



# Wedge Absorber

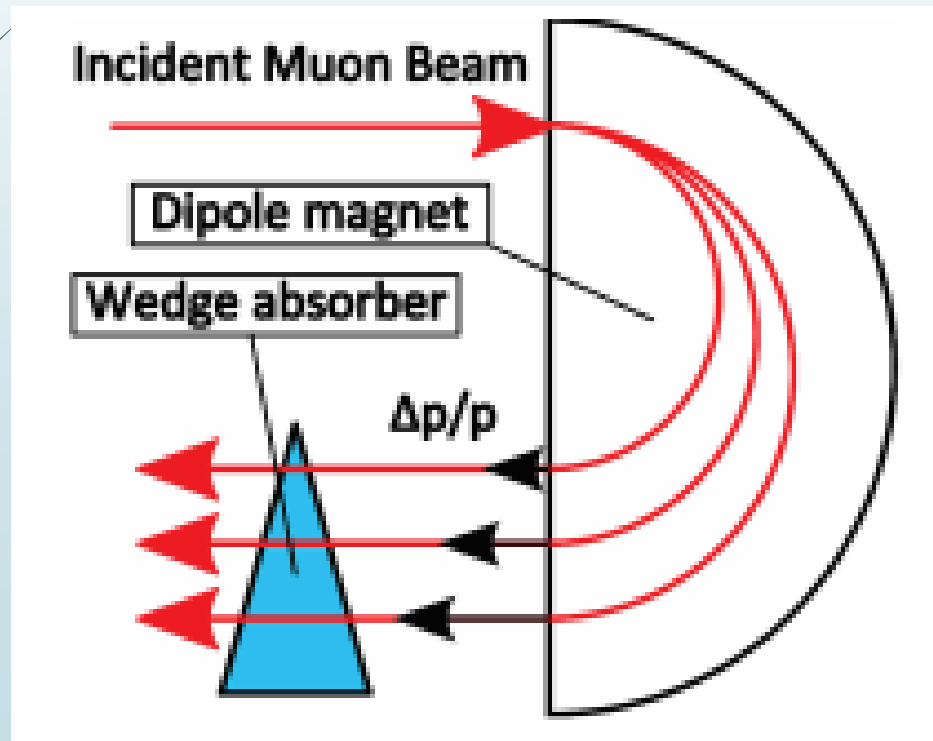
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# Aims

- Demonstrate Emittance Exchange in the Wedge using MICE data
- Number of techniques: KDE, KNN, Voronoi Tessellations, etc.
- Use beam reweighing techniques



# Weighting by beam moment

## EPAC08 – Chris Rogers

- Input Distribution  $f(\vec{x})$
- Desired Output Distribution  $g(\vec{x})$
- Multidimensional phase-space vector  $\vec{x}$
- Polynomial weighting function  $w(\vec{x})$ , where

$$w(\vec{x}) = 1 + \sum a_{i1}(x_{i1}) + \sum a_{i1i2}(x_{i1}x_{i2}) + \sum a_{i1i2i3}(x_{i1}x_{i2}x_{i3}) + \dots$$

Such that

$$g(\vec{x}) = N \left( 1 + \sum a_{i1}(x_{i1}) + \sum a_{i1i2}(x_{i1}x_{i2}) + \sum a_{i1i2i3}(x_{i1}x_{i2}x_{i3}) + \dots \right) f(\vec{x})$$

- Denote moments of distribution  $f(\vec{x})$ ,  $\langle x_{i1}x_{i2} \dots x_{in} \rangle$  by  $V_{i1i2\dots in}^f$

# Weighting by beam moment

## EPAC08 – Chris Rogers

- Denote moments of distribution  $f(\vec{x})$ ,  $\langle x_{i_1} x_{i_2} \dots x_{i_n} \rangle$  by  $V_{i_1 i_2 \dots i_n}^f$

- The  $n^{\text{th}}$  moment of the function  $g(\vec{x})$ ,  $V_{j_1 j_2 \dots j_n}^g$  can be written as

$$\frac{V_{j_1 \dots j_n}^f + \sum a_{i_1} (V_{i_1 j_1 \dots j_n}^f) + \sum a_{i_1 i_2} (V_{i_1 i_2 j_1 \dots j_n}^f) + \dots}{1 + \sum a_{i_1} (V_{j_1 \dots j_n}^f) + \sum a_{i_1 i_2} (V_{j_1 \dots j_n}^f) + \dots}$$

- Rearranging

$$V_{j_1 \dots j_n}^g - V_{j_1 \dots j_n}^f = \sum_{i_1 \dots i_m} a_{i_1 \dots i_m} (V_{i_1 \dots i_m j_1 \dots j_n}^f - V_{i_1 \dots i_m}^f V_{j_1 \dots j_n}^g)$$

- Define  $u_i = V_{j_1 \dots j_n}^g - V_{j_1 \dots j_n}^f$ ,  $a_i = a_{i_1 \dots i_m}$  and  $M_{ij} = (V_{i_1 \dots i_m j_1 \dots j_n}^f - V_{i_1 \dots i_m}^f V_{j_1 \dots j_n}^g)$

- Linear problem:  $\mathbf{M}\vec{a} = \vec{u}$

- Which can be solved for polynomial coefficients  $a_{i_1 \dots i_n}$

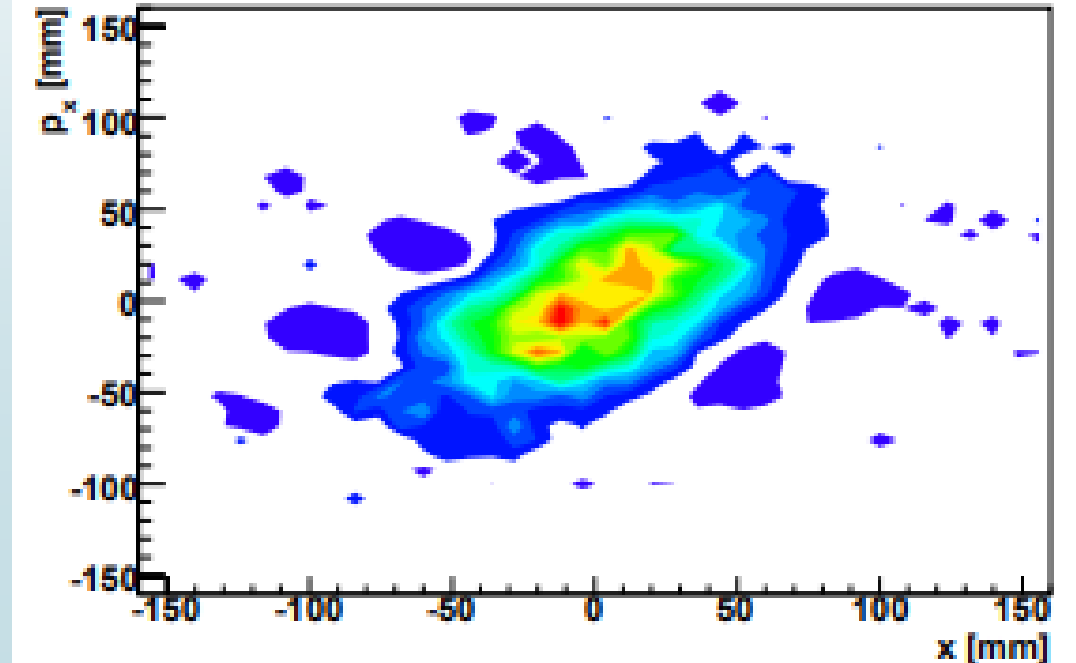
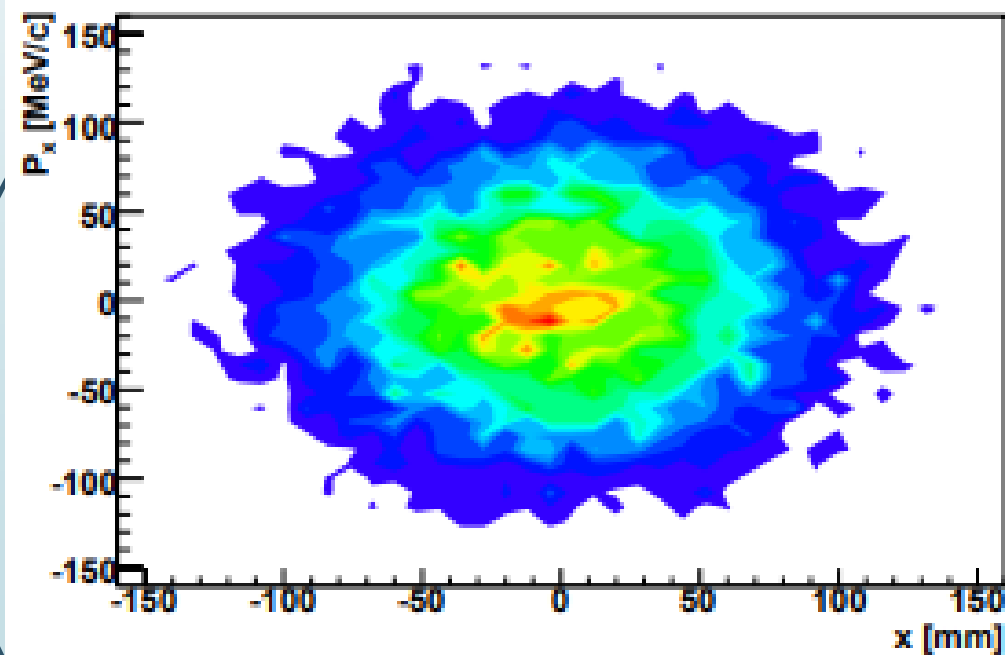
# Weighting by beam moment

## EPAC08 – Chris Rogers

- ▶ 10000 particles sampled from Gaussian Distribution (left)
- ▶ Statistical weighting applied (right)

Initial:  $\epsilon_x = 15\text{mm}$ ,  $\beta_x = 334\text{mm}$ ,  $\alpha_x = 0$

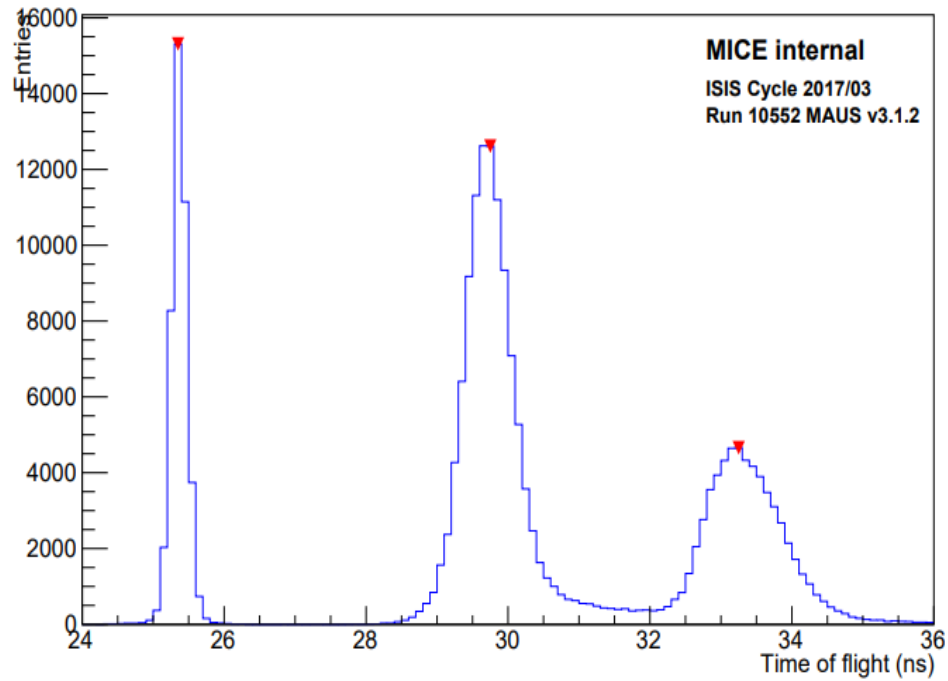
Final:  $\epsilon_x = 4.2\text{mm}$ ,  $\beta_x = 260\text{mm}$ ,  $\alpha_x = -0.75$



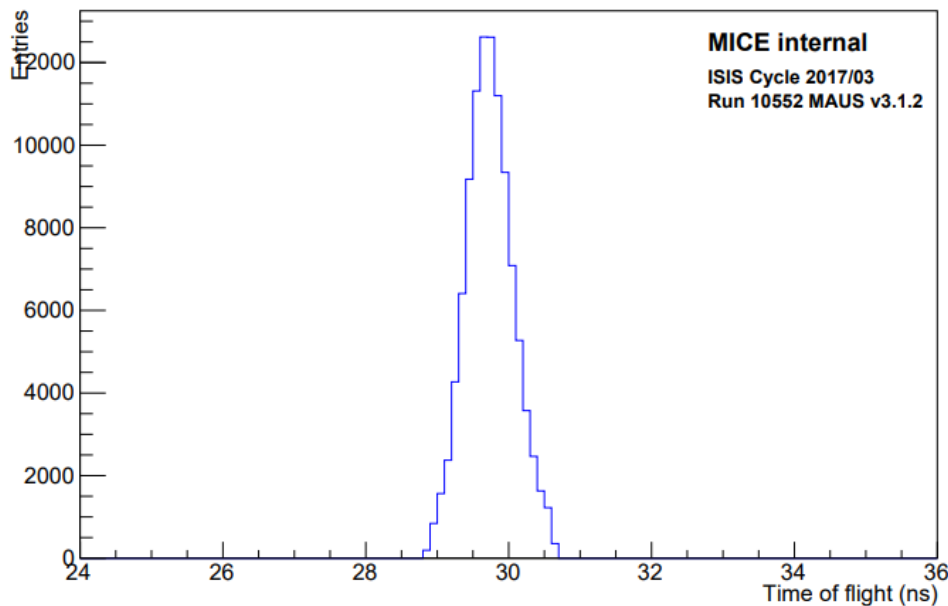
# Beam Moments through the Wedge

- ▶ Measurements at TKU and TKD reference plane
- ▶ Run Monte Carlo with Wedge and insert virtual planes
- ▶ See Evolution through Wedge and compare at TKD
  
