(SM)EFT, thoughts about what everybody has seen

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outline

Discussing several issues that arise in constructing an EFT, up to and including $\mathrm{dim} = 8$ operators

① Considerations on validity

② Local, non-local, hard, soft and all that or why you should not forget loopy EFT and Landau singularities: an "improved" review

③ Mixing or why SMEFT may not be as general as we think

④ Linear vs. quadratic EFT representation … I hate the scent of blood

I do not make any warranties about the completeness of this information

① Solomon:2017nlh

② Henning:2014wua, delAguila:2016zcb, Ellis:2016enq, Fuentes-Martin:2016uol, Ellis:2017jns, Donoghue:2017pgk, Henning:2016lyp

③ Gorbahn:2015gxa, Wells:2017aoy, Gabelmann:2018axh

④ Einhorn:2013kja, Gripaios:2015qya, Jiang:2016czg, Henning:2017fpj, Brivio:2017vri, Barzinji:2018xvu, Criado:2018sdb, Hays:2018zze, Helset:2018dht, Gripaios:2018zrz, Jiang:2018pbd, Quevillon:2018mfl, Bakshi:2018ics, Criado:2019ugp, Brehmer:2015rna, Biekotter:2016ecg, Degrande:2016dqg, Boggia:2016asg, Boggia:2017hyq, Alte:2017pme,2018xgc

derivative-coupled field theories $\mathscr{L} = \underbrace{\mathscr{L}_0}_{\text{propagator}} + \underbrace{\mathscr{L}_\mathrm{i}}_{\text{vertices}}$

$$\mathscr{L}_0 = \frac{1}{2}\,\phi\left(\Box - m^2 - \frac{a}{\Lambda^2}\,\Box^2\right)\phi \qquad\qquad \mathscr{L}_\mathrm{i} = -\frac{1}{4}\,\lambda\,\phi^4$$

- Validity of Matthews's theorem, i.e. the Feynman rules are just those obtained by using $\mathscr{L}_\mathrm{i}$ to determine the vertices and the covariant T^* product to determine the propagators (in other words, one can read Feynman rules from the Lagrangian).
- The validity of the theorem has been proven long ago in Bernard:1974st where an equivalent Lagrangian is obtained which contains only first derivatives but yields the same results (original method due to Ostrogradsky)

Spectrum of the theory : there are two masses, solutions of the equation

$$\frac{\mathrm{a}}{\Lambda^2}\mu_\pm^4 - \mu_\pm^2 + m^2 \quad = \quad 0$$

① To exclude **tachyons** we must have

$$\mathrm{a} > 0, \qquad \qquad \mathrm{a}\frac{m^2}{\Lambda^2} < \frac{1}{4}$$

② $\mu_-^2 \sim m^2(1 + \mathrm{a}\, m^2/\Lambda^2)$; however, there is a negative metric for the particle with the larger mass (μ_+), i.e. there is a **ghost** in the spectrum.

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$\mathrm{dim} = 8$ **à la** *Ostrogradsky*

$$\mathscr{L}_8 = -\frac{1}{2}\frac{\mathrm{a}}{\Lambda^4}\,\phi\,\Box^3\,\phi \quad \equiv \quad \frac{1}{\sqrt{\mathrm{a}}}\,\Lambda^2\,\psi_1\,\psi_2 - \psi_1\,\Box\,\psi_1 + \frac{1}{\sqrt{2}}\,\psi_2\,\Box\,\phi$$

- Mass spectrum: $\boldsymbol{\frac{a}{\Lambda^4}\,\mu^6 - \mu^2 + m^2 = 0}$
- All roots are real iff $0 < \mathrm{a} < \frac{4}{27}\,\frac{\Lambda^4}{m^4}$
- however the product of the roots is $-m^2$; therefore,

  there is at least one tachyon in the spectrum .

$\mathscr{G}$

EFT Option (first order in $1/\Lambda^2$): the "dangerous" term ($\Box^2$) is substituted by using EoMs where terms of $\mathcal{O}(\Lambda^{-2})$ are neglected.

(Of course one could work at second order in Λ^{-2}, including $\dim = 8$ operators).

EFT option is an effective realization of the original $\mathscr{L}$, where one assumes that $\mathscr{L}_{\text{eff}}$ will be replaced by a "well-behaved" $\mathscr{L}_{\text{new}}$ at some larger scale, therefore justifying a truncated perturbative expansion in $1/\Lambda^2$, even in the quadratic part of $\mathscr{L}^*$.

- Nevertheless, the **"Ostrogradsky" option** tells us something [†] about the range of validity of $\mathscr{L}_{\text{eff}}$, i.e.
  1. $0 < \frac{m^2}{\Lambda^2}\,\mathrm{a} < \frac{1}{4}$ (no tachyons),
  2. $\mathrm{E} \ll \mu_+$, where $\mu_+ \sim a^{-1/2}\,\Lambda$ is the upper real (positive) root and E is the scale at which we test the predictions of $\mathscr{L}_{\text{eff}}$, i.e. E must be well below the region where the (resummed) theory develops ghosts.

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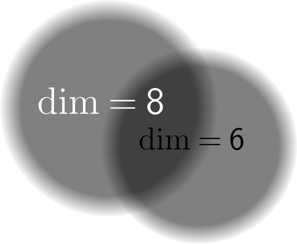
[*] EFT does not have a ghost while remaining within its regime of validity.

[†] If we were to start probing energies high enough then we would worry about producing the ghost.

I'm moving!!

to →



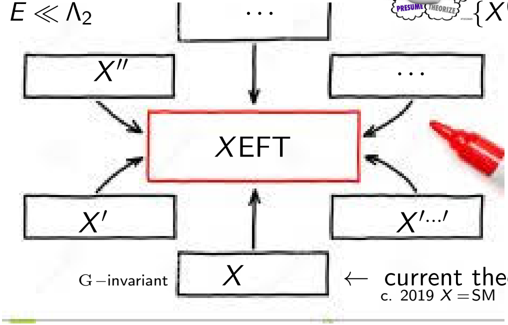
The X tree...

X' etc are UV completions of X or the next theory in a tower of theories

$\mathrm{Rep}_{\mathrm{G}} \supset$ heavy dof $\supset X'$ or X' is F-invariant and $\mathrm{G} \subset \mathrm{F}$

$\{X^{(\prime)}\} \to X^{\mathrm{fin}}$?

next theory $\Lambda_1 < E \ll \Lambda_2$



G −invariant

$\leftarrow$ current theory $E \ll \Lambda_1$

c. 2019 $X =$ SM

 $X' \to X\mathrm{EFT} \leftarrow X$ up to one loop

- X' described by $\mathscr{L}(\Phi)$, $\Phi = [\{\Phi_{\text{heavy}}\}, \{\Phi_{\text{light}}\}]$

① Expand $\Phi = \overset{\text{source}}{\Phi_c} + \phi$ (BFM)

② Use (heat kernel, a very convenient tool for studying various asymptotics of the effective action)

$$
\begin{aligned}
\mathscr{L}_{\mathrm{BFM}} &= \mathscr{L}_c + \langle \phi, \mathrm{D}\phi \rangle \\
\mathrm{Z}[\Phi_c] &= \int_{\text{both heavy and light}} [\mathscr{D}\phi] \exp\{i\,\mathrm{S}\} = \exp\{i\,\mathrm{S}_c\} \det{}^{-1/2}(\mathrm{D})
\end{aligned}
$$

Make sure that D is self-adjoint w.r.t. $\langle \dots \rangle$

Heat kernel  for

- one-loop divergences and counterterms;
- $1/\Lambda$ expansion of the effective action.

- (in the "naive" version) does not capture the finite $\ln p^2$ parts and there are tricky details

$$\mathscr{L}_{\mathrm{BFM}} = \mathscr{L}_c + \frac{1}{2}\,\phi^\dagger\,\mathrm{Q}\,\phi \qquad\qquad \mathrm{Q} = \Box - \mathrm{M} + \hat{\mathrm{Q}}(\Phi_c)$$

- M is the squared mass matrix. Heat kernel expansion requires computing

$$\mathrm{Tr}\,\ln \mathrm{Q}(x)\,\delta^4(x-y) = \int d^4x\,\frac{d^4q}{(2\pi)^4}\,\mathrm{tr}\,\ln\left[-q^2 - M + \Box + 2\,i\,q\cdot\partial + \hat{\mathrm{Q}}(x\,,\partial_x)\right]$$

- When there is one field or $\mathrm{M}_{ij} = \mathrm{M}^2\,\delta_{ij}$ we write

$$\ln\Big[-(q^2+\mathrm{M}^2)\,(\mathbb{I}+\mathrm{K})\Big]$$

expand $\ln(\mathbb{I}+\mathrm{K})$ in powers of K obtaining the large M-expansion of $\mathrm{S}_{\text{eff}} = 1/2\ln\det(\mathrm{D})$ in terms of (T) tadpole integrals.

