Measurability of pressure in the proton

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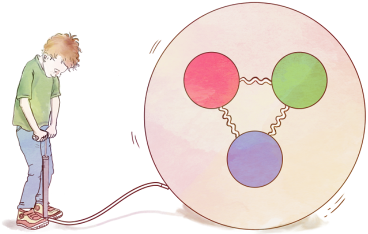


EUROPSKI STRUKTURNI I INVESTICIJSKI FONDOVI

Outline

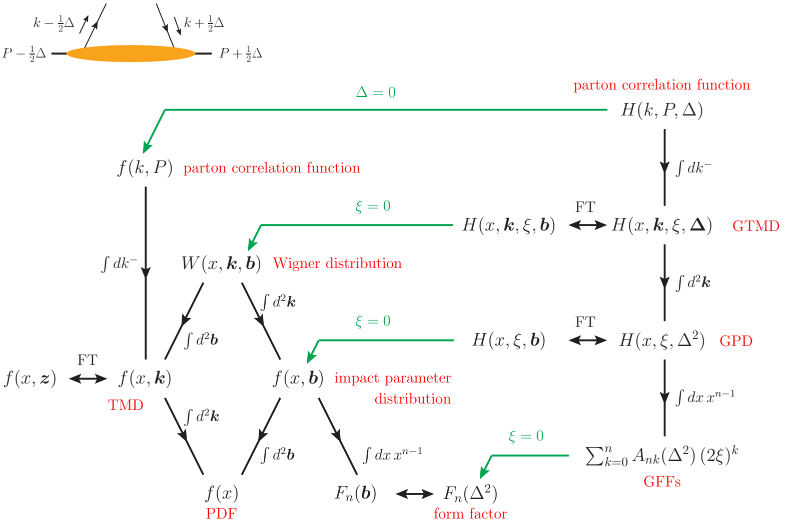
1. Introduction to measurement of pressure distribution

2. Neural nets CLAS results

3. Neural nets global results

4. Summary

Family tree of hadron structure functions



[Fig. by Markus Diehl]

($\xi \to \eta$ from now on)

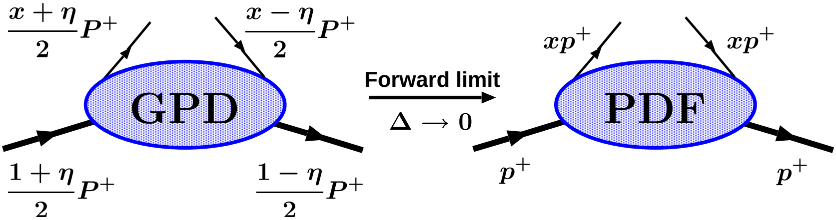
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$\widetilde{F}^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

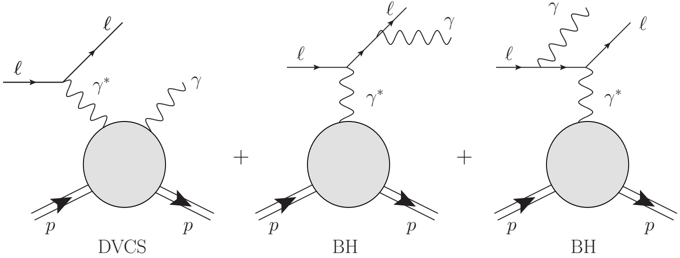
(and similarly for gluons F^g and $\widetilde{F}^g$).



$P = P_1 + P_2$; $\quad t = \Delta^2 = (P_2 - P_1)^2$; $\quad \eta = -\frac{\Delta^+}{P^+}$ (skewedness)

Access to GPDs via DVCS

- Deeply virtual Compton scattering (DVCS) — "gold plated" process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I} \; .$$

- where *e. g.* interference term is

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\},$$

- where *e. g.* $c_1^{\mathcal{I}}$ harmonic for unpolarized target is

$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1 \,\mathfrak{Re}\, \mathcal{H} - \frac{t}{4M_p^2} F_2 \,\mathfrak{Re}\, \mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}} (F_1+F_2) \,\mathfrak{Re}\, \widetilde{\mathcal{H}} \right]$$

