NLO corrections in the dimension–6 SMEFT ($h \to bb$)

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SMEFT Tools

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Outline of talk

We will look at results in the context of $h \to b\bar{b}$.

HOWEVER:
The focus will be on technical issues related to the renormalization at 1-loop. In particular

- Choice of renormalization scheme
- Tadpoles
- Higgs-Z mixing
- The use of decoupling relations

But first, a quick introduction to the $h \to b\bar{b}$ calculation.

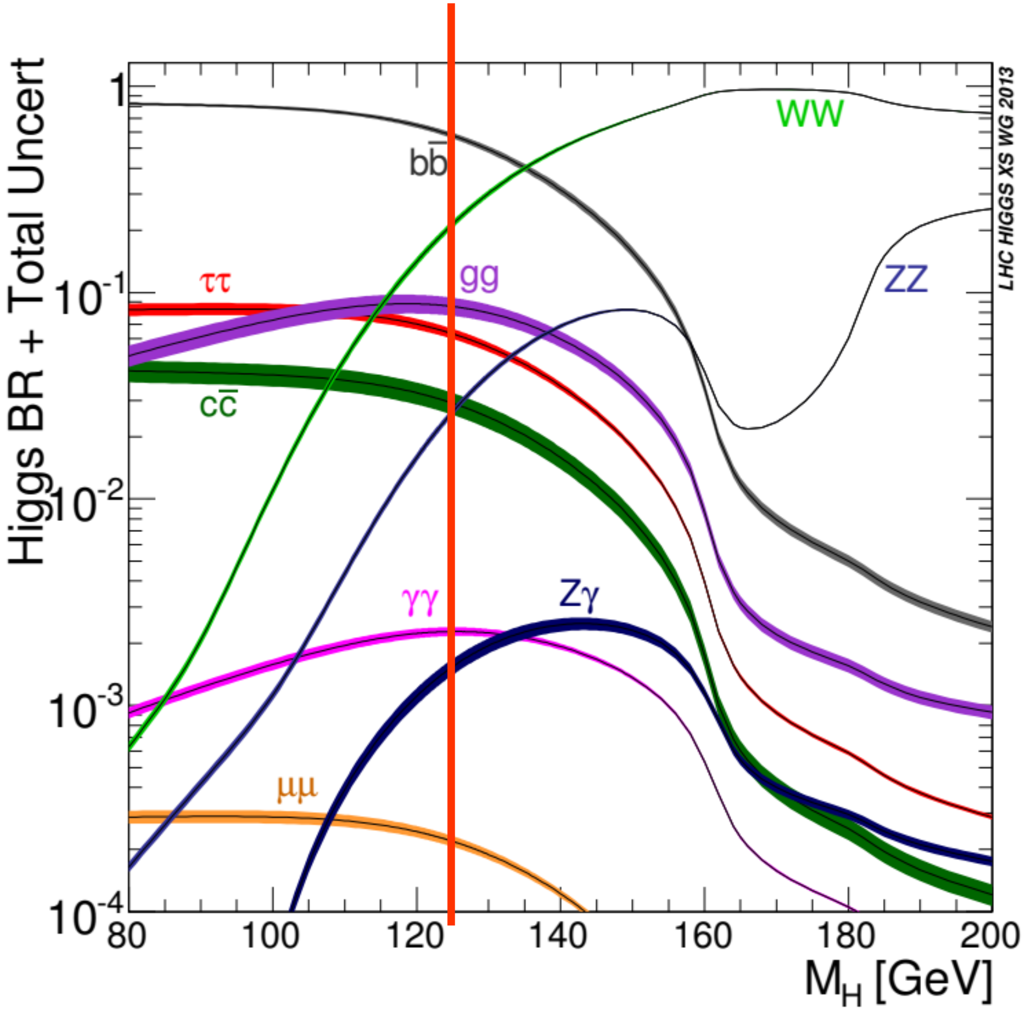
Why study h → bb?

Largest branching fraction of the Higgs:

$$\mathrm{Br}(h \to b\bar{b}) \sim 0.6$$

Newest particle in SM – many possible links to new physics.

Can use a “bottom up” EFT to parametrize possible new physics.



