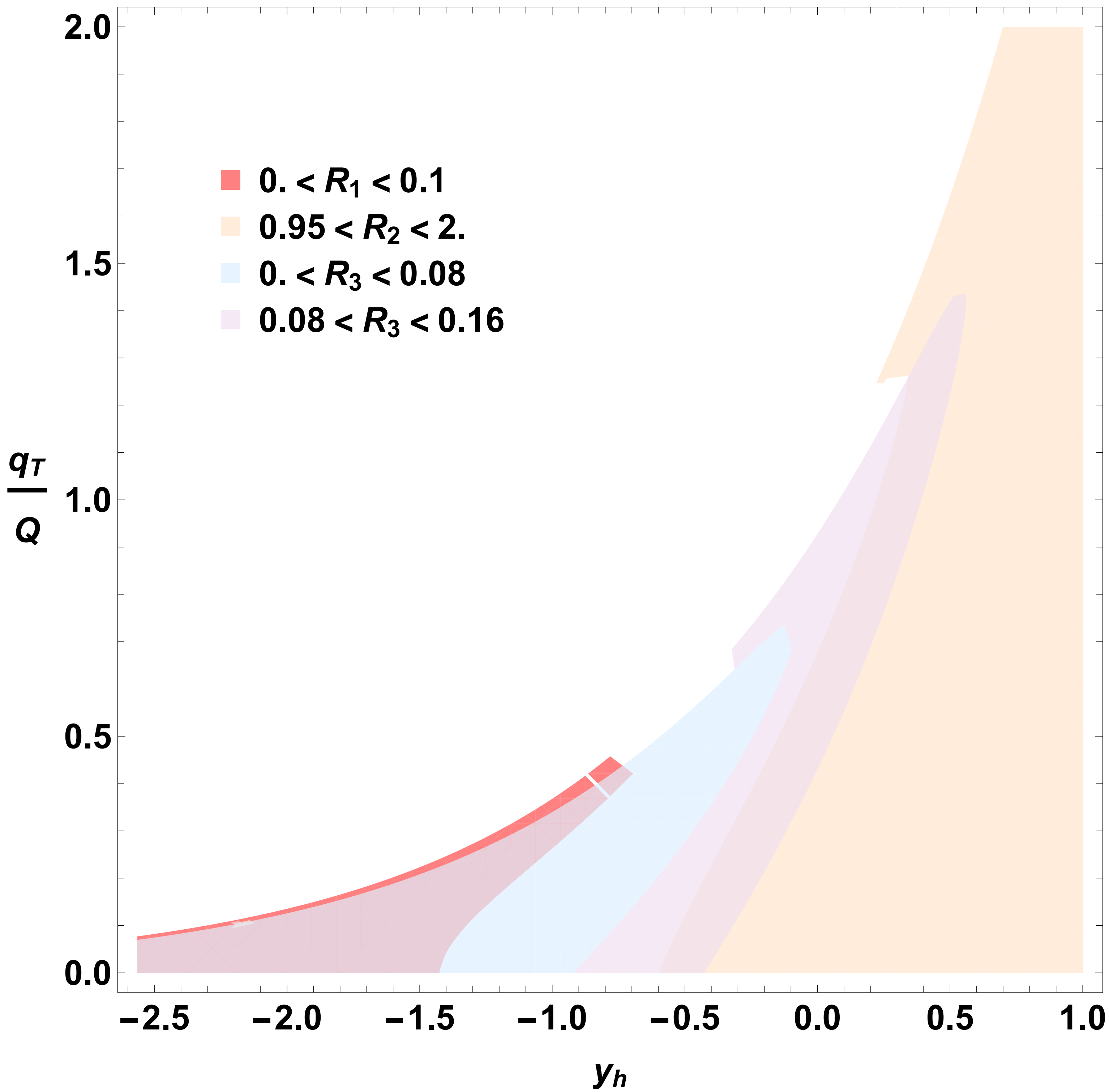
The size of these ratios determine partonic configurations (factorization theorem) and map to kinematical regions of the observables

$$\text{Collinearity} = R_1 \equiv \frac{P_{\rm B} \cdot k_{\rm f}}{P_{\rm B} \cdot k_{\rm i}}$$

