dave@walnuts: ~

```
dave@walnuts:~$ grep -i "eft" /usr/share/dict/british-english
Left
bereft
chieftain
chieftain's
chieftains
cleft
cleft's
clefts
deft
defter
deftest
deftly
deftness
deftness's
heft
heft's
hefted
heftier
heftiest
hefting
hefts
hefty
left
left's
lefter
leftest
leftie
leftie's
lefties
leftism
leftism's
leftist
leftist's
leftists
leftmost
leftover
leftover's
leftovers
lefts
leftwards
lefty
lefty's
theft
theft's
thefts
weft
weft's
wefts
dave@walnuts:~$
```

Basis construction and translation with DEFT

see B. Gripaios & DS, arXiv:1807.07546, code here

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Introduction

DEFT takes as input a set of fields and their irreps under a set of $SU(N)$-like symmetries. *In principle*, it can output:

- a list of all the contractions of products of these fields which are invariant under the symmetries, to a given order;
- a list of the redundancies between these operators (Fierz, IBPs, EOMs, &c.);
- an arbitrary operator basis;
- a matrix to convert into and between arbitrary operator bases.

This talk:

1. a look under the hood;
2. the extant obstacles to the practical use of DEFT.

Irreps of $SU(N)$

All irreps of $SU(N)$ can be encoded in terms of (anti-)symmetric and traceless combinations of upper and lower indices, $a, b, \ldots = 1, \ldots, N$. The former transforms in the defining irrep., the latter in the conjugate.

$$\begin{array}{lll} \phi^a \to U^a_b \phi^b; & \psi_a \to (U^\dagger)^b_a \psi_b; & A^a_b[A^a_a \equiv 0] \to \ldots . \\ N & \bar{N} & N^2 - 1 \end{array}$$

Conjugation raises/lowers indices

$$\phi^a \overset{\text{h.c.}}{\leftrightarrow} \phi^\dagger_a$$

There are only three invariant tensors

$$\delta^a_b; \quad \epsilon_{ab\ldots z}; \quad \epsilon^{ab\ldots z}.$$

Irreps of Lorentz group

The irreps of $SO(3,1)$ are the irreps of $SU(2)_{\text{lor,L}} \times SU(2)_{\text{lor,R}}$, encoded by combinations of undotted and dotted indices.

$$e_R^{\dot{\alpha}}, B_{\alpha\beta}$$

Conjugation dots/undots indices

$$\psi_\alpha \overset{\text{h.c.}}{\leftrightarrow} \psi^\dagger_{\dot{\alpha}}$$

More 'conventional' representations can always be repackaged in this form, e.g.,

$$\begin{aligned} D_{\alpha\dot{\alpha}} &= \sigma^\mu_{\alpha\dot{\alpha}} D_\mu , \\ F_{\alpha\beta} &= 2i(\sigma^{\mu\nu})_\alpha^\gamma \epsilon_{\gamma\beta} F_{\mu\nu} \,. \end{aligned}$$

‘Monomial operators’

a product of fields; each field may have some covariant derivatives

In DEFT’s internal machinery, all operators are expressed as linear combinations of ‘monomial operators’.

$\bar{W}^a_d \delta^d_a \equiv 0$

$$\delta^{\dot{\alpha}}_{\dot{\gamma}} \delta^{\dot{\beta}}_{\dot{\delta}} \epsilon_{ab} \epsilon^{de} \delta^A_B \; H^b \bar{Q}_L{}^{\dot{\gamma}}_{eA} d_R{}^{\dot{\delta}B} \; \bar{W}^a_{\dot{\alpha}\dot{\beta}d}$$

invariants are L-C epsilons and Kron. deltas

$\bar{W}_{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \equiv 0$

Irreps of $SU(N)$-like symmetries encoded by upper and lower fundamental indices.

Lagrangian redundancies

- Fierz relations $\epsilon^{\cdots}\epsilon_{\cdots} = \sum \delta^{\cdot}_{\cdot}\delta^{\cdot}_{\cdot}\dots\delta^{\cdot}_{\cdot}$ (also Schouten identities)

$$\epsilon^{\dot\alpha\dot\beta}\bar{H}_a \bar{e_R}^{\alpha} D_{\alpha\dot\beta} D_{\beta\dot\alpha} {L_L}^{\beta a} - \epsilon^{\dot\alpha\dot\beta}\bar{H}_a \bar{e_R}^{\alpha} D_{\beta\dot\beta} D_{\alpha\dot\alpha} {L_L}^{\beta a} + \epsilon_{\alpha\beta}\epsilon^{\gamma\delta}\epsilon^{\dot\alpha\dot\beta}\bar{H}_a \bar{e_R}^{\beta} D_{\delta\dot\beta} D_{\gamma\dot\alpha} {L_L}^{\alpha a} = 0$$