- Otherwise, with more heavy scales or mixed heavy-light scales,

…  more math and (Lt) log-tadpole and non-tadpole (finite $\ln p^2$ parts) integrals are needed

$$\mathrm{T}_j = \int \frac{d^{\mathrm{d}}q}{(q^2+M^2)^j}, \qquad \mathrm{Lt}_j = \int \frac{d^{\mathrm{d}}q}{(q^2)^j}\ln(q^2+M^2)$$

Short tour in LogMatLand...

$$\begin{aligned} \mathrm{tr}\,\ln(A+B) &= \mathrm{tr}\,\ln A + \mathrm{tr}\,\ln B, \qquad \text{if} \quad A, B \quad \text{are both positive} - \text{definite} \\ \ln(A\,B) &= \ln A + \ln B, \qquad \text{if} \quad A, B \quad \text{commute.} \end{aligned}$$

To be more precise: let $A, B \in \mathbb{C}^{n\times n}$ commute and have no eigenvalues on $\mathbb{R}^-$; if for every eigenvalue λ_i of A and the corresponding eigenvalue μ_i of B, $|\arg\lambda_i + \arg\mu_i| < \pi$, then $\ln AB = \ln A + \ln B$, the principal logarithm of AB. Expanding the log of a matrix :

$$\ln(A+B) - \ln A = \int_0^\infty d\mu^2 \left[(A+\mu^2\,\mathbb{I})^{-1} - (A+B+\mu^2\,\mathbb{I})^{-1}\right]$$

The *correct* Taylor expansion is Lashkari:2018tjh

$$\ln(A+B) - \ln A = \int_0^\infty d\mu^2 \left[A_+^{-1}\,B\,A_+^{-1} - A_+^{-1}\,B\,A_+^{-1}\,B\,A_+^{-1} + \dots\right],$$

where $A_+ = A + \mu^2\,\mathbb{I}$.

Ahem! For unbounded operators $\Delta = A + B$, the integral on the right-hand-side should be thought of as a limit of Riemann sums in the strong operator topology induced by the domain of the logarithm of Δ. In principle Δ should be a positive operator.

$\mathscr{G}$

$$\mathscr{L}_{\rm SM} - \frac{1}{2}\,\mathrm{M}_s^2\,\mathrm{S}^2 + \mu_s\,\Phi^\dagger\Phi\mathrm{S}$$

expansions, loopy EFT

- $p_i^2 = -\mathrm{M}_{\rm H}^2$ and $s = -(p_1+p_2)^2$.

$$\mathrm{I} \;=\; \mu_{\rm R}^{\varepsilon}\int d^{\rm d}q\,\frac{1}{(q^2+\mathrm{M}_{\rm H}^2)((q+p_1)^2+\mathrm{M}_s^2)((q+p_1+p_2)^2+\mathrm{M}_{\rm H}^2)}$$

① $$\rightarrow\quad \frac{1}{(q+p_1)^2+\mathrm{M}_s^2} = \frac{1}{\mathrm{M}_s^2}\,(\overbrace{1}^{\rm dim=4} - \overbrace{\frac{(q+p_1)^2}{\mathrm{M}_s^2}}^{\rm dim=6} + \dots)$$

$$\mathrm{I}\;\sim\;\frac{i\pi^2}{\mathrm{M}_s^2}\,(\underbrace{\frac{1}{\bar\varepsilon} - \ln\frac{\mathrm{M}_{\rm H}^2}{\mu_{\rm R}^2}}_{+\text{EFT C.T.}} + \underbrace{2-\beta\,\ln\frac{\beta+1}{\beta-1}}_{\text{soft}} + \dots)$$

$$\underbrace{\mathrm{M}_s^2 \gg |\,q^2\,| \sim |\,p_i^2\,|}_{\text{cancels out in the matching}}$$

② $$\rightarrow\quad \frac{1}{(q+p_1)^2+\mathrm{M}_s^2} = \frac{1}{q^2+\mathrm{M}_s^2}\,(1 - \frac{p_1^2+2\,p_1\cdot q}{q^2+\mathrm{M}_s^2} + \dots)$$ respects UV at one loop

$$\mathrm{I}\;\sim\;\frac{i\pi^2}{\mathrm{M}_s^2}\,(\underbrace{1+\ln\frac{\mathrm{M}_s^2}{\mathrm{M}_{\rm H}^2} - \beta\,\ln\frac{\beta+1}{\beta-1}}_{\text{soft + hard}} + \dots)$$

$$|\,q^2\,| \sim \mathrm{M}_s^2 \gg |\,p_i^2\,|$$

à la Mellin-Barnes $\rightarrow$ $\mathrm{I} \sim \frac{i\pi^2}{\mathrm{M}_s^2}\,(1+\ln\frac{\mathrm{M}_s^2}{\mathrm{M}_{\rm H}^2} - \beta\,\ln\frac{\beta+1}{\beta-1} + \dots)$

Low-energy limit, Mellin-Barnes expansion

- Consider a light-heavy-light triangle

$$\mathrm{I} \;=\; \int d^{\mathrm{d}}q\, \frac{1}{(q^2+m^2)\,((q+p_1)^2+M^2)\,((q+p_1+p_2)^2+m^2)}$$

where $p_i^2 = -m^2$ and $s = -(p_1+p_2)^2$. We introduce $M^2 = \lambda\, m^2$ and $s = 4\, m^2\, \mathrm{r}$.

- Mellin-Barnes representation :

$$\mathrm{I} \;=\; \frac{1}{2\,\pi\, i}\int_{-i\infty}^{+i\infty} d\mathrm{t}\, \lambda^{\mathrm{t}-1}\, \mathrm{B}(\mathrm{t}\,,1-\mathrm{t})\,\mathrm{B}(2-2\,\mathrm{t}\,,\mathrm{t})\int_0^1 dy\,\left[1-4\,\mathrm{r}\,y\,(1-y)\right]^{-\mathrm{t}}$$

- Limit $\lambda \to \infty$: the t-integral will be closed over the left-hand complex half-plane at infinity, with double poles at $\mathrm{t} = 0\,, -1\,, \dots$.

- The leading term in the expansion, $\mathscr{O}(1/\lambda)$, gives

$$\mathrm{I} \;=\; \frac{1}{M^2}\left\{1+\ln\lambda-\int_0^1 dy\,\ln\Big[1-4\,\mathrm{r}\,y\,(1-y)\Big]\right\}+\mathscr{O}\left(\frac{1}{M^4}\right),$$

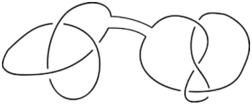
- For $s < 4\,m^2$ (unphysical region) we can expand the second logarithm in powers of r (Taylor expansion), as long as $m \neq 0$
- for $s > 4\,m^2$ we are above the normal threshold and the y-integral is the UV finite part of a two-point functions, showing the non-local, kinematic, logarithm

$$-\beta\,\ln\frac{\beta+1}{\beta-1},\;\beta^2 = 1-\frac{1}{\mathrm{r}} \qquad \left(\ln\frac{-s-i\,0}{\mu_{\mathrm{R}}^2}\ \text{when}\ m=0\right)$$

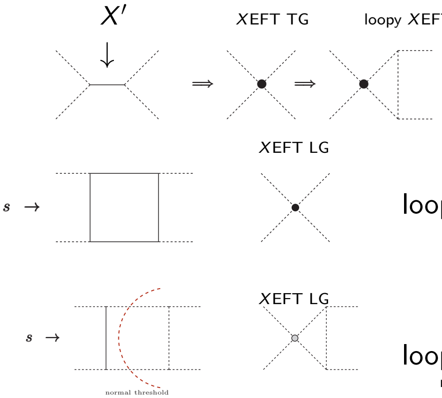
- To summarize, for $\sqrt{s}$, $m \ll M$, we can Taylor expand only in the region $s < 4\,m^2$

- the next term in the expansion is given by the residue of the pole at $\mathrm{t} = -1$ and gives

$$\frac{1}{M^4}\,\frac{2}{\beta^2-1}\left[1+\beta^2-2\,(1-3\,\beta^2)\,\ln\frac{M^2}{m^2}-4\,\beta^3\,\ln\frac{\beta+1}{\beta-1}\right]$$



$X' \to X\mathrm{EFT}$



tree → loop

loop / local

loop / non-local

normal threshold

$$(m \neq 0)\ \ \ln \frac{p^2}{\mathrm{M}^2} \to \beta \ln \frac{\beta+1}{\beta-1} \quad \beta^2 = 1 + \frac{4\,m^2}{p^2}$$

Having said that you start computing ...

