From lattice QCD correlators with static heavy quarks and dynamical light quarks to the unitary study of resonances

Pedro Bicudo*

Nuno Cardoso, Marco Cardoso*, Marc Wagner***

* CeFEMA, Instituto Superior Técnico, Universidade de Lisboa, Portugal
** ITP, Johann Wolfgang Goethe Universität Frankfurt am Main, Germany

June 2019

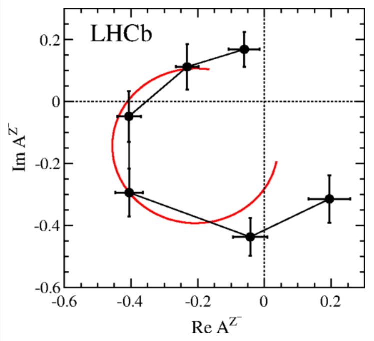
Outline

1. Introduction
   - Experimental observation of double heavy exotics
   - Applying the Born-Oppenheimer approximation
   - Previous study: prediction of $\bar{Q}\bar{Q}qq$ tetraquarks
2. The emergent wave method
   - Emergent and incident wavefunctions
   - Partial wave decomposition
   - Solving the differential equations for the emergent wave
   - Phase shifts and **S** and **T** matrix poles
3. $Q\bar{Q}$ and $Q\bar{Q}q\bar{q}$ phase shifts, **S** matrix poles and resonances
   - Correlators, string breaking and static potentials in quarkonium
   - With real energies : t Matrix, Argand plot and Phase shifts
   - Resonances as poles of the **S** and **T** matrices
4. Summary and outlook

Outline

Introduction

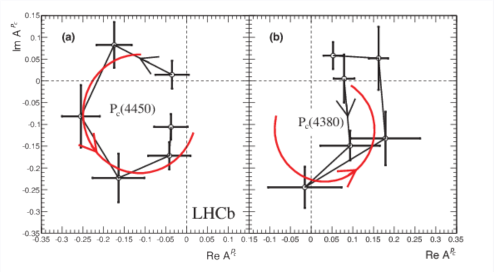
Confirmation of $Z_c(4430)^-$ by LHCb at CERN: π J/ψ Argand plot



- Exotic hadrons have been a *holy grail* of modern physics since the onset of QCD.
- In the 2010's, we have finally confirmed experimental double-heavy exotic hadronic resonances.
- There are two $Z_b^\pm$ observed by BELLE, slightly below $B\,B^*$ and B^*B^* thresholds, the $Z_b(10610)^+$ and $Z_b(10650)^+$.
- The two $Z_c(3940)^\pm$and $Z_c(4430)^\pm$ are clearly well above DD threshold, and have several confirmations, LHCb at CERN recently confirmed $Z_c(4430)^-$ resonance with a mass of 4475 MeV and width of 172 MeV.
- LHCb also clearly observed pentaquarks candidates p_c decaying to $J/\psi\ p$.

Introduction

LHCb $c\bar{c}$ pentaquark

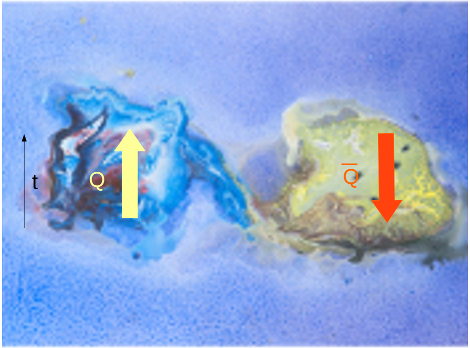


- LHCb has also observed two pentaquarks candidates with again an extremely large significance > 9.
- The two $p_c(4450)^{\pm}$and $P_c(4380)^{\pm}$ are clearly seen in the decay to a $J/\psi p$.
- The recent experimental success of resides in a very high luminosity and a good resolution since the exotics are produced in clear decays of hadrons.
- However these states are extremely hard to model because they may decay to many many channels (order of 30 for Zc'), being impractical for instance to apply techniques such as the Lüscher 's phase shift method.