Why stu

Largest bran
the Higgs:

$\mathrm{Br}(h$ –

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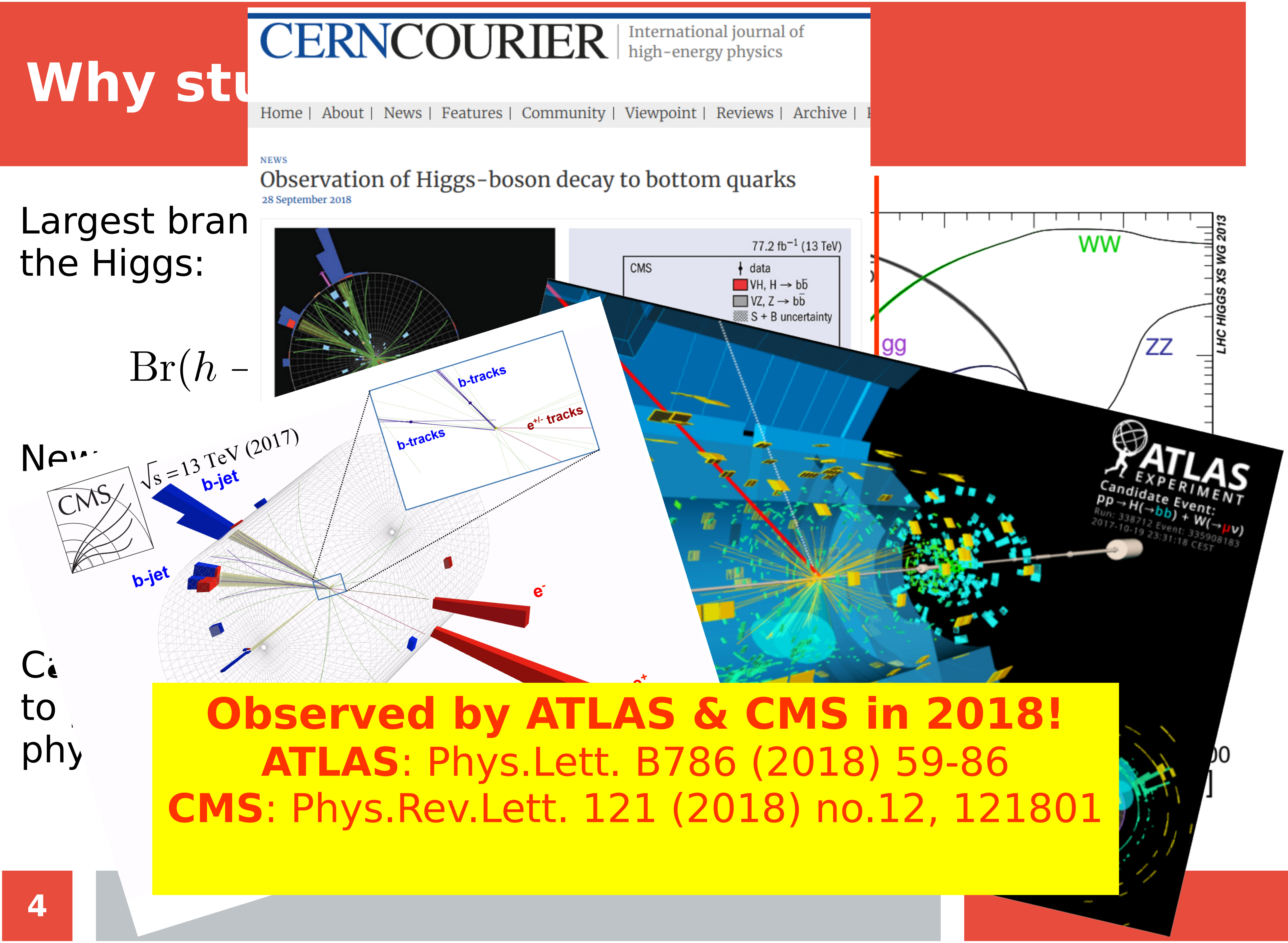

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NEWS

Observation of Higgs-boson decay to bottom quarks

28 September 2018

Observed by ATLAS & CMS in 2018!