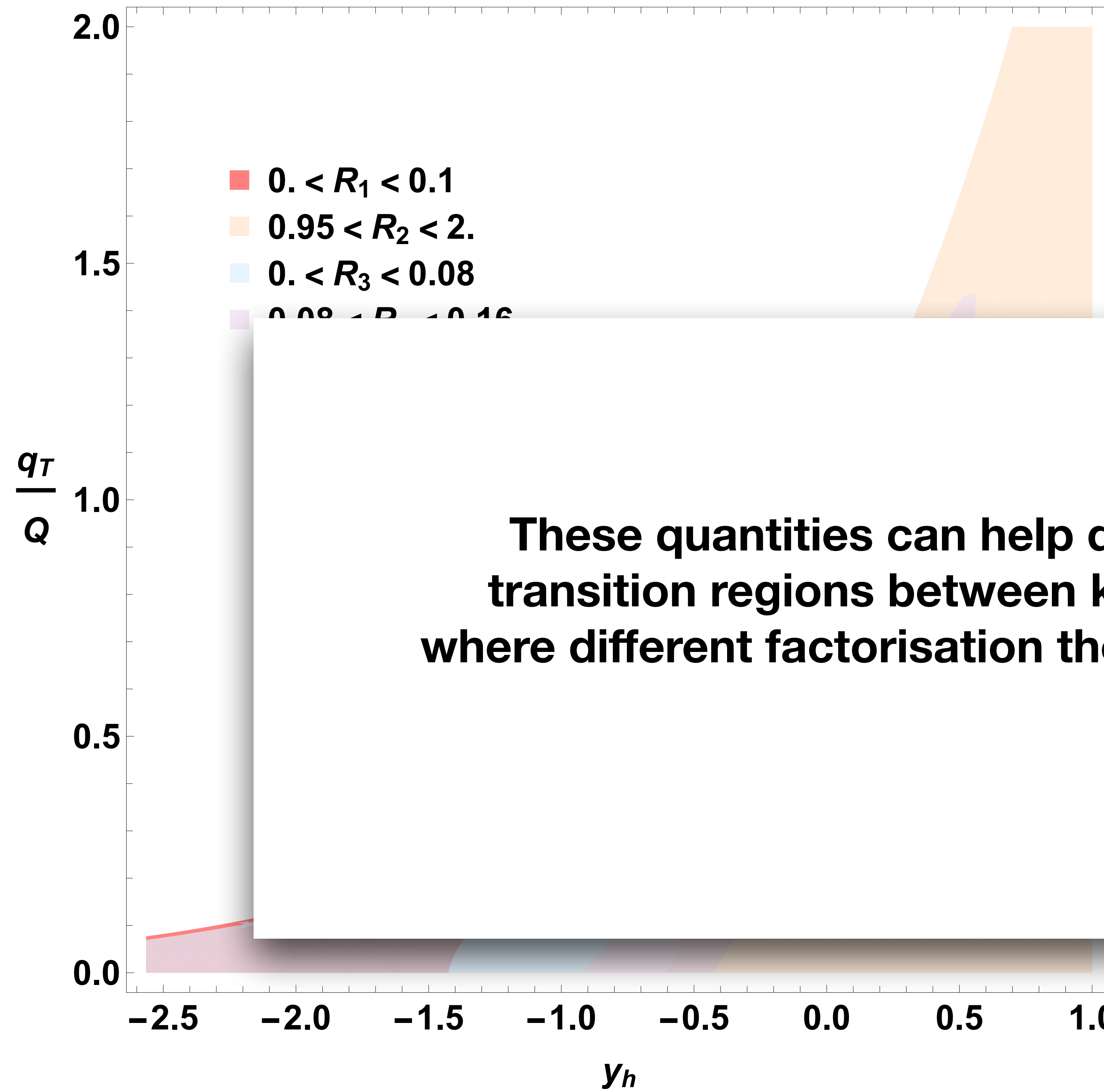
$$\text{Transverse Hardness Ratio} = R_2 \equiv \frac{|k^2|}{Q^2}$$

$$\text{Spectator Virtuality Ratio} = R_3 \equiv \frac{|k_{\rm X}^2|}{Q^2}$$

(caveat: Parton momenta have to be estimated, so this is really just an example)