Outline

Introduction

Born-Oppenheimer approximation



However resonances with many coupled channels are extremely hard for lattice QCD. We separate the problem adiabaticly:

- the valence gluons and light quarks are included in lattice QCD dynamical configurations,
- the constituent light quarks are in lattice propagators,
- with Wilson lines we approximate the heavy quarks to static ones,
- with correlation matrices, we compute lattice QCD potentials,
- finally we include the quantum kinetic energy of the heavy quarks, and we apply quantum mechanics techniques,
- we can study not only boundstates, also fully unitary resonances.

Introduction

Several sorts of hadrons, not just the Zc, Zb and Pc are in principle amenable to lattice QCD by the Born-Oppenheimer approximation.

- The $b - \bar{b}$ hybrid wave functions and spectra have been studied with lattice QCD and BO.
- Moreover the spin-dependent potentials can also be studied with heavy quark effective theories of lattice QCD.
- We first briefly review systems we recently predicted with lattice QCD potentials, a $ud\bar{b}\bar{b}$ tetraquark bound state with quantum numbers $I(J^P) = 0(1^+)$ and a resonance $I(J^P) = 0(1^-)$.
- Then our main goal here is to address higher bottomonium excitations bb, as an intermediate step before studying more difficult systems, cc or cb could also be studied.

Juge:1999ie, McNeile:2006bz, McNeile:2002az, Ader:1981db.
(using inpirehep.net code for references)

Outline

1. Introduction
   - Experimental observation of double heavy exotics
   - Applying the Born-Oppenheimer approximation
   - Previous study: prediction of $\bar{Q}\bar{Q}qq$ tetraquarks
2. The emergent wave method
   - Emergent and incident wavefunctions
   - Partial wave decomposition
   - Solving the differential equations for the emergent wave
   - Phase shifts and **S** and **T** matrix poles
3. $Q\bar{Q}$ and $Q\bar{Q}q\bar{q}$ phase shifts, **S** matrix poles and resonances
   - Correlators, string breaking and static potentials in quarkonium
   - With real energies : t Matrix, Argand plot and Phase shifts
   - Resonances as poles of the **S** and **T** matrices
4. Summary and outlook

Introduction

Also with K. Cichy, A. Peters, M. Pflaumer, J. Scheunert, B. Wagenbach.

- The lattice QCD results for the potentials of two static antiquarks $\bar{Q}\bar{Q}$ in the presence of two light quarks qq can be parametrized by a screened Coulomb potential,

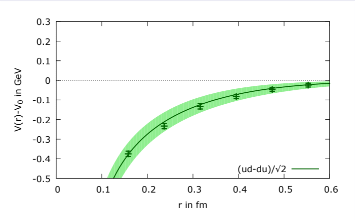
$$V(r) = -\frac{\alpha}{r} e^{-r^2/d^2} . \qquad (1)$$

- ansatz inspired by one-gluon exchange at small $\bar{Q}\bar{Q}$ separations r and a screening of the Coulomb potential by the two B mesons at large r.

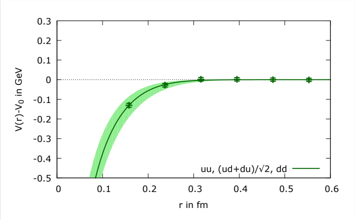
Wagner:2010ad, Wagner:2011ev, Bicudo:2015kna.

Fit with a screened Coulomb potential

scalar:

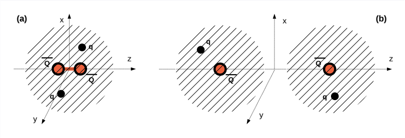


vector:



Introduction

Physics of heavy-heavy hadrons



(a) At small separations the static quarks (for instance in the figure $\bar{Q}\bar{Q}$) interact by perturbative one-gluon exchange.
(b) At large separations the light quarks screen the interaction and the four quarks form two rather weakly interacting heavy-light mesons (or barions).

Applying the Born-Oppenheimer approximation, we separate the problem:

- the light quarks are incorporated in lattice QCD with dynamical configurations and propagators,
- the heavy quarks can be approximated as static sources, and with Wilson lines we compute lattice QCD potentials,
- finally we include the quantum kinetic energy of the heavy quarks, and then we should be able to solve the full problem with quantum mechanics techniques,
- with adequate quantum mechanics we can study not only boundstates, also fully unitary resonances.

Introduction

Fit of the lattice QCD potential

I	j	α	d in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

Table: Parameters α and d of the potential of Eq. (1) for two static antiquarks $\bar{Q}\bar{Q}$, in the presence of two light quarks qq with quantum numbers I and j.