ATLAS: Phys.Lett. B786 (2018) 59-86

CMS: Phys.Rev.Lett. 121 (2018) no.12, 121801

“Minimal” SMEFT introduction

The Standard Model EFT (SMEFT) is the SM augmented with higher dimensional operators.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{d=5} \sum_{i} C_i^{(d)} Q_i^{(d)}$$

We will include only those operators of dimension-6

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{i} C_i Q_i \qquad C_i = \frac{\widetilde{C}_i}{\Lambda_{\mathrm{NP}}^2}$$

Scale of
\`New Physics’

Standard Model EFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i C_i Q_i \qquad\qquad C_i = \frac{\widetilde{C}_i}{\Lambda^2_{\text{NP}}}$$

Scale of
`New Physics'

We use the **Warsaw basis**

[Buchmuller, Wyler: Nucl.Phys. B268 (1986) 621-653]
[Grzadkowski, Iskrzynski, Misiak, Rosiek: JHEP 1010 (2010) 085]

* Work with a diagonal CKM matrix
* Consider only baryon number conserving operators

→ 59 independent operators

* Restrict the decay rate to $\mathcal{O}(1/\Lambda^2_{\text{NP}})$

→ At most one insertion of a dim-6 operator per diagram
→ Keep only interference between dim-6 amplitude and SM amplitude

$$|\mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{dim-6}}|^2 \sim \mathcal{M}^*_{\text{SM}}\mathcal{M}_{\text{SM}} + \mathcal{M}^*_{\text{SM}}\mathcal{M}_{\text{dim-6}} + \xcancel{\mathcal{M}^*_{\text{dim-6}}\mathcal{M}_{\text{dim-6}}}$$

SMEFT: Primer

Including dimension-6 terms induces a number of changes on the tree level Lagrangian.

- VEV altered: $C_H \left(H^\dagger H\right)^3$ $v_{\mathrm{SM}} \longrightarrow v_T$
- Kinetic terms not properly normalised: $C_W \left(H^\dagger H\right) W^I_{\mu\nu} W^{I\,\mu\nu}$
- Kinetic mixing terms introduced: $C_{HWB} \left(H^\dagger \tau^I H\right) W^I_{\mu\nu} B^{\mu\nu}$
- Rotation to mass basis complicated: $\sin\theta = s_w \propto s_w^{\mathrm{SM}} + \mathcal{O}\left(1/\Lambda^2_{\mathrm{NP}}\right)$
- Relation between Yukawa and mass terms changes: $y \neq \sqrt{2}m/v$
- Additional subtleties when gauge fixing (Talk on Wednesday by Mikolaj)
  [Misiak, Paraskevas, Rosiek, Suxho, Zglinicki: JHEP 1902 (2019) 051]
  [Helset, Paraskevas, Trott: Phys.Rev.Lett. 120]

We implemented our own version of gauge fixing, but I will not discuss it here.
Details can be found in the appendix of [Cullen, Pecjak, DJS: 1904.06358]
We find agreement with the Feynman rules in
[Dedes, Materkowska, Paraskevas, Rosiek, Suxho: JHEP 1706 (2017) 143]

SMEFT: Higgs primer

<u>Consequences</u>:
Details relevant for this talk..

- Higgs doublet normalisation:

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} i \phi^+(x) \\ \left[1 + C_{H,\text{kin}}\right] h(x) + i \left[1 - \frac{\hat{v}_T^2}{4} C_{HD}\right] \phi^0(x) + v_T \end{pmatrix}$$

$\sim C_{HD}, C_{H\Box}$

- Relation of Yukawa terms to masses:

$$y_f = \sqrt{2} \frac{m_f}{v_T} + \frac{v_T^2}{2} C_{fH}^*$$

$$\boxed{Q_{dH} = \left(H^\dagger H\right) \left(\bar{q}_p H d_r\right)}$$

- VEV in terms of physical parameters

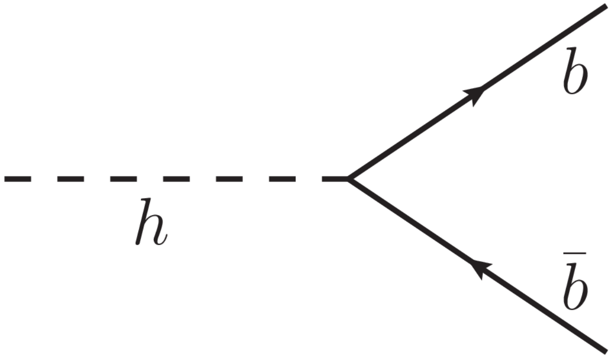
$$v_T = \hat{v}_T - \frac{\hat{c}_w}{\hat{s}_w} \left(C_{HWB} + \frac{\hat{c}_w}{4 \hat{s}_w} C_{HD} \right)$$

$$\boxed{\hat{v}_T = \frac{2 M_W \hat{s}_w}{e}, \qquad \hat{c}_w = \frac{M_W}{M_Z}}$$