- Example: Yukawa model + heavy scalar (SxYM). Local and non-local contributions to the higher-dimensional Lagrangian are better understood in diagrammatic language: let M be the heavy mass and m the light scalar mass (massless fermions).

- There will be terms like

$$\overbrace{\mathrm{L}}^{\text{non-tadpole}} = \int \frac{d^{\mathrm{d}}q\, d^{\mathrm{d}}p}{q^2\,(q^2+M^2)\,(q+p)^2}\, \exp\{i\,p\cdot x\}\,\phi_c(p)\,\overline{\psi}_c(x)\,\psi(x)$$

- giving local and non-local Yukawa couplings of $\mathscr{O}\left(1/M^2\right)$,

$$\mathrm{L}_{\mathrm{loc}} = \frac{i\,\pi^2}{M^2}\,\overline{\psi}_c(x)\,\psi(x)\,\phi_c(x),$$

$$\mathrm{L}_{\mathrm{nloc}} = -\frac{i\,\pi^2}{M^2}\int d^{\mathrm{d}}p\,\exp\{i\,p\cdot x\}\,\overline{\psi}_c(x)\,\psi(x)\,\phi_c(p)\,\underbrace{\ln\frac{p^2-i\,0}{\mathrm{M}^2}}_{\text{kinematic log}}$$

$$\text{soft region}\,(\Lambda \gg \text{all scales}) \quad \to \quad \frac{i\,\pi^2}{M^2}\,\Big(\frac{1}{\overline{\varepsilon}} - \ln\frac{p^2-i\,0}{\mu_{\mathrm{R}}^2} + 2\Big)$$

- with $1/\overline{\varepsilon} = 2/(4-\mathrm{d}) - \gamma - \ln\pi$ and where μ_{R} is the renormalization scale; as shown $\mathrm{L}_{\mathrm{nloc}}$ has a branch cut along the negative p^2-axis.

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local  non-local

Non-local EFT terms are present in loops with heavy and light (internal) lines and show the characteristic pattern of singularities (e.g. normal or anomalous thresholds) of 2(3 ...)-point functions.

- ▷ Consider a one-loop diagram in X' with one heavy internal line and several light internal lines:

① heavy-light contribution $\vdash$ **shrink the heavy line to a point** $\equiv$ **insertion** of one $\dim = 6$, tree-generated, operator inside a one-loop diagram of the theory where the heavy fields have been removed

② the latter is associated with **nloopy XEFT** ‡

③ therefore, when going to nloopy (NLO) EFT one has to be careful in treating heavy-light contributions while matching. Non-local EFT goes beyond the heat kernel tadpoles.

‡The set of one-loop diagrams derivable from $\mathscr{L}_{\mathrm{EFT}}$ containing, at most, one $\dim = 6$ vertex.

The upshot of this is that

- $\mathscr{L}_{\rm EFT}$ mimics the unknown UV by matching the **hard-local** part of the loops, i.e. the terms having a bounded number of derivatives.
  soft-non local components in loops cancel on both sides of the (loopy) matching condition but they are not a throwaway. N.B. we could also introduce a non-local in space, one-loop effective, $\mathscr{L} = \int d^{\rm d}x\, d^{\rm d}y\, \phi(x)\, {\rm L}(x-y)\, \phi(y),\ {\rm L}(z) = \mathscr{F}\{\ln(p^2)\}$

▷ Diagrams of X' with light external legs and heavy internal ones admit a local low-energy limit.

▷ Diagrams of X' with light external legs and mixed internal legs may show normal-threshold singularities in the low-energy region and give inherently non-local parts. Beware, UV logs and kinematic logs are not the same thing. Likewise non-local and "soft" are not synonyms.

kinematic $\Leftarrow$ $\Lambda^2 \gg s_{ij\dots k} = -(p_i + p_j + \dots + p_k)^2 > (m_1 + m_2 + \dots + m_n)^2$ or $\Lambda^2 \gg\mid t \mid\gg m^2$

Therefore: The key advantage of including the non-local behavior is the appearance of some important kinematic dependence

- Important predictions of the EFT are often related to non-analytic contributions which modify tails of distributions.

$X =$ **sigma-model** from Donoghue:2017pgk

- Low-energy behavior of $\mathrm{A}_{\mathrm{full}}(\pi^+\pi^0 \to \pi^+\pi^0)$

$$\boxed{\mathrm{A}_{\mathrm{full}}} \quad \mapsto \quad \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\mathrm{Polynomial}(s\,,\,t\,,\,u) - \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\,\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\,\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\,\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

- $\mathrm{A}_{\mathrm{EFT}}$ computed using $\Sigma_{\mu\nu} = \partial_\mu \mathrm{U}\,\partial_\nu \mathrm{U}^\dagger$ and

$$\boxed{\mathscr{L}_{\mathrm{EFT}}} \quad = \quad \frac{\mathrm{v}^2}{4}\,\mathrm{Tr}\,\Sigma^\mu_\mu + \mathrm{a}_1\left(\mathrm{Tr}\,\Sigma^\mu_\mu\right)^2 + \mathrm{a}_2\left(\mathrm{Tr}\,\Sigma_{\mu\mu}\right)^2$$

- Match "full" and EFT, obtained by including one-loop bubbles (loopy EFT);

$$\boxed{\mathrm{A}_{\mathrm{EFT}}} \quad = \quad \frac{t}{\mathrm{v}^2} + \frac{1}{\mathrm{v}^4}\,\mathrm{Polynomial}(s\,,\,t\,,\,u\,;\,\mathrm{a}_1\,,\,\mathrm{a}_2) - \frac{1}{96\,\pi^2\,\mathrm{v}^4}\left[3\,t^2\,\ln\frac{-t}{\mathrm{M}_\sigma^2} + s\,(s-u)\,\ln\frac{-s}{\mathrm{M}_\sigma^2} + u\,(u-s)\,\ln\frac{-u}{\mathrm{M}_\sigma^2}\right]$$

sigma-model Cont'd

- derive (renormalized) Wilson coefficients

$$a_1 = \frac{1}{8}\,\frac{v^2}{M_\sigma^2} + \frac{1}{384\,\pi^2}\,\Big(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{35}{6}\Big) \qquad\qquad a_2 = \frac{1}{192\,\pi^2}\,\Big(\ln\frac{M_\sigma^2}{\mu_R^2} - \frac{11}{6}\Big)$$

- Compare with the tree-level matching and conclude that we have taken into account an important kinematic feature,

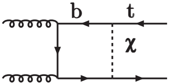
  the logarithmic dependence upon the characteristic momentum transfer in the problem

  $\mathscr{L}_{EFT}$ and A_{EFT} have a different meaning; the Lagrangian is local (as it should), the amplitude generates long-distance kinematic logarithms.

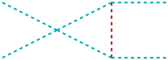
  However, it is a 

nothing prevents us from introducing a one-loop, effective, non-local Lagrangian including all processes up to a given order.

A more difficult case: CxSM



a



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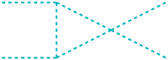

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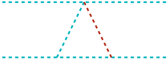

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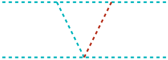


e

$\mathrm{g}\,\mathrm{g} \to \overline{\mathrm{t}}\,\mathrm{t}$ reducible to 4 scalars

After $1/M^2$ Mellin-Barnes expansion

heavy line

a,c) generate $\mathrm{Li}_2(\beta_-^{-1}) + \mathrm{Li}_2(\beta_+^{-1})$

$$\beta_\pm = \frac{1}{2}\,(1 \pm \beta) \quad \beta^2 = 1 - \frac{4\,m_{\mathrm{b}}^2}{s}$$

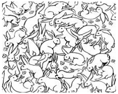
b) generates $\ln \frac{m_{\mathrm{b}}^2}{\mu_{\mathrm{R}}^2} + \beta \ln \frac{\beta+1}{\beta-1}$

d,e) generate $\ln \frac{m_{\mathrm{b}}^2}{\mu_{\mathrm{R}}^2}$

$\mathrm{g}\,\mathrm{g} \to \overline{\mathrm{b}}\,\mathrm{b}$, non-local only for $s > 4\,m_{\mathrm{t}}^2$

...... Summary

- Non-local effects correspond to long distance propagation and hence to reliable predictions at low energy.
- Local terms by contrast summarize the unknown effects from high energy.
- Having both local and non-local terms allows us to implement the full (one-loop) EFT program.
- Heavy-light terms describe a multi scale scenario: the light masses, the Mandelstam invariants characterizing the process and the heavy scale; on the whole they are the leading term in the Mellin-Barnes expansion of X'



Proliferation of scalars and mixing.