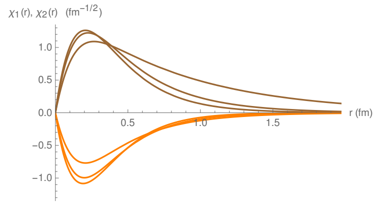
- There are both attractive and repulsive channels.
- Most promising with respect to the existence of tetraquark bound states or resonances are light quarks $q \in \{u, d\}$ together with $(I = 0, j = 0)$ or $(I = 1, j = 1)$,
- the corresponding potentials $V(r)$ are not only attractive, but also rather wide and deep

Bicudo:2012qt, Brown:2012tm.
Bicudo:2015vta, Bicudo:2016ooe.

Introduction

- Using the Born-Oppenheimer approximation (very good for $\bar{b}$, fair for $\bar{c}$ quarks), we provide a quantum kinetic energy $p^2/2\mu$ to the heavy quarks.
- Solving the Schrödinger equation, we found evidence for the existence of ONLY ONE double heavy tetraquark boundstate $ud\bar{b}\bar{b}$ with $I=0$ and $J^P=1^+$ equivalent to a $B\,B^* \oplus B^*\,B^*$ state.
- We found several non-existence evidences of I=1 $ud\bar{b}\bar{b}$, nor of $u/ds\bar{b}\bar{b}$, $ss\bar{b}\bar{b}$, $u/dc\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$, $cc\bar{b}\bar{b}$, $ud\bar{c}\bar{b}$, $ss\bar{c}\bar{b}$, $u/dc\bar{c}\bar{b}$, $sc\bar{c}\bar{b}$, $cc\bar{c}\bar{b}$, $ud\bar{c}\bar{c}$, $u/ds\bar{c}\bar{c}$, $ss\bar{c}\bar{c}$, $u/dc\bar{c}\bar{c}$, $sc\bar{c}\bar{c}$, $cc\bar{c}\bar{c}$ tetraquarks.

Wavefunctions of the $ud\bar{b}\bar{b}$ $I=0,\ J^P=1^+,\ E_b=59^{+30}_{-38}$ MeV

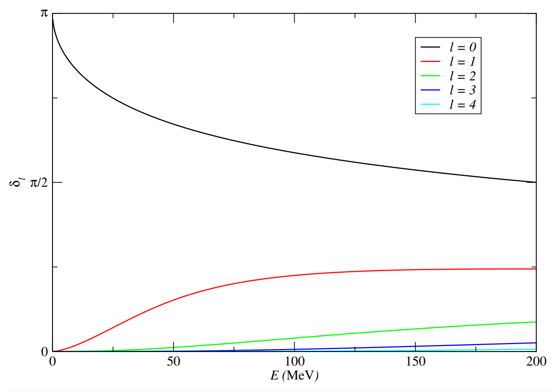


The two components $B\,B^*$ and $B^*\,B^*$ of the wave function, including the error bars.

Bicudo:2015vta, Bicudo:2015kna, Bicudo:2016ooe.

Introduction

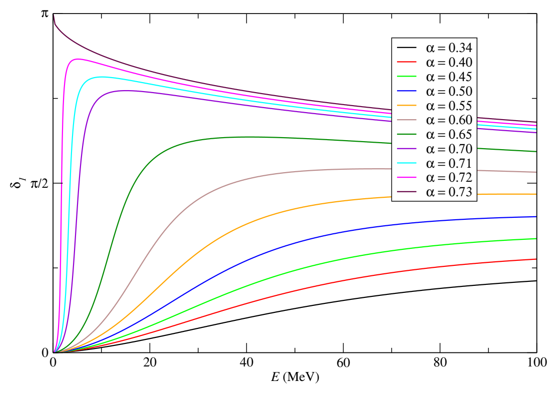
Using the emergent wave method we also compute the phase shifts δ_l



Phase shift δ_l as a function of the energy E for different angular momenta $l = 0, 1, 2, 3, 4$ for the $(I = 0, j = 0)$ potential $(\alpha = 0.34,\ d = 0.45\,\mathrm{fm})$.