- ▶ Measurements after cuts are made at TKU used as input for MC
- ▶ TOF cut for muons
- ▶ TKU momentum cut 130 – 150 MeV/c
- ▶ Radius Cut upstream and downstream < 150 mm
  
- ▶ TKD momentum cut added (< 200MeV/c) to prevent incorrectly reconstructed particles skewing the calculated moments

Time of flight (TOF0 to TOF1)

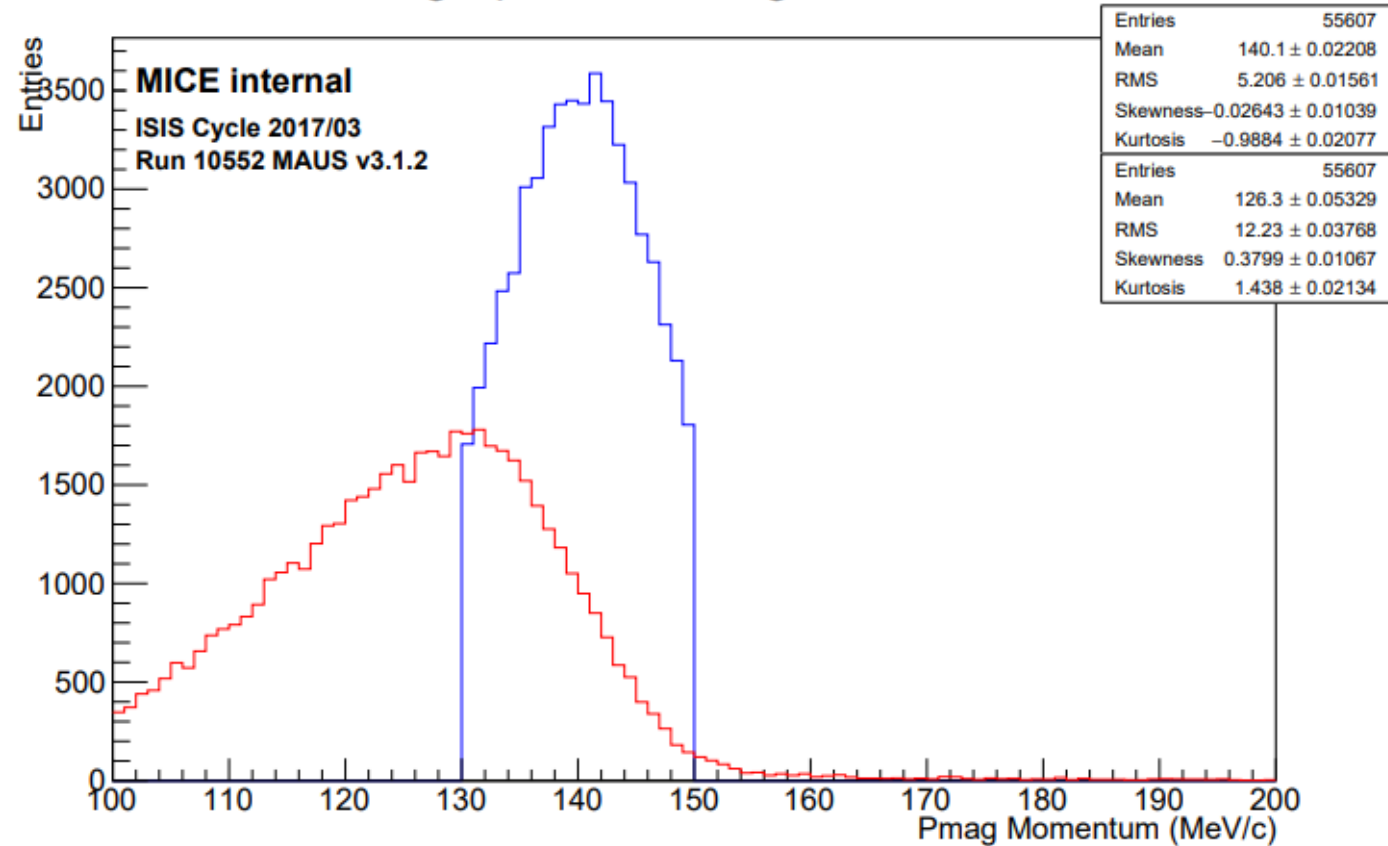


Time of flight (TOF0 to TOF1)



# TOF and momentum cut

Pmag Upstream Pmag Downstream



# Beam Moments through the Wedge

- Use Scipy to calculate the moments, where  $E$  is the expectation operator:

$$\mu_n = E[(X - E[X])^n] = \int_{-\infty}^{+\infty} (x - \mu)^n f(x) dx$$

- Will display data in terms of Mean, RMS, Skew and Kurtosis, where the coefficients of Skewness and Kurtosis are given by:

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}$$

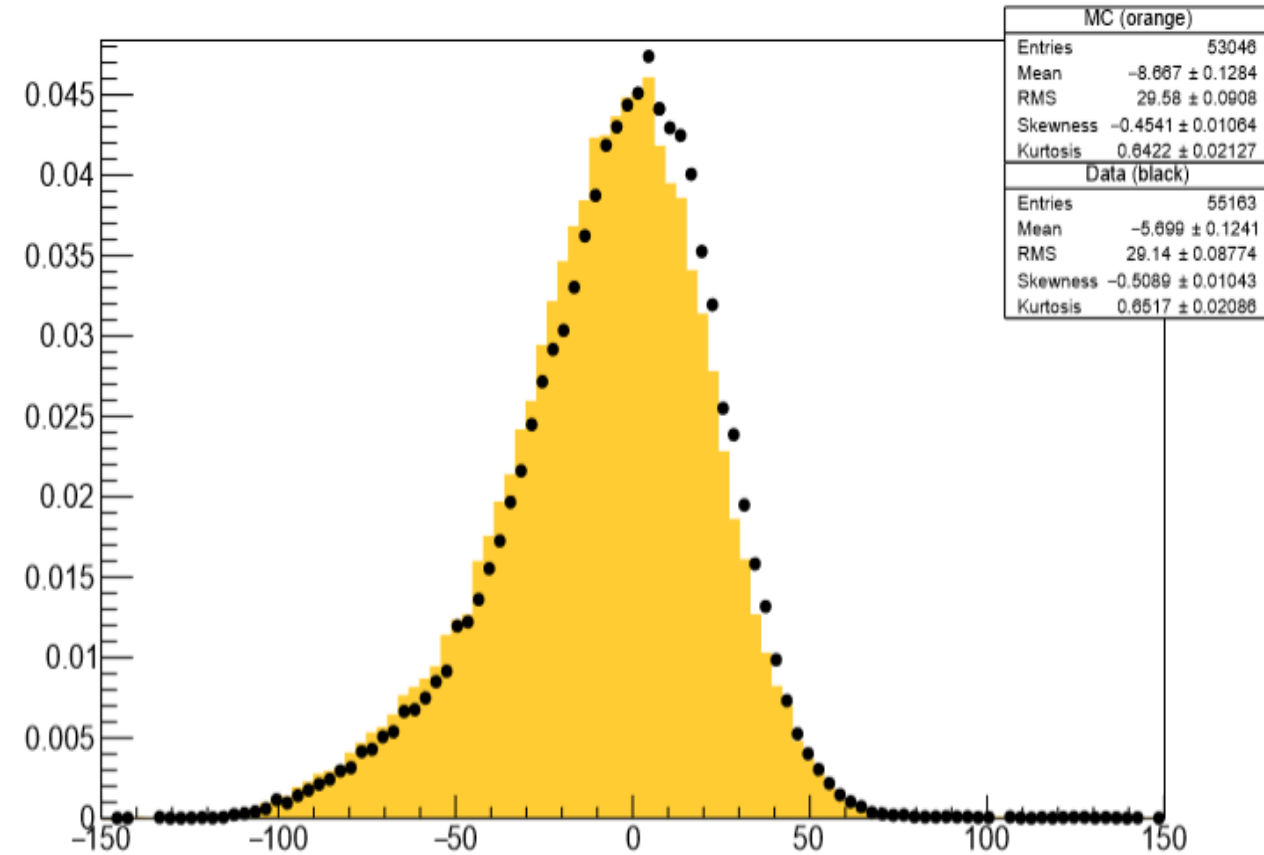
$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$$

