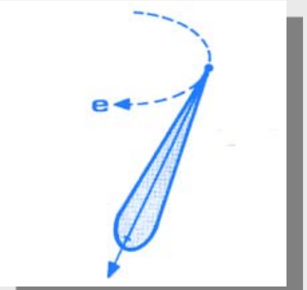


SYNCHROTRON RADIATION

Lenny Rivkin

Ecole Polythechnique Federale de Lausanne (EPFL)

and Paul Scherrer Institute (PSI), Switzerland

African School on Fundamental Physics and its Applications

August 2010, NITheP at Stellenbosch, South Africa

Useful books and references

A. Hofmann, *The Physics of Synchrotron Radiation*
Cambridge University Press 2004

H. Wiedemann, *Synchrotron Radiation*
Springer-Verlag Berlin Heidelberg 2003

H. Wiedemann, *Particle Accelerator Physics I and II*
Springer Study Edition, 2003

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

CERN Accelerator School Proceedings

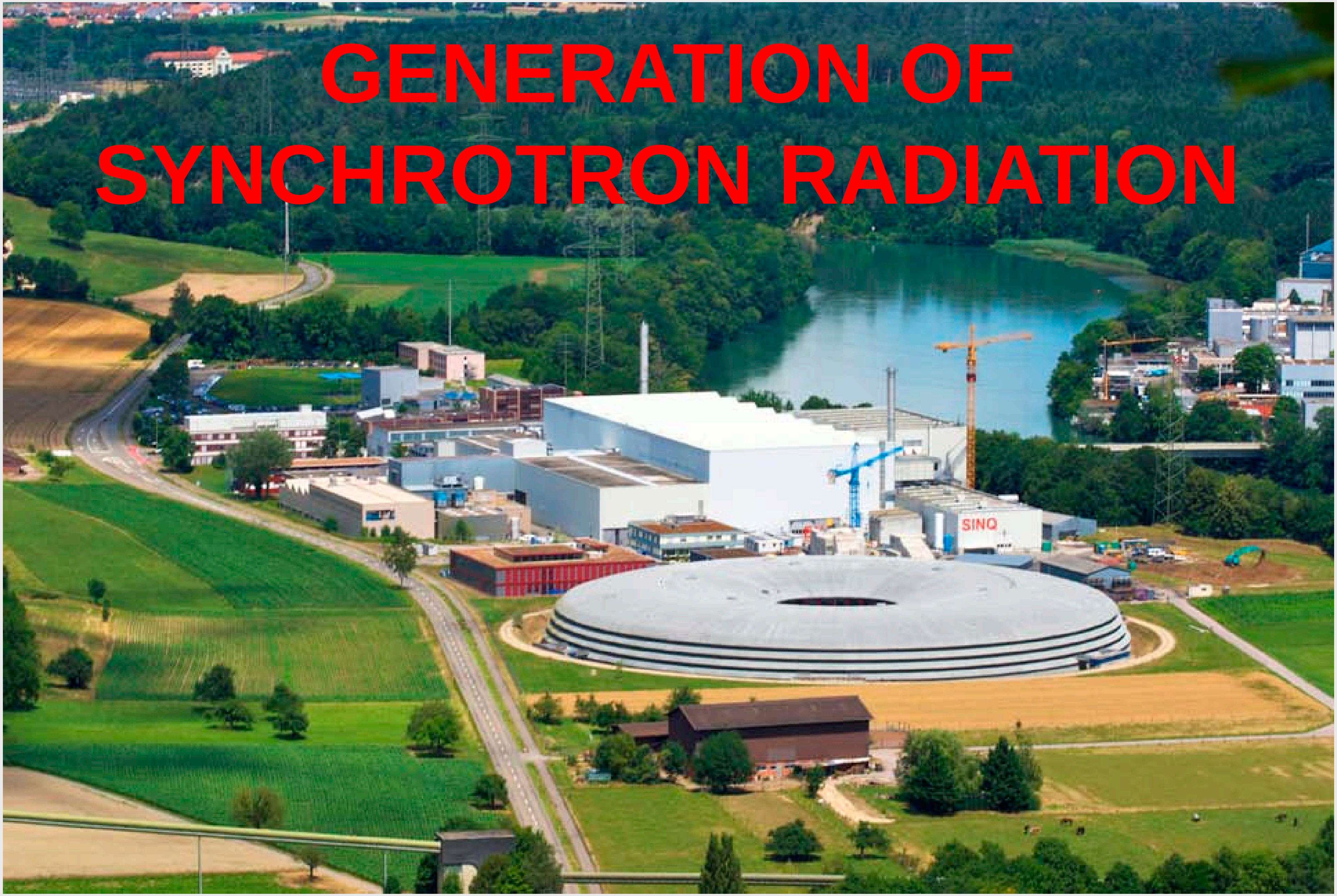
Synchrotron Radiation and Free Electron Lasers

Grenoble, France, 22 - 27 April 1996
(A. Hofmann's lectures on synchrotron radiation)
CERN Yellow Report 98-04

Brunnen, Switzerland, 2 – 9 July 2003
CERN Yellow Report 2005-012

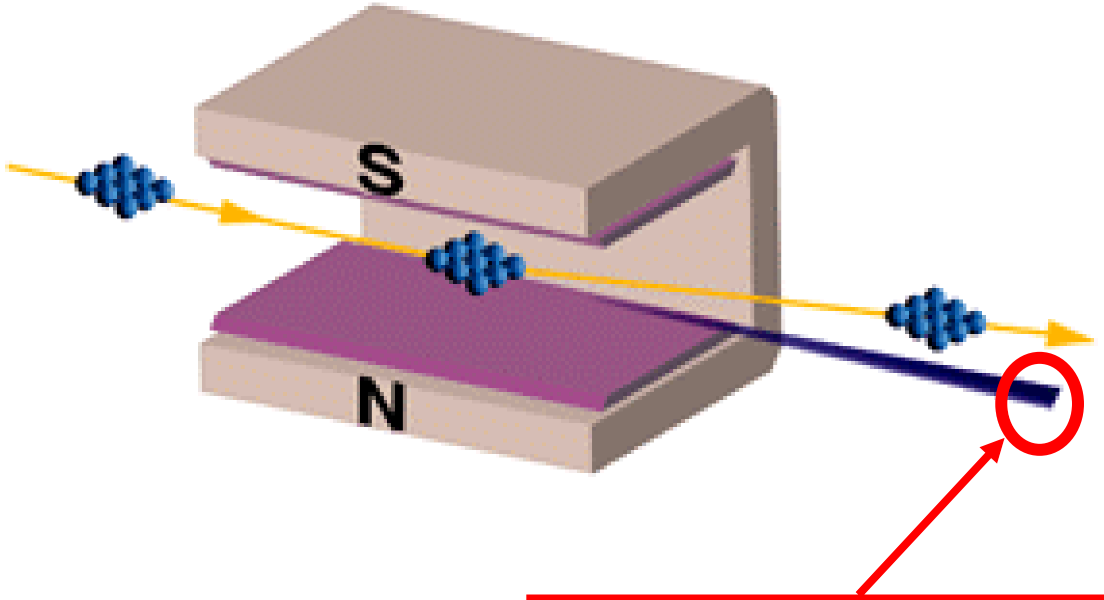
http://cas.web.cern.ch/cas/Proceedings.html

GENERATION OF SYNCHROTRON RADIATION



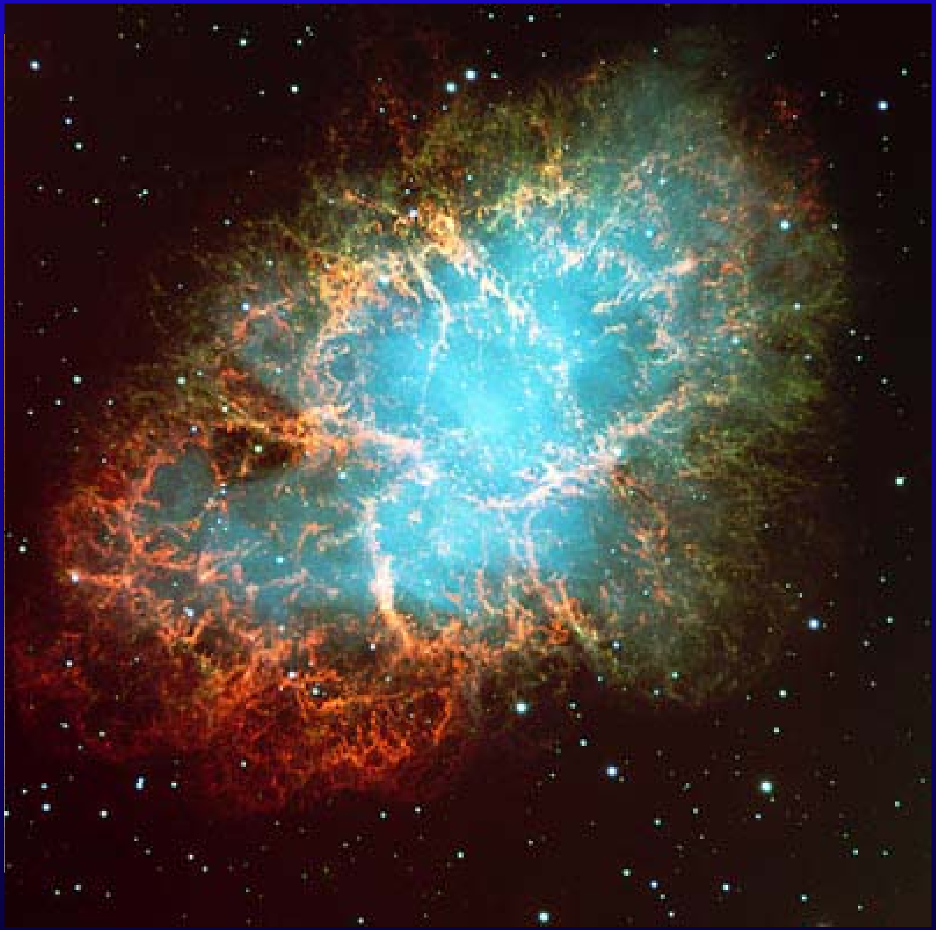
Swiss Light Source, Paul Scherrer Institute, Switzerland

Curved orbit of electrons in magnet field



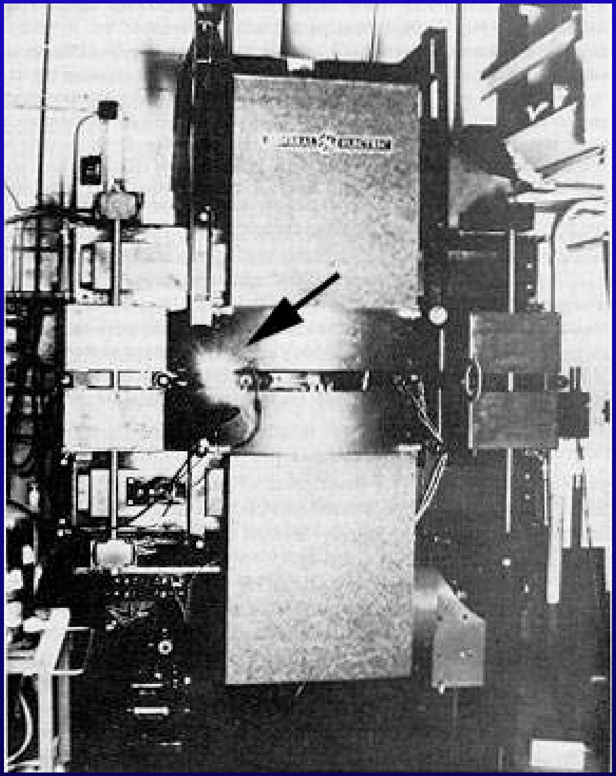
Accelerated charge → Electromagnetic radiation

Crab Nebula
6000 light years away



First light observed
1054 AD

GE Synchrotron
New York State



First light observed
1947

Synchrotron radiation: some dates

- 1873 Maxwell's equations
- 1887 Hertz: electromagnetic waves
- 1898 Liénard: retarded potentials
- 1900 Wiechert: retarded potentials
- 1908 Schott: Adams Prize Essay

... waiting for accelerators …

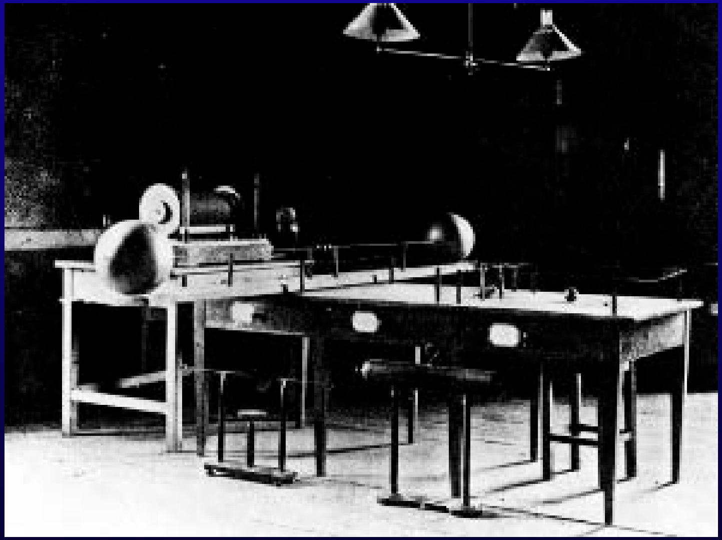
1940: 2.3 MeV betatron,Kerst, Serber

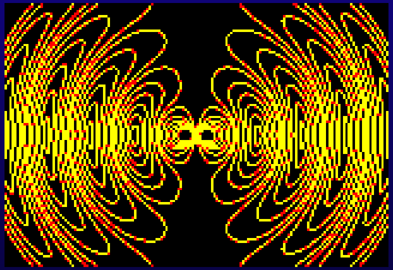
THEORETICAL UNDERSTANDING →

1873 Maxwell's equations

→ made evident that changing charge densities would result in electric fields that would radiate outward