(caveat: Parton momenta have to be estimated, so this is really just an example)



These quantities can help determine transition regions between kinematics where different factorisation theorems apply

Final remarks

Simple model like gaussian in momentum space are useful “snapshots” of TMDs at a given scale. They may also help assessing signals of evolution when used as a benchmark.

This is important since several issues remain in the extraction of TMDs.

Fits in CSS formalism (TMD evolution) undershoot the data.

One can probably study “shape” and evolution of TMDs separately, although ultimately, the normalisation issues should be resolved.

The issues propagate to other extractions.

Among other things, assessing the kinematics validity of different factorisation theorems seems important.

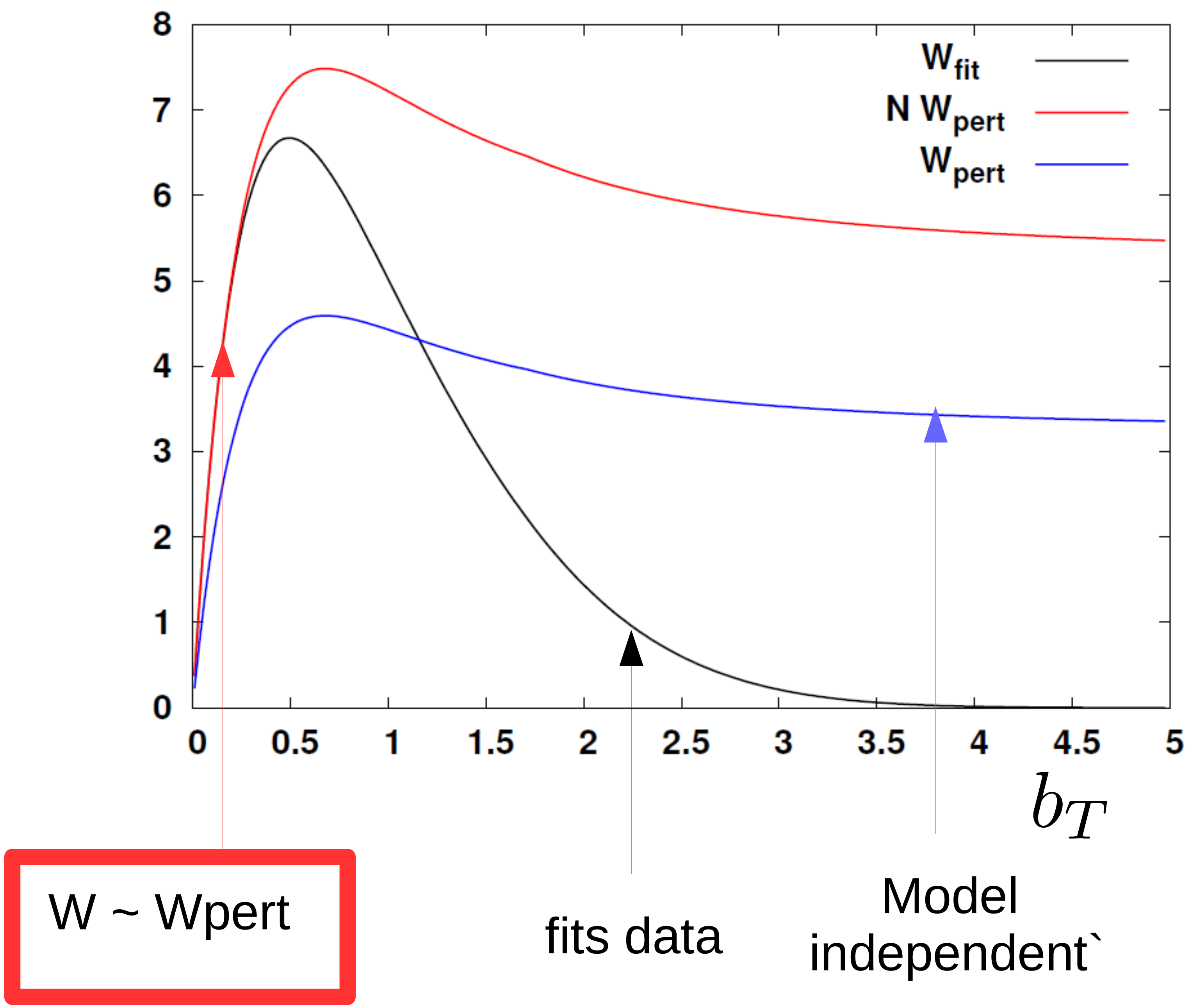
Quantities R1, R2, R3 presented here can serve as tool to do this.