- The lack of discovery of beyond-the-SM (BSM) physics suggests that the SM is "isolated" (Wells:2017aoy), including small mixing between light and heavy scalars. The small mixing scenario raises the following question:

if there are many scalars then we have to assume that there is at least the same small mixing for every one of them.

- This is no longer accidental but systematic, and so must involve a principle; this principle is unknown

NB SMEFT assumes a Higgs doublet, so any mixing among scalars (in general among heavy and light degrees of freedom) in the high-energy theory brings us to the HEFT/SMEFT dichotomy. Although there is a wide class of BSM models that support the linear SMEFT description, this realization does not always provide the appropriate framework.

$\mathscr{G}$

Mixing, low-energy behavior of $X' \Longleftrightarrow X$EFT and *residual* **gauge invariance**

- This question can be illustrated by starting with $\mathscr{L}_{\mathrm{SM}}$

$$\mathrm{A}_\mu(\mathrm{Z}_\mu) \quad \to \quad \mathrm{A}_\mu(\mathrm{Z}_\mu) + i\,g\,\mathrm{s_W}\,(\mathrm{c_W})\,(\Lambda^-\,\mathrm{W^+}_\mu - \Lambda^+\,\mathrm{W^-}_\mu) - \partial_\mu\,\Lambda^{\mathrm{A(Z)}}$$

- and by integrating out the massive electroweak gauge bosons, the Higgs boson, and the top quark fields.
- The gauge group of the resulting low-energy effective field theory (LEFT) is QCD $\times$ QED (Jenkins:2017jig,Jenkins:2017dyc), $\mathrm{A}_\mu \to \mathrm{A}_\mu - \partial_\mu\,\Lambda$ and the photon is not the $U(1)$ field in $SU(3) \times SU(2) \times U(1)$.
- Stated differently, $\mathrm{W/Z}$ are integrated out, *not the* $SU(2)$ *fields*.

Mixing and low-energy

① The choice of Λ is crucial when going from X' to X; Λ is (generally) a ratio of masses and powers of couplings.

② The low energy behavior of X' should be computed in the mass eigenbasis, not in the weak eigenbasis (mixing angles (Λ) $\mapsto$) Large number of $1/\Lambda^2$ terms due to expansion of mixing angles, not to the integration of heavy fields.

For Example: Consider the singlet extension of the SM (RxSM) where we have one scalar doublet and one singlet; the SM scalar field Φ is

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} h_2 + \sqrt{2}\,v + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix}$$

while the singlet is $\chi = 1/\sqrt{2}(h_1 + v_s)$. There is a mixing angle, N.B. $\alpha(\Lambda)$, such that

$$h = \cos\alpha\, h_2 - \sin\alpha\, h_1 \qquad H = \sin\alpha\, h_2 + \cos\alpha\, h_1$$

are the mass eigenstates, one light Higgs (h) and one heavy Higgs (H).

❍ Gauge invariance of the low energy limit is complicated since we integrate the H field, and h does not transform as the SM Higgs boson. SM decoupling limit can only be achieved by imposing further assumptions on the couplings.

▷ For instance $\mathscr{L}(\mathrm{h}\,,\mathrm{H}=0)$ alone is not invariant; the full $\mathscr{L}(\mathrm{h}\,,\mathrm{H})$ is $SU(2)\times U(1)$ invariant, but $\mathscr{L}(\mathrm{h}\,,0)$ is not.

❍ Working at $\mathscr{O}\left(1/\Lambda^2\right)$ we can split the total Lagrangian into

$$\mathscr{L}_{\mathrm{H}=0} = \mathscr{L}_{\mathrm{SM}}(\mathrm{h}) + \sum_{n=0,2} \Lambda^{2n-2}\,\delta\mathscr{L}_{6-2n}\,, \qquad \mathscr{L}_{\mathrm{H}} \underset{\text{generates}}{\rightarrow} \mathscr{L}^{\mathrm{TG}} + \mathscr{L}^{\mathrm{LG}} + \mathscr{L}^{\beta}$$

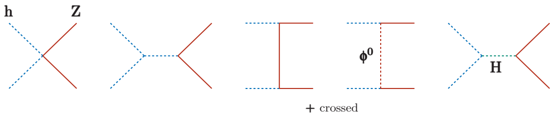
❍ The sum over n in $\mathscr{L}_{\mathrm{H}=0}$ is due to the expansion of $\sin\alpha(\cos\alpha)$ in terms of Λ. After integrating out H in $\mathscr{L}_{\mathrm{H}}$ we will have

① a tree generated $\mathscr{L}_{\text{eff}}$, i.e. $\mathscr{L}^{\mathrm{TG}} = \mathscr{L}_0^{\mathrm{TG}} + \mathscr{O}\left(1/\Lambda^2\right)$,

② a loop generated one, $\mathscr{L}^{\mathrm{LG}}$, and the tadpole contributions

❍ The sum of $\mathscr{L}_{\mathrm{H}=0}$ and of $\mathscr{L}_0^{\mathrm{TG}}$ restores invariance at $\mathscr{O}(1)$. The procedure can be iterated order-by-order 

please read Boggia:2016asg



RxSM color map

$$\sin\alpha = \lambda_2 \frac{M}{M_s} \qquad \cos\alpha = 1 - \frac{1}{2}\left(\lambda_2 \frac{M}{M_s}\right)^2 \qquad M_H^2 = \lambda_1 \left(1 + \lambda_2^2 \frac{M^2}{M_s^2}\right) M_s^2$$

$$A_{\mu\nu} = \left(1 - 2\lambda_2^2 \frac{M^2}{M_s^2}\right) A^{MSM}_{\mu\nu} \qquad T^{sh}_{\mu\nu} = M_Z^2\,\delta_{\mu\nu} + p_{1\mu}\,p_{2\nu}$$

$$+ \frac{g^2}{c_W^2} \lambda_2^2 \frac{M^2}{M_s^2}\left[\frac{1}{2}\,\mathrm{TG} - 1\,\delta_{\mu\nu} + \frac{T^{sh}_{\mu\nu}}{t - M_Z^2} + + \frac{T^{sh}_{\nu\mu}}{u - M_Z^2}\right]$$

❍ SM, one-doublet , Mixing , TG , $M_s^2 = 1/4\, g^2\,(\text{singlet VEV})^2$, depending on RxSM parameters

$$\mathrm{A}^{\mathrm{SMEFT}}_{\mu\nu} = \Big[1 + \frac{1}{3\sqrt{2}\,G_{\mathrm{F}}\,\Lambda^2}\,\big(6\,a_{\phi\mathrm{W}} - a_{\phi\mathrm{D}} + 10\,a_{\phi\Box}\big)\Big]\,\mathrm{A}^{\mathrm{MSM}}_{\mu\nu}$$

$$+ \quad \frac{1}{\sqrt{2}\,G_{\mathrm{F}}\,\Lambda^2}\,\frac{g^2}{\mathrm{c}^2_{\mathrm{W}}}\,\Big[\mathrm{F}_1\,\delta_{\mu\nu} + \mathrm{F}_2\,\mathrm{T}^{sh}_{\mu\nu} + \mathrm{F}_3\,\mathrm{T}^{sh}_{\nu\mu} + \mathrm{F}_4\,\mathrm{T}^{sZ}_{\mu\nu} + \mathrm{F}_5\,\mathrm{T}^{\mathrm{t}}_{\mu\nu} + \mathrm{F}_6\,\mathrm{T}^{\mathrm{u}}_{\mu\nu}\Big]$$

$$\mathrm{F}_1 = 12\,\frac{\mathrm{M}^2}{s-\mathrm{M}^2_{\mathrm{h}}}\,a_\phi + \frac{1}{4}\,\frac{s}{s-\mathrm{M}^2_{\mathrm{h}}}\,\big(a_{\phi\mathrm{D}} - 4\,a_{\phi\Box}\big) - \frac{1}{6}\,\big(7\,a_{\phi\mathrm{D}} - 4\,a_{\phi\Box}\big)$$

$$\mathrm{F}_2 = \frac{1}{6}\,\frac{1}{\mathrm{t}-\mathrm{M}^2_{\mathrm{Z}}}\,\big(5\,a_{\phi\mathrm{D}} - 8\,a_{\phi\Box}\big) \qquad \mathrm{F}_3 = \frac{1}{6}\,\frac{1}{\mathrm{u}-\mathrm{M}^2_{\mathrm{Z}}}\,\big(5\,a_{\phi\mathrm{D}} - 8\,a_{\phi\Box}\big)$$

$$\mathrm{F}_4 = \frac{1}{\mathrm{M}^2_{\mathrm{Z}}}\,\Big(3\,\frac{\mathrm{M}^2_{\mathrm{h}}}{s-\mathrm{M}^2_{\mathrm{h}}} + 1\Big)\,a_{\mathrm{ZZ}} \qquad \mathrm{F}_5 = \frac{2}{\mathrm{t}-\mathrm{M}^2_{\mathrm{Z}}}\,a_{\mathrm{ZZ}} \qquad \mathrm{F}_6 = \frac{2}{\mathrm{u}-\mathrm{M}^2_{\mathrm{Z}}}\,a_{\mathrm{ZZ}}$$

$$\mathrm{T}^{sZ}_{\mu\nu} = p_{3\mu}\,p_{4\nu} + \Big(\frac{1}{2}\,s - \mathrm{M}^2_{\mathrm{Z}}\Big)\,\delta_{\mu\nu}$$