1887 Heinrich Hertz demonstrated such waves:



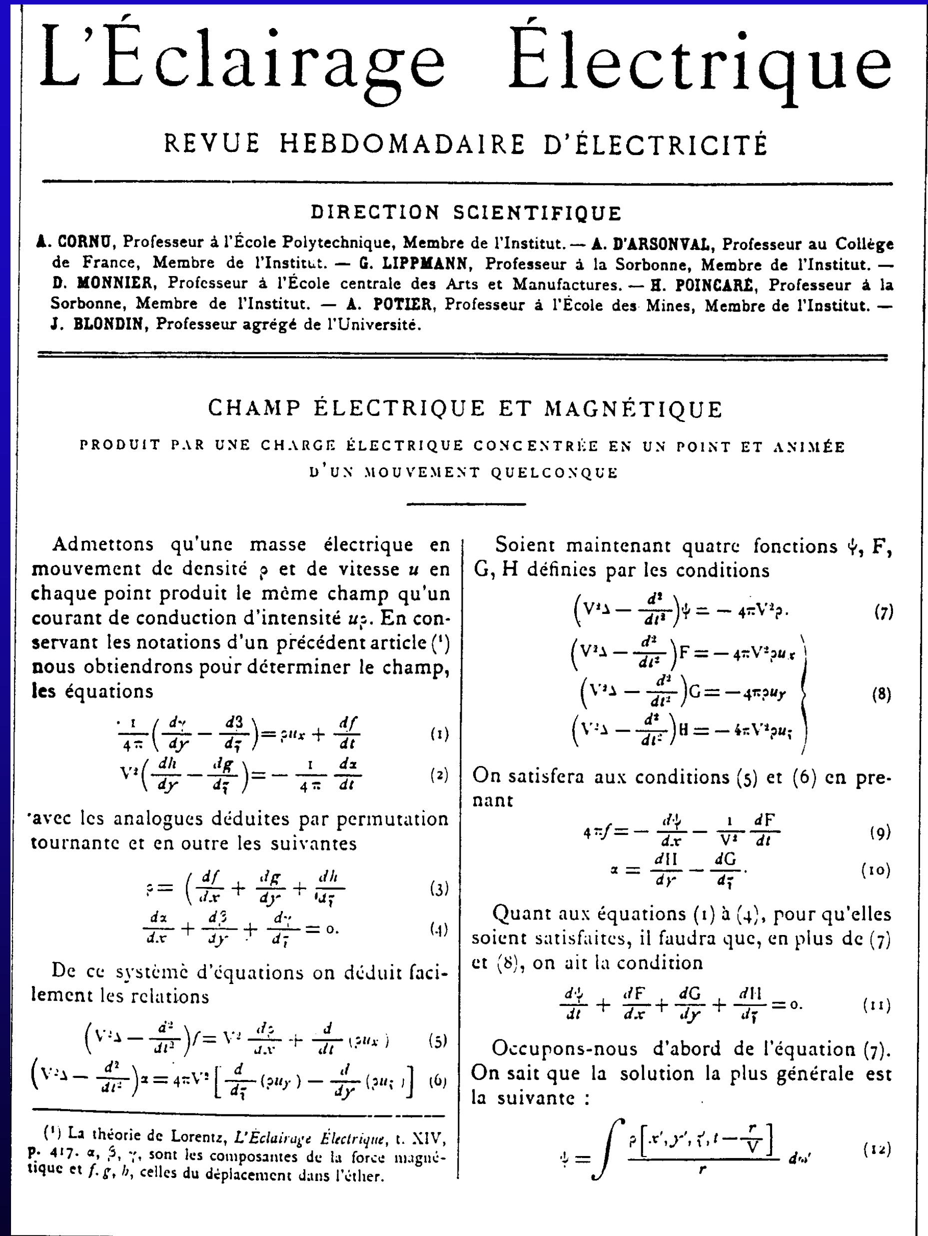


….. this is of no use whatsoever !

1898 Liénard:

ELECTRIC AND MAGNETIC FIELDS PRODUCED BY A POINT CHARGE MOVING ON AN ARBITRARY PATH

**(by means of retarded potentials …
proposed first by Ludwig Lorenz in 1867)**



L'Éclairage Électrique

REVUE HEBDOMADAIRE D'ÉLECTRICITÉ

DIRECTION SCIENTIFIQUE

A. CORNU, Professeur à l'École Polytechnique, Membre de l'Institut. — **A. D'ARSONVAL**, Professeur au Collège de France, Membre de l'Institut. — **G. LIPPMANN**, Professeur à la Sorbonne, Membre de l'Institut. — **D. MONNIER**, Professeur à l'École centrale des Arts et Manufactures. — **H. POINCARÉ**, Professeur à la Sorbonne, Membre de l'Institut. — **A. POTIER**, Professeur à l'École des Mines, Membre de l'Institut. — **J. BLONDIN**, Professeur agrégé de l'Université.

CHAMP ÉLECTRIQUE ET MAGNÉTIQUE

PRODUIT PAR UNE CHARGE ÉLECTRIQUE CONCENTRÉE EN UN POINT ET ANIMÉE D'UN MOUVEMENT QUELCONQUE

Admettons qu'une masse électrique en mouvement de densité ρ et de vitesse u en chaque point produit le même champ qu'un courant de conduction d'intensité $u\rho$. En conservant les notations d'un précédent article (¹) nous obtiendrons pour déterminer le champ, les équations

$$\frac{1}{4\pi}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)=\rho u_x+\frac{df}{dt} \quad (1)$$

$$V^2\left(\frac{dh}{dy}-\frac{dg}{dz}\right)=-\frac{1}{4\pi}\frac{d\alpha}{dt} \quad (2)$$

avec les analogues déduites par permutation tournante et en outre les suivantes

$$\rho=\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right. \quad (3)$$

$$\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}=0. \quad (4)$$

De ce système d'équations on déduit facilement les relations

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)f=V^2\frac{d\rho}{dx}+\frac{d}{dt}(\rho u_x) \quad (5)$$

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)\alpha=4\pi V^2\left[\frac{d}{dz}(\rho u_y)-\frac{d}{dy}(\rho u_z)\right] \quad (6)$$

(¹) La théorie de Lorentz, *L'Éclairage Électrique*, t. XIV, p. 417. α, β, γ, sont les composantes de la force magnétique et f, g, h, celles du déplacement dans l'éther.

Soient maintenant quatre fonctions ψ, F, G, H définies par les conditions

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)\psi=-4\pi V^2\rho. \quad (7)$$

$$\left.\begin{aligned}\left(V^2\Delta-\frac{d^2}{dt^2}\right)F&=-4\pi V^2\rho u_x\\ \left(V^2\Delta-\frac{d^2}{dt^2}\right)G&=-4\pi\rho u_y\\ \left(V^2\Delta-\frac{d^2}{dt^2}\right)H&=-4\pi V^2\rho u_z\end{aligned}\right\} \quad (8)$$

On satisfera aux conditions (5) et (6) en prenant

$$4\pi f=-\frac{d\psi}{dx}-\frac{1}{V^2}\frac{dF}{dt} \quad (9)$$

$$\alpha=\frac{dH}{dy}-\frac{dG}{dz}. \quad (10)$$

Quant aux équations (1) à (4), pour qu'elles soient satisfaites, il faudra que, en plus de (7) et (8), on ait la condition

$$\frac{d\psi}{dt}+\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz}=0. \quad (11)$$

Occupons-nous d'abord de l'équation (7). On sait que la solution la plus générale est la suivante :

$$\psi=\int\frac{\rho\left[x',y',z',t-\frac{r}{V}\right]}{r}\,d\omega' \quad (12)$$

Fig. 1. First page of Liénard's 1898 paper.

1912 Schott:

COMPLETE THEORY OF SYNCHROTRON RADIATION IN ALL THE GORY DETAILS (327 pages long)

… to be forgotten for 30 years

(on the usefulness of prizes)

ELECTROMAGNETIC RADIATION

AND THE MECHANICAL REACTIONS ARISING FROM IT

BEING AN ADAMS PRIZE ESSAY IN THE UNIVERSITY OF CAMBRIDGE

by

G. A. SCHOTT, B.A., D.Sc.

Professor of Applied Mathematics in the University College of Wales, Aberystwyth
Formerly Scholar of Trinity College, Cambridge

Cambridge:
at the University Press
1912

Donald Kerst: first betatron (1940)

"*Ausserordentlichhochgeschwindigkeitelektronen entwickelndenschwerarbeitsbeigollitron*"

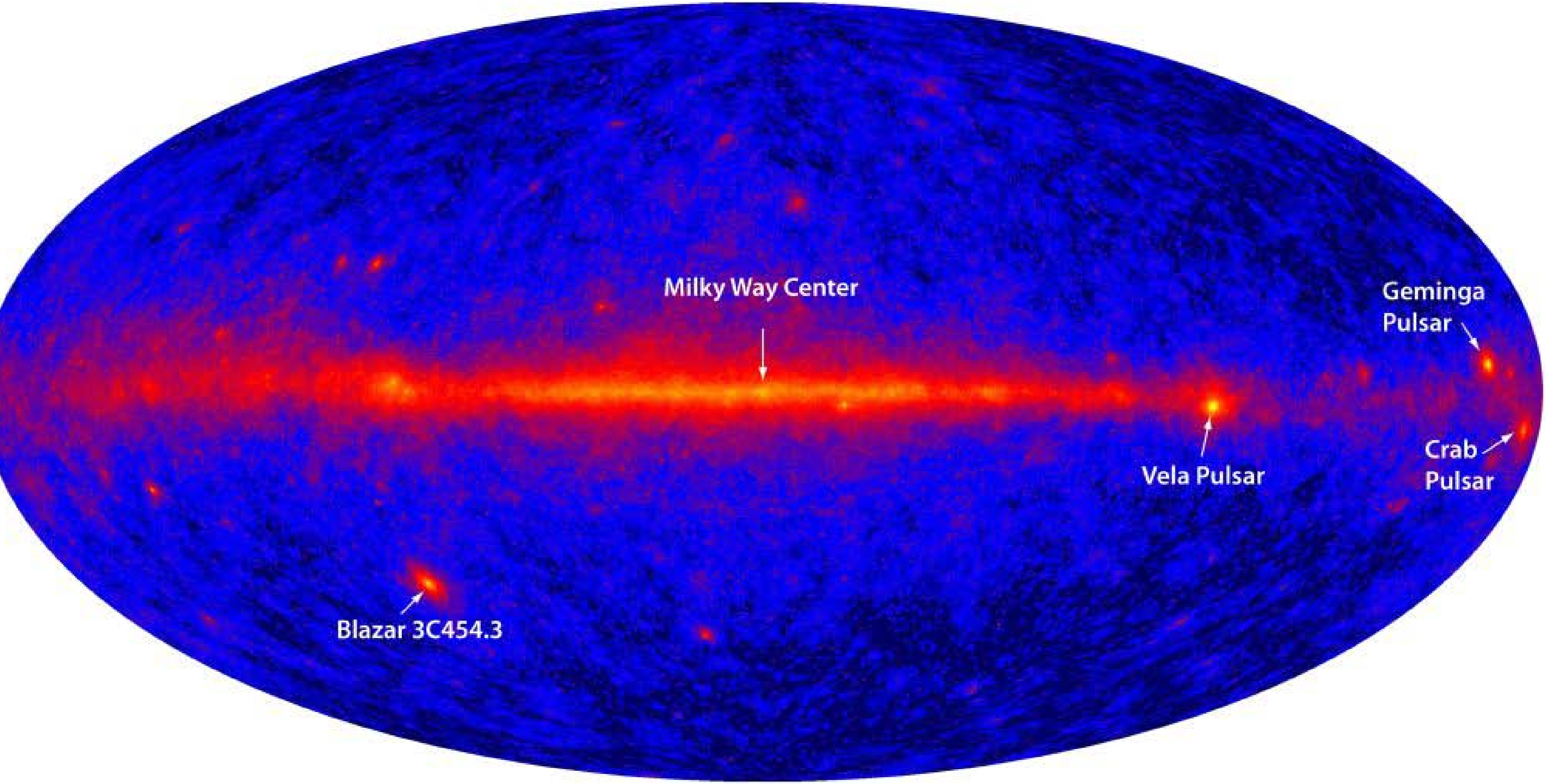
Synchrotron radiation: some dates

- 1946 Blewett observes energy loss due to synchrotron radiation
  100 MeV betatron
- 1947 First visual observation of SR
  70 MeV synchrotron, GE Lab
- 1949 Schwinger PhysRev paper