Tree level results: Amplitude

We can now compute the tree level result

$$i\mathcal{M} = -i\overline{u}(p_b)\left[\mathcal{M}_L P_L + \mathcal{M}_R P_R\right] v(p_{\bar{b}})$$



SM

dim-6

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

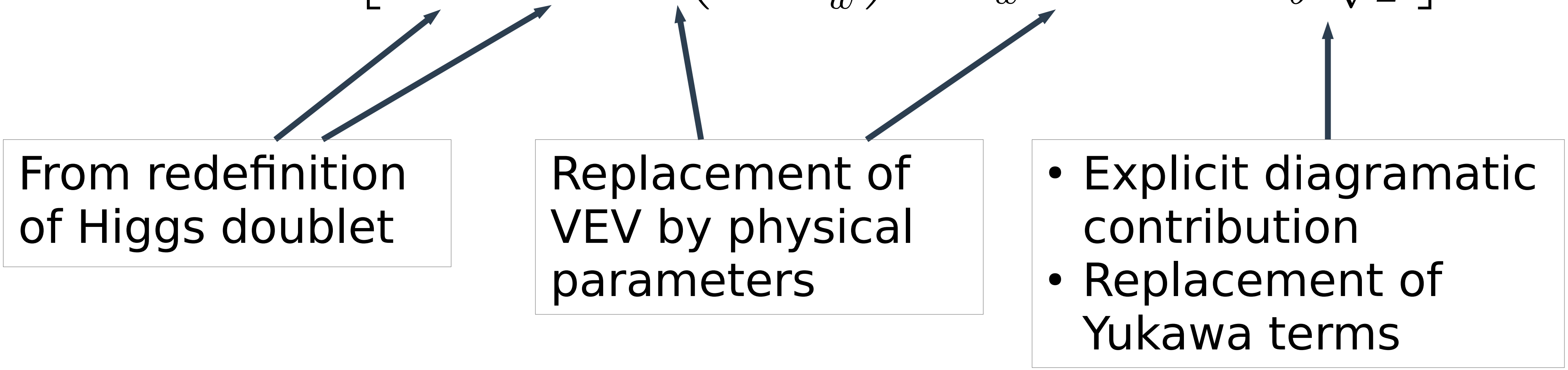
$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T}$$

$$\mathcal{M}_L^{(6,0)} = \frac{m_b}{\hat{v}_T}\left[C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}^*}{\sqrt{2}}\right]$$

Tree level results: Decay rate

$$\Gamma^{(4,0)} = \frac{N_c m_H m_b^2}{8\pi \hat{v}_T^2}$$

$$\Gamma^{(6,0)} = 2\Gamma^{(4,0)}\left[C_{H\Box} - \frac{C_{HD}}{4}\left(1-\frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}}{\sqrt{2}}\right]\hat{v}_T^2$$



From redefinition of Higgs doublet

Replacement of VEV by physical parameters

- Explicit diagramatic contribution
- Replacement of Yukawa terms

We will look at numerics later.

Next: higher order corrections

Beyond tree level

Why?

- Loop effects important elsewhere in QFT (especially for Higgs). What is the impact for dimension-6 coefficients?
- Coefficients appearing for the first time at NLO & new topologies.

Set up

- Specify a set of input parameters:

$$\alpha, \alpha_s, m_f, M_W, M_Z, m_H, C_i$$

- We make use of *FeynRules*, *FeynArts*, and *FormCalc* for our calculation.
- Calculate in both Feynman and Unitary gauge as a check.

Renormalization scheme

One could also use G_F as an input instead of α.

- G_F introduces tree level dependence on Wilson coefficients which contribute to muon decay.
- Also requires full NLO SMEFT muon decay calculation.

[Dawson, Giardino: Phys. Rev. D 97, 093003]

It is relatively straightforward to switch between the schemes however.

Pick a renormalization scheme: $X_{\text{bare}} = X + \delta X$

Wave function factors, $\delta Z_{b,L}$: on-shell scheme

Wilson coefficients, C_i : $\overline{\text{MS}}$ scheme

Renormalization scheme

We will be flexible with the scheme for the masses and electric charge.

