

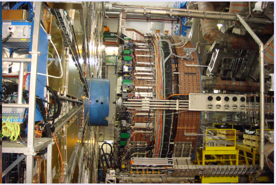

Foundations of Nuclear and Particle Physics.

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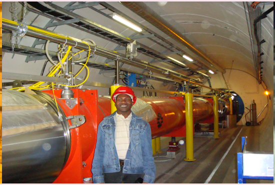
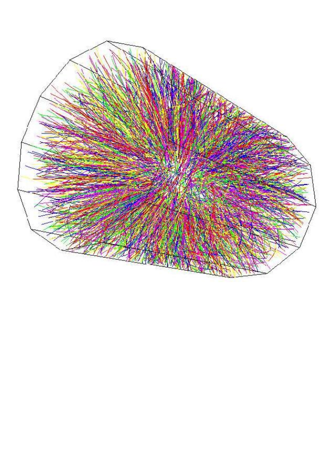
2/3 August 2010 / Stellenbosch, South Africa


Outline

1. High Energy Physics
2. Excluded Volume Corrections
3. Canonical Corrections

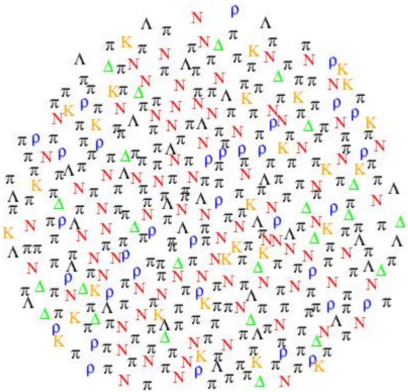
Hgh Energy Physics



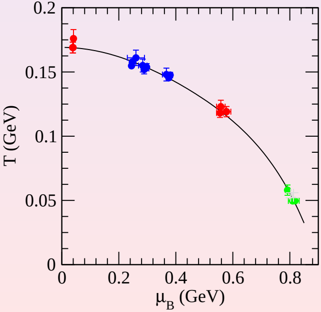
Finite-Temperature Field Theory
Pronciples and Applications.

J.I. Kapusta and C. Gale

Hadronic Gas before Chemical Freeze-Out



Chemical Freeze-Out: Status in 2010



Relation between grand canonical and canonical ensembles:

$$Z_{GC}(T, V, \mu) = \sum_{N=0}^{\infty} e^{\frac{\mu N}{T}} Z_C(T, V, N)$$

Relation between grand canonical and pressure ensembles:

$$Z_p(T, P, \mu) = \int_0^{\infty} dV \; e^{\frac{PV}{T}} Z_{GC}(T, V, \mu)$$

Excluded Volume Corrections.

$$\begin{aligned} Z &= \exp\left\{ V \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}+\frac{\mu}{T}} \right\} \\ &= \sum_{N=0}^{\infty} \frac{V^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \end{aligned}$$

with excluded volume corrections

$$\begin{aligned} Z &\rightarrow \sum_{N=0}^{\infty} \frac{(V - V_0 N)^N}{N!} e^{\mu N/T} \\ &\quad \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \theta(V - V_0 N) \end{aligned}$$

Excluded Volume Corrections.

It is more convenient to consider these corrections in the pressure ensemble:

$$Z_p \equiv \int_0^\infty dV\; e^{-PV/T} \sum_{N=0}^{\infty} \frac{V^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

$$\begin{aligned} Z_p \;\; \to \;\; & \sum_{N=0}^{\infty} \int_0^\infty dV\; e^{-PV/T} \\ & \frac{(V - V_0 N)^N}{N!} e^{\mu N/T} \\ & \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \theta(V - V_0 N) \end{aligned}$$

introduce $x \equiv V - V_0 N$.

Excluded Volume Corrections.

$$
\begin{aligned}
Z_p &= \sum_{N=0}^{\infty} \int_0^{\infty} dx\; e^{-Px/T} \\
&\quad \frac{x^N}{N!} e^{-PV_0 N/T} e^{\mu N/T} \\
&\quad \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N
\end{aligned}
$$

a new variable $\bar{\mu} \equiv \mu - PV_0$

Excluded Volume Corrections.

$$Z_p \rightarrow \sum_{N=0}^{\infty} \int_0^{\infty} dx\ e^{-Px/T}$$

$$\frac{x^N}{N!} e^{\bar{\mu}N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

which is the original partition function with the he replacement

$$\boxed{\bar{\mu} = \mu - P\ V_0}$$

Excluded Volume Corrections.