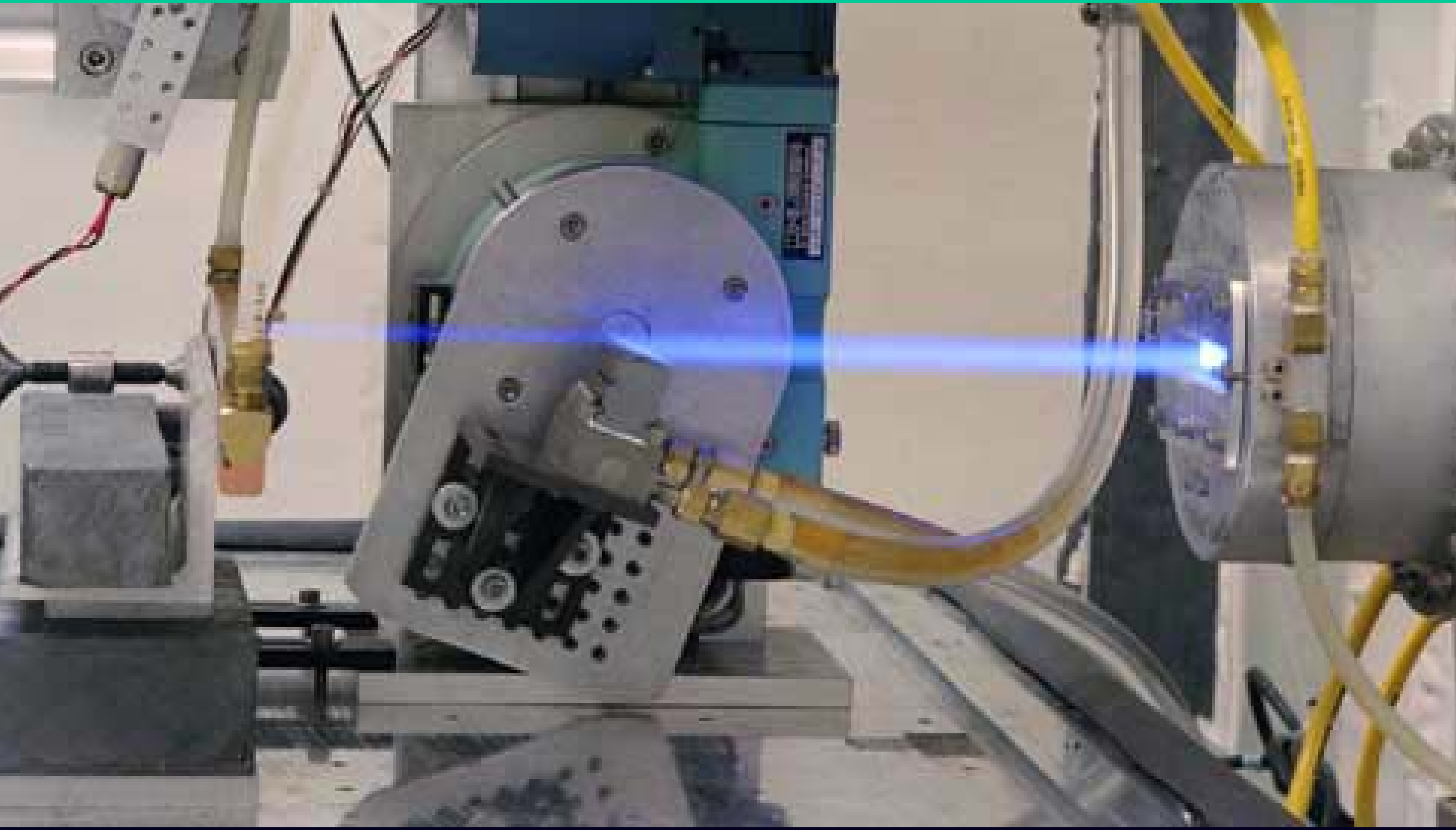
…

- 1976 Madey: first demonstration of Free Electron laser

A larger view

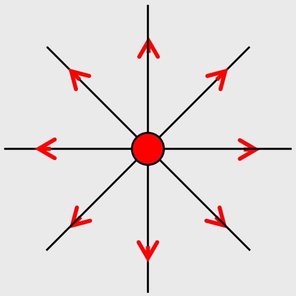


Storage ring based synchrotron light source

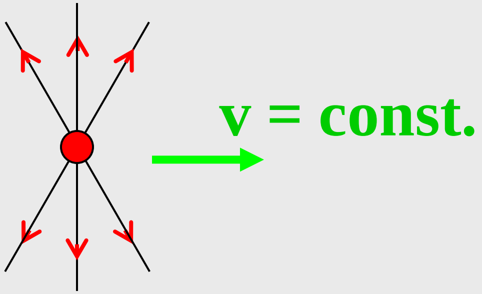


Why do they radiate?

Charge at rest: Coulomb field, no radiation

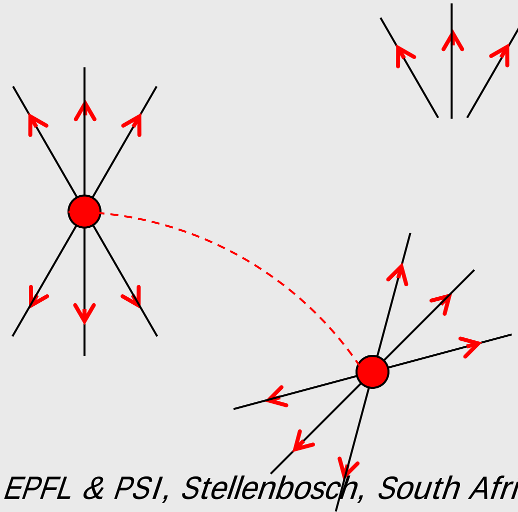


Uniformly moving charge
does not radiate



But! Cerenkov!

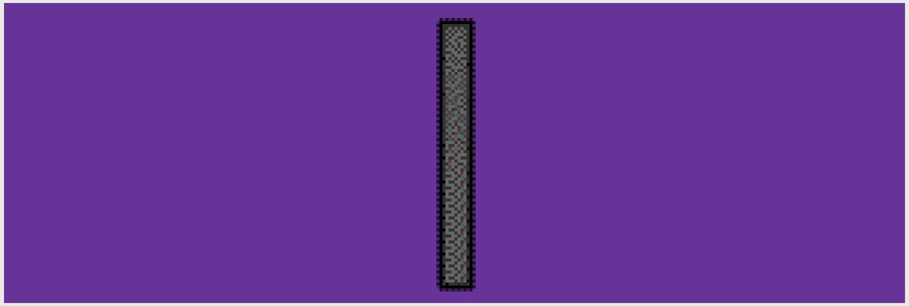
Accelerated charge



Bremsstrahlung

or

breaking radiation



Liénard-Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left[r\left(1 - \vec{\mathbf{n}} \cdot \vec{\beta}\right)\right]_{ret}}$$

$$\vec{\mathbf{A}}(t) = \frac{q}{4\pi\varepsilon_0 c^2} \left[\frac{\vec{\mathbf{v}}}{r\left(1 - \vec{\mathbf{n}} \cdot \vec{\beta}\right)} \right]_{ret}$$

and the electromagnetic fields:

$$\nabla \cdot \vec{\mathbf{A}} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

(Lorentz gauge)

$$\vec{\mathbf{B}} = \nabla \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\nabla \varphi - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

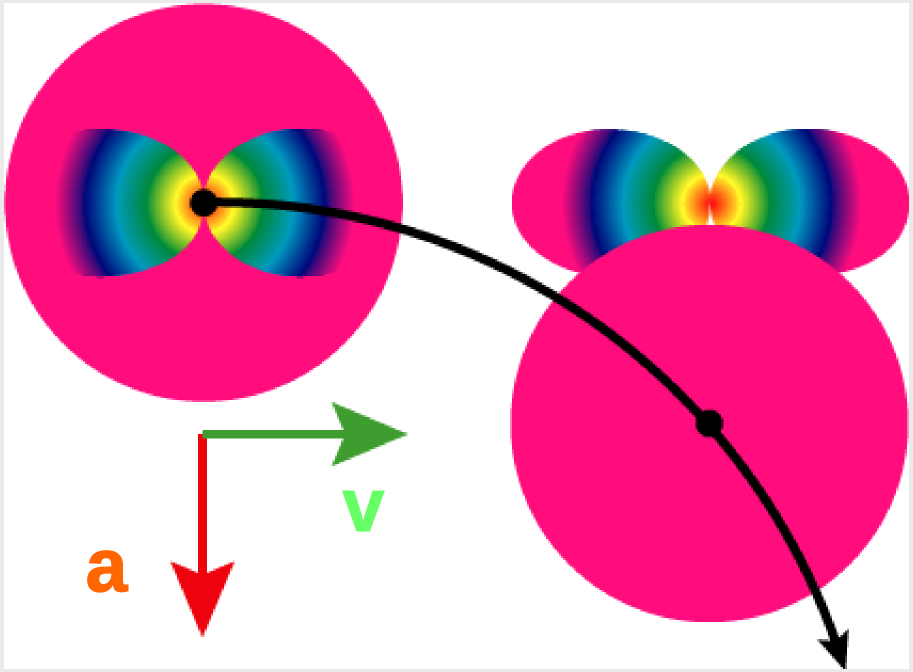
Fields of a moving charge

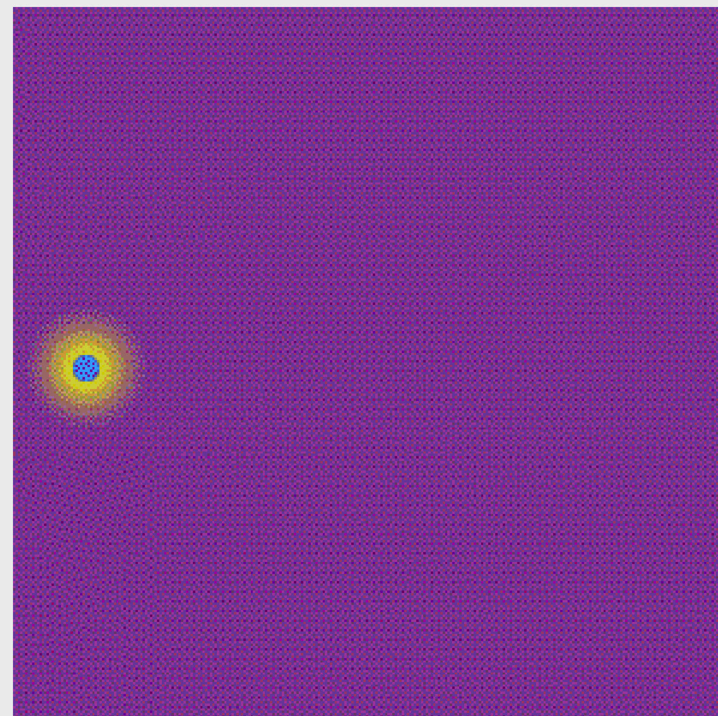
$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}^2}}\right]_{ret} +$$

$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}} \times \left[\left(\vec{\mathbf{n}} - \vec{\boldsymbol{\beta}}\right) \times \boxed{\dot{\vec{\boldsymbol{\beta}}}}\right]}{\left(1 - \vec{\mathbf{n}} \cdot \vec{\boldsymbol{\beta}}\right)^3 \gamma^2} \cdot \boxed{\frac{\mathbf{1}}{\mathbf{r}}}\right]_{ret}$$

$$\vec{\mathbf{B}}(\mathrm{t}) = \frac{1}{\mathrm{c}}\left[\vec{\mathbf{n}} \times \vec{\mathbf{E}}\right]$$