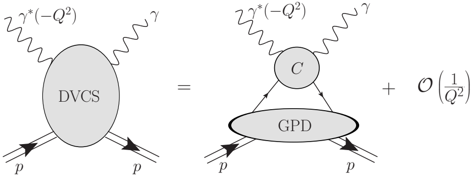
- and at leading order everything depends on four complex

Compton form factors (CFFs)

$$\mathcal{H}(\xi,t,Q^2), \quad \mathcal{E}(\xi,t,Q^2), \quad \widetilde{\mathcal{H}}(\xi,t,Q^2), \quad \widetilde{\mathcal{E}}(\xi,t,Q^2)$$

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- CFFs are convolution:

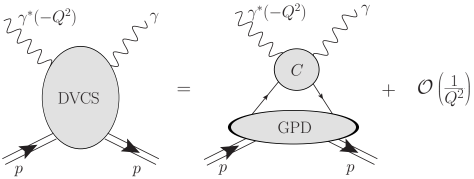
$$ {}^{a}\mathcal{H}(\xi, t, Q^2) = \int \mathrm{d}x \; C^{a}(x, \xi, \frac{Q^2}{Q_0^2}) \; H^{a}(x, \xi, t, Q_0^2) $$

$a = q, G$

- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

Factorization of DVCS ⟶ GPDs

- [Collins et al. '98]



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- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)

- GPDs nontrivially depend on three variables: $H^a(x, \eta, t, Q^2)$
- CFFs nontrivially depend on two variables: $\mathcal{H}^a(\xi, t, Q^2)$

Three "classical" objectives of GPD studies

- Both meanings are valid:
  - "classical" = well known, venerable
  - "classical" = understandable from non-quantum viewpoint

❶ Ji's "sum rule"

$$J^a = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^a(x, \eta, t) + E^a(x, \eta, t) \Big]_{t \to 0}$$ [Ji '96]

- Mellin moments of GPD are generally difficult to access [Polyakov '07]
- E is particularly poorly constrained by present data

❷ 3D tomography

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H(x, 0, -\vec{\Delta}_\perp^2)$$ [Burkardt '00]

- experiments are mostly sensitive to $H(x, x, t)$
- "deskewing" to $H(x, 0, t)$ — model dependent

Pressure distribution in the nucleon

3. Pressure distribution in the nucleon
   Nucleon form-factors of energy-momentum tensor

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\Big(A^a(t)\frac{P_\mu P_\nu}{4M} + J^a(t)\frac{iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4M} + d^a_1(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M} + \tilde{c}^a(t)Mg_{\mu\nu}\Big)u$$

- Since pressure p is part of T^{ii}, one derives [Polyakov,Schweitzer '18]

$$p(r) = \frac{1}{30\pi^2 M}\int_{-\infty}^{0} dt\, \frac{\sin(r\sqrt{-t})}{r}\, t d_1(t)$$

- This is in principle valid only for total $p = p^q + p^g$. Some studies show that quark↔gluon flow may be small [Polyakov,Son '18]. Then one uses this formula for quark subsystem as well.

Pressure related to GPDs and DVCS

- Form-factor $d_1(t)$ can be measured in DVCS [Polyakov '03], since it is related to GPD D-term $D(t) = \frac{4}{5} d_1(t)$ [Polyakov, Weiss '99]

$$\int_{-1}^{1} dx\, x\, H^a(x, \eta, t) = A^a(t) + \eta^2 \frac{4}{5} d_1^a(t)$$

- It is also directly related to subtraction constant of CFF dispersion relation [Teryaev '05]

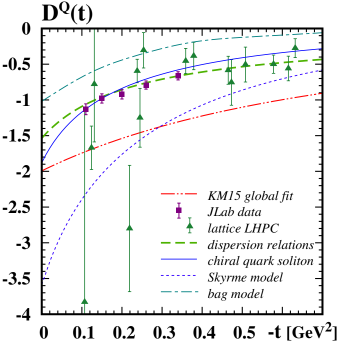
$$\Delta(t) = \mathfrak{Re}\, \mathcal{H}(\xi, t) - \frac{1}{\pi} \mathrm{P.V.} \int_0^1 dx \frac{2x}{\xi^2 - x^2}\, \mathfrak{Im}\, \mathcal{H}(x, t)$$

via expansion

$$\Delta(t) = 4 \sum_q Q_q^2 \left(d_1^q(t) + d_3^q(t) + \cdots \right)$$

- D-term should be easier to extract than moments of GPDs.