The particle number density now becomes:

$$\begin{aligned} n &= \frac{T}{V}\frac{\partial}{\partial\mu}\ln\ Z \\ &= \frac{T}{V}\frac{\partial\bar{\mu}}{\partial\mu}\frac{\partial}{\partial\bar{\mu}}\ln\ Z \\ &= \frac{\partial\bar{\mu}}{\partial\mu}n_0 \\ &= [1 - V_0\ n]\,n_0 \end{aligned}$$

$$\boxed{n = \frac{n_0}{1 + V_0 n_0}}$$

Effects Cancel Out in Ratios.

J.C., K. Redlich, H. Satz, E. Suhonen, ZfP C33, 151, (1986)

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, ZfP C51, 485 (1991).

Exact Strangeness Conservation.

For a small system at low temperatures ($T \approx 50$ MeV), e.g. at GSI canonical corrections are necessary.
Instead of

$$N_K \approx \exp -M_K/T$$

one gets

$$N_K \approx \exp -2M_K/T$$

Extra suppression is due to strangeness conservation and lack of a large heat bath. This correction disappears quickly at higher energies and is already small at AGS energies.

Exact Strangeness Conservation.

$$Z = \mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}$$

Insert a Kronecker delta in the trace:

$$\sum_i n_i(S=1) + 2\sum_j n_j(S=2) + 3\sum_k n_k(3) =$$

$$\sum_i \bar{n}_i(S=-1) + 2\sum_j \bar{n}_j(S=-2) + 3\sum_k \bar{n}_k(S=-3)$$

and rewrite it as

$$\delta\left(\sum_i n_i(S=1) + ...,\sum_i \bar{n}_i(S=-1) + ...\right)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\phi$$

$$\exp\left(i\phi\sum_i n_i(S=1) + ..\ - i\phi\sum_i \bar{n}_i(S=-1)\right)$$

Exact Strangeness Conservation.

$$
\begin{aligned}
Z = \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp & \left\{ Z_1 e^{i\phi} + Z_{-1} e^{-i\phi} \right. \\
+ & \quad Z_2 e^{2i\phi} + Z_{-2} e^{-2i\phi} \\
+ & \quad \left. Z_3 e^{3i\phi} + Z_{-3} e^{-3i\phi} \right\}
\end{aligned}
$$

$$
\begin{aligned}
= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp & \left\{ \sqrt{Z_1 Z_{-1}} \left[\sqrt{\frac{Z_1}{Z_{-1}}} e^{i\phi} + \sqrt{\frac{Z_{-1}}{Z_1}} e^{-i\phi} \right] \right. \\
+ & \quad \sqrt{Z_2 Z_{-2}} \left[\sqrt{\frac{Z_2}{Z_{-2}}} e^{2i\phi} + \sqrt{\frac{Z_{-2}}{Z_2}} e^{-2i\phi} \right] \\
+ & \quad \left. \sqrt{Z_3 Z_{-3}} \left[\sqrt{\frac{Z_3}{Z_{-3}}} e^{3i\phi} + \sqrt{\frac{Z_{-3}}{Z_3}} e^{-3i\phi} \right] \right\}
\end{aligned}
$$

Z_1: sum of all particles with strangeness 1, e.g. K^+

Exact Strangeness Conservation.

Use

$$\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\} = \sum_{n=-\infty}^{\infty} I_m(x)t^m$$

to obtain

$$Z = \frac{1}{2\pi}\int_0^{2\pi} e^{3im\phi+2in\phi+ip\phi} \sum_{p=-\infty}^{\infty} I_p(x_1) \sum_{n=-\infty}^{\infty} I_n(x_2) \sum_{m=-\infty}^{\infty} I_m(x_3)\, y_1^p y_2^n y_3^m$$

where

$$y_i = \sqrt{\frac{Z_i}{Z_{-i}}} \qquad x_i = 2\sqrt{Z_i Z_{-i}}$$

Exact Strangeness Conservation.

$$Z = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_{3m+2n}(x_1)\, y_1^{-3m-2n}\, I_n(x_2)\, y_2^n\, I_m(x_3)\, y_3^m$$