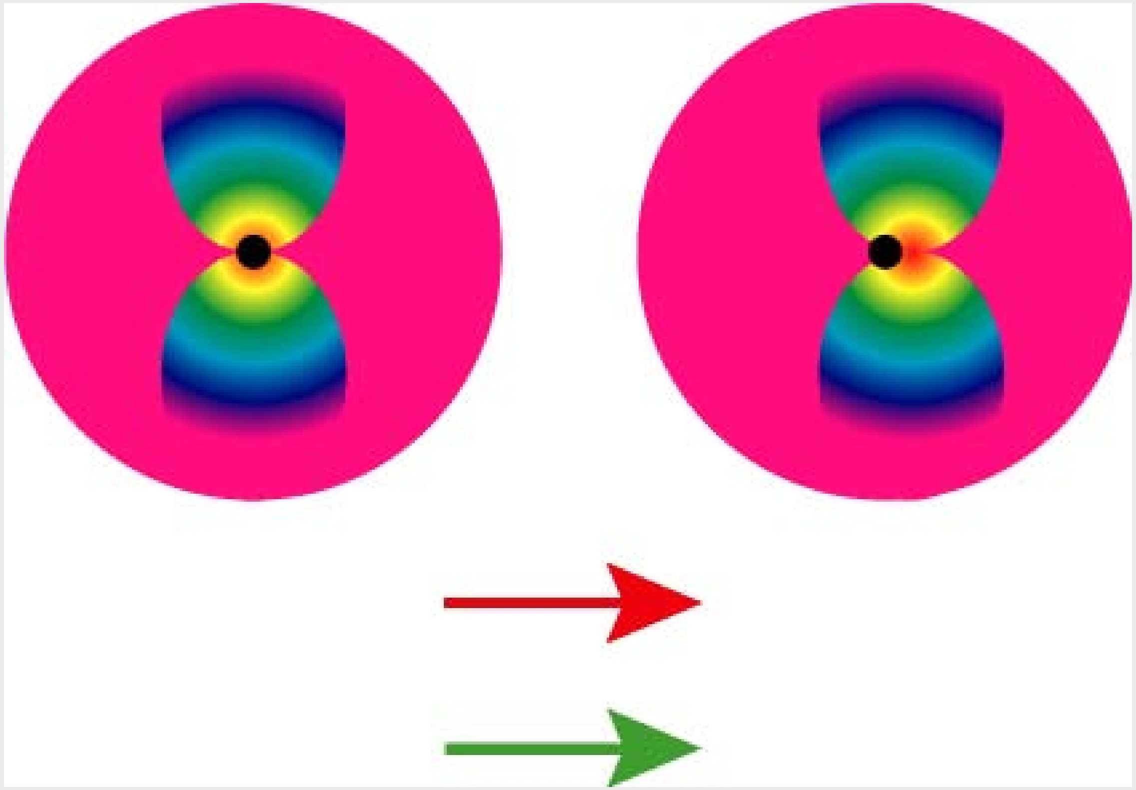
Transverse acceleration





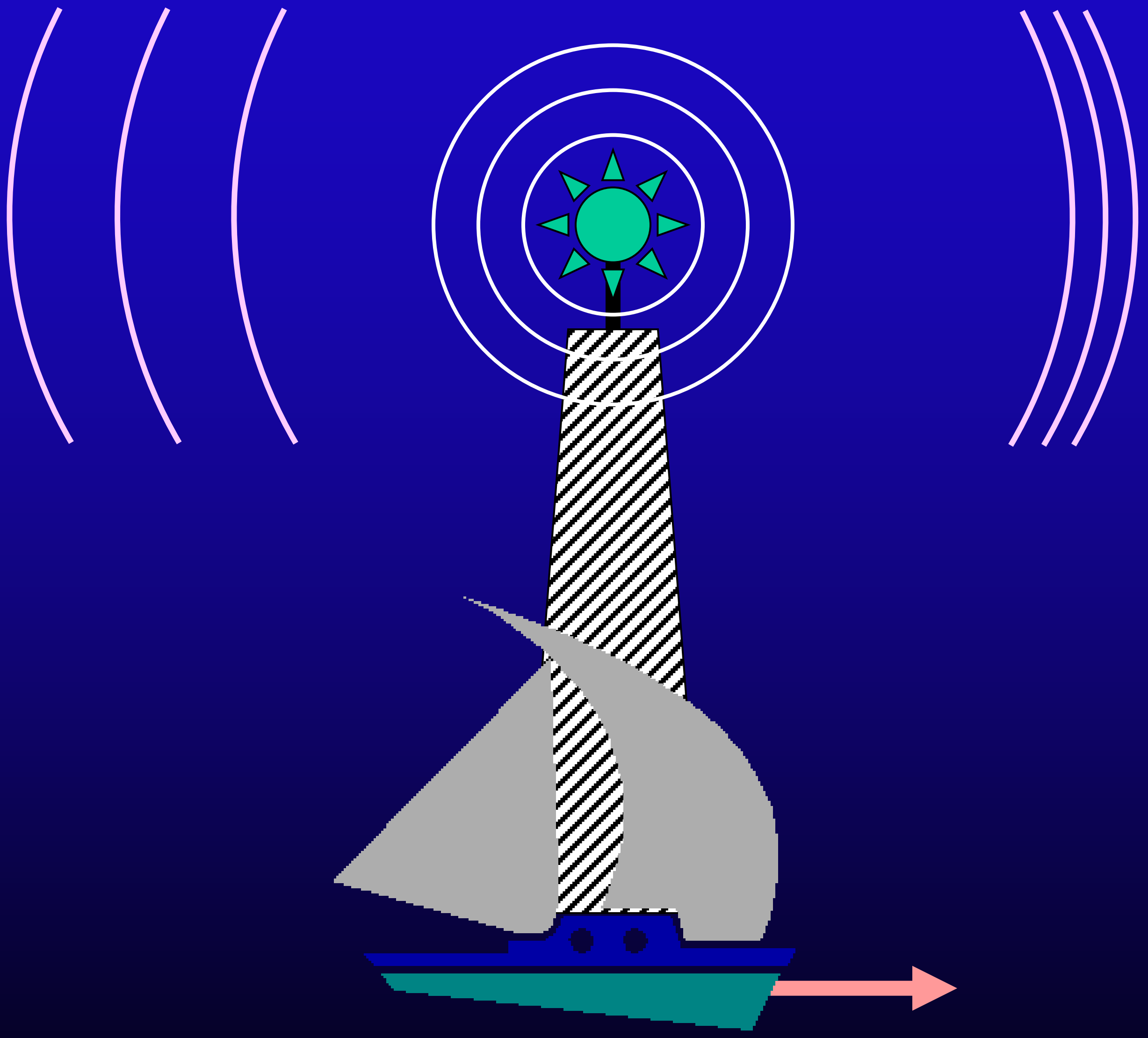
Radiation field quickly separates itself from the Coulomb field

Longitudinal acceleration



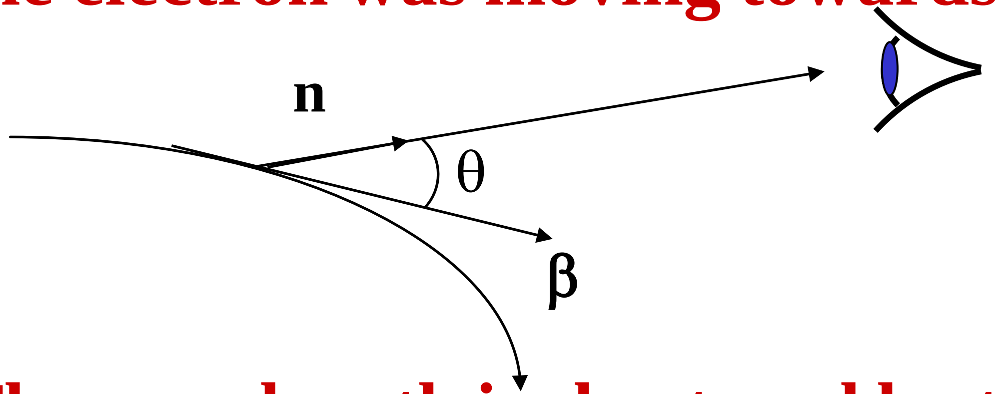
Radiation field cannot separate itself from the Coulomb field

Moving Source of Waves



Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta})\, T_{emit}$$

The wavelength is shortened by the same factor

$$\lambda_{obs} = (1 - \beta \cos\theta)\, \lambda_{emit}$$

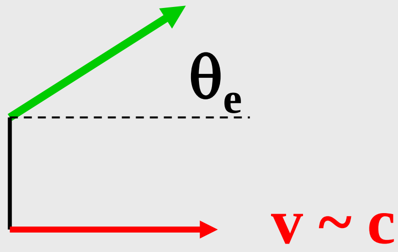
in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}$$

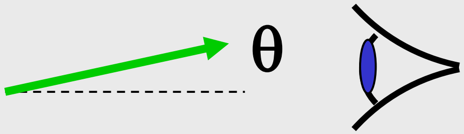
since

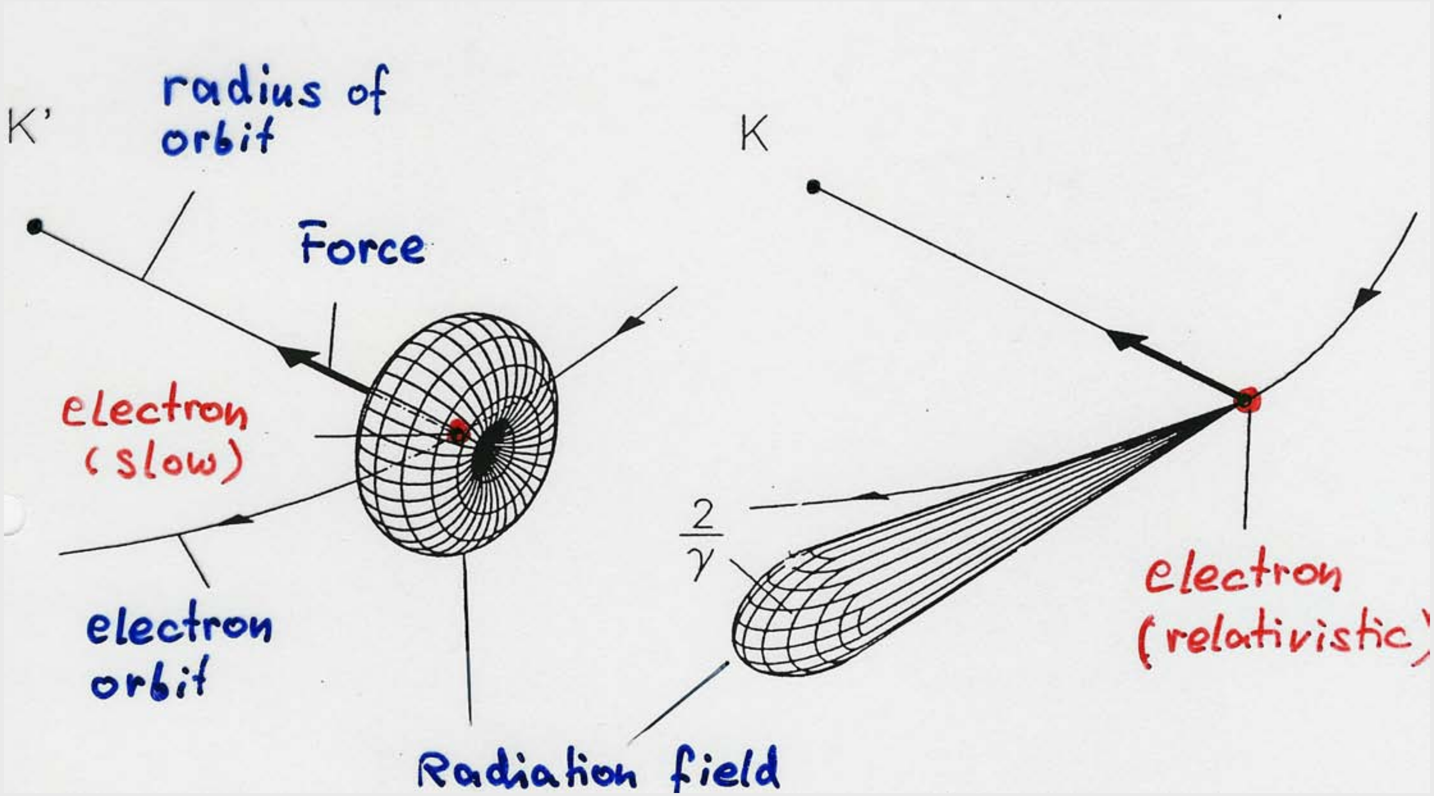
$$1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}$$

Radiation is emitted into a narrow cone



$$\theta = \frac{1}{\gamma} \cdot \theta_e$$



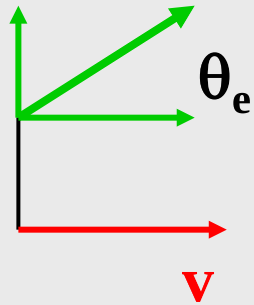


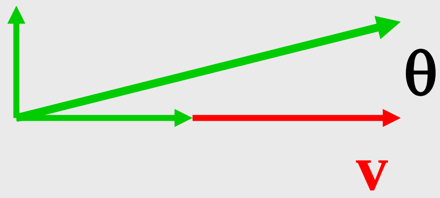
v << c

v ≈ c

Sound waves (non-relativistic)