Extractions of D-term

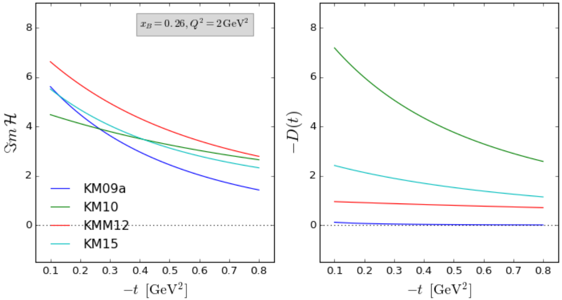


[Polyakov, Schweitzer '18]

- Many model calculations agree that $D^Q(t) < 0$ as is required by the stability of nucleon
- QCD on lattice shows a lot of promise [Shanahan, Detmold '18]
- But can we measure it?
- [Burkert, Elouadrhiri, Girod '18 (Nature)] use CLAS DVCS data to extract D-term with great precision!

Extractions of D-term in KM global fits

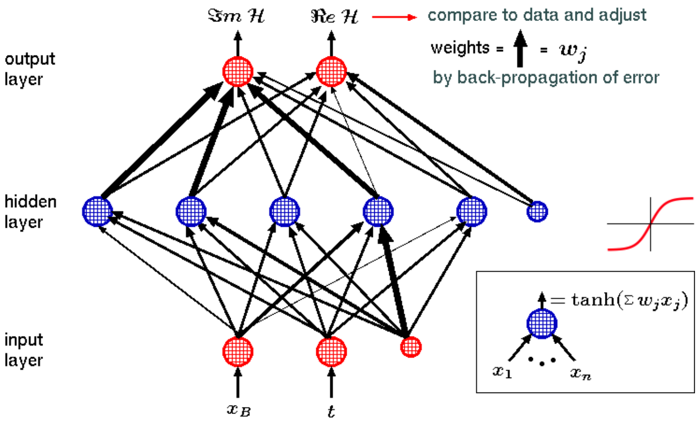
- In KM fits [K.K., D. Müller], $D(t) = D/(1 - t/M_D^2)^2$, where D and M_D are fit parameters



- Fit parameter uncertainties of $D(t)$ are $\sim 20\%$, but systematic uncertainty due to model selection is unknown and presumably much larger!
- To study the model uncertainty, we turn to neural nets method.

Neural nets CLAS fits

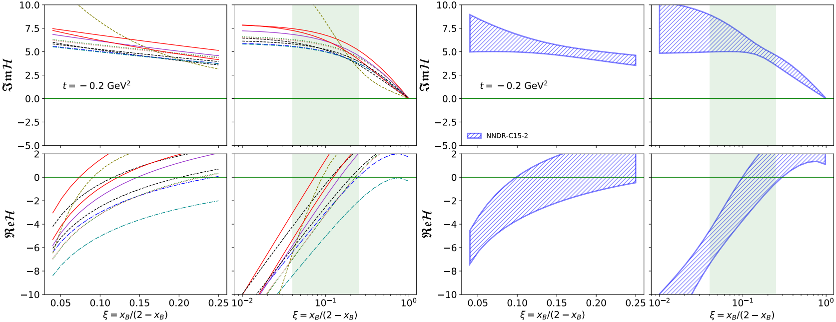
Fitting with neural networks



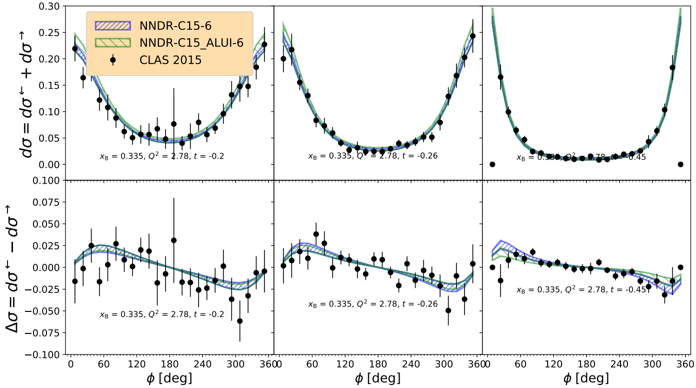
- Essentially a least-square fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$ $\Rightarrow$ no theory bias