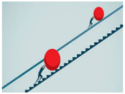
$$\mathrm{T}^{\mathrm{t}}_{\mu\nu} = \big(\mathrm{M}^2_{\mathrm{h}} - \mathrm{M}^2_{\mathrm{Z}} - \mathrm{t}\big)\,\delta_{\mu\nu} - p_{1\mu}\,p_{4\nu} - p_{2\nu}\,p_{3\mu}$$

$$\mathrm{T}^{\mathrm{u}}_{\mu\nu} = \big(\mathrm{M}^2_{\mathrm{h}} - \mathrm{M}^2_{\mathrm{Z}} - \mathrm{u}\big)\,\delta_{\mu\nu} - p_{1\nu}\,p_{3\mu} - p_{2\mu}\,p_{4\nu}$$

extra terms not reproduced even in higher orders in $1/\mathrm{M}_s$

compare with SMEFT...

- Experiments $\Longrightarrow$ $\vec{\mathrm{a}}_{\mathrm{SMEFT}} \equiv$ fitted SMEFT Wilson coeff.
- How to ***compare*** with the low-energy limit of a theory with mixing (X)?



- Eventually fit (directly) $\vec{\mathrm{a}}_X$ ($\forall X$?)

Q : model independent parametrization of mixing

with A. David

In case deviations are observed, one needs to compare at the observable level (O) when interpreting the X parameters. Comparison at the O level implies:

1. In X, with parameters $\vec{p}$, compute an observable $\mathrm{O}^i = \mathrm{O}^i_X(\vec{p})$.
2. Perform a fit of the SMEFT coefficients ($\vec{\mathrm{a}}$) to a set of observables ($\vec{\mathrm{O}}$), that may but does not need to include O^i.
3. Take the best-fit coefficients ($\hat{\vec{\mathrm{a}}}$) from the fit above and compute the SMEFT-predicted observable, $\hat{\mathrm{O}}^i{}_{\mathrm{SMEFT}} = \mathrm{O}^i_{\mathrm{SMEFT}}(\hat{\vec{a}})$.
4. Perform a fit for the X parameters $\vec{p}$ from the comparison to the SMEFT-predicted observable: $\hat{\mathrm{O}}^i{}_{\mathrm{SMEFT}} \sim \mathrm{O}^i_X(\vec{p})$

Should no deviations appear to arise from the SMEFT fit to data, X comes into play.
The result of a global SMEFT fit may yield $\hat{\vec{a}} \sim 0$.

This can *only* be interpreted as no deviations from SM under the SMEFT assumptions, viz. that of a one doublet scalar sector. In a scenario where a X is realized in nature, it is possible that the effects in the set observables used is such that the SMEFT result *seems* to be null. This can come about via an averaging effect, with some observables pulling Wilson coefficients in opposite directions.

Linear vs. quadratic representation (short version)

- Most of the SMEFT calculations include the extra term, i.e.

$$\left|\mathrm{A}^{(4)} + \frac{1}{\Lambda^2}\,\mathrm{A}^{(6)}\right|^2 \quad\to\quad \left|\mathrm{A}^{(4)}\right|^2 + 2\,\frac{1}{\Lambda^2}\,\mathrm{Re}\left[\mathrm{A}^{(4)}\right]^*\mathrm{A}^{(6)} + \frac{1}{\Lambda^4}\left|\mathrm{A}^{(6)}\right|^2$$

  making positive definite (by construction) all the observables.

- $\mathrm{dim} = 8$ operators are (yet) unavailable, but there is more than neglecting the $\mathrm{dim} = 4\,/\,\mathrm{dim} = 8$ interfence:

  we construct S-matrix elements at $\mathscr{O}\left(1/\Lambda^4\right)$ using a canonically transformed $\mathscr{L}$ truncated at $\mathscr{O}\left(1/\Lambda^2\right)$

▷ What we have is

$$\mathscr{L} \quad = \quad -\frac{1}{2}\,(1 + \frac{M^2}{\Lambda^2}\,\delta\mathrm{Z}^6_{\mathrm{H}} + \boxed{\frac{M^4}{\Lambda^4}\,\delta\mathrm{Z}^8_{\mathrm{H}}}\,)\,\partial_\mu\mathrm{H}\,\partial_\mu\mathrm{H} + \dots + \frac{1}{\Lambda^2}\left[\overbrace{\mathrm{a}\,M^3\,\mathrm{H}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu}^{\text{pick at random}} + \dots\right] + \boxed{\frac{1}{\Lambda^4}\,\sum_i\,\mathrm{a}^8_i\,\mathscr{O}^{(8)}_i}$$

where the frame box indicates that the terms are not available.

Linear vs. quadratic representation

- We should write

$$\mathrm{H} \;=\; (1 + \frac{M^2}{\Lambda^2}\,\eta^6_{\mathrm{H}} + \frac{M^4}{\Lambda^4}\,\eta^8_{\mathrm{H}})\,\hat{\mathrm{H}},$$

select

$$\eta^6_{\mathrm{H}} = -\frac{1}{2}\,\delta \mathrm{Z}^6_{\mathrm{H}}, \qquad \eta^8_{\mathrm{H}} = \frac{3}{8}\,\left[\delta \mathrm{Z}^6_{\mathrm{H}}\right]^2,$$

obtaining

$$\hat{\mathscr{L}} \;=\; -\frac{1}{2}\,\partial_\mu \hat{\mathrm{H}}\,\partial_\mu \hat{\mathrm{H}} + \mathrm{a}\,\frac{M^3}{\Lambda^2}\,(1 - \boxed{\frac{1}{2}\,\frac{M^2}{\Lambda^2}\,\delta \mathrm{Z}^6_{\mathrm{H}}}\,)\,\hat{\mathrm{H}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu + \;\dots$$

where the round box gives terms that are neglected in the "naive" quadratic approach.

Linear vs. quadratic representation (longer version)

- To summarize, the proper definition of "quadratic" EFT is as follows: given a "truncated" Lagrangian

$$\mathscr{L} \quad = \quad \mathscr{L}^{(4)} + \frac{1}{\Lambda^2}\,\mathscr{L}^{(6)} + \frac{1}{\Lambda^4}\,\mathscr{L}^{(8)}$$

- we distinguish between redundant and non-redundant operators:

$$\mathscr{L}^{(6,8)} \quad = \quad \mathscr{L}^{(6,8)}_{\mathrm{NR}} + \sum_{i\in\mathrm{R}} \mathscr{O}^{(6,8)}_i\,\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}$$

- redefine fields according to

$$\phi \quad \to \quad \phi - \sum_{n=2,4}\frac{1}{\Lambda^n}\sum_{i\in\mathrm{R}}\mathscr{O}^{(n+4)}_i$$

- The corresponding shift in $\mathscr{L}$ will eliminate redundant operators leaving a (neglected) term

$$\Delta\mathscr{L} \quad = \quad -\frac{1}{\Lambda^4}\Big[\frac{\delta\mathscr{L}^{(4)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}^{(8)}_i + \frac{\delta\mathscr{L}^{(6)}}{\delta\phi}\sum_{i\in\mathrm{R}}\mathscr{O}^{(6)}_i + \frac{1}{2}\,\frac{\delta^2\mathscr{L}^{(4)}}{\delta\phi^2}\sum_{i,j\in\mathrm{R}}\mathscr{O}^{(6)}_i\,\mathscr{O}^{(6)}_j\Big]$$

- Once again, $\Delta\mathscr{L}$ will never generate terms that are not present in $\mathscr{L}^{(8)}$ (symmetry)

Linear vs. quadratic representation (longer version)