- For Gaussian distribution  $\gamma_1 = 0$  and  $\gamma_2 = 0$

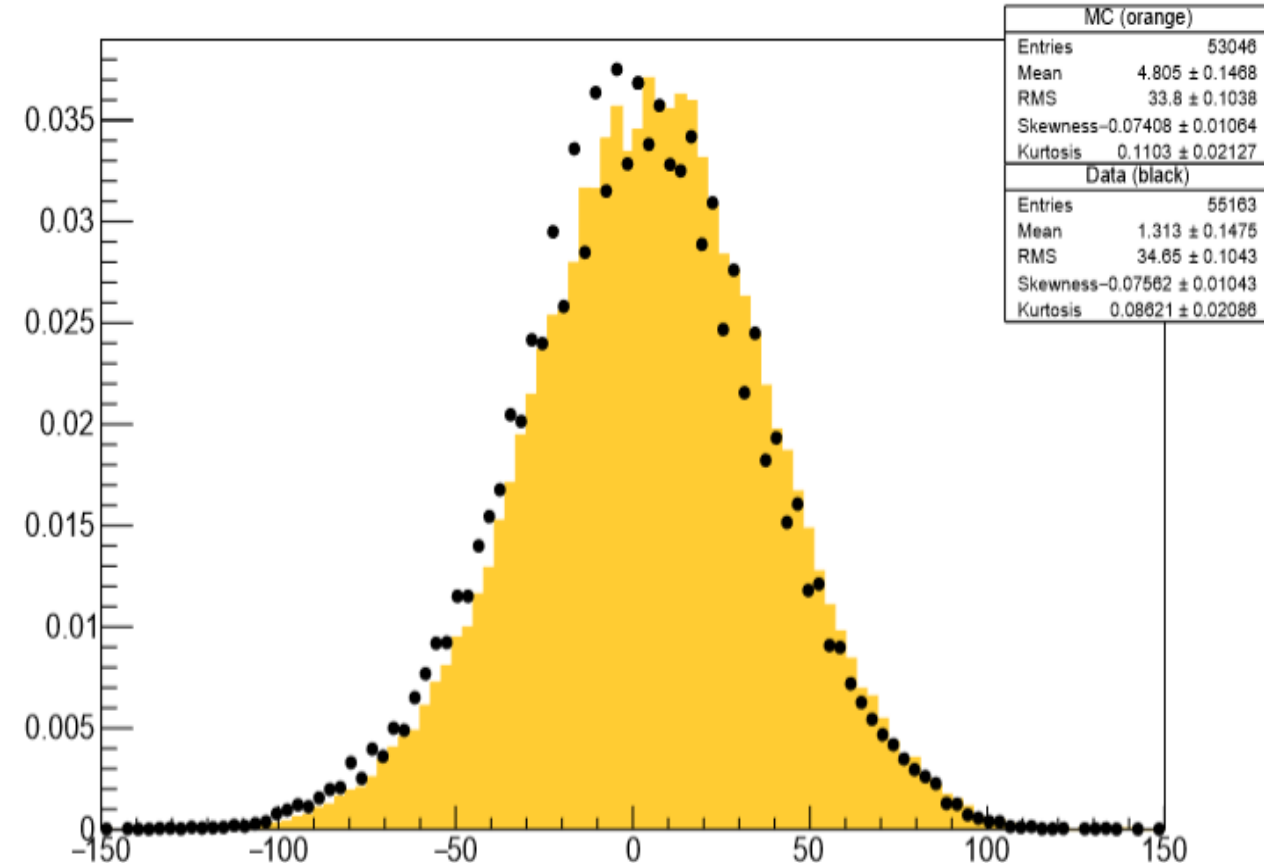


## 6 – 140 beam TKD MC vs Data 10552

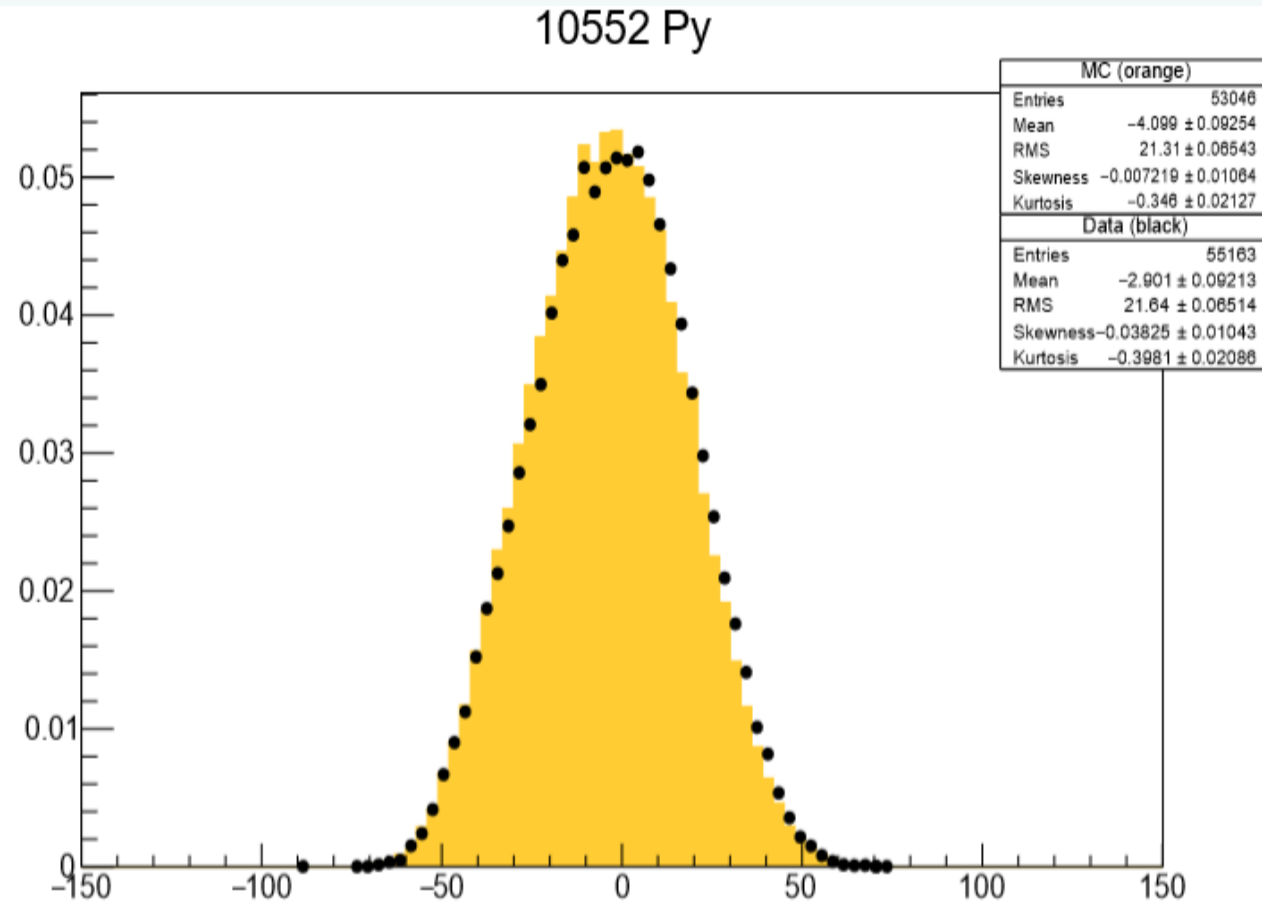
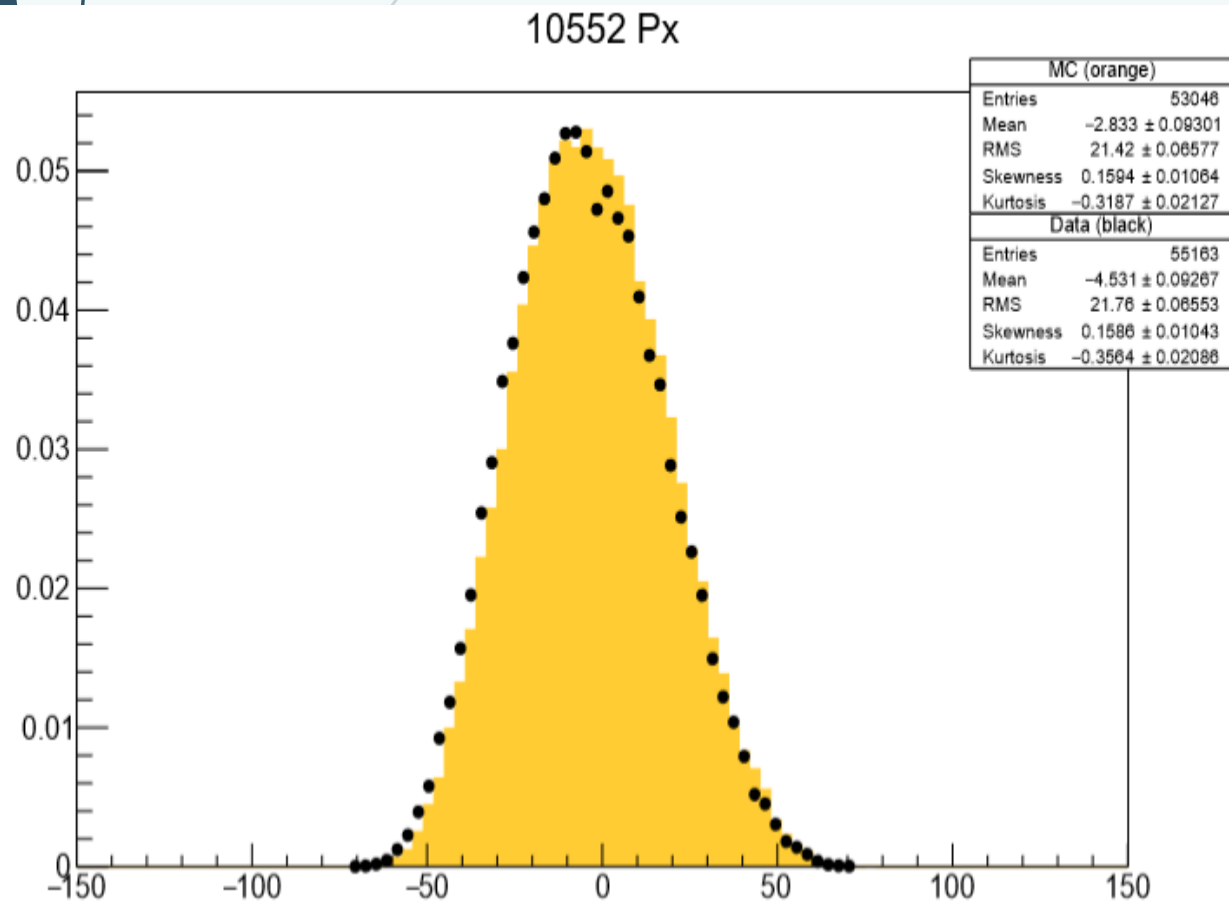
10552 X