Study A: NN fit to CLAS 2015 data

- We start by fitting just to the CLAS 2015 $d\sigma$ and $\Delta\sigma$ measurements [Jo et al. '15], and just $\mathcal{H}$
- We utilize dispersion relations (one NNet represents $\mathfrak{Im}\,\mathcal{H}$, another represents $D(t)$)
- Uncertainty is estimated by averaging over ensemble of neural nets:



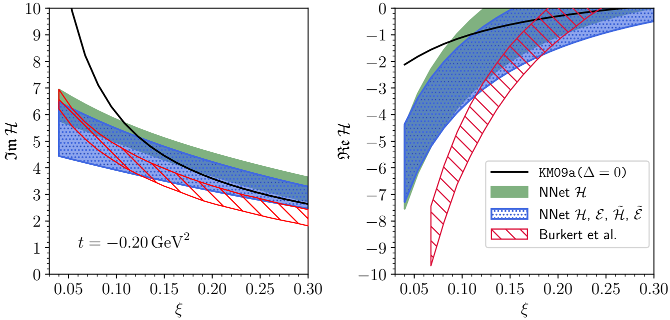
Propagating uncertainties back to $d\sigma$ and $\Delta\sigma$



- Small propagated error is due to small sensitivity of these observables to CFFs (and D-term).

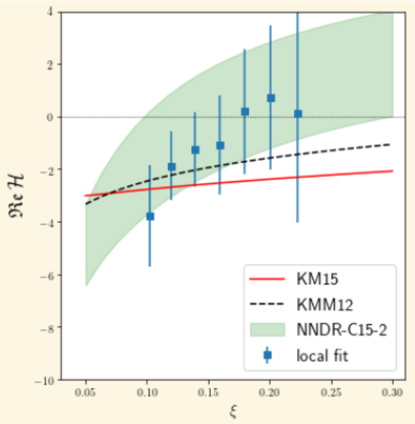
Comparison to [Burkert et al., Nature '18]

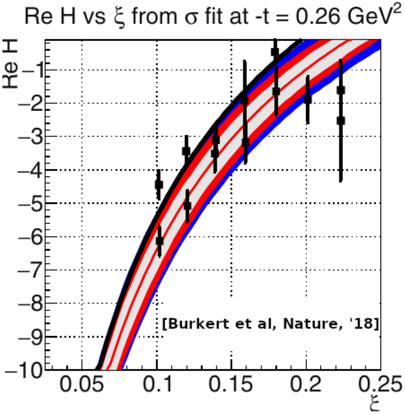
- $\mathfrak{Im}\,\mathcal{H}$ — good agreement
- $\mathfrak{Re}\,\mathcal{H}$ — only qualitative agreement



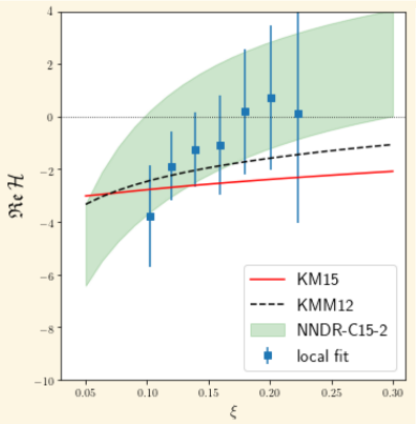
- Resulting $\Delta(t) = 0.78 \pm 1.5$, with almost no dependence on t! So D-term (and pressure) are consistent with zero in this model-independent approach! [K.K., Nature '19]

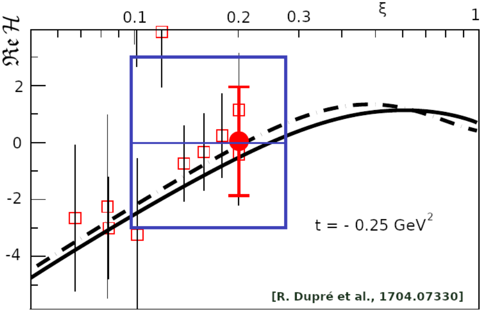
More detailed comparison of $\Re e\,\mathcal{H}$