Going from on-shell scheme to $\overline{\text{MS}}$ scheme involves dropping the finite part of the counterterm.

$$\delta X = \delta X^{\text{div}} + c_X \delta X^{\text{O.S., fin.}}$$

$c_X = 0 \implies$ $\overline{\text{MS}}$ scheme

$c_X = 1 \implies$ On-shell scheme

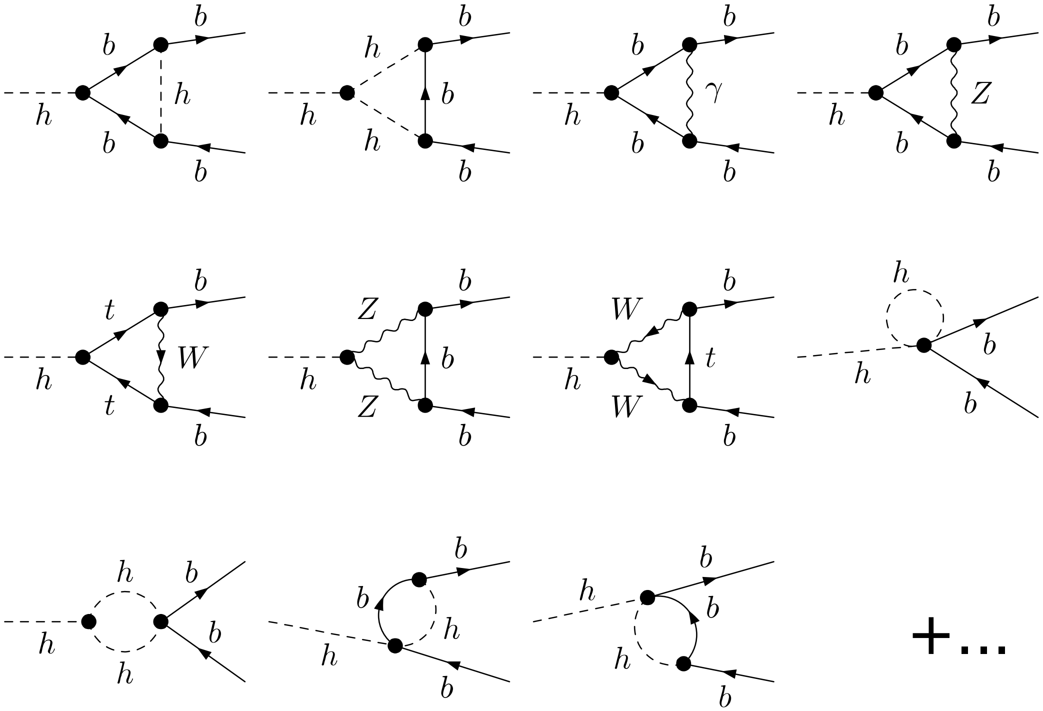
Look at structure of corrections and decide an appropriate scheme.

Obtaining NLO predictions

To compute the NLO result:

1) Compute corrections from virtual & real emission graphs
2) Derive and compute necessary objects for the counterterm
3) Put everything together!

<u>1) NLO contributions</u>



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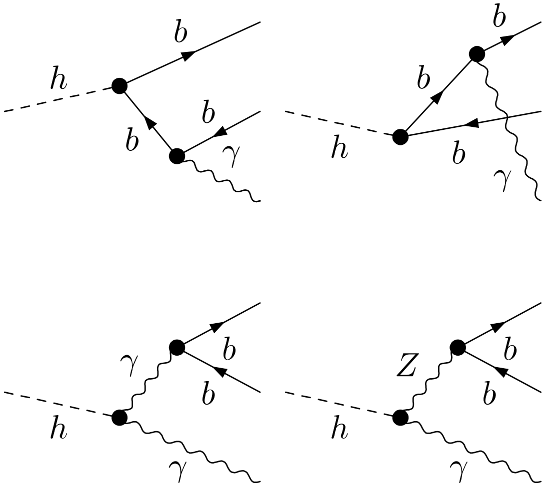