- IBPs

$$-\epsilon^{\dot\alpha\dot\beta} D_{\alpha\dot\beta} \bar{L_L}^{\dot\gamma}_{b} {L_L}^{\alpha a} \bar{W}^{b}_{\dot\alpha\dot\gamma a} - \epsilon^{\dot\alpha\dot\beta} \bar{L_L}^{\dot\gamma}_{b} D_{\alpha\dot\beta} {L_L}^{\alpha a} \bar{W}^{b}_{\dot\alpha\dot\gamma a} + \epsilon^{\dot\alpha\dot\beta} \bar{L_L}^{\dot\gamma}_{b} {L_L}^{\alpha a} D_{\alpha\dot\alpha} \bar{W}^{b}_{\dot\beta\dot\gamma a} = 0$$

- Commuting derivatives

$$-\frac{1}{2} i g' \epsilon^{\alpha\beta} B_{\beta\gamma} D_{\alpha\dot\alpha} \bar{e_R}^{\gamma} {e_R}^{\dot\alpha} + \frac{1}{2} i g' \epsilon^{\dot\alpha\dot\beta} \bar{B}_{\dot\beta\dot\gamma} D_{\alpha\dot\alpha} \bar{e_R}^{\alpha} {e_R}^{\dot\gamma} - \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} D_{\gamma\dot\beta} D_{\beta\dot\gamma} D_{\alpha\dot\alpha} \bar{e_R}^{\gamma} {e_R}^{\dot\gamma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} D_{\beta\dot\gamma} D_{\gamma\dot\beta} D_{\alpha\dot\alpha} \bar{e_R}^{\gamma} {e_R}^{\dot\gamma} = 0$$

EOM relations[2]

Local field redefinitions don't affect S-matrix elements

$$\phi(x) \to \phi(x) + \frac{U[\phi](x)}{\Lambda^2}$$

$$m_\phi^2 S_2 + S_4 + \frac{S_6}{\Lambda^2} \to m_\phi^2 S_2 + \left(S_4 + \frac{m_\phi^2}{\Lambda^2}\int \mathrm{d}x U \frac{\delta S_2}{\delta\phi}\right) + \frac{1}{\Lambda^2}\left(S_6 + \int \mathrm{d}x\, U\, \frac{\delta S_4}{\delta\phi}\right) + \mathrm{O}\left(\frac{1}{\Lambda^4}\right)$$

We calculate the marginal EOM, whose parts plus the remainder we find in higher dimensional terms[1]

$$-\frac{1}{2}\epsilon_{\alpha\beta}\,\epsilon^{\gamma\delta}\epsilon^{\dot\alpha\dot\beta}\,\epsilon_{ab}\,D_{\delta\dot\beta}D_{\gamma\dot\alpha}H^b\,{Q_L}^{\beta a A}\bar{u_R}^{\alpha}_{A}$$
$$+y_e^\dagger\,\epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta}\,\epsilon_{ab}\,\bar{e_R}^{\delta}{L_L}^{\gamma b}\,{Q_L}^{\beta a A}\bar{u_R}^{\alpha}_{A} + y_u\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\bar{Q_L}^{\dot\beta}_{aB}\,{Q_L}^{\beta a A}\bar{u_R}^{\alpha}_{A}\,{u_R}^{\dot\alpha B}$$
$$+y_d^\dagger\,\epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta}\,\epsilon_{ab}\,{Q_L}^{\delta b A}\,{Q_L}^{\beta a B}\bar{u_R}^{\alpha}_{B}\,\bar{d_R}^{\gamma}_{A} - 2\lambda\,\epsilon_{\alpha\beta}\,\epsilon_{ab}\,\bar{H}_d H^b H^d\,{Q_L}^{\beta a A}\bar{u_R}^{\alpha}_{A}$$
$$= \text{operators of different dimension, which we ignore}$$

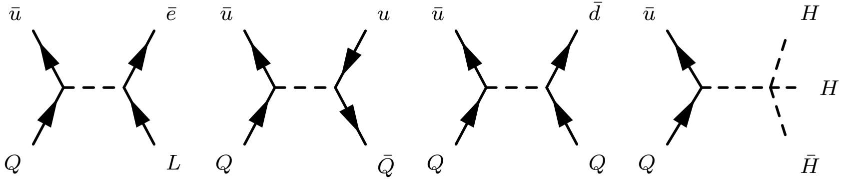
---

[1] taking care over the contractions between the two

[2] Because they look similar, we include Bianchi identities $D_\mu \tilde{F}^{\mu\nu} = 0$ in this class.

EOM relations at the amplitude level

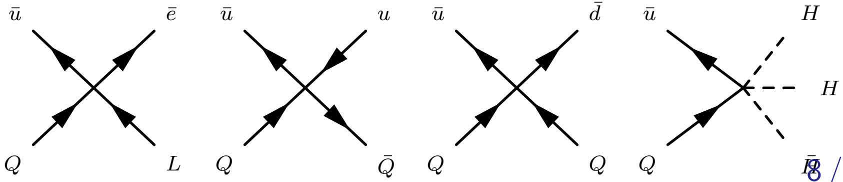
$$Q\bar{u}(-\Box-m_H^2)H = Q\bar{u}(y_e^\dagger L\bar{e}+y_u\bar{Q}u+y_d^\dagger Q\bar{d}+2\lambda HH^\dagger H)+\text{O(dim 8)}$$



with a internal factor of

$$\frac{s}{s-m_H^2} - \frac{m_H^2}{s-m_H^2} = 1$$

is the same as the contact terms



Put all the relations in a matrix

Each relation is a vector of Wilson coefficients $\sum_i c_i \mathcal{O}_i = 0$. Collect these as the rows of a matrix.