- however, we will see a difference when interpreting "fitted" Wilson coefficients in terms of the low-energy behavior of some X'

$$\mathscr{L} \quad = \quad -\frac{1}{2} Z_{\phi}^{ij}\, \partial_\mu \phi_i\, \partial_\mu \phi_j - \frac{1}{2} Z_m^{ij}\, \phi_i\, \phi_j + \mathscr{L}_{\text{rest}},$$

$$\begin{aligned} Z_{\phi}^{ij} &= \delta^{ij} + \frac{1}{\Lambda^2}\, \delta Z_{\phi}^{(6)\,;\,ij} + \frac{1}{\Lambda^4}\, \delta Z_{\phi}^{(8)\,;\,ij}, \\ Z_m^{ij} &= m_i^2\, \delta^{ij} + \frac{1}{\Lambda^2}\, \delta Z_m^{(6)\,;\,ij} + \frac{1}{\Lambda^4}\, \delta Z_m^{(8)\,;\,ij} \end{aligned}$$

- We rescale fields and masses (and possibly couplings) in order to reestablish canonical normalization.
  - ▷ This additional transformation will affect $\mathscr{L}_{\text{rest}}$
- Actually, this is not the end of the story since we have to link the Lagrangian parameters to a given set of experimental data.
  - ▷ These relations will, once again, change $\mathscr{L}_{\text{rest}}$

Linear vs. quadratic representation (longer version)

- Once we have obtained the Lagrangian, up to $\mathscr{O}\left(1/\Lambda^4\right)$, we can obtain Feynman rules and amplitudes. Furthermore, a given $\mathrm{A}_{\mathrm{tree}}$ containing terms up to $\mathscr{O}\left(1/\Lambda^4\right)$ has single and double insertions of $\dim = 6$ operators in the tree diagrams $\in \mathscr{L}^{(4)}$ (plus set of diagrams having new structures, $\notin \mathscr{L}^{(4)}$). Given

$$\mathrm{A} \quad = \quad \mathrm{A}^{(4)} + \frac{1}{\Lambda^2}\,\mathrm{A}^{(6)} + \frac{1}{\Lambda^4}\,\mathrm{A}^{(8)}$$

  - **linear** means including the interference between $\mathrm{A}^{(4)}$ and $\mathrm{A}^{(6)}$,
  - **quadratic** "currently" means including the square of $\mathrm{A}^{(6)}$ and **Not**
  - the **complete inclusion** of all terms giving $1/\Lambda^4$ (before considering $\mathrm{A}^{(8)}$). N.B. heavy-light turn on $\mathrm{Im}\,\mathrm{A}^{(6)}$, i.e. π^2 terms; without $\dim = 8$ the $1/\Lambda^4$ terms are basis-dependent

A continuum EFT is not a model, but a sequence of low-energy effective actions $\mathrm{S}_{\mathrm{eff}}(\Lambda)$, for all $\Lambda < \infty$ [§].

EFT **theories** [¶] are being widely used in an effort to interpret experimental measurements of SM processes. In this scenario, various consistency issues arise; one should critically examine the issues and we argue for the necessity to learn more general lessons about new physics within the EFT approach;

inconsistent results usually attributed to the EFTs are in fact the consequence of unnecessary further approximations.

[§] as highlighted in Costello, Renormalization and Effective Field Theory.

[¶] A theory is aimed at a generalized statement aimed at explaining a phenomenon. A model, on the other hand, is a purposeful representation of reality.

Thank you for your attention


Backup Slides

$\dim = 8$

Field transformations, aka EoM

- Consider a Lagrangian $\mathscr{L}^{(4)} + \mathscr{L}^{(6)}$ containing one real scalar field,

$$\mathscr{L}^{(4)} = -\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}\,m^2\phi^2 - \frac{1}{4}\,\lambda\phi^4, \qquad \mathscr{L}^{(6)} = -\frac{1}{2}\,\frac{a}{\Lambda^2}\,\phi\,\Box^2\,\phi = -\frac{1}{\Lambda^2}\,\mathscr{O}\,\Box\phi$$

- If we perform the transformation

$$\phi \quad\rightarrow\quad \phi + \frac{1}{2}\,\frac{a}{\Lambda^2}\,\Box\phi = \phi + \frac{1}{\Lambda^2}\,\mathscr{O},$$

the Lagrangian transforms as $\mathscr{L} \rightarrow \mathscr{L}^{t} = \mathscr{L} + \Delta\mathscr{L}$, with

$$\begin{aligned}\Delta\mathscr{L} &= \left[\frac{\delta\mathscr{L}^{(4)}}{\delta\phi} - \frac{a}{\Lambda^2}\,\Box^2\phi\right]\frac{1}{\Lambda^2}\,\mathscr{O} + \frac{1}{2}\,\frac{\delta^2\mathscr{L}}{\delta\phi^2}\,\frac{1}{\Lambda^4}\,\mathscr{O}^2 + \cdots = \frac{\delta\mathscr{L}^{(4)}}{\delta\phi}\,\frac{1}{\Lambda^2}\,\mathscr{O} \quad + \quad \text{higher orders}\\ &= \frac{1}{2}\,\frac{a}{\Lambda^2}\,\phi\,\Box^2\,\phi - \frac{1}{2}\,\frac{a}{\Lambda^2}\,(m^2\,\phi + \lambda\,\phi^3)\,\Box\phi \quad + \quad \underbrace{\text{higher orders}}_{\text{compensation}}\end{aligned}$$

- The term of second order in the derivatives cancels in $\mathscr{L}^{t}$ and the S-matrix remains unchanged.
- Note that we have neglected higher order terms since the goal was constructing the $\dim = 6$ Lagrangian. Of course one could work at second order in Λ^{-2}, including $\dim = 8$ operators, etc.

- Usually we find statements like “by using EoM we can remove …”, meaning that many linear combinations of operators “vanish by the EoMs”.

If one is interested in the $\mathrm{dim} = 6$ basis then the necessary EoMs are going to be used at $\mathcal{O}(1)$, i.e. we can derive them from $\mathscr{L}^{(4)}$ alone

This last statement, taken out of context, creates the impression that the $\mathrm{dim} = 8$ basis requires EoMs used naively at $\mathcal{O}\left(1/\Lambda^2\right)$, which is

Scherer:1994wiu

No elimination without compensation e.g. in Warsaw basis

$\mathrm{dim} = 6$ redundant operator

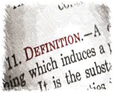
$$\mathcal{O}_{\mathrm{R}}^{(6)} = \overline{\mathrm{Q}}_{\mathrm{L}}\,(\mathrm{D}_\mu\,\mathrm{D}_\mu\,\Phi)\,\mathrm{u}_{\mathrm{R}} + \overline{\mathrm{u}}_{\mathrm{R}}\,(\mathrm{D}_\mu\,\mathrm{D}_\mu\,\Phi)^\dagger\,\mathrm{Q}_{\mathrm{L}}$$

field transformation

$$\Phi \to \Phi - g\,\frac{\mathrm{a}_{\mathrm{R}}^{6}}{\Lambda^2}\,\overline{\mathrm{u}}_{\mathrm{R}}\,\mathrm{Q}_{\mathrm{L}}$$

$\mathrm{dim} = 8$ compensation

$$\to (\overline{\mathrm{Q}}_{\mathrm{L}}\,\Phi\,\mathrm{u}_{\mathrm{R}})(\overline{\mathrm{Q}}_{\mathrm{L}}\,\Phi^c\,\mathrm{d}_{\mathrm{R}})$$



Higher order compensation of redundant operators.

- Removing a redundant $\mathscr{O}^{(6)}$ with a Wilson coefficient a^6_R will propagate a^6_R into the Wilson coefficients of $\dim = 8$ operators.
- In the bottom-up approach it does not matter since we only "measure" combinations of Wilson coefficients, linear in the a^8_i and quadratic in a^6_R ‖. **Thus, the shift due to the field redefinition can be absorbed into the coefficients of operators that are already present in the theory**.
- However, the low energy limit of X' may contain some $\mathscr{O}^{(8)}_\mathrm{a}$ as well as some $\mathscr{O}^{(6)}_R$ whose $\dim = 8$ compensations contain $\mathscr{O}^{(8)}_\mathrm{a}$; the Wilson coefficient a^8_a is now computable in terms of the parameters of X' **but what we have "measured" at low energy is not a^8_a**.

$$
\begin{array}{ccc}
X' & \longrightarrow & \mathscr{O}^{(8)}_\mathrm{a} \\
\downarrow & \nearrow & \\
\mathscr{O}^{(6)}_R & \longrightarrow & \mathscr{O}^{(6)}_\mathrm{b}, \mathscr{O}^{(6)}_\mathrm{c} \ldots
\end{array}
$$

‖ Indeed, when constructing the original EFT, one must include all possible operators consistent with the symmetries at every order in the $1/\Lambda$ expansion.

example

① Start by considering the Lagrangian

$$\mathscr{L}^{(4)} = -\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}\,m^2\phi^2 - \frac{1}{4\,!}\,g\phi^4$$

with a symmetry $\phi \to -\phi$. How to construct $\mathscr{L}^{(6)}$ and $\mathscr{L}^{(8)}$?