Introduction

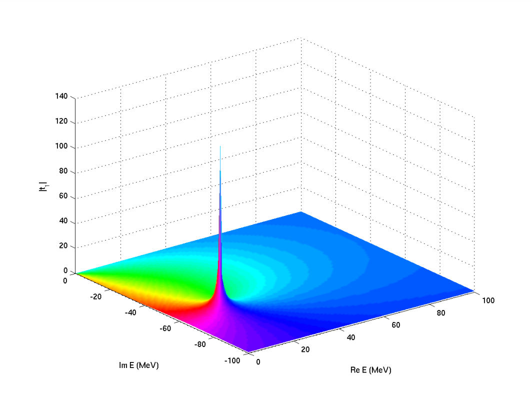
δ_1 for different α parameters



Phase shift δ_1 as a function of the energy E for different parameters α for the $(I = 0, j = 0)$ potential $(d = 0.45\,\mathrm{fm})$.

Introduction

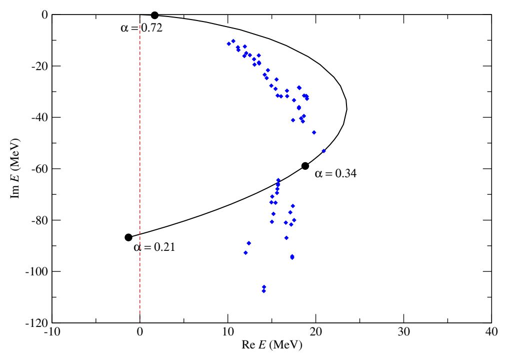
Pole in the complex plane of $E \in \mathbb{C}$: a $I_Q = 1$ $ud\bar{b}\bar{b}$ tetraquark resonance



3D plot of t_1 as a function of the complex energy E . The vertical axis shows the norm $|t_1|$, the colours represent the phase $\arg(t_l)$.

Introduction

Resonance mass $m = 10\,576^{+4}_{-4}\,\mathrm{MeV}$, decay width $\Gamma = 112^{+90}_{-103}\,\mathrm{MeV}$



Dependence on parameter α of the S matrix pole, and cloud of diamond points illustrating the systematic error. Bicudo:2015vta, Bicudo:2017szl .

Outline

Emergent wave method

- Our goal now is to study resonances, a 1st technical step to address the exotics such as Z_b, Z_c and P_c observed at BELLE, BESIII, LHCb... and predict the future resonances observed at PANDA.
- Notice systematic error bars come from the ansatz to fit (or interpolate) the potentials, and in the heavy quark $1/m_Q$ expansion.
- We tried several techniques, first with a toy model. Typically momentum space techniques are used in effective theories, but a position space technique is more convenient for lattice QCD potentials.
- It turns out the best approach is to get back to fundamental quantum mechanics. We adopt a simple technique, we call it the emergent wave method.

Bicudo:2015bra, Bicudo:2017szl

Emergent wave method

The first step in the emergent wave method is to split the wave function of the Schrödinger Eq. $\Big(H_0 + V(r) - E\Big)\psi = 0$, into two parts,

$$\Psi = \Psi_0 + X \,, \tag{2}$$

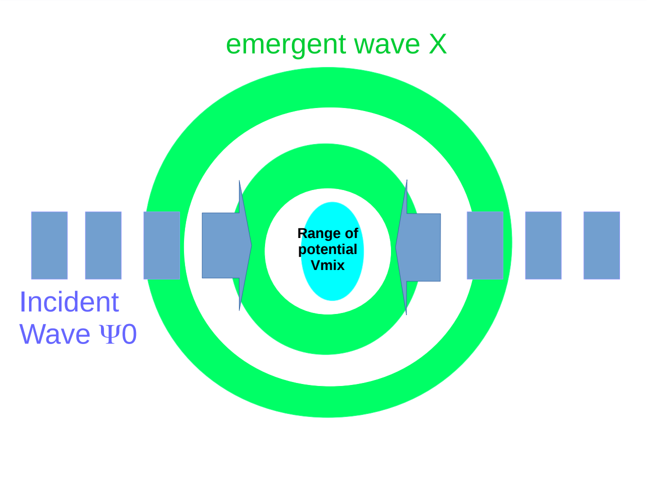
where Ψ_0 is the incident wave, a solution of the free Schrödinger equation, $H_0\Psi_0 = E\Psi_0$, and X is the emergent wave. We obtain

$$\Big(H_0 + V(r) - E\Big)X = -V(r)\Psi_0 \,. \tag{3}$$

- For any energy E we calculate the emergent wave X by providing the corresponding Ψ_0 and fixing the appropriate boundary conditions.
- From the asymptotic behaviour of the emergent wave X we then determine the phase shifts δ_l , the S matrix and the T matrix.
- Continuing to complex energies $E \in \mathbb{C}$ we find the poles of the S matrix and the T matrix in the complex plane.
- We identify a resonance with a pole of S in the second Riemann sheet at $m - i\Gamma/2$, where m is the mass and Γ is the resonance decay width.