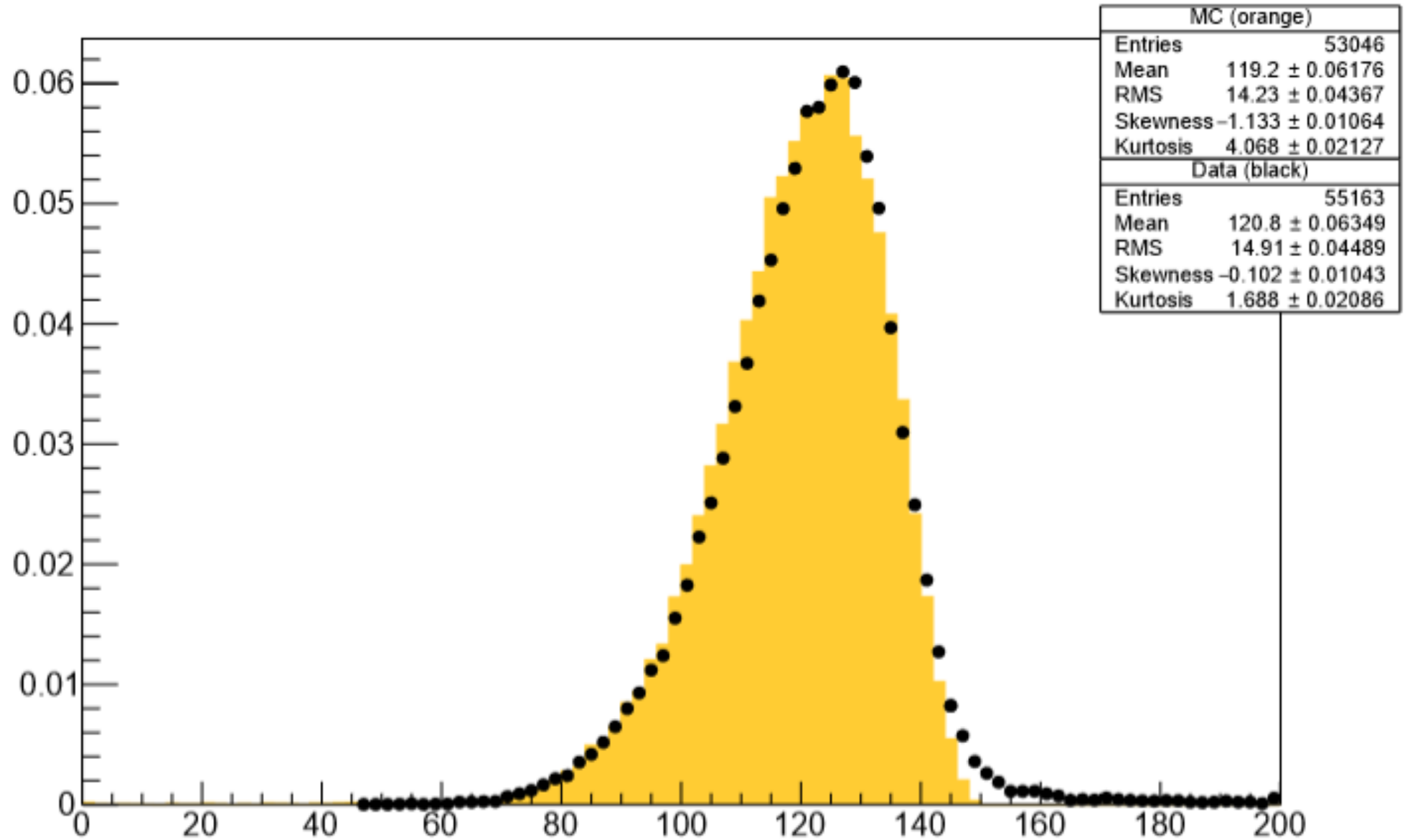
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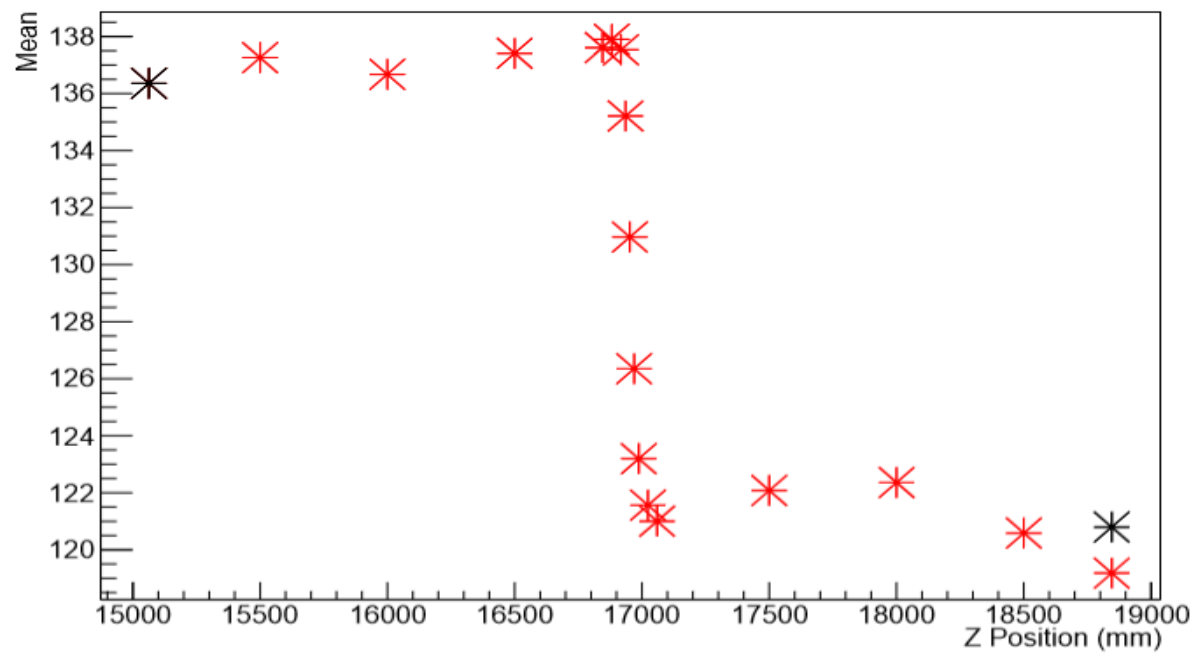
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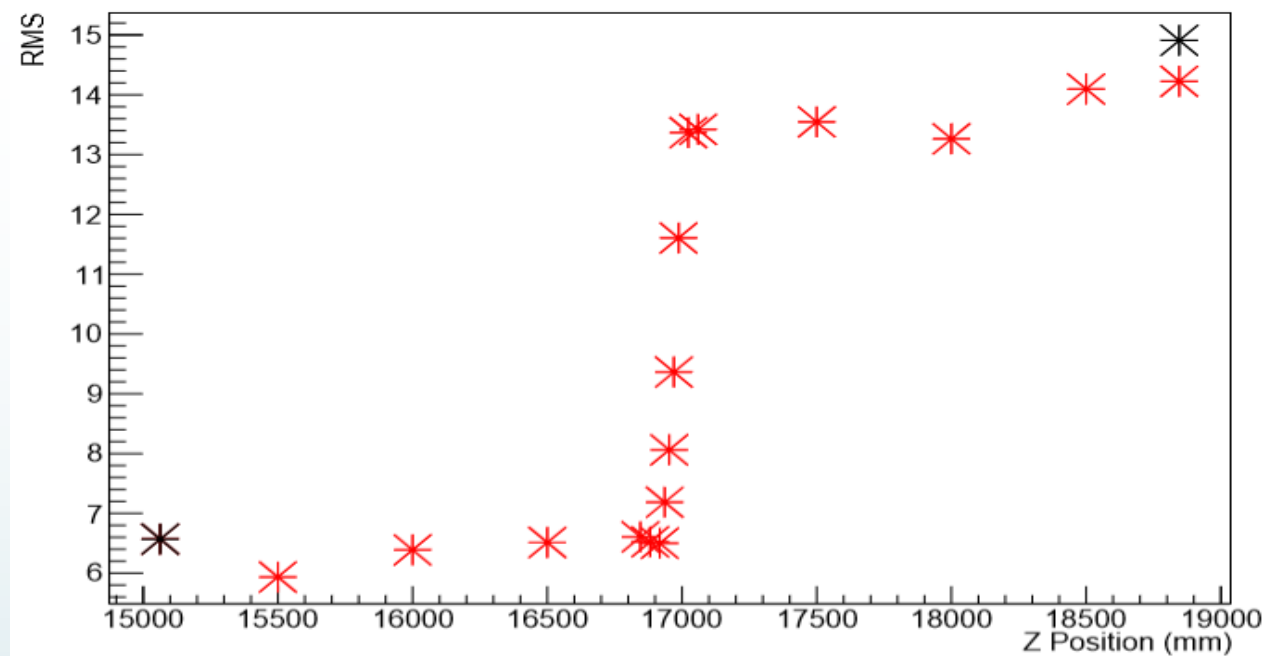
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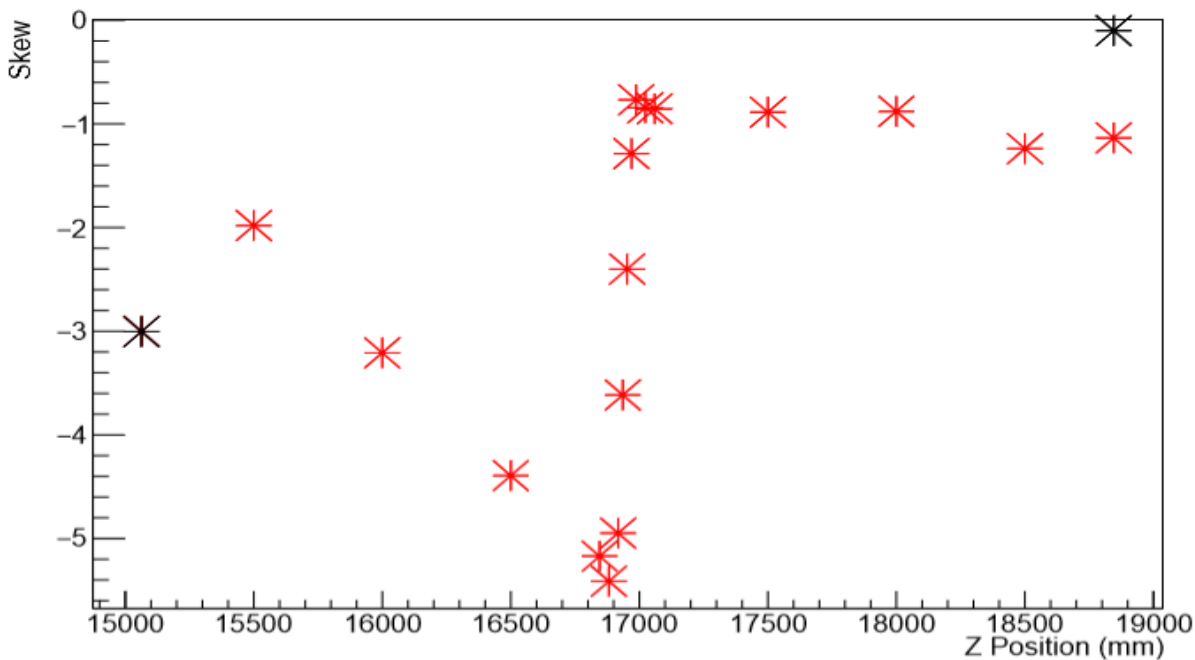
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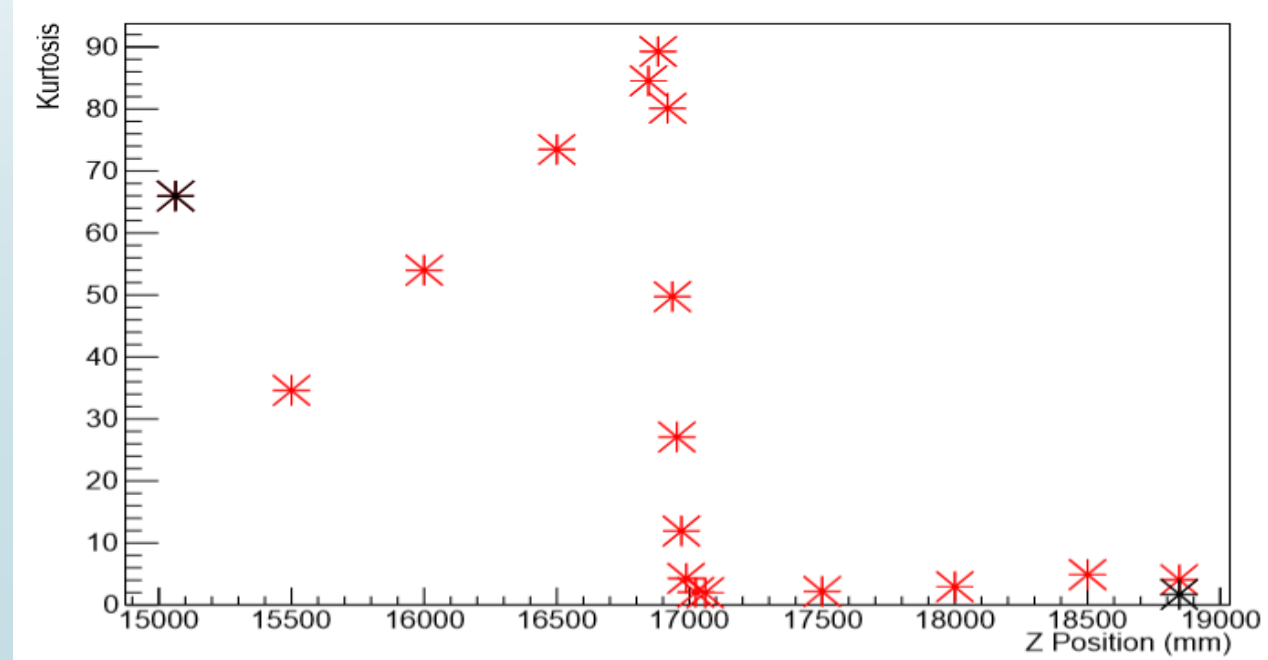
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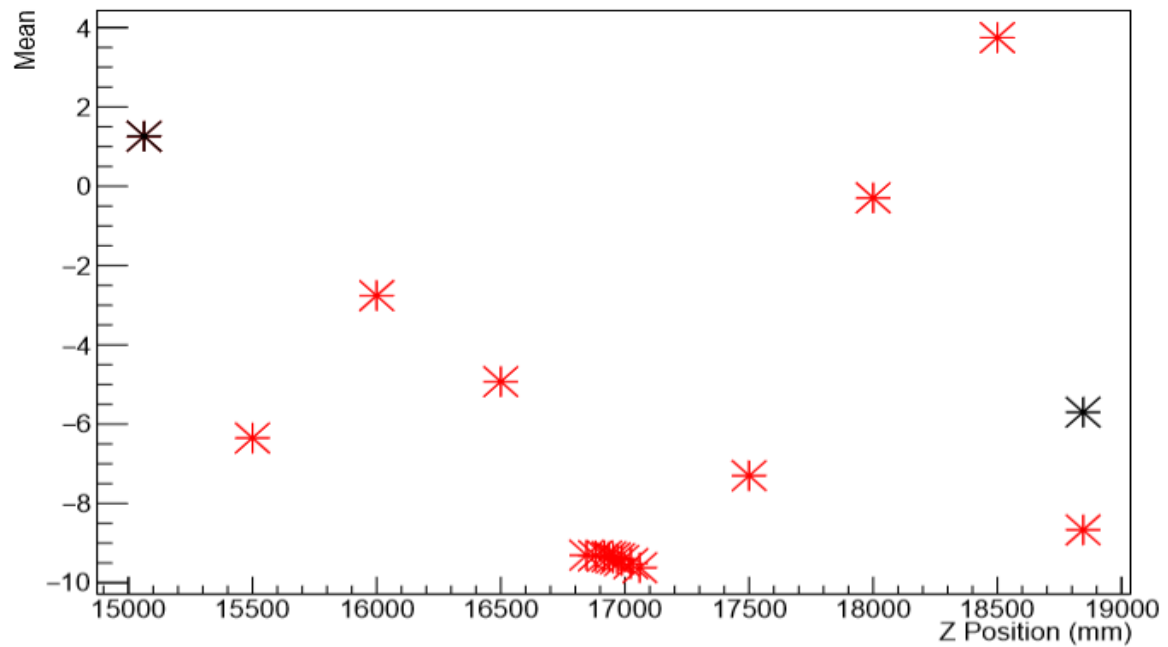
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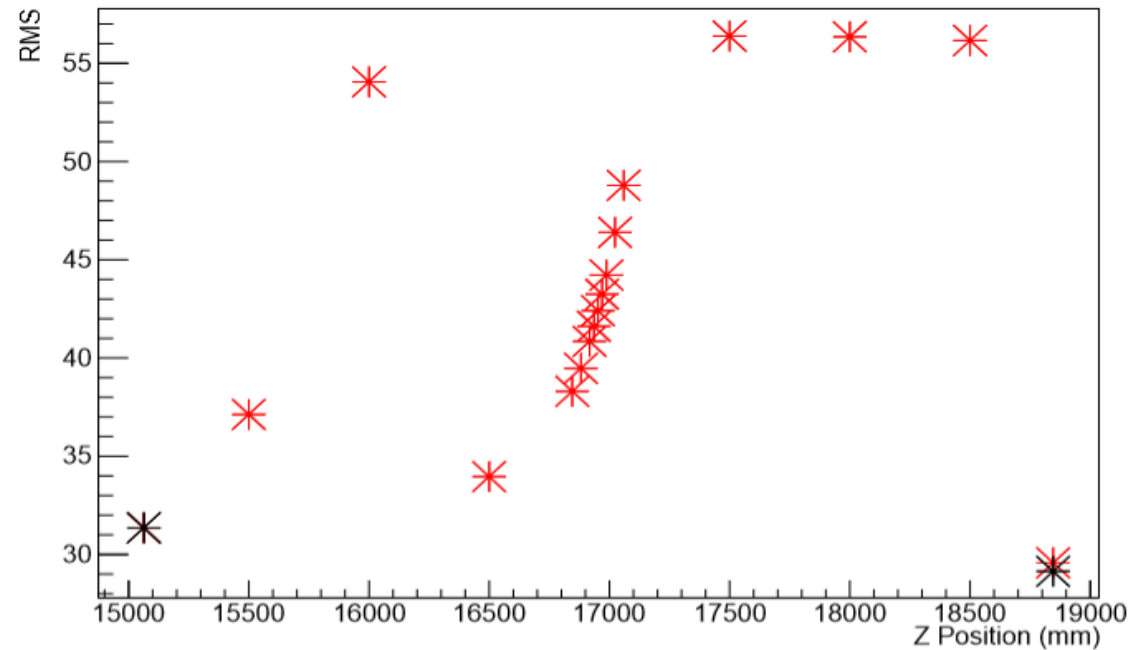