② For $\mathrm{dim} = 6$ we have 7 operators, reducible by IBP identities to

$$\mathscr{L}^{(6)} = \frac{1}{\Lambda^2}\left[g^4\,\mathrm{a}_0^6\,\phi^6 + \mathrm{a}_1^6\,\phi\,\Box^2\phi + g^2\,\mathrm{a}_2^6\,\phi^3\,\Box\phi\right]$$

③ For $\mathrm{dim} = 8$ we get

$$\begin{aligned}\mathscr{L}^{(8)} &= \frac{1}{\Lambda^4}\Big\{g^6\,\mathrm{a}_0^8\,\phi^8 + g^4\,\mathrm{a}_1^8\,\phi^5\,\Box\phi + \mathrm{a}_2^8\,(\Box\phi)\,\Box^2\phi \\ &+ \; g^2\left[\mathrm{a}_3^8\,\phi^3\,\Box^2\phi + \mathrm{a}_4^8\,\phi^2\,(\partial_\mu\partial_\nu\phi)\,(\partial_\mu\partial_\nu\phi) + \mathrm{a}_5^8\,\phi^2\,(\Box\phi)^2\right]\Big\}\end{aligned}$$

example

① We eliminate all the operators containing $\Box^n \phi$; this can be achieved by transforming ϕ.

② After the transformation the Lagrangian becomes

$$\mathscr{L} = \ldots + \frac{g^2}{\Lambda^4}\,\phi^2\,(\partial_\mu \partial_\nu \phi)^2 \left[a_4^8 + 6\, a_3^8 + a_2^8 - 9\, a_1^6\, a_2^6 - 2\,(a_1^6)^2 \right]$$

- In fitting the data we constrain the combinations of coefficients appearing in $\mathscr{L}$; after that Definition: the Wilson coefficients are the pseudo-data.
- When interpreting the results we should remember that the coefficient of $\phi^2\,(\partial_\mu \partial_\nu \phi)^2$ is not a_4^8, etc.

▷ ∴ caution should be used in constructing the coefficients in the $\dim = 8$ part of the basis if we want to extract the parameters of X' from the pseudo-data.

example

Perform canonical normalization (little more than LSZ, normalize sources, make the propagators fully diagonal, with residue 1) and recombine Wilson coefficients

$$\bar{a}^6_0 = a^6_0 - \frac{1}{12}\,a^6_1 - \frac{1}{2}\,a^6_2, \qquad \bar{a}^6_1 = a^6_1,$$

$$\bar{a}^8_0 = a^8_0 + \frac{1}{216}\,(2\,a^8_5 - 6\,a^8_3 - a^8_2 + 12\,a^8_1), \qquad \bar{a}^8_1 = \frac{1}{36}\,(5\,a^8_5 - 15\,a^8_3 - 2\,a^8_2 + 12\,a^8_1),$$

$$\bar{a}^8_2 = 2\,a^8_2 + 12\,a^8_3, \qquad \bar{a}^8_3 = \frac{1}{4}\,(6\,a^8_3 - \bar{a}^8_2 - 6\,a^8_5),$$

and obtain

$$\mathscr{L} \to \overline{\mathscr{L}} = \ldots + \frac{\bar{g}^2}{\Lambda^4}\,\bar{\phi}^2\,(\partial_\mu \partial_\nu \bar{\phi})\,(\partial_\mu \partial_\nu \bar{\phi})\,\left[\bar{a}^8_4 - \frac{1}{2}\,(\bar{a}^6_1 - 12\,\bar{a}^6_2)\,\bar{a}^6_1\right]$$

$$= \ldots + \frac{\bar{g}^2}{\Lambda^4}\,c^8_4\,\bar{\phi}^2\,(\partial_\mu \partial_\nu \bar{\phi})\,(\partial_\mu \partial_\nu \bar{\phi})$$

...... Summary

- Suppose that we use $\mathscr{L}^{\mathrm{XEFT}}_{\text{Abasis}}$ to fit the data and that c^8_4 results to be compatible with zero.
- Next, we consider $X'(\{\mathrm{p}\})$; imagine that, after computing the low energy limit, we obtain
  ① a set of $\dim = 8$ operators, including $\bar{\phi}^2 (\partial_\mu \partial_\nu \bar{\phi})(\partial_\mu \partial_\nu \bar{\phi})$ with coefficient $\mathrm{d}(\{\mathrm{p}\})$;
  ② a set of $\dim = 6$ operators, some of them redundant in A-basis

not the same basis 

▷ Different extensions turn on different bases, to compare we need to change basis, including the higher order compensations; therefore, we **cannot conclude that** $\mathrm{d}(\{p\}) = 0$.

Functional integration and non-tadpole integrals

$$\begin{aligned}\mathscr{L}_{\rm SxYM} &= -\frac{1}{2}\,\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}\,m^2\phi^2 - \frac{1}{4}\,g\phi^4 - \overline{\psi}\,(\partial\!\!\!/ - \lambda_1\,\phi)\,\psi \\ &- \frac{1}{2}\,\partial_\mu S\,\partial_\mu S - \frac{1}{2}\,M^2\,S^2 - \frac{1}{4}\,\lambda_4\,S^4 + \lambda_2\,\overline{\psi}\psi\,S - \lambda_3\,\phi^2\,S^2\end{aligned}$$

- Apply BFM, define $\eta = (\lambda_1\,\phi + \lambda_2\,S)\,\psi_c$ and generate non-tadpoles

$$Z_f = \exp\{-{\rm Tr}\,\ln\partial\!\!\!/\}\left[1 - i\,\lambda_1\int d^4z\,\phi_c(z)\,\frac{\delta^2}{\delta\eta(z)\,\delta\overline{\eta}(z)} + \dots\right]\exp\{i\int d^4x d^4y\,\overline{\eta}(x)\,S_F(x-y)\,\eta(y)\}$$

- $\mathscr{L}_{\rm eff}$ derived from (Φ contains both (light) ϕ and (heavy) S)

$$\begin{aligned}Z &= \exp\{-{\rm Tr}\,\ln(\partial\!\!\!/)\}\int[\mathscr{D}S]\,[\mathscr{D}\phi]\,\exp\{i\int d^4xd^4y\,\mathscr{L}_\Phi(x,y)\} \\ &\times \left[1 + \text{ loops } + i\int d^4x_1\,d^4x_2\,\overline{\eta}(x_1)\,\Gamma(x_1,x_2)\,\eta(x_2) + \dots\right]\end{aligned}$$

$$\mathscr{L}_\Phi = \frac{1}{2}\,\Phi^\dagger(x)\,D(x)\,\Phi(x)\,\delta^4(x-y) + \overline{\eta}(x)\,S_F(x-y)\,\eta(y) \qquad \Gamma(x,y) = \sum_{i,j}\lambda_1^i\,\lambda_2^j\,\Gamma_{ij}(x,y)$$

- where Γ_{ij} are open strings of γ-matrices and of propagators S_F while "loops" indicates closed strings, generating loop diagrams with internal fermion lines

More on SxYM

- Non-local terms at the Lagrangian level (spectral decomposition)

$$\int d^4x\,\mathrm{L}_{\mathrm{nloc}} = -\frac{i\pi^2}{\mathrm{M}^2}\int d^4x d^4y\,\overline{\psi}_c(x)\,\psi(x)\,\Sigma_{\mathrm{M}}(x-y)\,\phi_c(y) \qquad \Sigma_{\mathrm{M}}(\mathrm{z}) = \int_0^\infty d\mu^2\left[\frac{\delta^4(\mathrm{z})}{\mu^2+\mathrm{M}^2} - \Delta_{\mathrm{F}}(\mathrm{z};\mu^2)\right]$$

$$\Delta_{\mathrm{F}}(\mathrm{z};\mu^2) = \frac{1}{4\pi^2}\,\frac{\mu}{x}\,\mathrm{K}_1(\mu x)$$