Emergent wave method

We decompose our wave function Ψ in an incident wave Ψ_0 and emergent wave X



Outline

Emergent wave method

We consider an incident plane wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$, which can be expressed as a sum of spherical waves,

$$\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l (2l+1) i^l j_l(kr) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \ , \tag{4}$$

where j_l are spherical Bessel functions, P_l are Legendre polynomials and the relation between energy and momentum is $\hbar k = \sqrt{2\mu E}$. For a spherically symmetric potential $V(r)$ as in Eq. (1) and an incident wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$ the emergent wave X can also be expanded in terms of Legendre polynomials P_l,

$$X = \sum_l (2l+1) i^l \frac{\chi_l(r)}{kr} P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \ . \tag{5}$$

Inserting Eq. (4) and Eq. (5) into Eq. (3) leads to a set of ordinary differential equations for χ_l,

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r) kr j_l(kr) \ . \tag{6}$$

Outline

Emergent wave method

The potentials $V(r)$, say in Eq. (1), are exponentially screened, i.e. $V(r) \approx 0$ for $r \geq R$, where $R \gg d$. For large separations $r \geq R$ the emergent wave is, hence, a superposition of outgoing spherical waves, i.e.

$$\frac{\chi_l(r)}{kr} = i\, t_l h_l^{(1)}(kr), \tag{7}$$

where $h_l^{(1)}$ are the spherical Hankel functions of first kind.
Our aim is now to compute the complex prefactors t_l, which will eventually lead to the phase shifts. To this end we solve the ordinary differential equation (6). The corresponding boundary conditions are the following:

- At $r = 0$: $\chi_l(r) \propto r^{l+1}$.
- For $r \geq R$: Eq. (7).

The boundary condition for $r \geq R$ fixes t_l as a function of E.
We solve it numerically, with two different numerical techniques approaches: (1) a fine uniform discretization of the interval $[0, R]$, which reduces the differential equation to a large set of linear equations, which can be solved rather efficiently, since the corresponding matrix is tridiagonal; (2) a standard 4-th order Runge-Kutta shooting method.

Outline

Emergent wave method

The quantity t_l is a T matrix eigenvalue. From t_l we directly calculate the phase shift δ_l and also read off the corresponding S matrix eigenvalue s_l, [1]

$$s_l \equiv 1 + 2it_l = e^{2i\delta_l} \ . \tag{8}$$

Moreover, note that both the S matrix and the T matrix are analytical in the complex plane. They are well-defined for complex energies $E \in \mathbb{C}$.

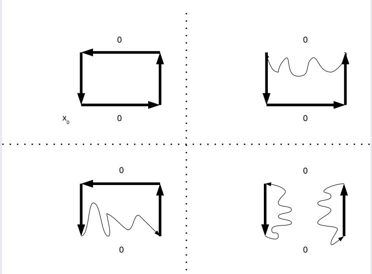
- Thus, our numerical method can as well be applied to solve the differential Eq. (6) for complex $E \in \mathbb{C}$.
- We find the S and T matrix poles by scanning the complex plane $(\mathrm{Re}(E), \mathrm{Im}(E))$ and applying Newton's method to find the roots of $1/t_l(E)$. The poles of the S and the T matrix correspond to complex energies of resonances.
- Note the resonance poles must be in the second Riemann sheet with a negative imaginary part both for the energy E and the momentum k.

[1] At large distances $r \geq R$, the radial wavefunction is $kr[j_l(kr) + i\, t_l h_l^{(1)}(kr)] = (kr/2)[h_l^{(2)}(kr) + e^{2i\delta_l} h_l^{(1)}(kr)]$.