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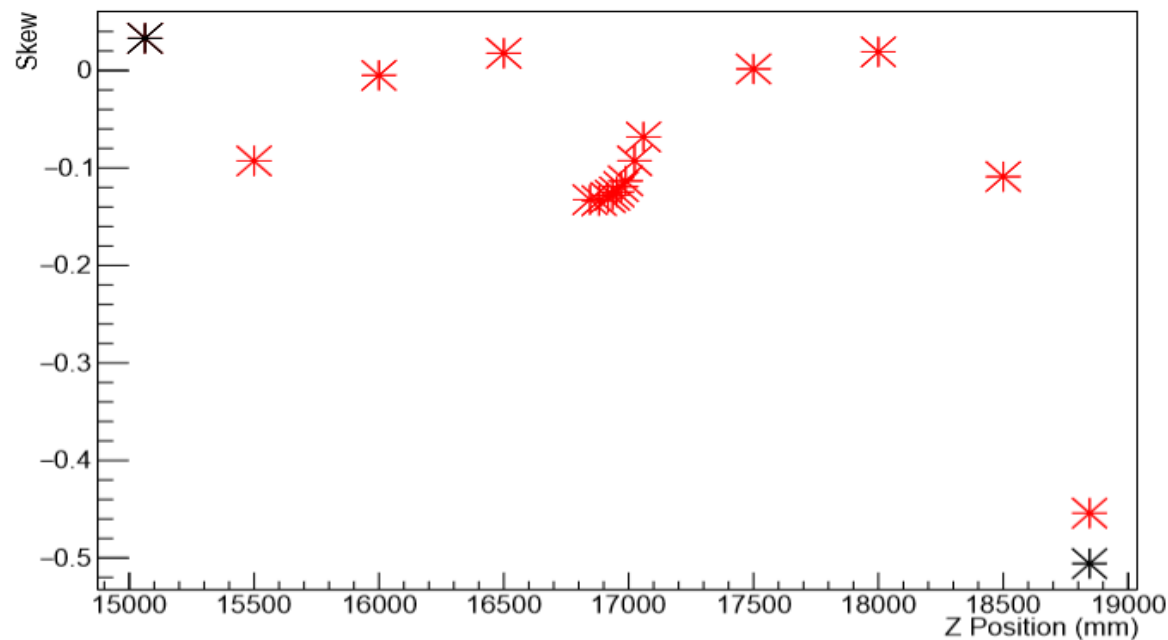
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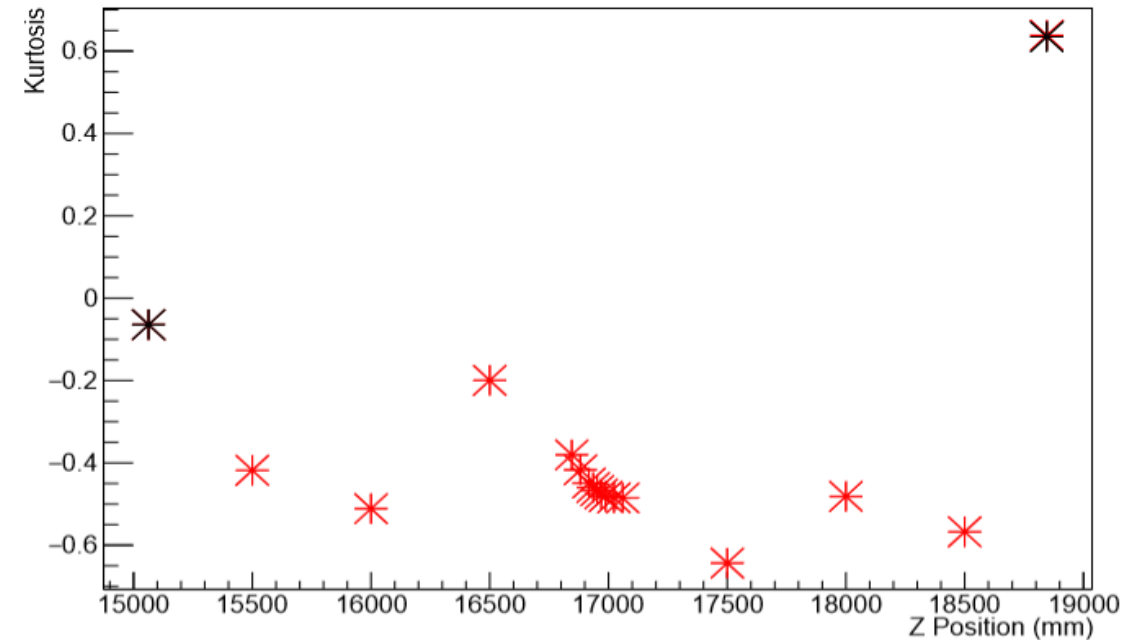
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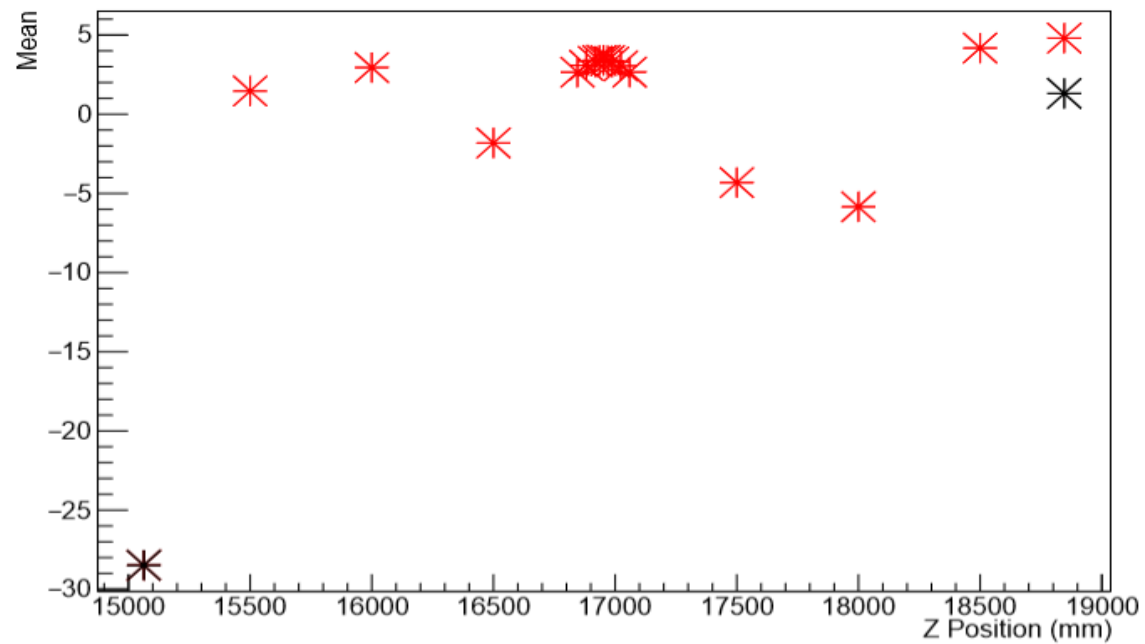
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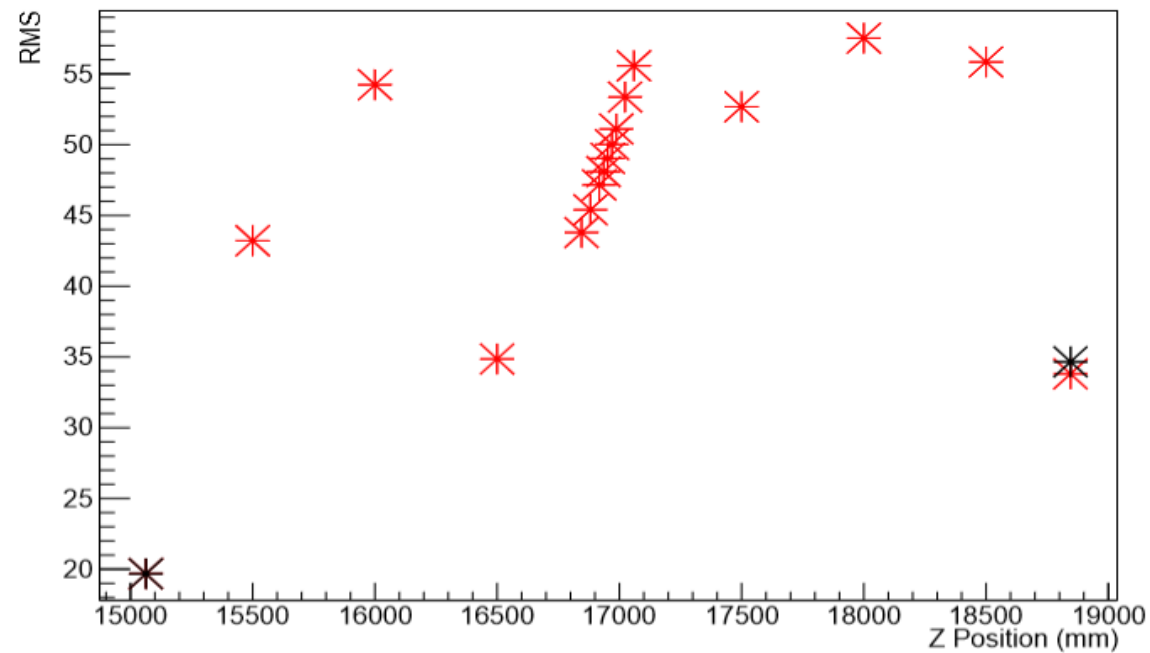
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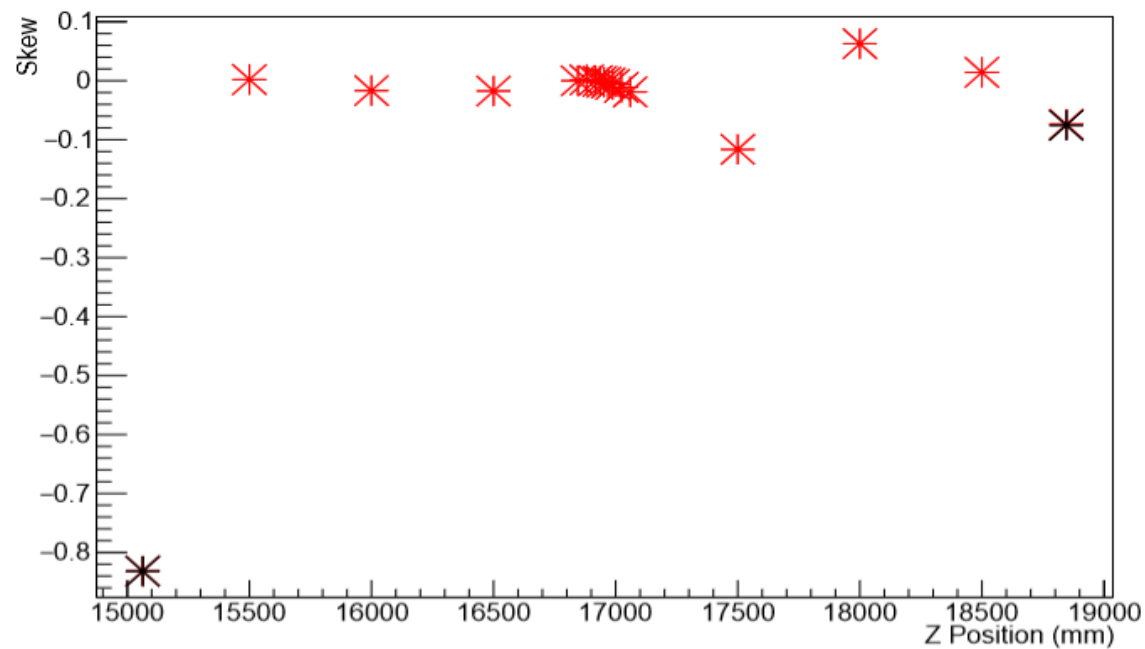
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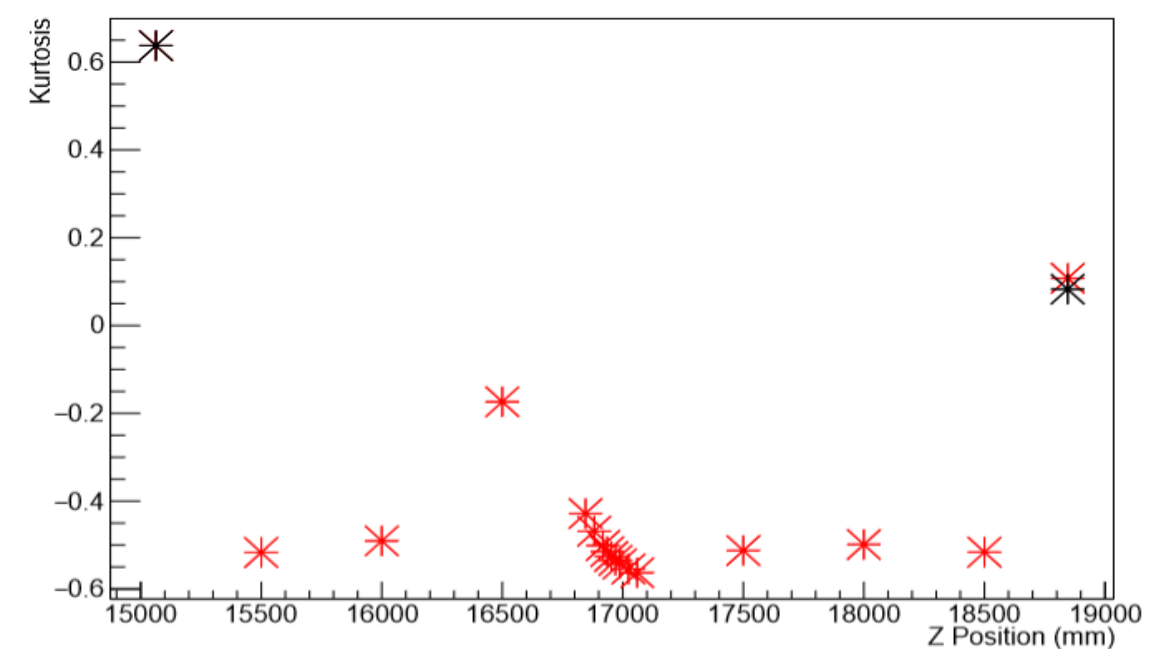
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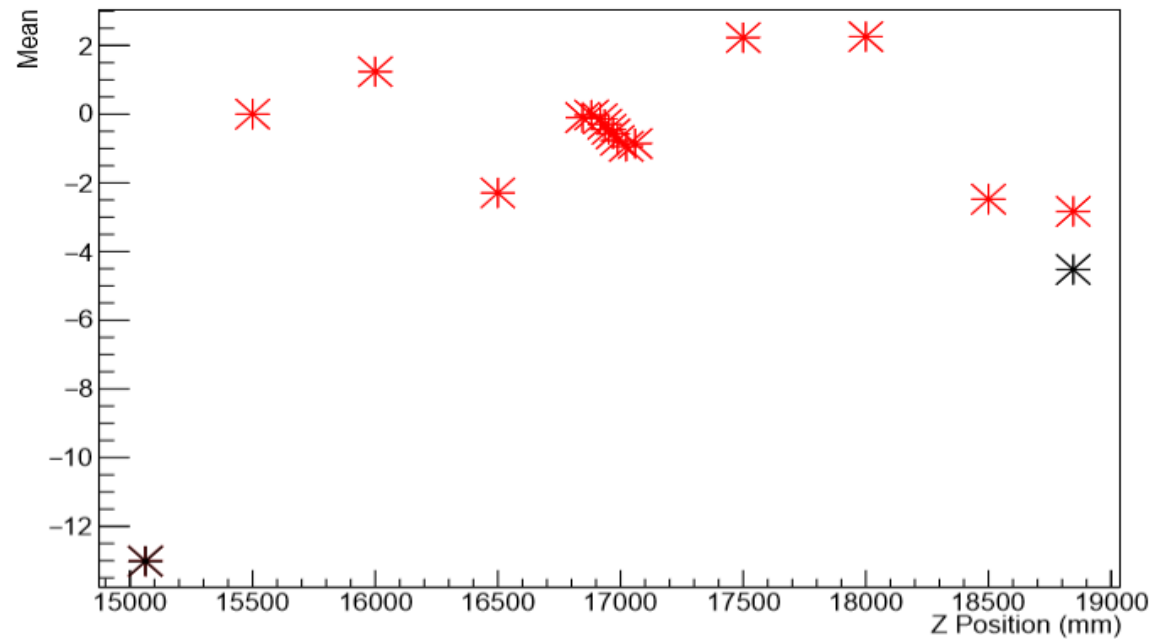