Angular collimation





$$\theta = \frac{v_{s\perp}}{v_{s\parallel} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\parallel}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Doppler effect (moving source of sound)

$$\lambda_{heard} = \lambda_{emitted}\left(1 - \frac{\mathbf{v}}{v_s}\right)$$

Synchrotron radiation power

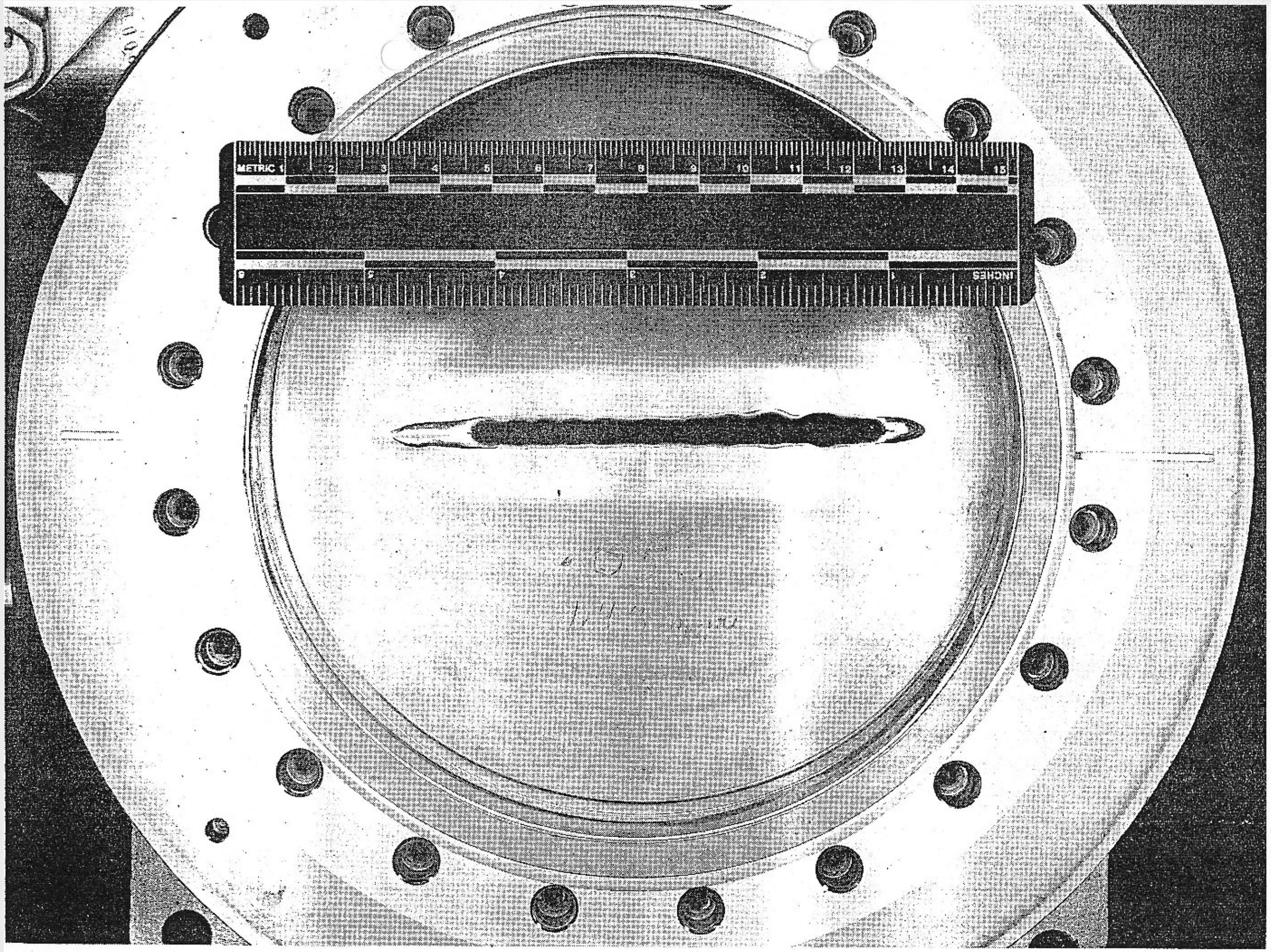
Power emitted is proportional to:

$$P \propto E^2 B^2$$

$$P_\gamma = \frac{c C_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

The power is all too real!



ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Synchrotron radiation power

Power emitted is proportional to:

$$\boldsymbol{P \propto E^2 B^2}$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3} \alpha \hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\text{m}}{\text{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197 \text{ Mev} \cdot \text{fm}$$

Energy loss per turn:

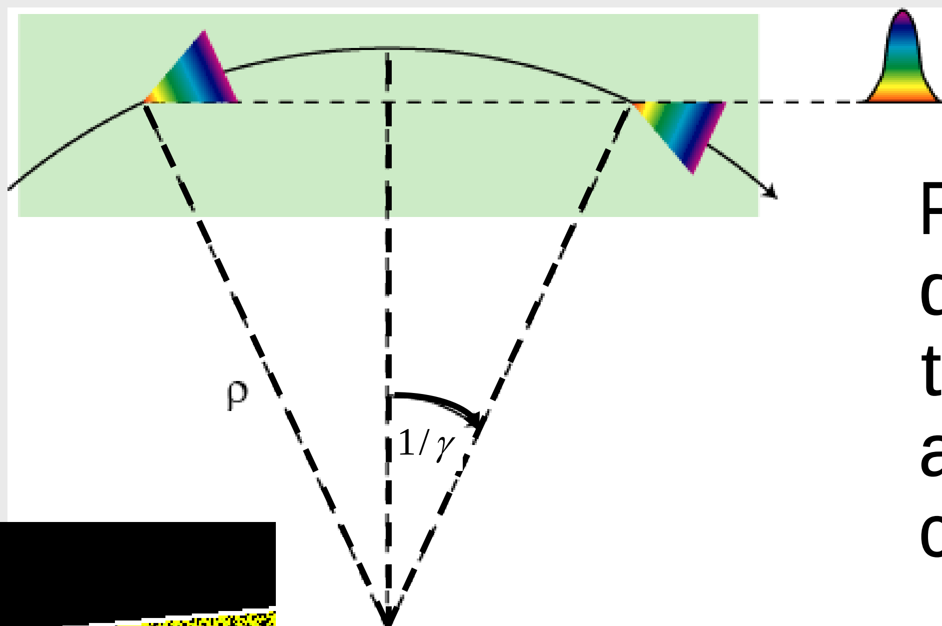
$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho}$$

Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

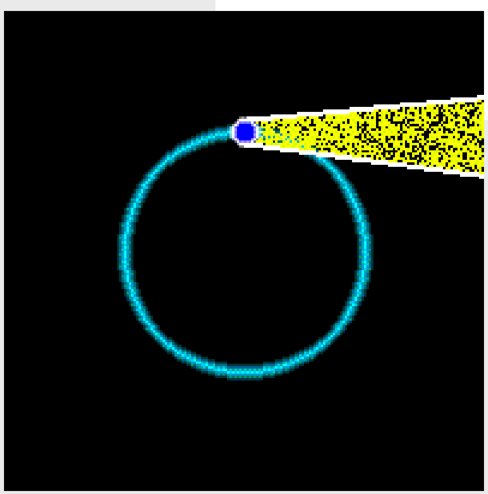
$$l \sim \frac{2\rho}{\gamma}$$



Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

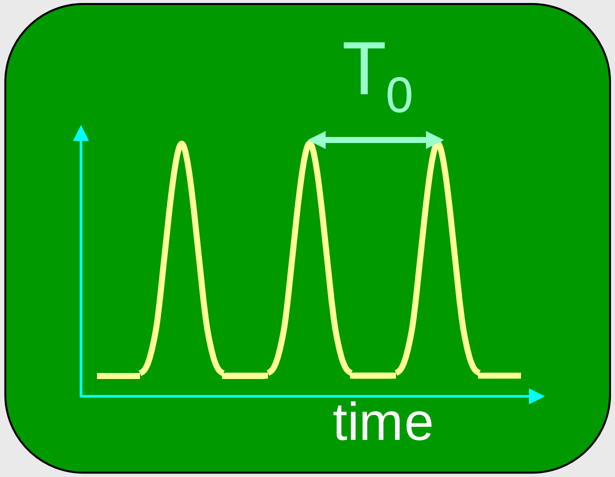
$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$



$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every T_0 (revolution period)



- the spectrum consists of harmonics of

$$\omega_0 = \frac{1}{T_0}$$

- flashes are extremely short: harmonics reach up to very high frequencies

$$\omega_{typ} \cong \gamma^3 \omega_0$$

$$\omega_0 \sim 1 \text{ MHz}$$
$$\gamma \sim 4000$$
$$\omega_{\text{typ}} \sim 10^{16} \text{ Hz} \,!$$

- At high frequencies the individual harmonics overlap

continuous spectrum !

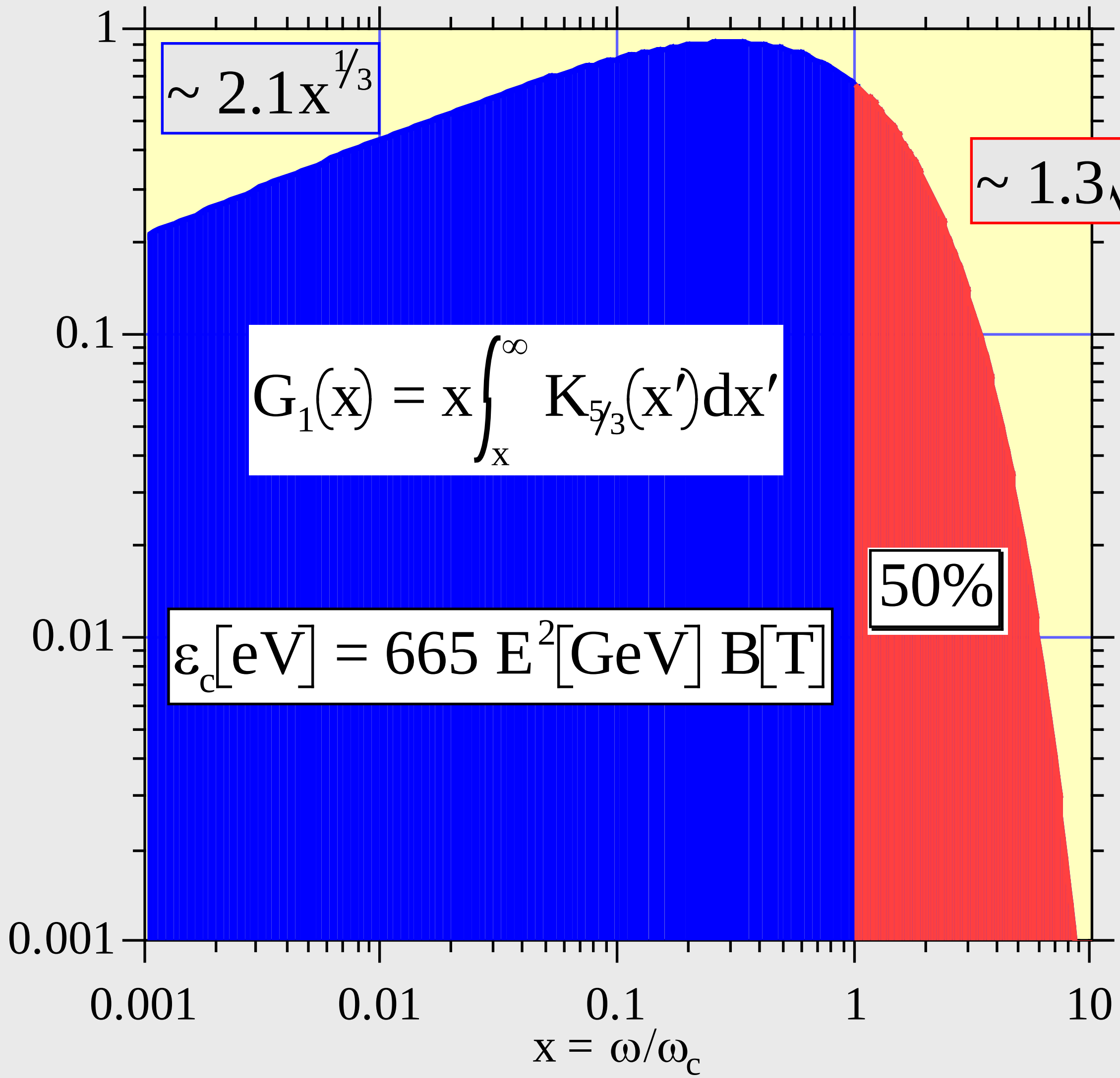
$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')dx'$$

$$\int_0^\infty S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$



A useful approximation

Spectral flux from a dipole magnet with field B

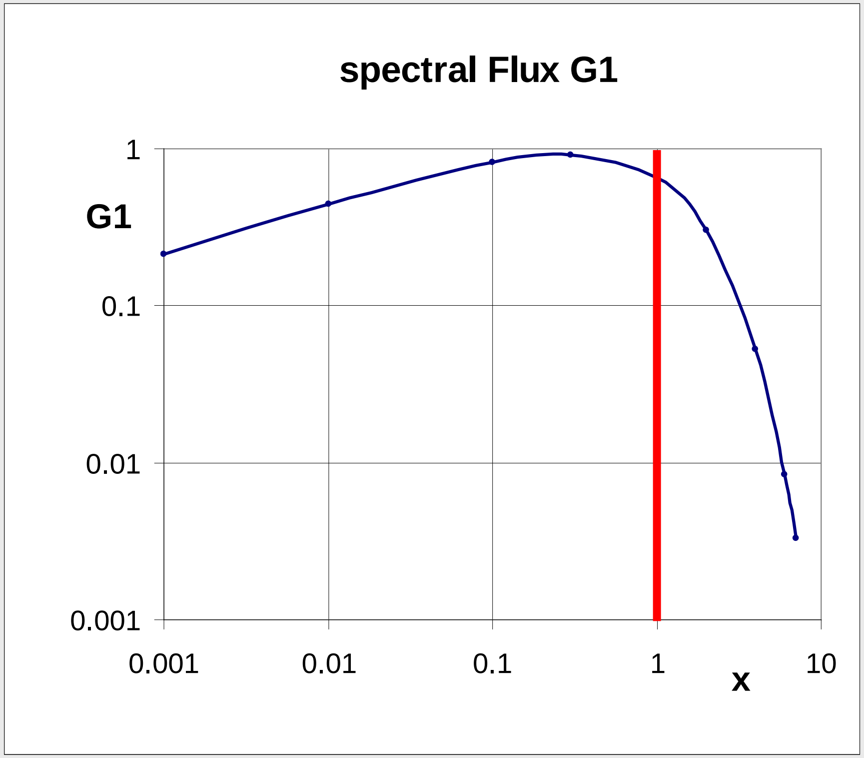
$$\text{Flux}\left[\frac{\text{photons}}{\text{s}\cdot\text{mrad}\cdot 0.1\%\text{BW}}\right] = 2.46\cdot 10^{13}\,E[\text{GeV}]\ I[\text{A}]\,G_1(x)$$