Exact Strangeness Conservation.

$$
\begin{aligned}
Z &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{Z_1 e^{i\phi} + Z_{-1} e^{-i\phi}\right\} \\
&= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{\sqrt{Z_1 Z_{-1}}\left[\sqrt{\frac{Z_1}{Z_{-1}}} e^{i\phi} + \sqrt{\frac{Z_{-1}}{Z_1}} e^{-i\phi}\right]\right\}
\end{aligned}
$$

Z_1: sum of all particles with strangeness 1, e.g. K^+
Z_{-1}: sum of all particles with strangeness -1, e.g. Λ

Exact Strangeness Conservation.

Use

$$\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\} = \sum_{n=-\infty}^{\infty} I_m(x)t^m$$

to obtain

$$Z = \frac{1}{2\pi}\int_0^{2\pi} e^{ip\phi} \sum_{p=-\infty}^{\infty} I_p(x_1)y_1^p$$

where

$$y_1 = \sqrt{\frac{Z_1}{Z_{-1}}} \qquad x_1 = 2\sqrt{Z_1 Z_{-1}}$$

$$Z = I_0(x_1)$$

Exact Strangeness Conservation.

In more detail, e.g. the multiplicity of K^+

$$N_{K^+} = \frac{T}{Z}\frac{\partial I_0(x_1)}{\partial \mu_{K^+}}\bigg|_{\mu_{K^+}=0}$$

Use

$$\frac{d}{dz} I_0(z) = I_1(z)$$

Exact Strangeness Conservation.

$$
\begin{aligned}
N_{K^+} &= \frac{T}{Z}\frac{\partial}{\partial \mu_{K^+}} I_0(x_1) \\
&= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial x_1}{\partial \mu_{K^+}} \\
&= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial 2\sqrt{Z_1 Z_{-1}}}{\partial \mu_{K^+}} \\
&= \frac{I_1(x_1)}{I_0(x_1)} \sqrt{\frac{Z_{-1}}{Z_1}} N^0_{K^+}
\end{aligned}
$$

where $N^0_{K^+}$ refers to the "unmodified" kaon multiplicity.

Exact Strangeness Conservation.

In the small volume limit this becomes

$$\lim_{z\to 0} I_0(z) = 1$$

and

$$\lim_{z\to 0} I_1(z) = \frac{z}{2}$$

$$\begin{aligned}
\lim_{V\to 0} &= N^0_{K^+} Z_{-1} \\
\lim &= N^0_{K^+} Z_{-1} \\
&= N^0_{K^+} \left[N^0_{K^-} + N^0_{\Lambda} + \cdots \right]
\end{aligned}$$

i.e., the particle multiplicity is

- proportional to V^2, and not V^1.
- proportional to $\exp(-2m_K/T)$ or to $\exp(-(m_K + m_\Lambda)/T)$ and not simply $\exp(-m_K/T)$, i.e. there is additional suppression of strange particles.

Exact Strangeness Conservation.

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Exact Strangeness Conservation

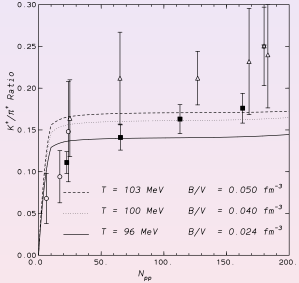

Figure 1

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Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion

After integration over m_T

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

where N_i^0 is the particle yield
as calculated in a fireball **AT REST!**

Effects of hydrodynamic flow cancel out in ratios.

	Chemical Equilibrium	No Chem. Equil.
π	$\exp\left[-\frac{E_\pi}{T}\right]$	$\exp\left[-\frac{E_\pi}{T}+\frac{\mu_\pi}{T}\right]$
N	$\exp\left[-\frac{E_N}{T}+\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_N}{T}\right]$
$\overline{N}$	$\exp\left[-\frac{E_N}{T}-\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_{\overline{N}}}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_\Lambda}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_{\overline{\Lambda}}}{T}\right]$
K	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_K}{T}\right]$
$\overline{K}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_{\overline{K}}}{T}\right]$

The number of particles of type i is determined by:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

For bosons:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) - 1}$$

For fermions:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T} - \frac{\mu_i}{T}\right) + 1}$$