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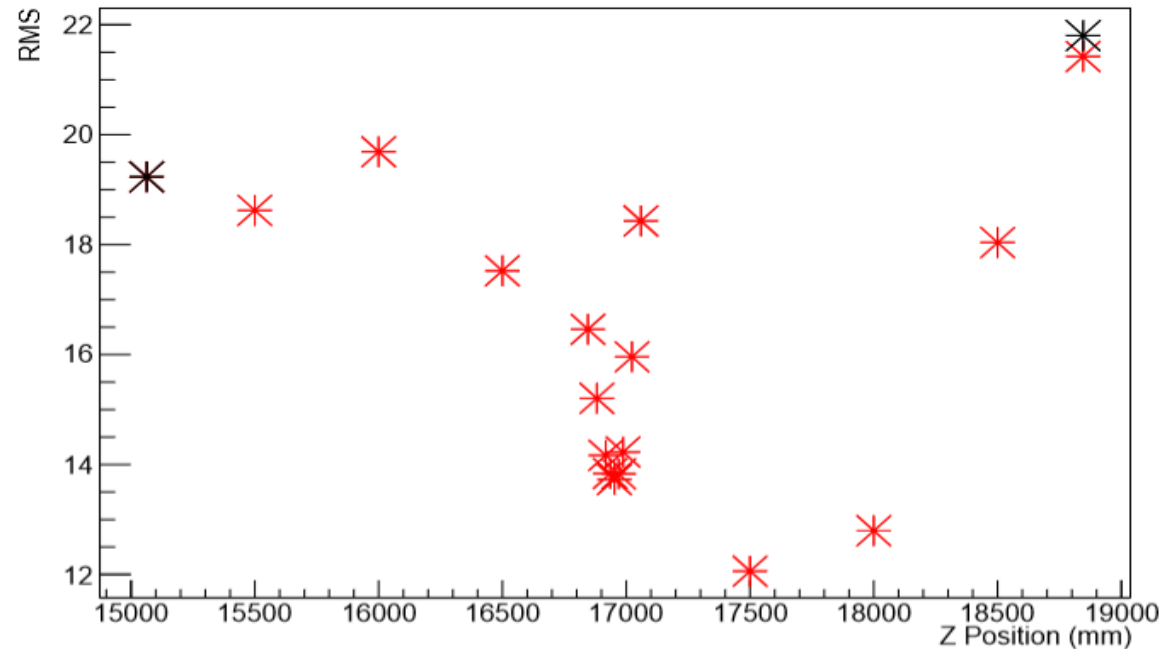
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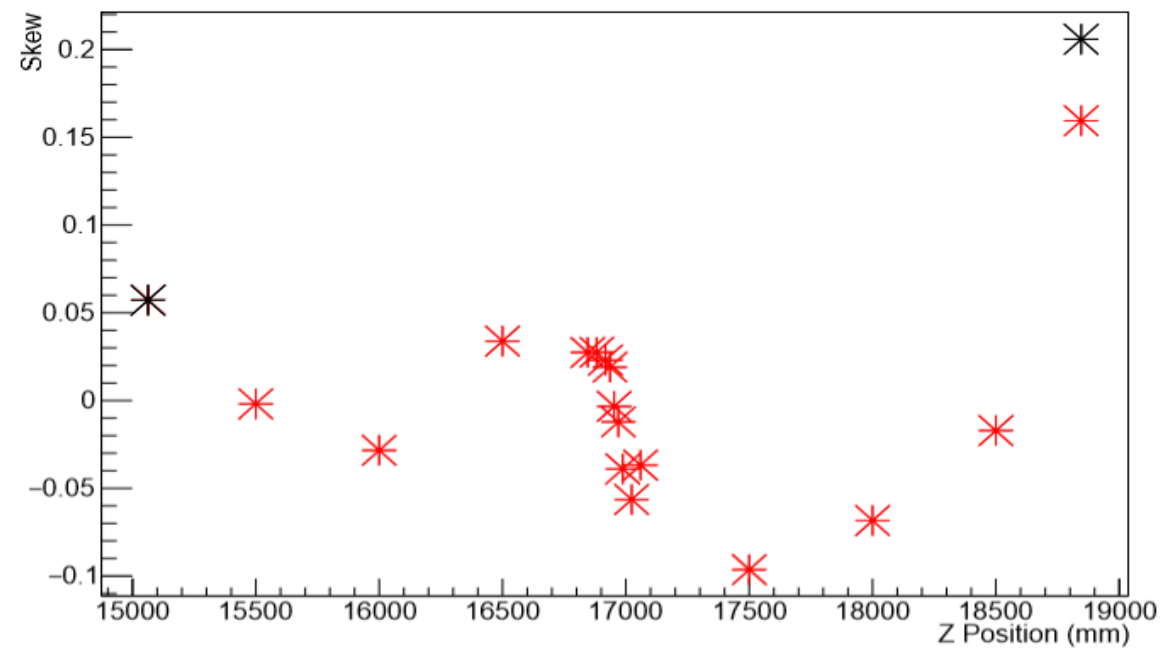
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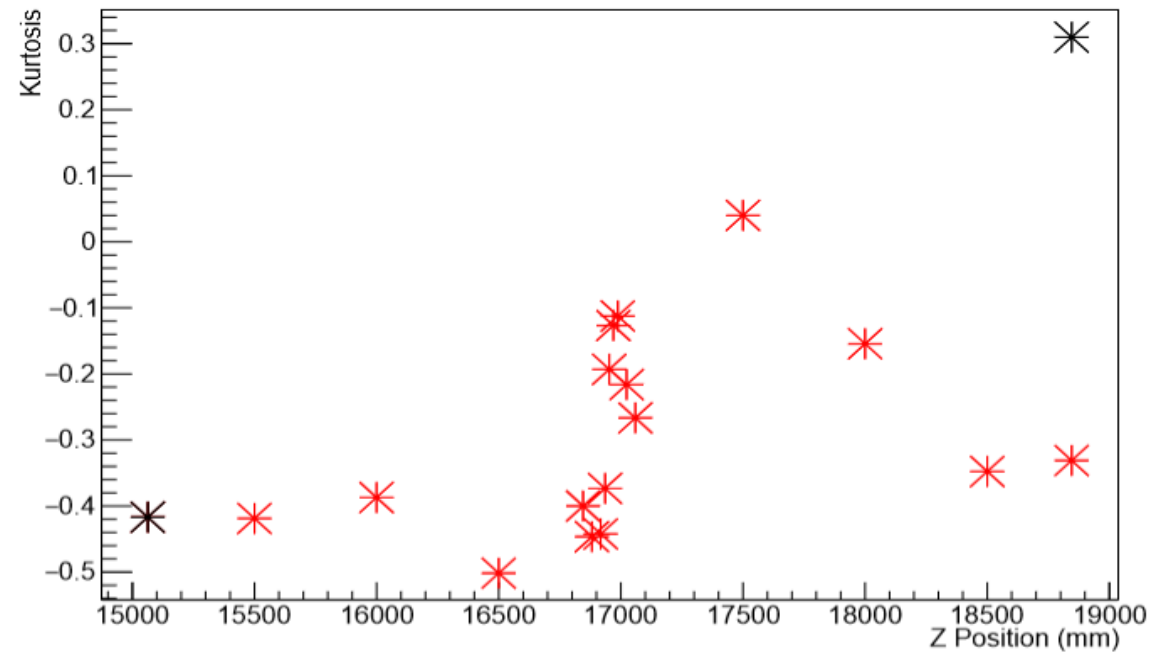
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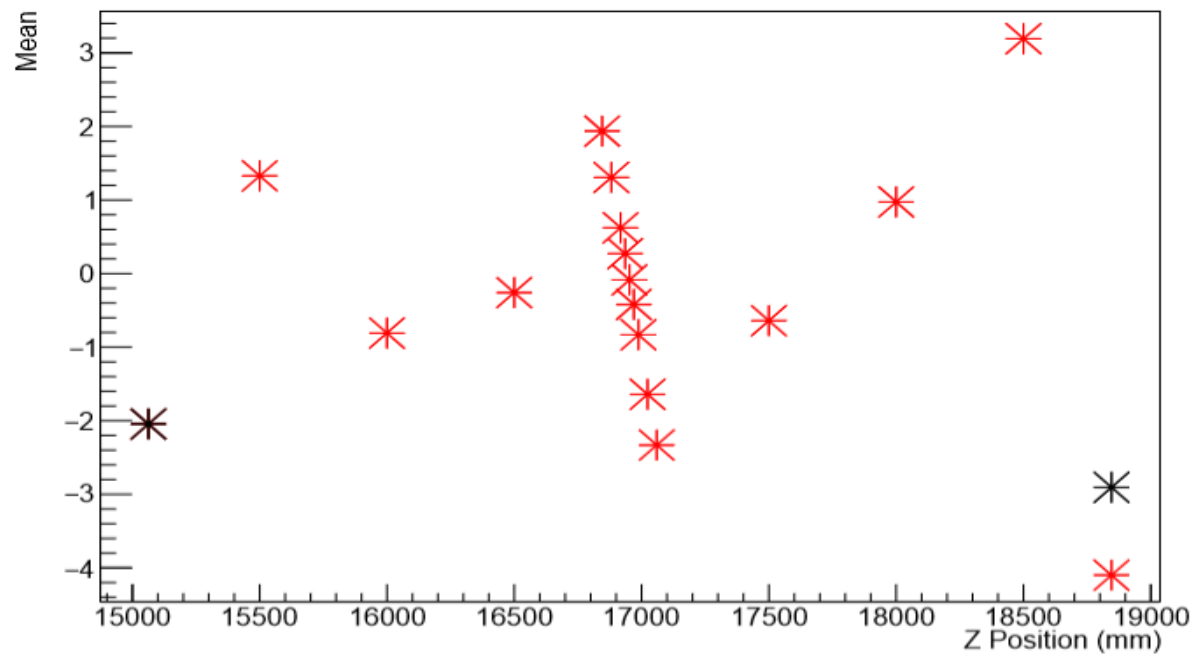
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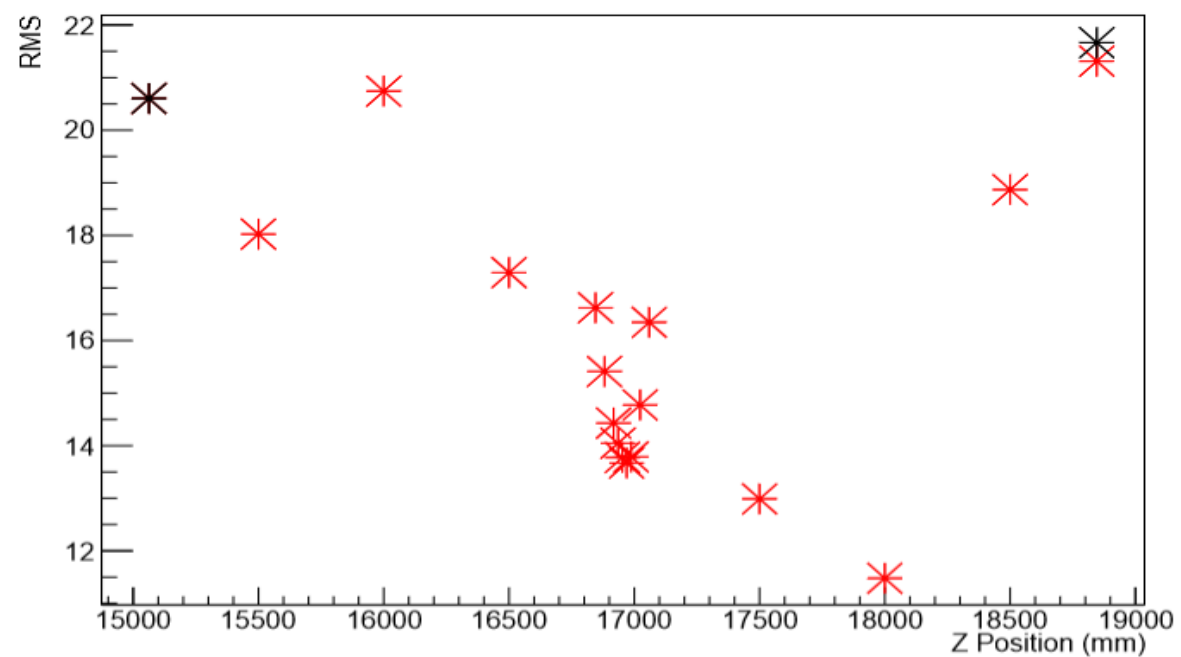
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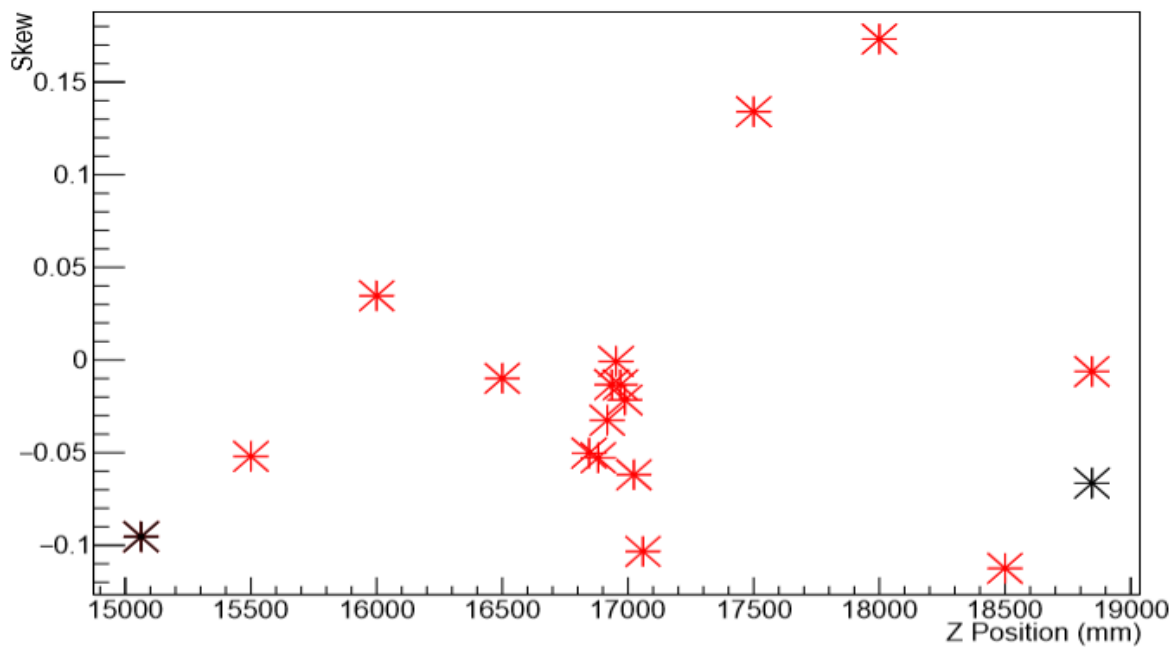
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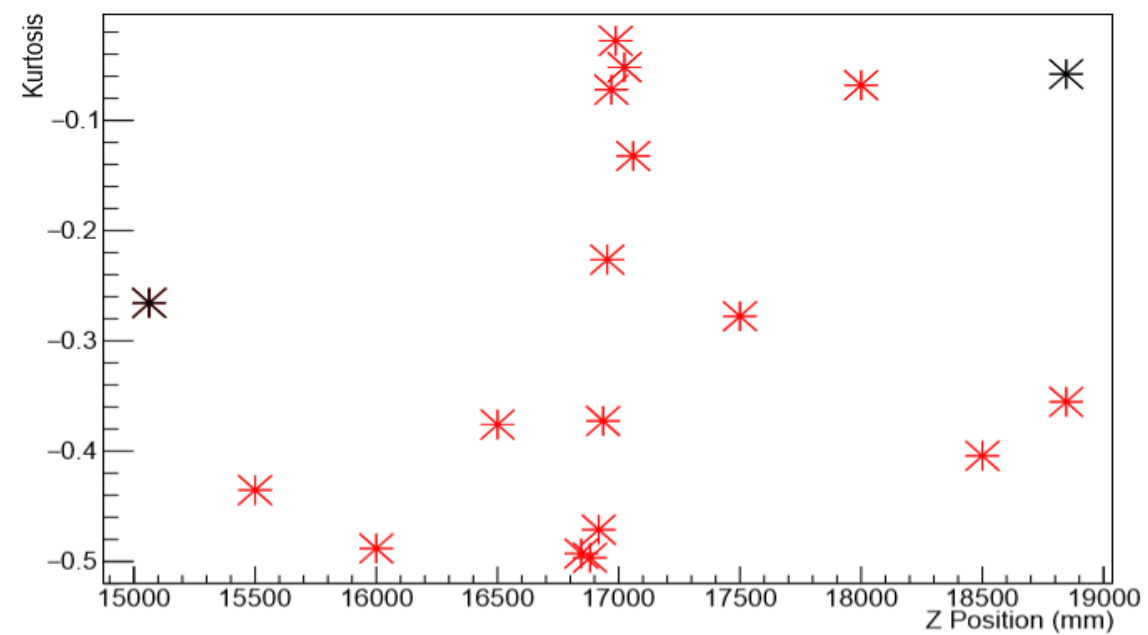
10552 Py