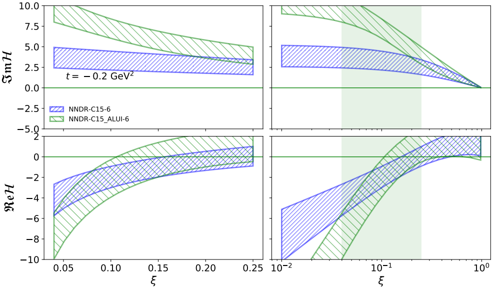
More detailed comparison of $\mathfrak{Re}\,\mathcal{H}$





Adding more data points

- Adding HERMES $A_{LU,I}$ data. (Model includes $\mathcal{H}$ and $\mathcal{E}$)



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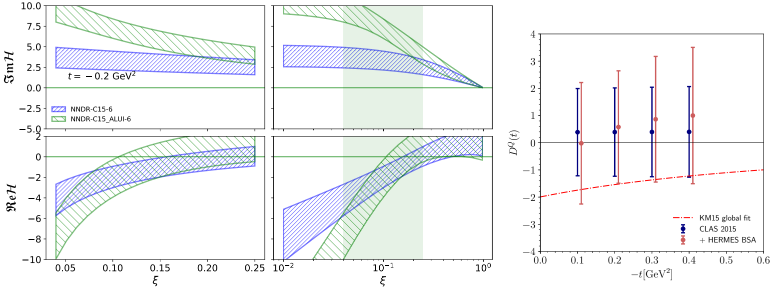
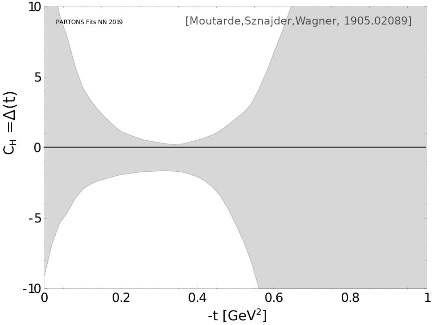


- Even this dataset is still consistent with zero D-term.
- We need measurements more sensitive to $\mathfrak{Re}\,\mathcal{H}$ (BCA, DDVCS, ...)

Subtraction constant by PARTONS

- Global neural net fit (including BCA) still results in a D-term consistent with zero ☹



- [Moutarde, Sznajder, Wagner '19], see talk by Pawel Sznajder

Neural nets global fits

Study B: NN fit to world fixed target data

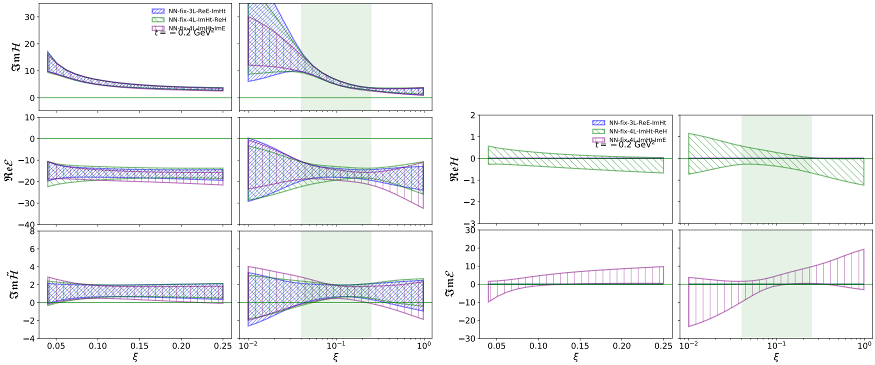
- Representative subset of world DVCS fixed target data:

```
npt x obs    collab  harm.  ref.
-----------------------------------------
 6 x ALUI   HERMES  -1.0    arXiv:1203.6287
12 x AUTDVCS  HERMES  0     arXiv:0802.2499
12 x AUTI   HERMES  1.0     arXiv:0802.2499
 6 x BCA    HERMES  0.0     arXiv:1203.6287
 6 x BCA    HERMES  1.0     arXiv:1203.6287
12 x BSDw   CLAS    -1      arXiv:1504.02009
15 x BSDw   HALLA   -1      arXiv:1504.05453
12 x BSSw   CLAS    0.0     arXiv:1504.02009
12 x BSSw   CLAS    1.0     arXiv:1504.02009
10 x BSSw   HALLA   0.0     arXiv:1504.05453
10 x BSSw   HALLA   1.0     arXiv:1504.05453
 6 x BTSA   HERMES  0.0     arXiv:1004.0177v1
 3 x TSA    CLAS    -1      arXiv:hep-ex/0605012
 6 x TSA    HERMES  -1.0    arXiv:1004.0177v1
-----------------------------------------
TOTAL = 128
```