Approximation: $G_1 \approx A\, x^{1/3}\, g(x)$

$$g(x) = [(1-(\frac{x}{x_L})^N]^{\frac{1}{S}}$$

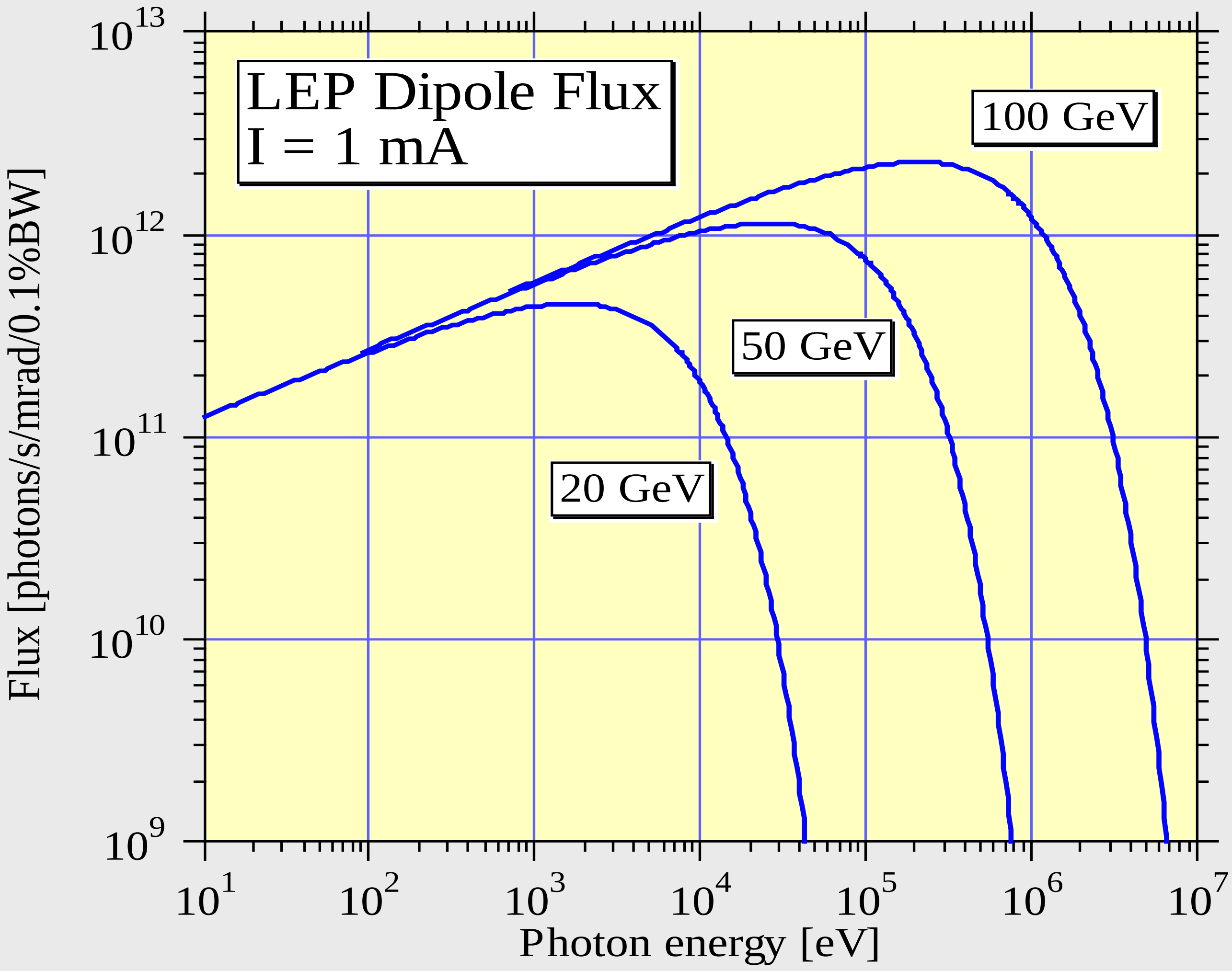
$A = 2.11$, $N = 0.848$

$x_L = 28.17$, $S = 0.0513$



Werner Joho, PSI

Synchrotron radiation flux for different electron energies



Angular divergence of radiation

The rms opening angle R'

- **at the critical frequency:**

$$\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$$

- **well below**

$$\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4\left(\frac{\lambda}{\rho}\right)^{1/3}$$

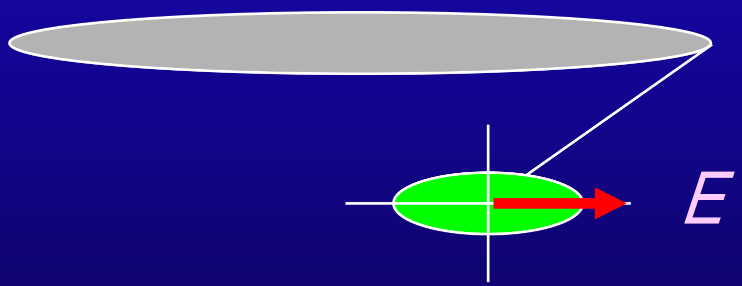
independent of γ !

- **well above**

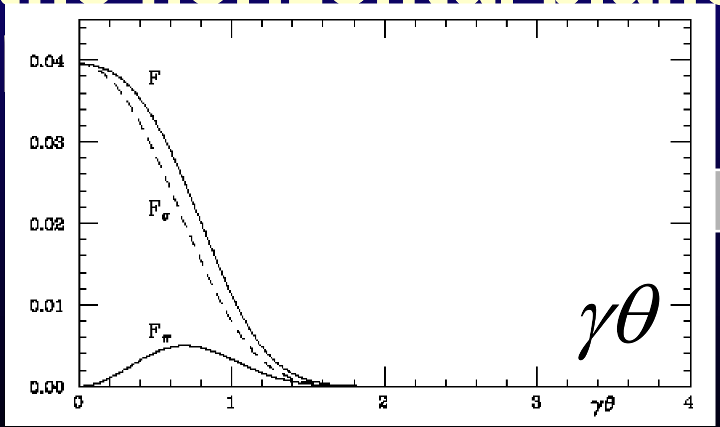
$$\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$$

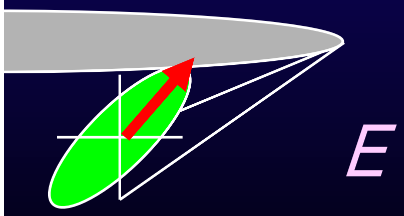
Polarisation

Synchrotron radiation observed in the plane of the particle orbit is horizontally polarized, i.e. the electric field vector is horizontal



Observed out of the horizontal plane, the radiation is elli



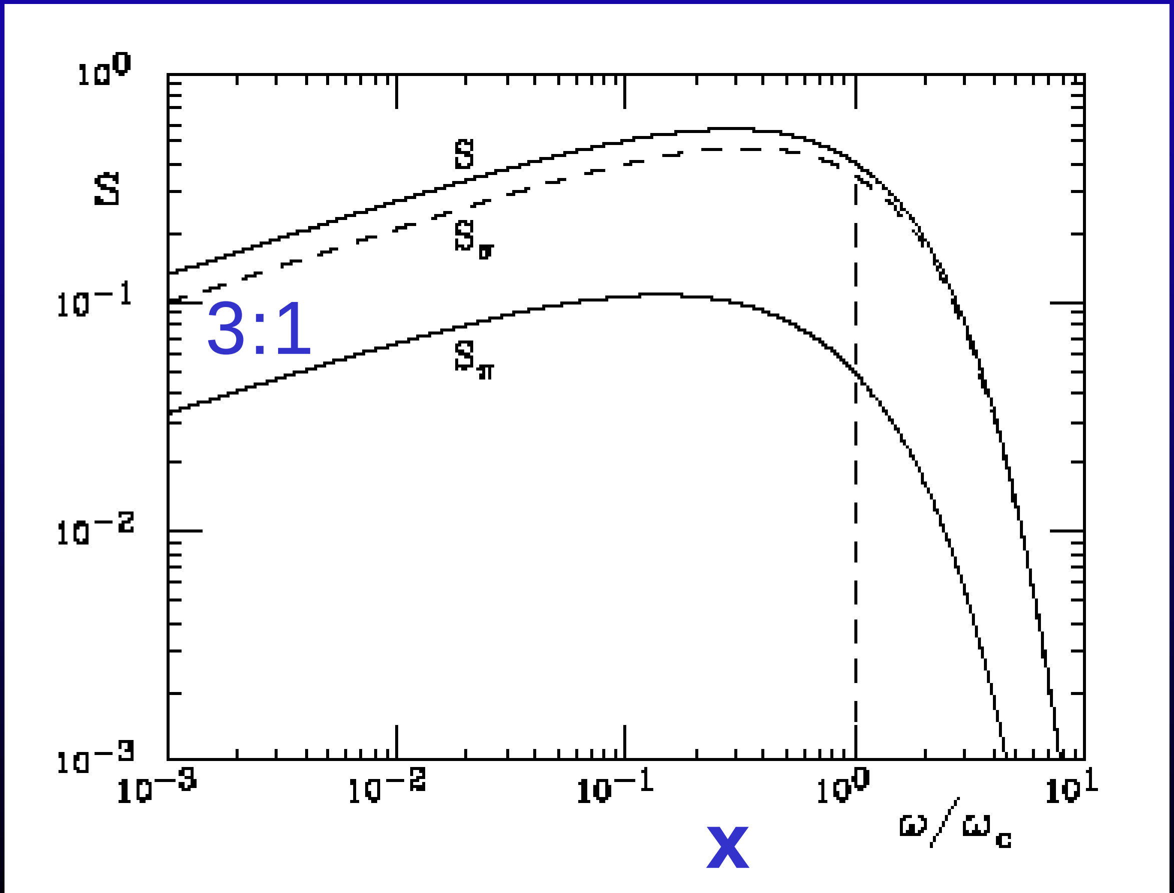


Polarisation: spectral distribution

$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S(x) = \frac{P_{tot}}{\omega_c} \left[S_\sigma(x) + S_\pi(x) \right]$$