$$
\begin{array}{rc}
 & \begin{array}{cccccc} c_1 & c_2 & c_3 & & & c_N \\ \downarrow & \downarrow & \downarrow & & & \downarrow \end{array} \\
\begin{array}{r} \text{relation 1} \longrightarrow \\ \text{relation 2} \longrightarrow \\ \\ \\ \\ \end{array} &
\begin{pmatrix} i & 0 & y_u & 0 & \dots & 0 \\ 1 & 0 & 0 & 1 & \dots & g_s \\ \vdots & & & & & \\ . & & & & & \\ . & & & & & \end{pmatrix}
\end{array}
$$

The rest is linear algebra, for which we use `sympy`. We've checked the ranks of these matrices for various subsets and simple extensions of the one generation SM, using Hilbert series methods.

Making a non-redundant basis

1. Perform column operations[3] to express the relations in terms of the coefficients of the operators we want to keep, $\{\mathcal{B}_1, \mathcal{B}_2\}$, and the ones we want to eliminate, $\{\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3\}$
2. Perform row operations to reduce to reduced row echelon form

$$\begin{array}{ccccc} c_{\mathcal{D}_1} & c_{\mathcal{D}_2} & c_{\mathcal{D}_3} & c_{\mathcal{B}_1} & c_{\mathcal{B}_2} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$
$$\left(\begin{array}{ccc|cc} 1 & 0 & 0 & * & * \\ 0 & 1 & 0 & * & * \\ 0 & 0 & 1 & * & * \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Iff. the matrix can be put in this form, $\{\mathcal{B}_1, \mathcal{B}_2\}$ is a valid basis.

---

[3] By default, the monomial operators are partially ordered by number of derivatives and epsilon tensors.

Converting into and between non-redundant bases

The components of the RREF matrix give expressions for any operator $\mathcal{O}_i \in \{\vec{\mathcal{D}}, \vec{\mathcal{B}}\}$ in terms of the $\mathcal{B}_i$:

$$\mathcal{O}_i = R_{ij}\mathcal{B}_j$$

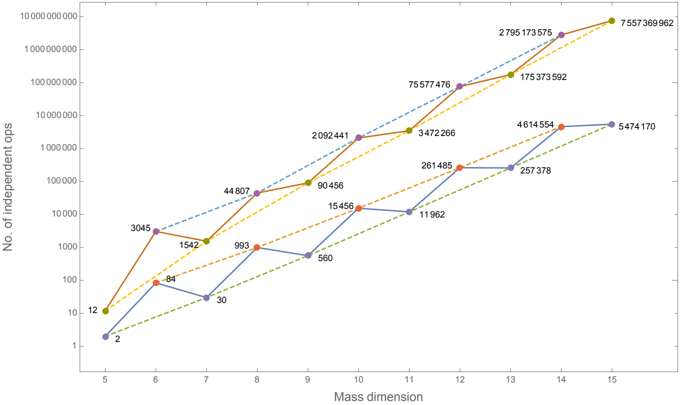
If we can write another basis $\mathcal{B}'_i$ in terms of the $\mathcal{O}_i$ (where the $\mathcal{O}_i$ are by default the monomial terms)

$$\mathcal{B}'_i = S_{ij}\mathcal{O}_j$$

then

$$\mathcal{B}'_i = S_{ij}R_{jk}\mathcal{B}_k$$

DEFT presently runs out of steam pretty quickly



(From Henning, Lu, Melia, Murayama arXiv:1512.03433)

For the one generation Standard Model on a laptop:

- ~ 45 minutes to generate dim 6 operators and convert from Warsaw to SILH basis
- ~ 1 day to generate a basis of dim 8 operators

#TODO

- **Generational indices** Leave them uncontracted or contracted with 0-dimensional spurions in the irreps of the flavour symmetries
- **Convert between invariants** $\gamma^\mu, \lambda^A, \ldots \leftrightarrow \epsilon, \delta$
- **Interfaces** Print-out to FeynRules, ... (although note built-in AllYourBases module)
- **SM in broken phase** Substitute B, W, H for A, Z, W, h
- **Transdimensional interactions** Expand EOM relations to higher orders. Systematically calculate corrections to lower order params.
- **Tabulate higher irreps of $SU(N)$** Currently has the rules for (anti)-fundamental, adjoint and all-symmetric tensors
- **Refactoring** Speed-ups, especially in row reduction methods and memory management

SM applications?

- **Matching** Matching a UV theory to the SMEFT is typically done off-shell. One needs to define a 'Green's function basis' and expressions to convert into the usual 'S-matrix bases'.
- **Phenomenology of** $d > 6$ Can study novel processes and define precisely the validity of the EFT expansion.

Summary

Fields and symmetries in, operators and bases out. In particular, it's been tested against existing results for the one-generation Standard Model.

`DEFT` is very much a 'proof of concept'. All feedback welcome!