Outline

Quarkonium correlators, string breaking, potentials

$Q\bar{Q}$ and $Q\bar{l}l\bar{Q}$ correlator matrix



- A similar correlation matrix has been computed for string breaking, where a correlation matrix with $Q\bar{Q}$ and MM, say $b\bar{b}$ and $B\bar{B}$, channels are coupled, since 2015 in Bali:2005fu and very recently in Koch:2018puh .
- Analysing the quantum numbers of the light quarks in the string breaking correlation matrix, we extract from it the coupled channel Schrödinger equation, with potential,

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r)\Big(1 \otimes \mathbf{e}_r\Big) \\ V_{\text{mix}}(r)\Big(\mathbf{e}_r \otimes 1\Big) & V_{\bar{M}M,\parallel}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\bar{M}M,\perp}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big) \end{pmatrix} \quad (9)$$

Quarkonium correlators, string breaking, potentials

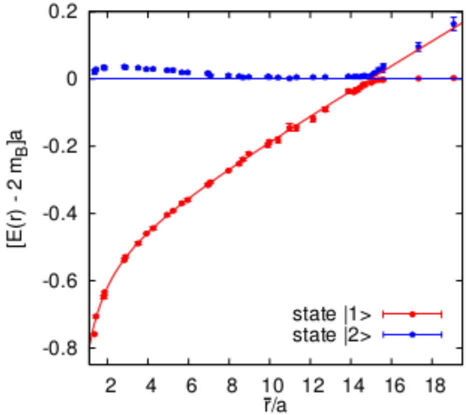
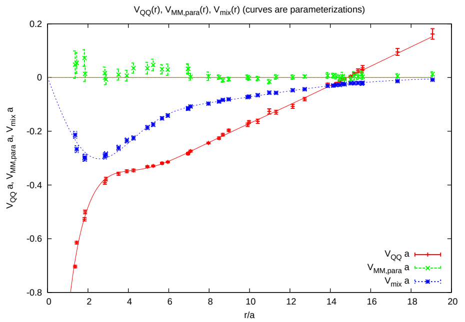


FIG. 13: The two energy levels, as a function of $\bar{r}$, normalized with respect to $2m_B$ (horizontal line). The curve corresponds to the three parameter fit to $E_1(\bar{r})$, Eqs. (80)–(82), for $0.2\,\text{fm} \leq \bar{r} \leq 0.9\,\text{fm} < r_c$.

Quarkonium correlators, string breaking, potentials

Matrix elements of potential



Potentials $V_{Q\bar{Q}}(r)$, $V_{\rm mix}(r)$, $V_{MM}(r)$ extracted from the string breaking potentials.

For instance for l_Q=0 and $l_M = 1$ we get the potential,

$$\begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\rm mix}(r) \\ V_{\rm mix}(r) & V_{\bar{M}M,\parallel}(r) \end{pmatrix} \tag{10}$$

we fit with just a constant $2m_M$ in the meson-meson channel,
with a funnel and two Gaussian in the $Q\bar{Q}$ channel,
and a combination of two Gaussian in the mixing matrix element.

Bali:2005fu, Koch:2018puh

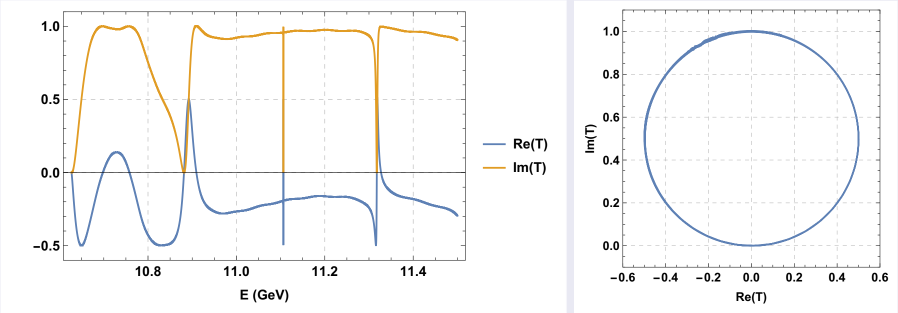
Outline

1. Introduction
   - Experimental observation of double heavy exotics
   - Applying the Born-Oppenheimer approximation
   - Previous study: prediction of $\bar{Q}\bar{Q}qq$ tetraquarks
2. The emergent wave method
   - Emergent and incident wavefunctions
   - Partial wave decomposition
   - Solving the differential equations for the emergent wave
   - Phase shifts and **S** and **T** matrix poles
3. $Q\bar{Q}$ and $Q\bar{Q}q\bar{q}$ phase shifts, **S** matrix poles and resonances
   - Correlators, string breaking and static potentials in quarkonium
   - With real energies : t Matrix, Argand plot and Phase shifts
   - Resonances as poles of the **S** and **T** matrices
4. Summary and outlook

Results for the phase shifts and resonances

Using the emergent wave method we compute the T and S matrices.