- General form for non-local terms

$$\begin{aligned}
\text{non-local} \quad &= \quad (2\pi)^4\,\mathrm{N}_1\int d^4p d^4q_1 d^4q_2\,\delta^{(4)}(p+q_1+q_2)\,\overline{\psi}_c(q_1)\,\psi_c(q_2)\,\phi_c(p)\\
&+ \quad (2\pi)^4\sum_{i=1,2}\mathrm{N}_{2i}\int d^4p_1 d^4p_2 d^4q_1 d^4q_2\,\delta^{(4)}(p_1+p_2+q_1+q_2)\\
&\times \quad \overline{\psi}_c(q_1)\,\not{p}_i\,\psi_c(q_2)\,\phi_c(p_1)\,\phi_c(p_2)
\end{aligned}$$

$$\mathrm{N}_1 = \lambda_1\,\lambda_2^2\,\frac{p^2}{M^2}\,\ln(p^2) \qquad \mathrm{N}_{2i} = \lambda_1^2\,\lambda_2^2\,\frac{\mathrm{F}_i(p_1,p_2)}{M^2}$$

- where the functions F_i are combinations of three-point tensor integrals and $\lambda_{1,2}$ are SxYM couplings

$$\lambda_1\,\overline{\psi}\psi\phi \qquad \lambda_2\,\overline{\psi}\psi\mathrm{S}$$

SMEFT, HEFT and mixing  can we distinguish mixed from inert?

$$\underbrace{\mathrm{H} \to \mathrm{V}^{\mu}(p_1)\,\mathrm{V}^{\nu}(p_2)}_{\text{simplest test}} = \mathrm{F}_{\mathrm{D}}^{\mathrm{VV}}\,\delta^{\mu\nu} + \mathrm{F}_{\mathrm{T}}^{\mathrm{VV}}\,\underbrace{\left(p_1 \cdot p_2\,\delta^{\mu\nu} - p_1^{\nu}\,p_2^{\mu}\right)}_{\mathrm{T}^{\mu\nu}}$$

- SMEFT prediction is

$$M_{\mathrm{W}}\,\mathrm{F}_{\mathrm{D}}^{\mathrm{WW}} = -g\,M_{\mathrm{W}}^2\left[1 + \frac{g_6}{\sqrt{2}}\left(\mathrm{a}_{\phi\mathrm{W}} + \mathrm{a}_{\phi\Box} - \frac{1}{4}\,\mathrm{a}_{\phi\mathrm{D}}\right)\right],$$

$$M_{\mathrm{W}}\,\mathrm{F}_{\mathrm{D}}^{\mathrm{ZZ}} = -g\,M_{\mathrm{Z}}^2\,\rho\left[1 + \frac{g_6}{\sqrt{2}}\left(\mathrm{a}_{\phi\mathrm{W}} + \mathrm{a}_{\phi\Box} + \frac{1}{4}\,\mathrm{a}_{\phi\mathrm{D}}\right)\right],$$

$$M_{\mathrm{W}}\,\mathrm{F}_{\mathrm{T}}^{\mathrm{WW}} = -2g\,\frac{g_6}{\sqrt{2}}\,\mathrm{a}_{\phi\mathrm{W}}\,, \qquad M_{\mathrm{W}}\,\mathrm{F}_{\mathrm{T}}^{\mathrm{ZZ}} = -2g\,\frac{g_6}{\sqrt{2}}\,\mathrm{a}_{\mathrm{ZZ}}\,,$$

where $\mathrm{a}_{\mathrm{ZZ}} = \mathrm{s}_{\mathrm{W}}^2\,\mathrm{a}_{\phi\mathrm{B}} + \mathrm{c}_{\mathrm{W}}^2\,\mathrm{a}_{\phi\mathrm{W}} - \mathrm{s}_{\mathrm{W}}\,\mathrm{c}_{\mathrm{W}}\,\mathrm{a}_{\phi\mathrm{WB}}$, $\rho = M_{\mathrm{W}}^2/(\mathrm{c}_{\mathrm{W}}^2\,M_{\mathrm{Z}}^2)$ and $\sqrt{2}\,g_6 = 1/(G_{\mathrm{F}}\,\Lambda^2)$.

▷ As a consequence, SMEFT predicts a change in the normalization of the *SM*-like term and the appearance of the transverse term. $\mathscr{O}_{\phi\mathrm{D}}$ is Custodial Symmetry breaking.

Relevant quantities to be constrained, e.g. in $\mathrm{H} \to 4\mathrm{l}$

$\left(\frac{\mathrm{F}_{\mathrm{D}}^{\mathrm{WW}}}{M_{\mathrm{W}}} - \frac{\mathrm{F}_{\mathrm{D}}^{\mathrm{ZZ}}}{M_{\mathrm{Z}}}\right)$ the forbidden even to speak of $\boldsymbol{POs}$ $\mathrm{F}_{\mathrm{T}}^{\mathrm{VV}}$

- a "measure" of $\mathrm{a}_{\phi\mathrm{D}}/\Lambda^2$ and a "measure" of a non-SM tensor structure at $\mathcal{O}\left(1/\Lambda^2\right)$ (i.e. a "measure" of $\mathrm{a}_{\phi\mathrm{W}}$ and a_{ZZ} in SMEFT or of the corresponding operators in HEFT).
- Rewrite

$$\begin{aligned}
\mathrm{H}\,\mathrm{W}_{\mu}^{-}(p_1)\,\mathrm{W}_{\nu}^{+}(p_2) &= \kappa_{\mathrm{H}}^{\mathrm{WW}}\,(1+\frac{g_6}{\sqrt{2}}\,\delta\kappa_{\mathrm{H}}^{\mathrm{WW}})\,\delta_{\mu\nu} - \sqrt{2}\,\frac{g}{M_{\mathrm{W}}}\,g_6\,\mathrm{a}_{\phi\mathrm{W}}\,\mathrm{T}_{\mu\nu},\\
\mathrm{H}\,\mathrm{Z}_{\mu}(p_1)\,\mathrm{Z}_{\nu}(p_2) &= \kappa_{\mathrm{H}}^{\mathrm{ZZ}}\,(1+\frac{g_6}{\sqrt{2}}\,\delta\kappa_{\mathrm{H}}^{\mathrm{ZZ}})\,\delta_{\mu\nu} - \sqrt{2}\,\frac{g}{M_{\mathrm{W}}}\,g_6\,\mathrm{a}_{\mathrm{ZZ}}\,\mathrm{T}_{\mu\nu},
\end{aligned}$$

$$\kappa_{\mathrm{H}}^{\mathrm{WW}} = -g\,M_{\mathrm{W}}\,,\;\; \kappa_{\mathrm{H}}^{\mathrm{ZZ}} = -g\,\frac{M_{\mathrm{Z}}^2}{M_{\mathrm{W}}}\,\rho, \qquad \delta\kappa_{\mathrm{H}}^{\mathrm{WW(ZZ)}} = \mathrm{a}_{\phi\mathrm{W}} + \mathrm{a}_{\phi\Box} \mp \frac{1}{4}\,\mathrm{a}_{\phi\mathrm{D}},$$

Include HHVV and fit

$$(1+\frac{c_1^{\mathrm{VV}}}{\Lambda^2}\,\mathrm{H}+\frac{c_2^{\mathrm{VV}}}{\Lambda^2}\,\mathrm{H}^2+\,\ldots)\,\mathrm{V}_\mu\,\mathrm{V}_\mu+\,\ldots\quad\in\quad\mathscr{L}_{\mathrm{HEFT}}$$

- $c_{1,2}^{\mathrm{VV}}$ give some information on the doublet structure of the scalar field, e.g. SMEFT gives

$$\frac{c_2^{\mathrm{WW}}}{c_1^{\mathrm{WW}}}=\frac{1}{2}\,\frac{g}{M_{\mathrm{W}}}\,(1-\frac{g_6}{\sqrt{2}}\,\delta\kappa_{\mathrm{H}}^{\mathrm{WW}})\qquad\frac{c_2^{\mathrm{ZZ}}}{c_1^{\mathrm{ZZ}}}=\frac{1}{2}\,\frac{g}{M_{\mathrm{W}}}\left[1+\frac{g_6}{\sqrt{2}}\,(a_{\phi\mathrm{D}}-\delta\kappa_{\mathrm{H}}^{\mathrm{ZZ}})\right]$$

with $\delta\kappa_{\mathrm{H}}^{\mathrm{WW}}=\delta\kappa_{\mathrm{H}}^{\mathrm{ZZ}}$ if $a_{\phi\mathrm{D}}=0$ (custodial symm.), i.e. $r_{\mathrm{Z}}=r_{\mathrm{W}}\,(1+\frac{g_6}{\sqrt{2}}\,a_{\phi\mathrm{D}})$

NB the c coefficients can be computed in any X' theory.
$(1+\frac{d_1^{\mathrm{VV}}}{\Lambda^2}\,\mathrm{H}+\frac{d_2^{\mathrm{VV}}}{\Lambda^2}\,\mathrm{H}^2)\,\mathrm{F}^a_{\mu\nu}\,\mathrm{F}^a_{\mu\nu}\;\Rightarrow$ consequences to the kinematics