Thanks.

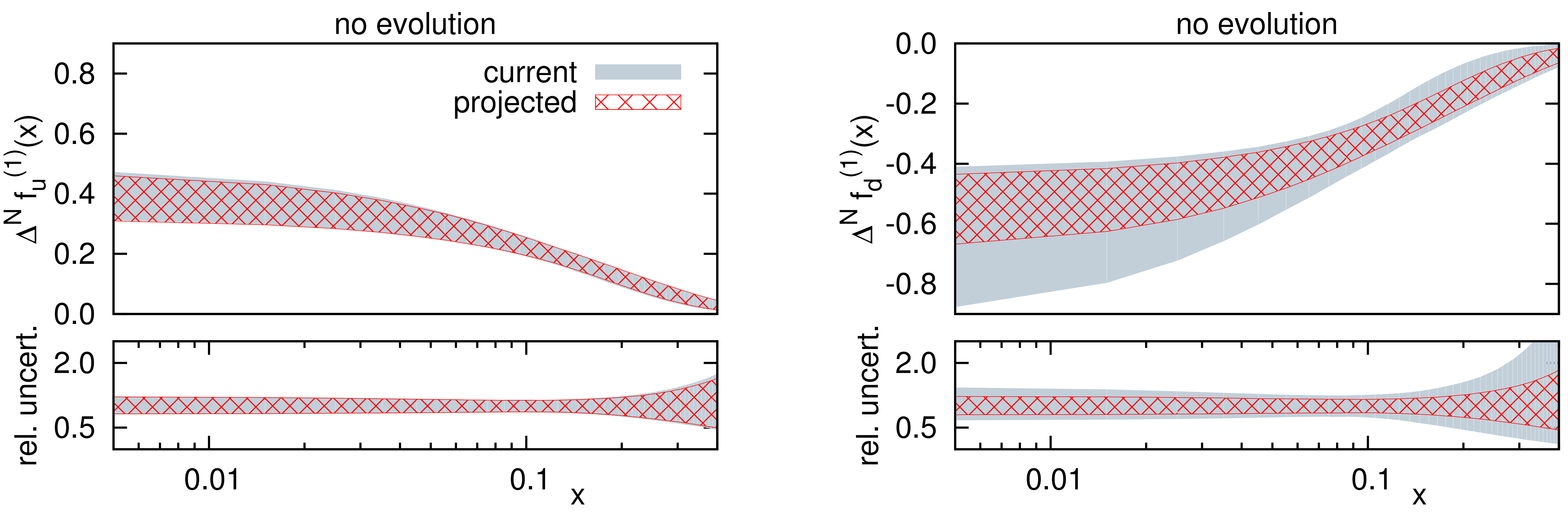
backup.

Some challenges:

These normalizations are hard to justify, but they do have and impact in the “shape” of the TMDs.



Sivers asymmetry in SIDIS



Uncertainty bands corresponding to projected errors for future COMPASS run on Deuteron target