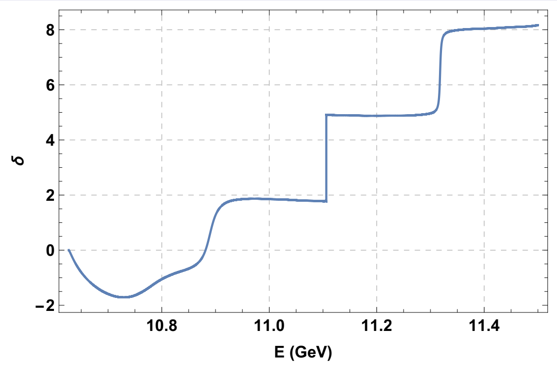
Real and Imaginary part of the T Matrix for the median parameters of the potential



(Left) real and Imaginary part of the $T/2$ matrix up to E= 11.5 GeV. (Right) Argand plot, with only one open channel, from S matrix unitarity we expect a perfect circle .

Results for the phase shifts and resonances

Phase shifts for the median parameters of the potential (real energy E)

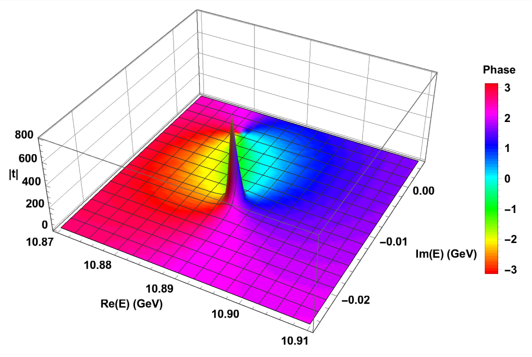


Phase shift δ_l as a function of the energy E. We clearly see 3 resonances at E=10.9 GeV, 11.2 GeV and 11.3 GeV, perhaps another one below 10.8 GeV.

Outline

Results for the phase shifts and resonances

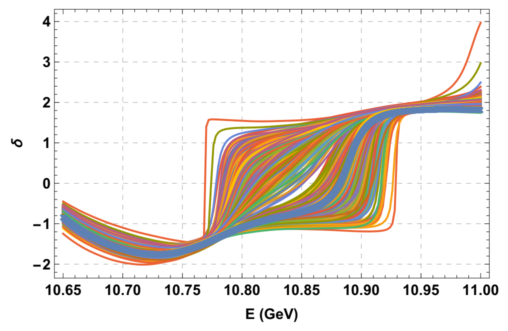
Pole in the complex plane of $E \in \mathbb{C}$ for the median parameters of the potential



3D plot of t_1 as a function of the complex energy E close to the 1st resonance at re$(E) \simeq 10.9$ GeV. The vertical axis shows the norm $|t_1|$, the colours represent the phase $\arg(t_l)$.

Results for the phase shifts and resonances

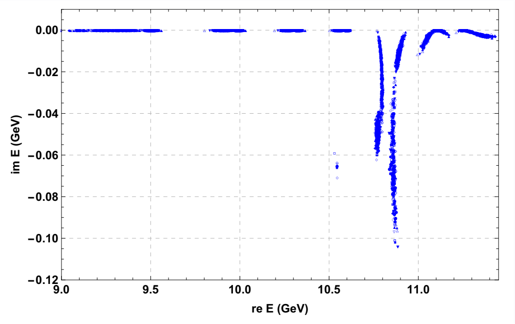
Error bars: phase shifts for a sample of 1000 sets of potentials.



Determination of the error of the phase shifts close to the first resonance, considering a sample of 1000 sets of potential parameters representing the error bars of the lattice QCD potentials. This is quite non-perturbative.