10552 Py



10552 Py





# Beam Weighting: Next Steps

- ▶ Beam Moments look reasonable through cooling channel at TKD
- ▶ Will look at beam Selection
- ▶ Beam has imperfections => Selection routine
- ▶ Select an ideal distribution from the data using a weighted algorithm at either TKU or absorber and see the results at TKD

# Kernel Density Estimation (KDE)

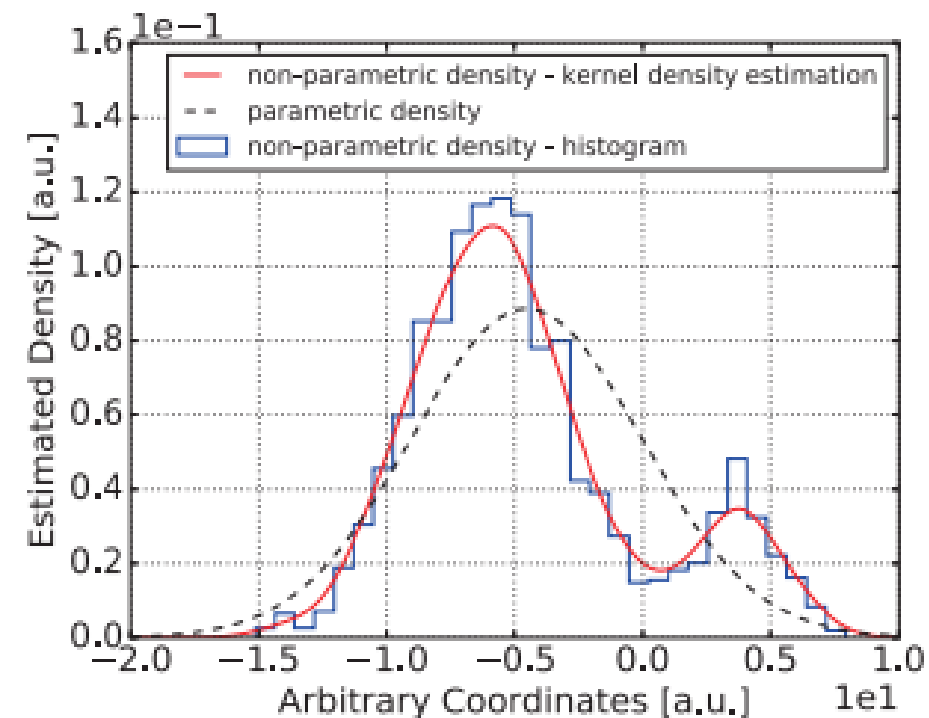
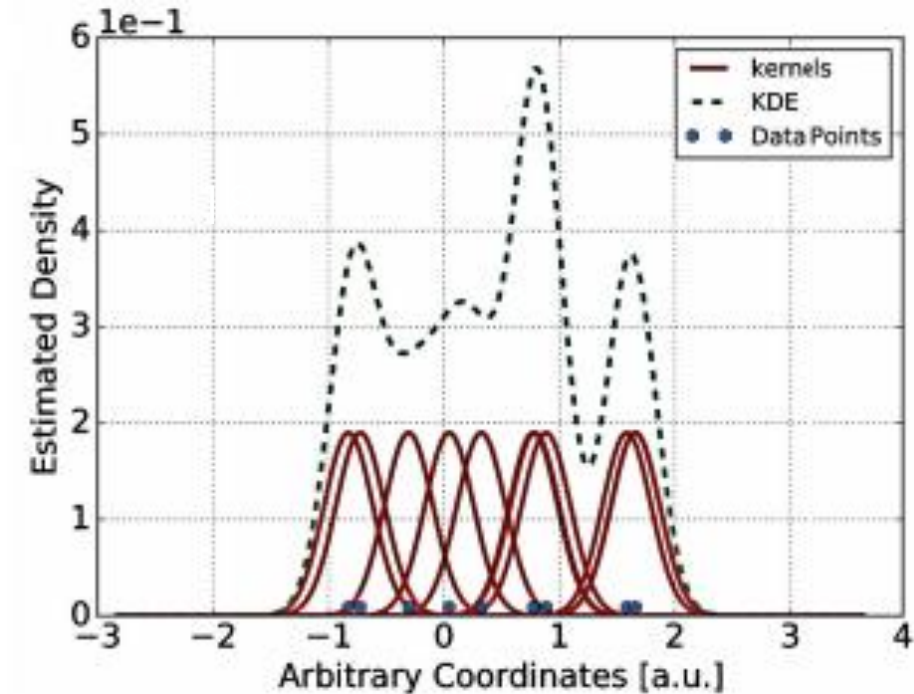
## Tanaz – IPAC 2018

- Calculate the kernel, a multivariate Gaussian for each data point
- Sum all the kernels to get the KDE

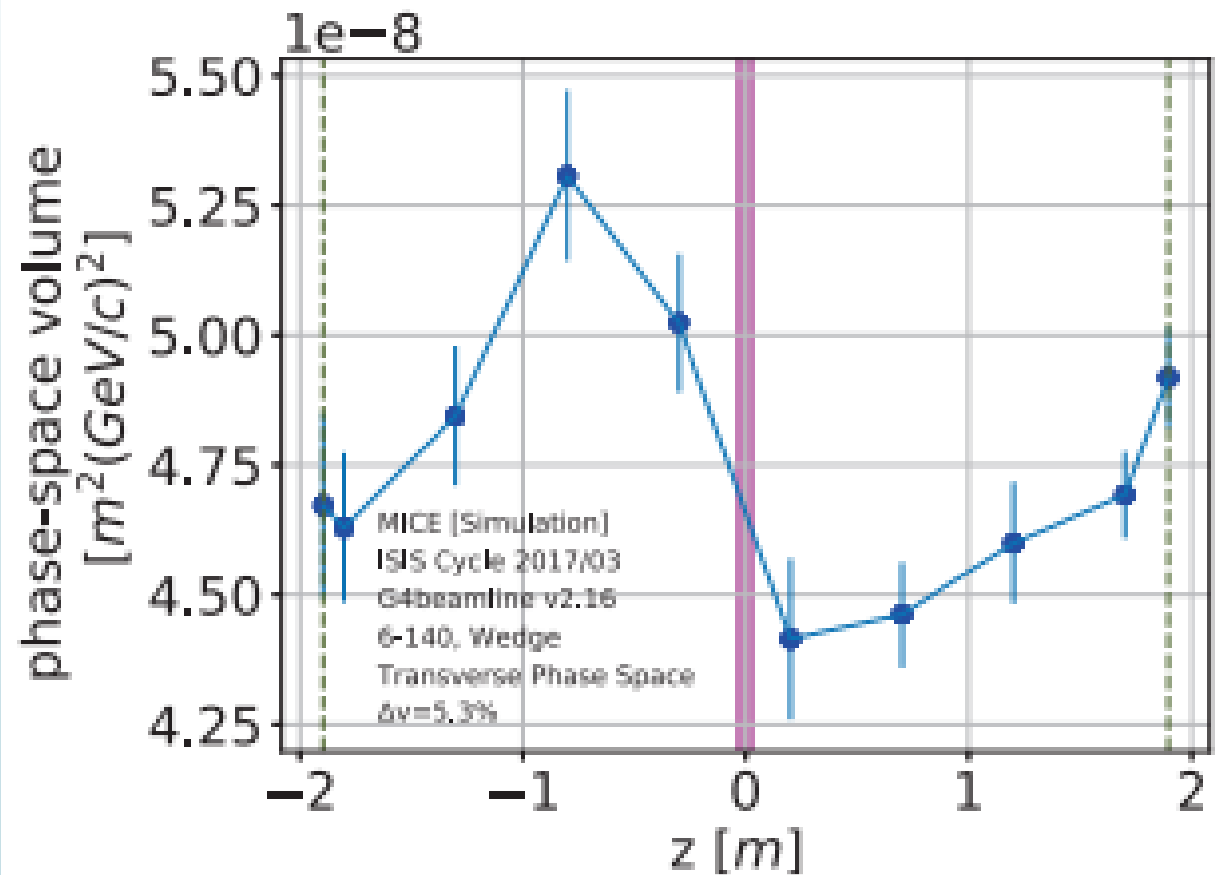
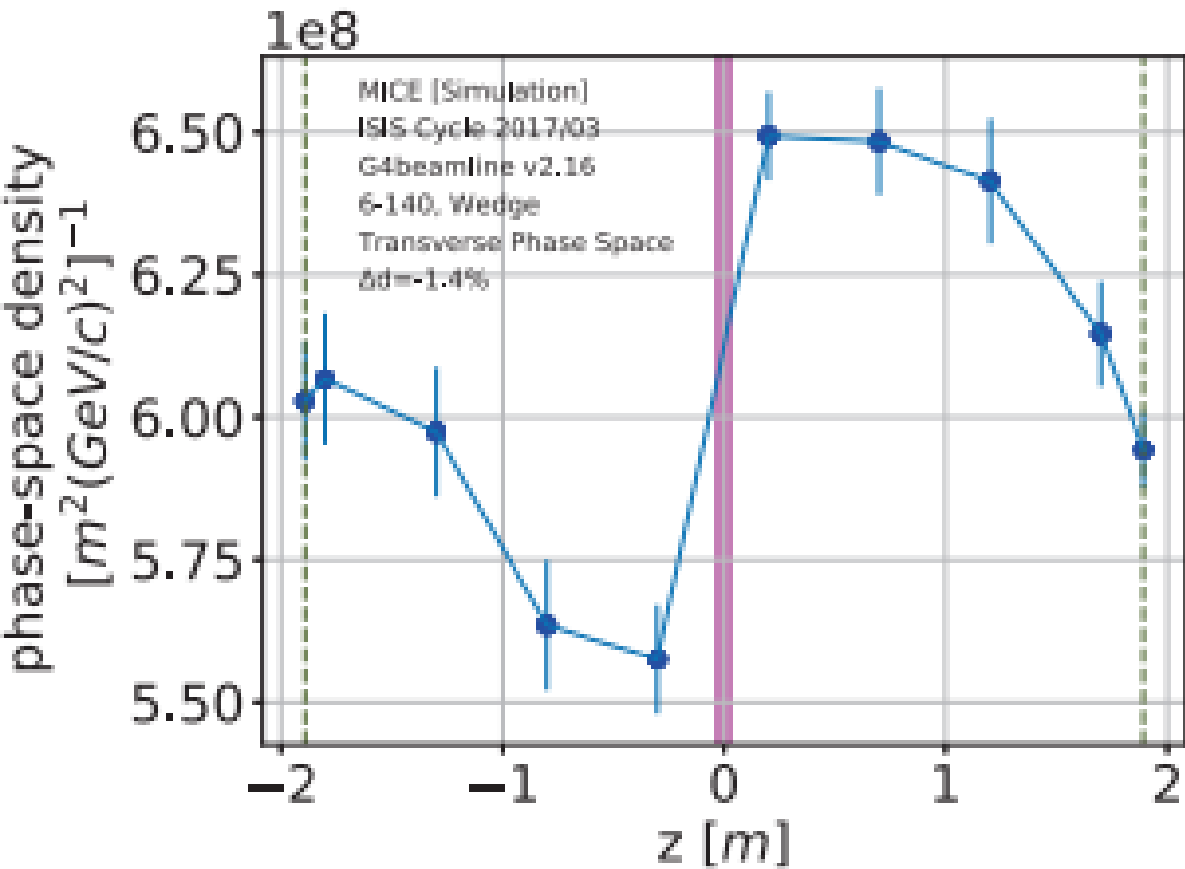
$$\hat{f}(\vec{x}) = \frac{1}{nh^d \sqrt{2\pi}} \sum_{i=1}^n k\left(\frac{-|\vec{x} - \vec{X}_i|^2}{2h^2}\right)$$