Renormalization

2) Counterterm & relevant contributions

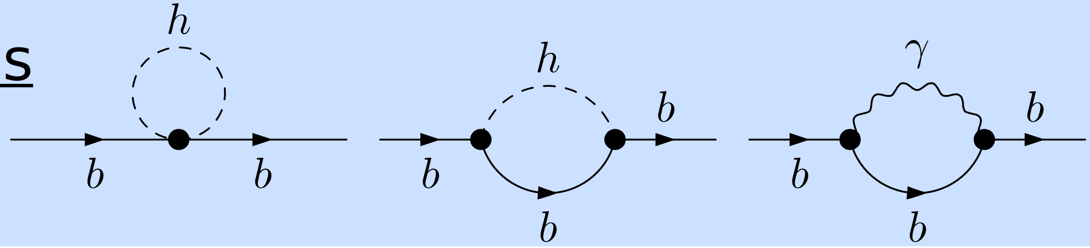
$$\begin{aligned}
\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right) \\
&+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right) \\
&- \frac{\hat{v}_T^2}{\sqrt{2}}C_{bH}^*\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} \\
&+ m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)
\end{aligned}$$

Renormalization

2) Counterterm & relevant contributions

$$
\begin{aligned}
\delta\mathcal{M}_L^{(6)} = & \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right) \\
& + \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right) \\
& - \frac{\hat{v}_T^2}{\sqrt{2}}C_{bH}^*\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} \\
& + m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)
\end{aligned}
$$

Self energy graphs



Renormalization

2) Counterterm & relevant contributions

$$
\begin{aligned}
\delta\mathcal{M}_L^{(6)} = & \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right) \\
& + \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right) \\
& - \frac{\hat{v}_T^2}{\sqrt{2}}C_{bH}^{*}\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} \\
& + m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^{*}}{\sqrt{2}}\right)
\end{aligned}
$$

$$\hat{v}_T = \frac{2M_W\hat{s}_w}{e} \qquad \delta\hat{v}_T \sim \delta M_W,\ \delta\hat{s}_w,\ \delta e$$

δe - Related to 3-point vertex & wavefunction renormalization

Renormalization

2) Counterterm & relevant contributions

$$\begin{aligned}
\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right) \\
&+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right) \\
&- \frac{\hat{v}_T^2}{\sqrt{2}} C_{bH}^* \left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w} C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} \\
&+ m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)
\end{aligned}$$

δC_i – Extract from 1-loop anomalous dimension

[Jenkins, Manohar, Trott: JHEP 1310 (2013) 087, JHEP 1401 (2014) 035]

[Alonso, Jenkins, Manohar, Trott: JHEP 1404 (2014) 159]

Final result

Combining all the results, at NLO the decay rate becomes

$$\Gamma^{(1)} = \Gamma^{(4,1)} + \Gamma^{(6,1)}$$

$\Gamma^{(6,1)}$ ~ Depends on 45 Wilson coefficients.

We will analyse major contributions to this at the end.

First, we will check the dependence on the renormalization scheme.

Decay rate

Decompose decay rate into three separate pieces:

$$\Gamma^{(d,1)} = \Gamma^{(d,1)}_{g,\gamma} + \Gamma^{(d,1)}_{t} + \Gamma^{(d,1)}_{\text{rem}} \qquad d = \{4, 6\}$$

$\Gamma^{(d,1)}_{g,\gamma}$: Virtual & real radiation involving gluons or photons. (QCD part previously computed)

[Gauld, Pecjak, DJS: Phys.Rev. D94 (2016) no.7, 074045]

$\Gamma^{(d,1)}_{t}$: Virtual weak corrections in large m_t limit (Calculated in on-shell scheme previously)

[Gauld, Pecjak, DJS: JHEP 1605 (2016) 080]

$\Gamma^{(d,1)}_{\text{rem}}$: Everything else (Including four-fermion operators – previously calculated as above)

Renormalization of the electric charge

In the **on-shell** scheme, one can make use of relations (resulting from SM Ward identities), allowing one to express $\delta e^{(4)}$ in terms of two-point functions.

See for ex: [Denner: Fortsch.Phys. 41 (1993) 307-420]

$$\frac{\delta e^{(4)}}{e} = \frac{1}{2} \frac{\partial \Sigma_T^{AA(4)}(k^2)}{\partial k^2}\bigg|_{k^2=0} - \frac{(v_f^{(4)} - a_f^{(4)})}{Q_f} \frac{\Sigma_T^{AZ(4)}(0)}{M_Z^2}$$

$\Sigma_T^{IJ}(k^2)$ - transverse component of I → J two-point function.