Results for the phase shifts and resonances

Error bars: pole position for a sample of 1000 sets of potentials.



Considering a sample of 1000 sets of potential parameters representing the error bars of the lattice QCD potentials, we show the corresponding cloud of points for the position of the boundstates and resonances.

Results for the phase shifts and resonances

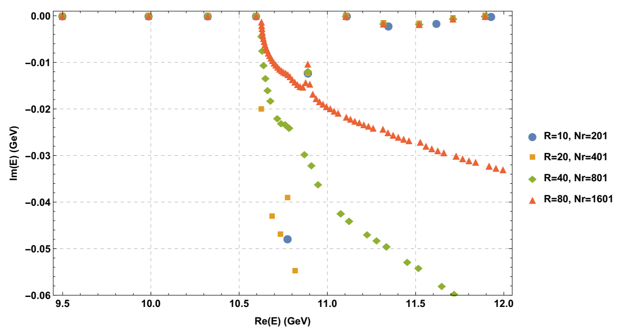
Poles of the $S(E)$ matrix, $E \in \mathbb{C}$, using the sample of potential parameters

$E_{Q\bar{Q}}$ (GeV	state	E_{exp} (MeV)	$E_{\text{pole}}(GeV)$	Δ Re(E) (GeV)	Γ (GeV)
9.6091(1407)	$\Upsilon(1S)$	$9.4603(3) - 0(0)i$	9.4458(1070) - 0(0) i	-0.1633(1768)	0(0)
10.0956(256)	$\Upsilon(2S)$	$10.0233(3) - 0(0)i$	9.9684(408) - 0(0) i	-0.1272(482)	0(0)
10.4163(217)	$\Upsilon(3S)$	$10.3552(5) - 0(0)i$	10.3100(290) - 0(0) i	-0.1063(362)	0(0)
10.6853(222)	$\Upsilon(4S)$	$10.5794(12) - 0.0103(13)i$	10.5875(217) - 0(0) i	-0.0978(310)	0(0)
			10.7772(91) -0.0438(121) i		0.0876(242)
10.9279(284)	$\Upsilon(5S\,?)$	$10.8899(32) - 0.0255(35)i$	10.8834(177) -0.0267(255) i	-0.0445(335)	0.0534(510)
11.1579(371)	$\Upsilon(6S\,?)$	$10.9929(10) - 0.0245(75)i$	11.0993(312) -0.0011(18) i	-0.0586(485)	0.0022(36)
11.4014(452)	$\Upsilon(7S\,?)$		11.3288(400) -0.0017(10) i	-0.0726(604)	0.0034(20)
11.6839(524)	$\Upsilon(8S\,?)$		11.5928(523) -0.0023(14) i	-0.0911(740)	0.0046(28)

Poles of the S matrix in the complex energy space computed with the emergent wave method for R =30 GeV^{-1} and $Nr = 601$, considering considering the 1000 sets of potential parameters in the sample. We mark with horizontal lines the opening of the average of the $D - \bar{D}$ and $D* - \bar{D}*$ thresholds.

Results for the phase shifts and resonances

A detail: Breit-Wigner complex poles versus density of real states.



We show the dependence on the poles of the boundary R. Notice that, not only the resonance poles exist, also the meson-meson poles are present, corresponding to the branch cut of the open channel.

Summary and outlook

- For more details on the emergent wave method and on $ud\bar{b}\bar{b}$ resonances, please see the recent Phys.Rev. D96, 054510 (2017), Pedro Bicudo, Marco Cardoso (CeFEMA, IST, Lisbon Univ.) , Antje Peters, Martin Pflaumer, Marc Wagner (Frankfurt Univ.), and our preprints to appear briefly on the arXiv.
- In what concerns bottomonium, we use lattice QCD string-breaking correlation matrix published in the litterature, and with a sample of 1000 error bar parametrizations we first find a dynamical resonance, and for $\Upsilon(4S)$ our pole is at E_{pole}= 10.8834(177) -0.0267(255) i GeV, corresponding to a width similar to the experimental one.
- The heavier resonances are much narrower, since we do not have yet the potentials to couple the next meson-meson channels.
- As an outlook we plan to compute the potentials, with the necessary precision, to couple more channels to the system.
- In the future, it should be possible to address exotic resonances such as Z_b, further developing the emergent wave method.

Bicudo:2017szl