- Bottom left: Comparison between KDE, Histogram and a parametric approach

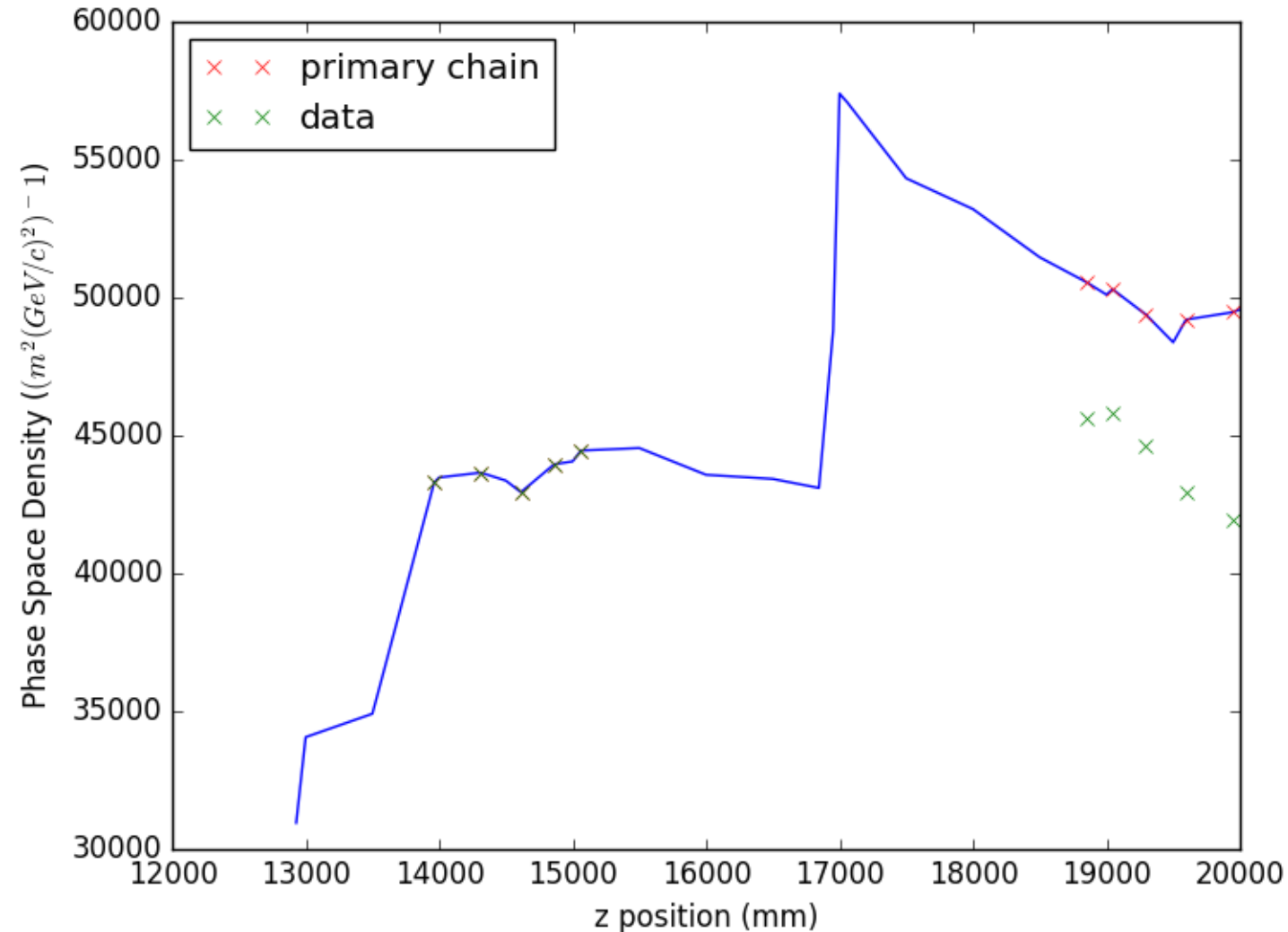
Parametric methods make an assumption of the underlying distribution



## Tanaz's 6-140 transverse 4D results – IPAC2018



# Try to recreate Tanaz's results (Analysis Workshop)



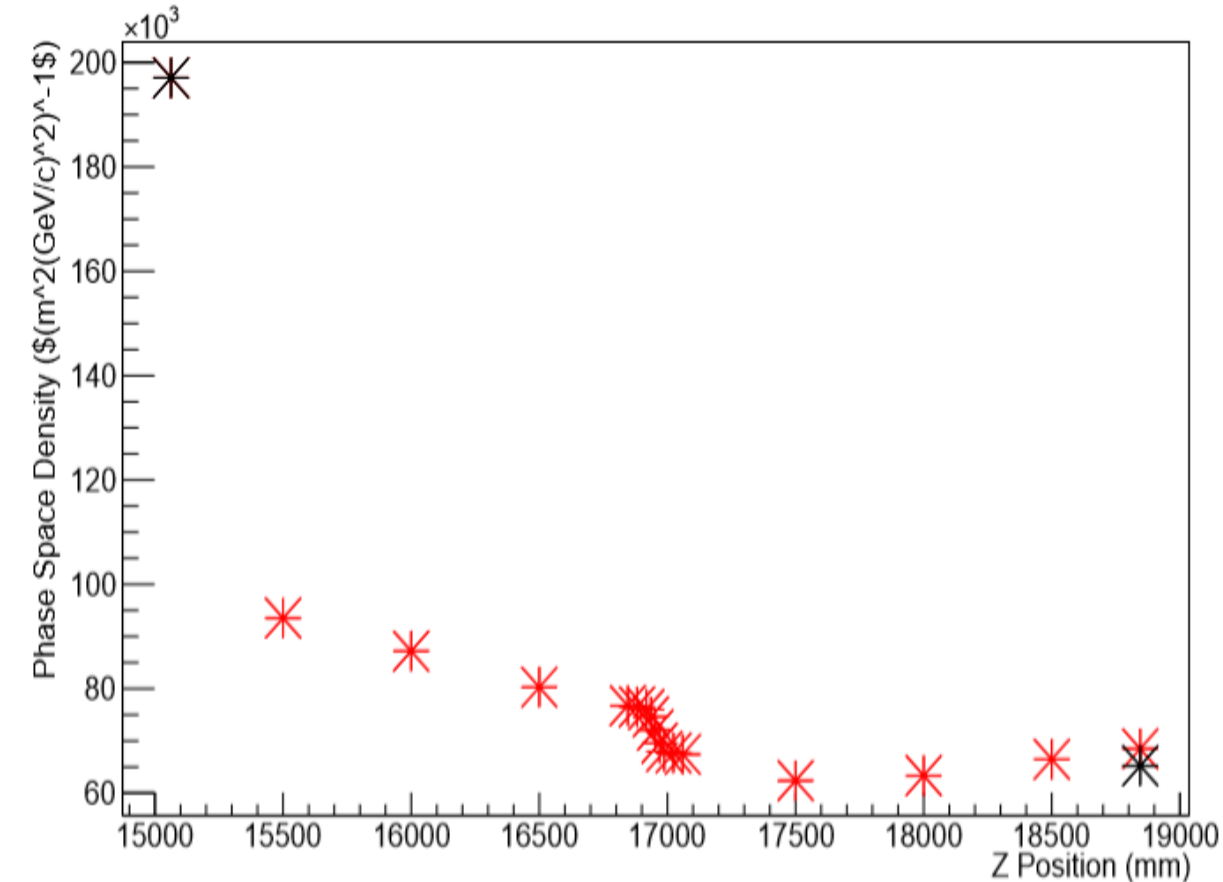
## Problems

- Only started looking at Tanaz's work, so I have likely made some mistakes
- Axial look-up, cylinder not very Wedge shaped, primary chain extrapolation doesn't reflect Wedge
- Need to run extrapolation with Wedge geometry
- I am off by a factor of  $>10^4$
- No Cuts on data
- Only require a matched upstream track to a downstream track

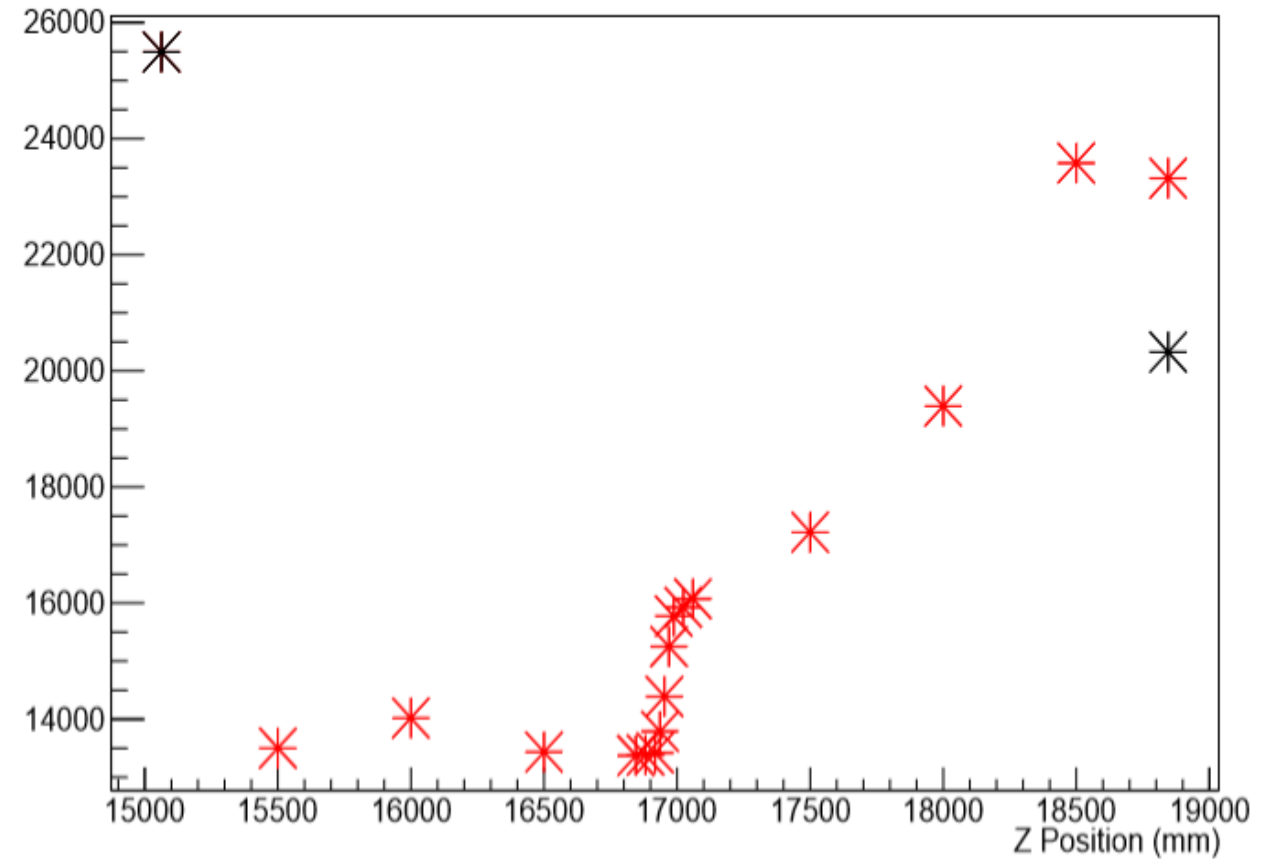


## 3-140 (left) and 10-140 (right)

10532



10535



# Summary

- ▶ Beam moment evolution through cooling channel looks reasonable
- ▶ Will use Chris Rogers beam weighing algorithm to select a smooth distribution
- ▶ Use that as input to calculate KDE
- ▶ KDE analysis needs more work
- ▶ Will try to get further clarifications from Tanaz, and how she arrived at her results, I believe I am missing something
- ▶ Write own code to compare with Tanaz's results

The End