In the SM, we have: $v_f^{(4)} - a_f^{(4)} = -Q_f \hat{s}_w / \hat{c}_w$

→ Independence on fermion flavour

Renormalization of the electric charge

For class-7 operators $(\psi^2 H^2 D)$ we find that:

$$v_f^{(6)} - a_f^{(6)} = C_{Hf}\hat{v}_T^2/2\hat{c}_w\hat{s}_w$$

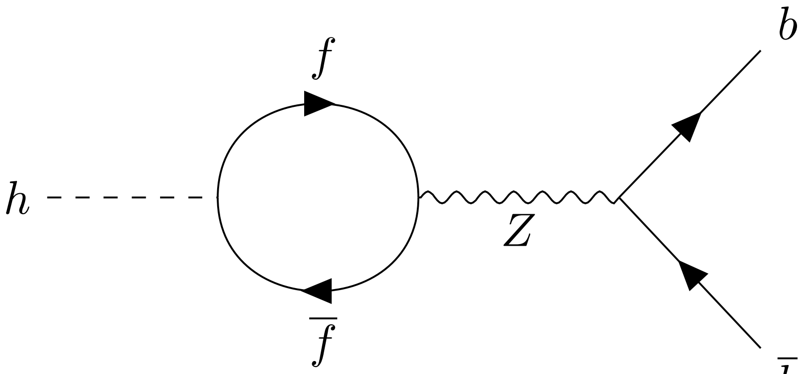
Charge renormalization appears to depend on fermion type..

We renormalize the $ff\gamma$-vertex directly (using 3-point functions) and find agreement with:

$$\frac{\delta e^{(6)}}{e} = \frac{1}{2}\frac{\partial\Sigma_T^{AA(6)}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \frac{1}{M_Z^2}\left(\frac{\hat{s}_w}{\hat{c}_w}\Sigma_T^{AZ(6)}(0) - \frac{\hat{v}_T^2}{4\hat{c}_w\hat{s}_w}C_{HD}\Sigma_T^{AZ(4)}(0)\right)$$

Higgs-Z mixing

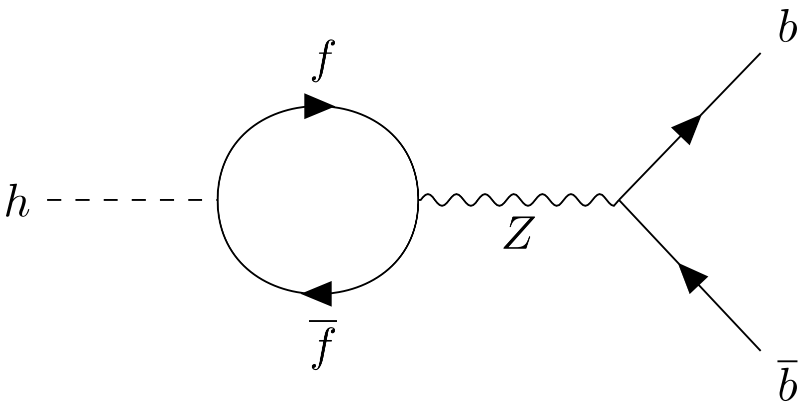
In the SM contributions of the type:



do not contribute.

Such diagrams contribute to the renormalization of the class-5 operators $(H^3\psi^2)$ cancelling divergences related to the imaginary parts of the Wilson coefficients (even after rotation to the mass basis).