- We now use completely unconstrained neural nets representing $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$, $\mathfrak{Im}\,\mathcal{E}$, $\mathfrak{Re}\,\mathcal{E}$, ... (do not assume dispersion relations)

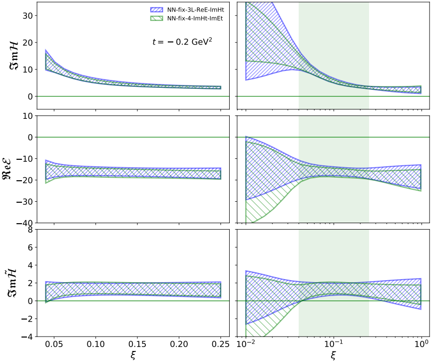
Results (1/2)

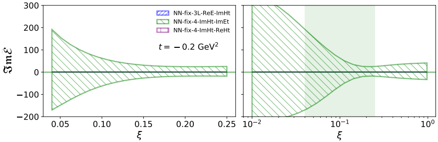
- Only $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Im}\,\widetilde{\mathcal{H}}$ and $\mathfrak{Re}\,\mathcal{E}$ consistently extracted as different from zero, and, with somewhat smaller significance, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\mathcal{E}$:



Results (2/2)

- Other CFFs come out consistent with zero. Only bounds on their size are obtained. *E. g.* for $\mathfrak{Im}\,\widetilde{\mathcal{E}}$:





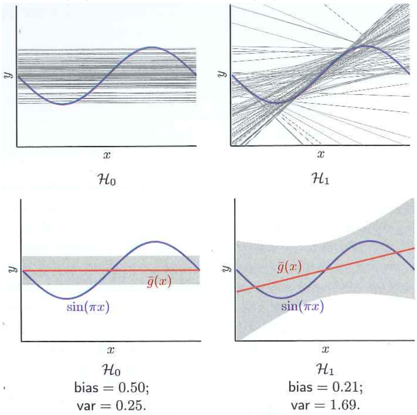
- See next talk by Pawel Sznajder for a more extensive global neural net analysis

Bias-variance tradeoff: toy example

- "Unknown" $f(x) = sin(\pi x)$ "measured" at two points.

$\mathcal{H}_0$ - rigid (biased) $\quad$ $\mathcal{H}_1$ - flexible $\quad$ [Abu-Mostafa et al. '12]

$h(x) = a$ $\quad$ $h(x) = ax + b$



error = bias + variance

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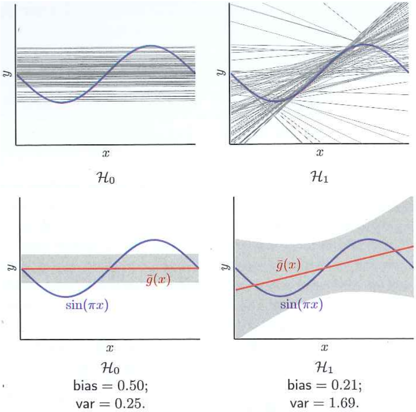

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- In DVCS situation is the opposite! We need to decrease bias.

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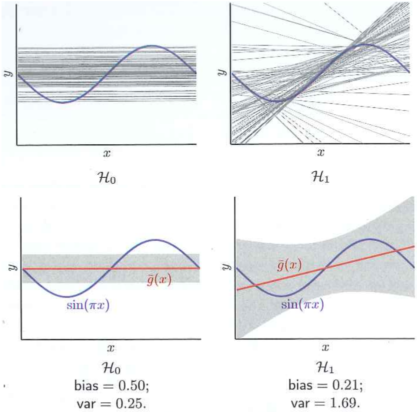
[Abu-Mostafa et al. '12]



error = bias + variance

- In DVCS situation is the opposite! We need to decrease bias.
- Neural networks are proven to be unbiased

Summary

- Neural network method has a unique capability of extraction of Compton form factors (and, later, GPDs) with reliable uncertainties
- More experimental and phenomenological work is needed to determine pressure distribution in a nucleon in a reliable and model-independent way.

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The End