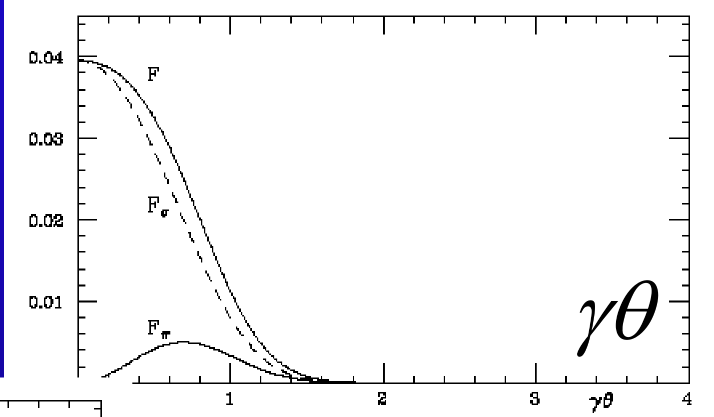
$$S_\sigma = \frac{7}{8} S$$

$$S_\pi = \frac{1}{8} S$$

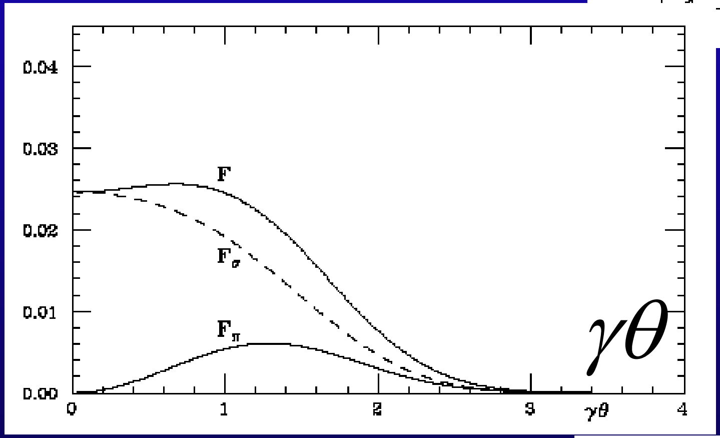


Angular divergence of radiation

- **at the critical frequency**



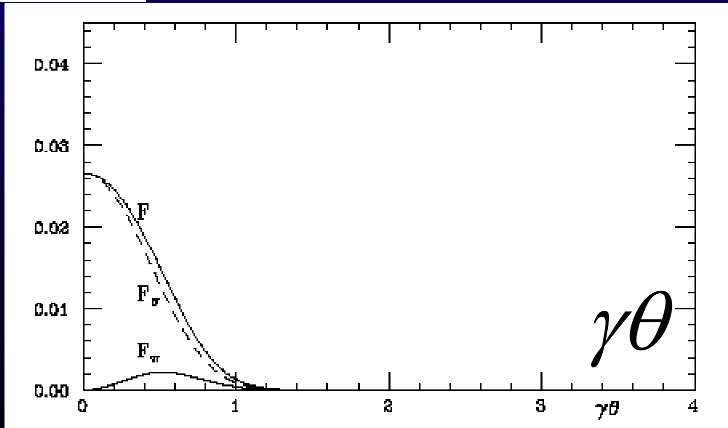
- **well below**



$$\omega = 0.2\,\omega_c$$

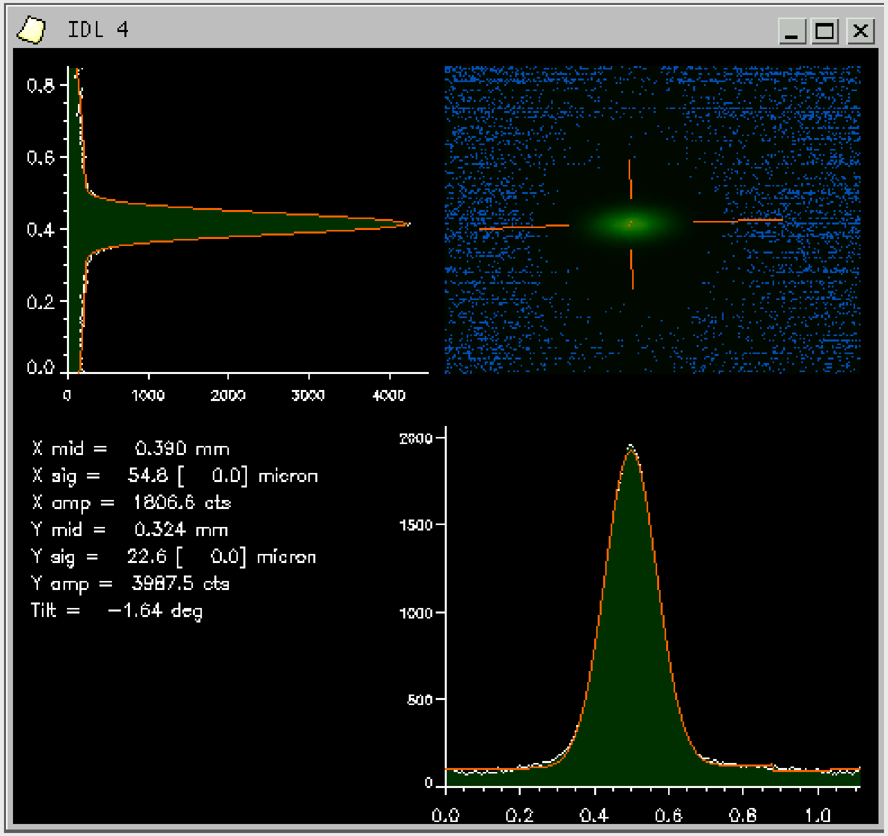
- **well above**

$$\omega = 2\,\omega_c$$



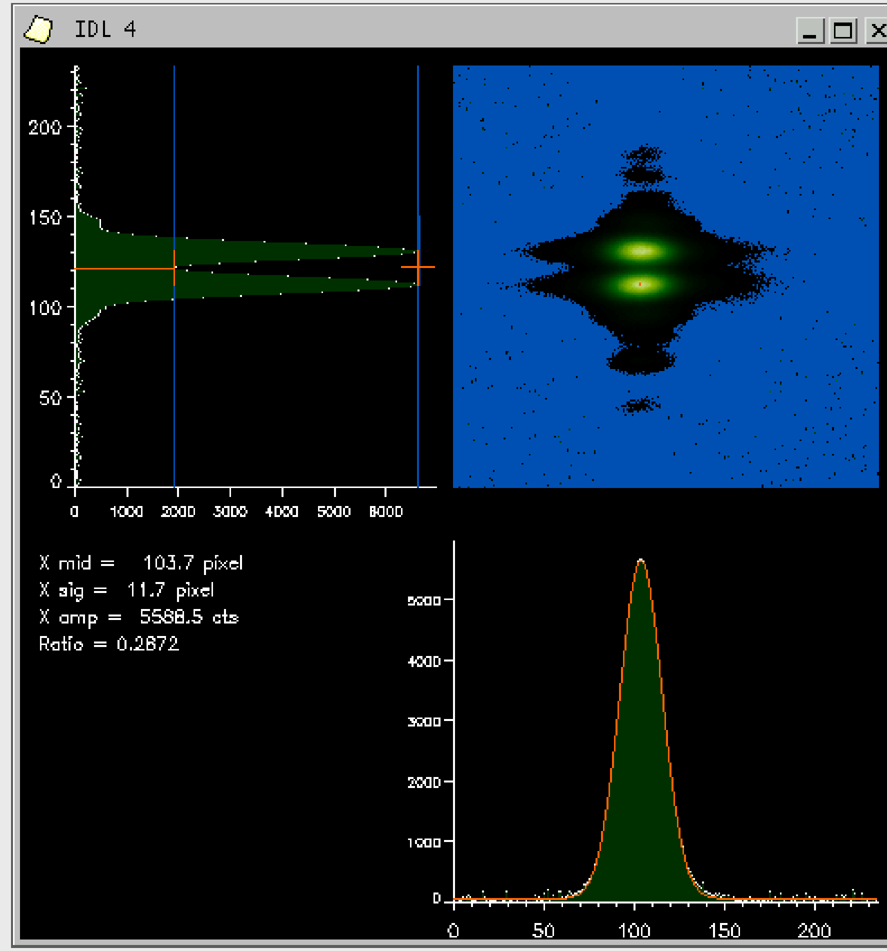
Seeing the electron beam (SLS)

X rays

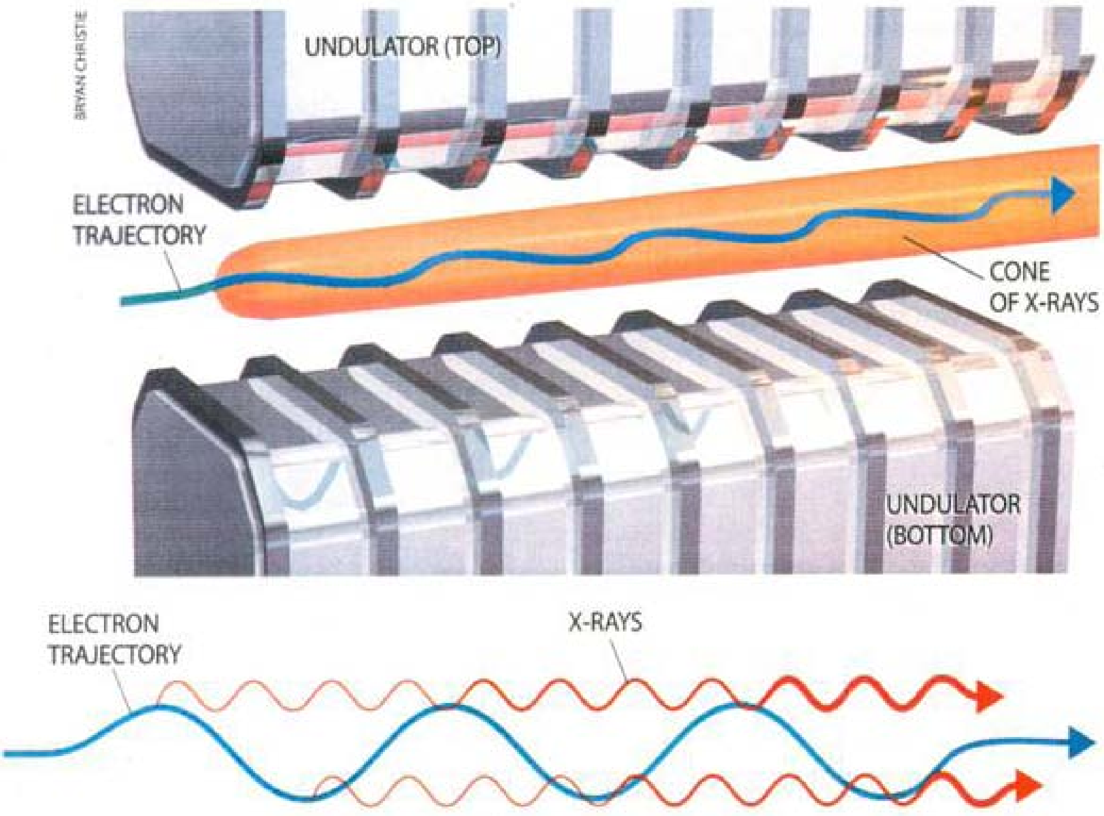


$$\sigma_x \sim 55\mu m$$

visible light, vertically polarised



Undulators

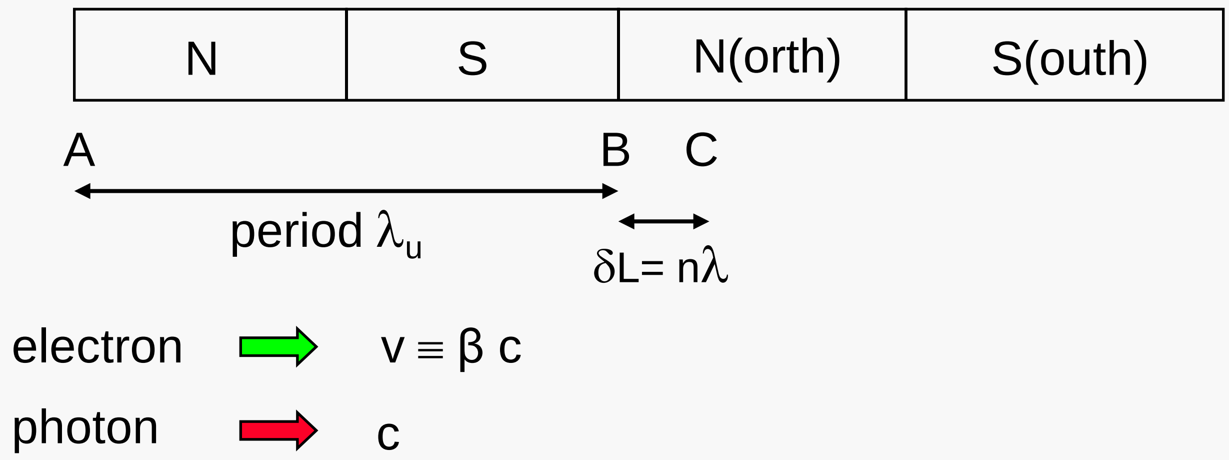


$$T_{obs} = T_{emit}(1 - \beta)$$

$$\lambda_{light} \approx \frac{\lambda_u}{2\gamma^2}$$

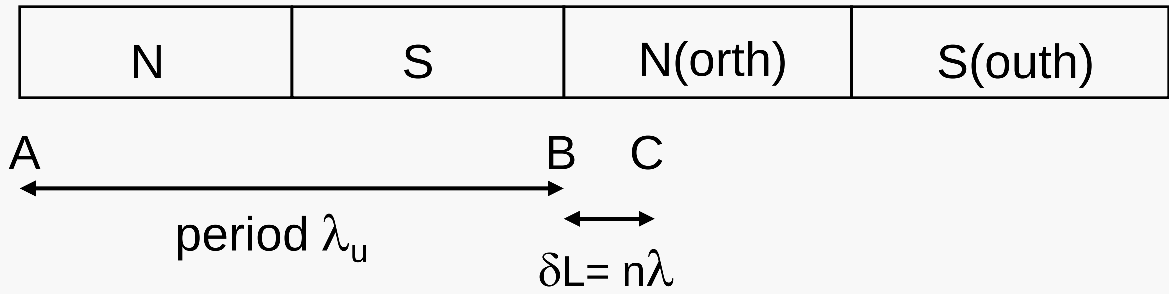
Selection of wavelength in an undulator

In an undulator
an electron
(on a slalom)
races an emitted
photon



at A an electron emits a photon with wavelength λ and flies one period λ_u ahead to B with velocity $v = \beta c$. There it emits another photon with the same wavelength λ. At this moment the first photon **is already at C. If the path difference δL corresponds to n** wavelengths, then we have a positive interference between the two photons. This enhances the intensity at this wavelength.

Selection of wavelength in an undulator II



electron → $v \equiv \beta\, c$

photon → c

The path difference $\delta L \equiv n\lambda \approx (1-\beta)\,\lambda_u\ , \quad 1-\beta \approx \dfrac{1}{2\gamma^2}$

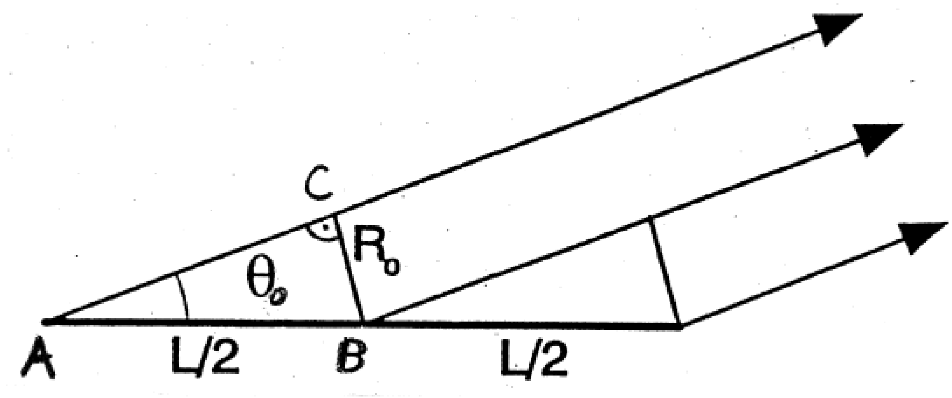
$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{K^2}{2}\right)$$

detour through slalom

$$K = 0.0934 \cdot \lambda_u[mm] \cdot B[T]$$

Radiation cone of an undulator

Undulator radiates from ist whole length L into a narrow cone.

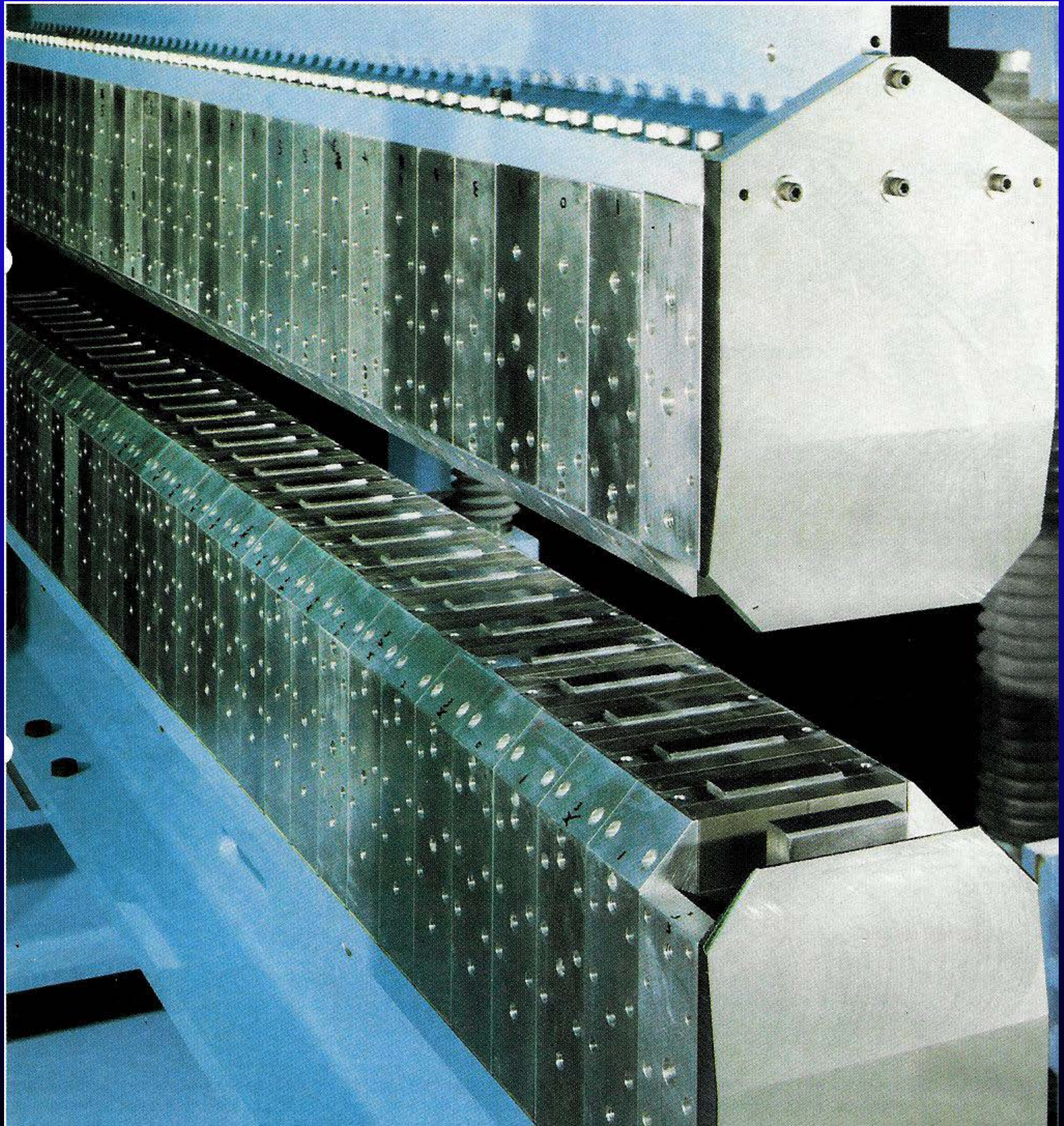


Propagation of the wave front BC is suppressed under an angle θ_0, if the path length AC is just shorter by a half wavelength compared to AB (negative interference). This defines the central cone.

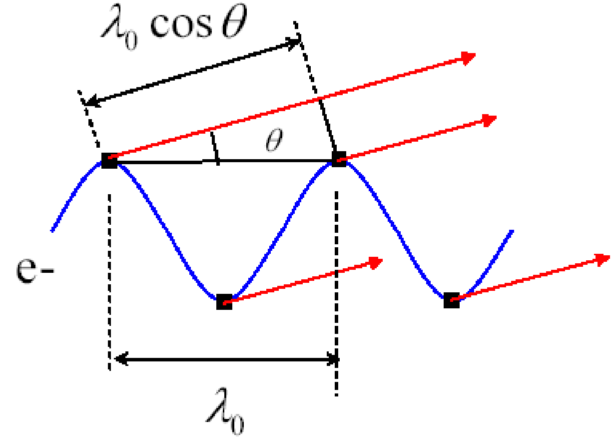
$$\Delta L = AB - AC = \frac{1}{2} L \left(1 - \cos\theta_0\right) \approx \frac{1}{4} L \theta_0^2$$

Negative interference for $\Delta L = \frac{\lambda}{2}$

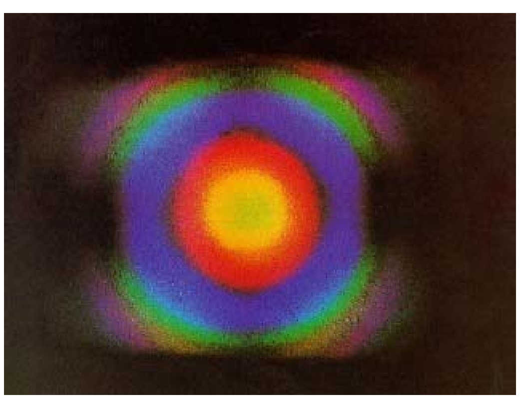
$$\theta_0 = \sqrt{\frac{2\lambda}{L}} \qquad R_0 = \sqrt{\frac{\lambda \cdot L}{2}} \qquad \varepsilon_0 = \theta_0 R_0 = \lambda$$

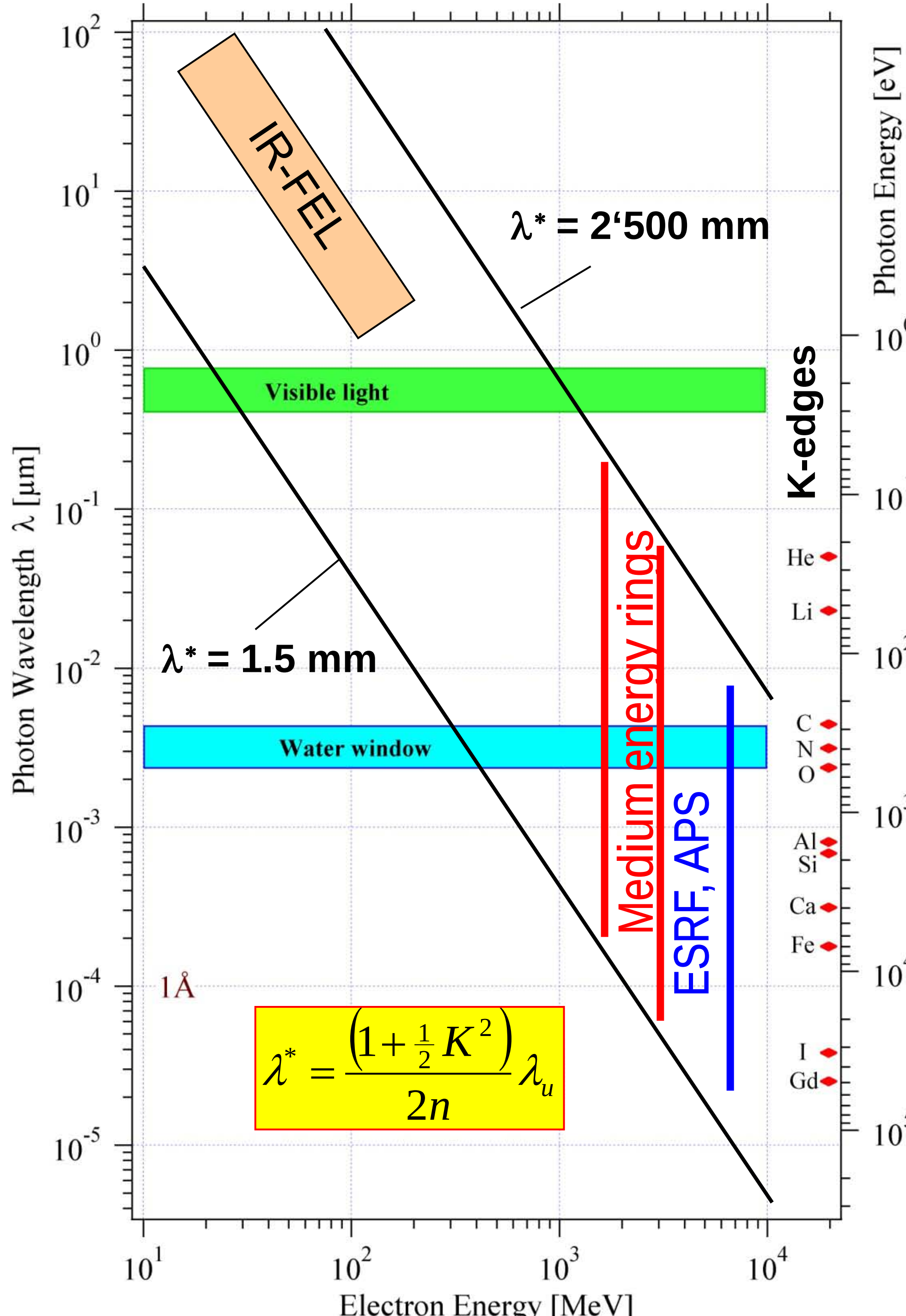


Undulator radiation

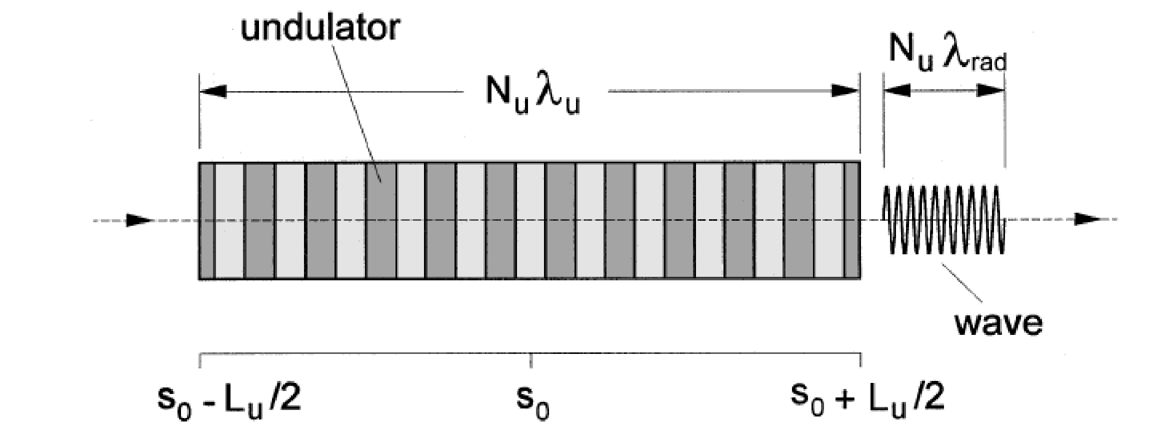


$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$





Undulator line width



Undulator of infinite length

$$N_u = \infty \quad \Rightarrow \quad \frac{\Delta\lambda}{\lambda} = 0$$

Finite length undulator

- radiation pulse has as many periods as the undulator
- the line width is

$$\frac{\Delta\lambda}{\lambda} \sim \frac{1}{N_u}$$

Due to the electron energy spread

$$\frac{\Delta\lambda}{\lambda} = 2\frac{\sigma_E}{E}$$

END