Higgs-Z mixing



+ Goldstones if in Feynman gauge

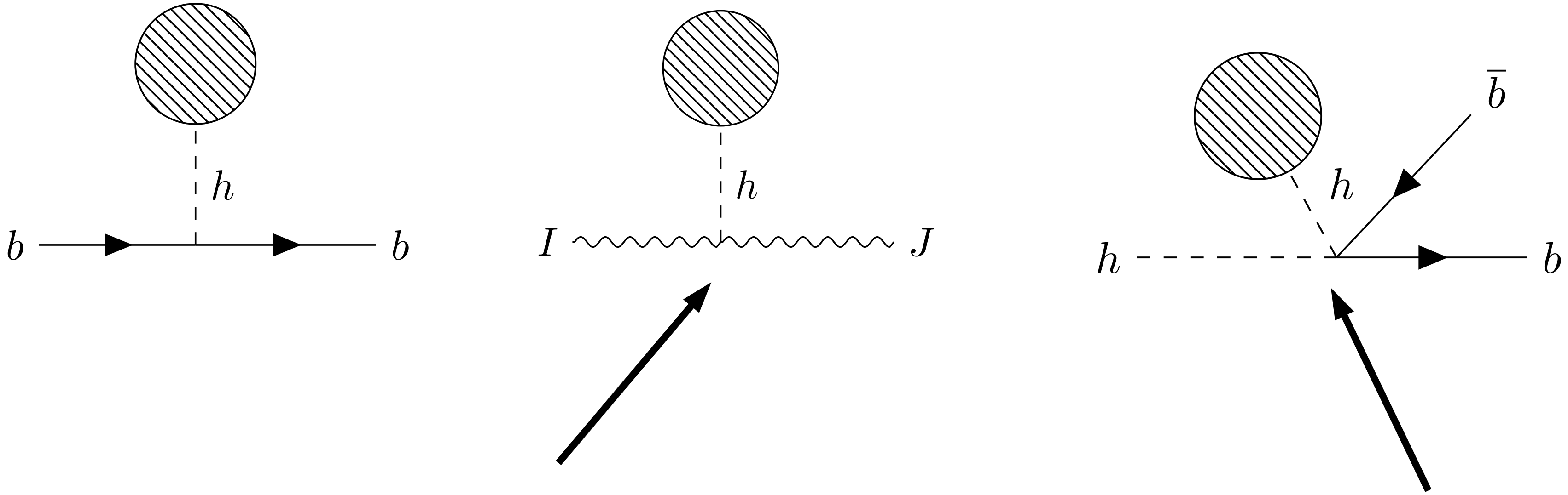
Sum over diagrams gives result proportional to

$$\frac{\sqrt{2}}{\hat{v}_T}\,\mathrm{Im}\left[N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H}\right]$$

UV-divergent part cancelled exactly by η_5 term in δC_{bH}.

Tadpoles

Graphs of type:



Appears in SMEFT for $I = J = \gamma$ due to operators

$$C_{HB} H^\dagger H B_{\mu\nu} B^{\mu\nu}$$

$$C_{HW} H^\dagger H W^I_{\mu\nu} W^{I\,\mu\nu}$$

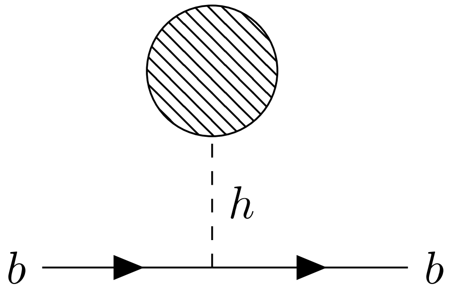
$$C_{HWB} H^\dagger \sigma^I H W^I_{\mu\nu} B^{\mu\nu}$$

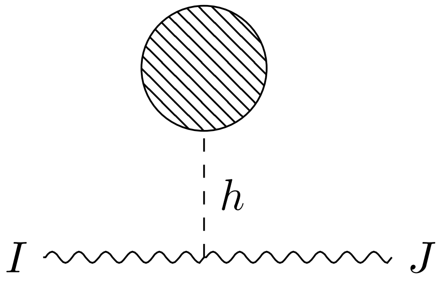
Graph entirely from dimension-6 operators of type:

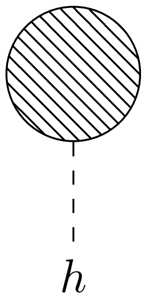
$$C_{dH} \left(H^\dagger H\right) \left(\bar{Q} H d\right)$$

Tadpoles

Graphs of type:







$= T$
Tadpole
Function

- Cancel in the on-shell scheme
- If we use $\overline{\text{MS}}$ for some parameters, this cancellation will no longer happen
- Necessary to include for gauge invariance

We use the FJ tadpole scheme:

[Fleischer, Jegerlehner: Phys. Rev. D 23 (1981) 2001

→ Include tadpoles in diagramatic calculations
→ No need to add explicit tadpole counterterm

Tadpoles

Tadpole function (unitary gauge):

$$T^{(4)} = \frac{1}{32\pi^2 \hat{v}_T}\left\{6\left(1-\frac{2\epsilon}{3}\right)\left[2M_W^2 A_0(M_W^2) + M_Z^2 A_0(M_Z^2)\right] + 3M_H^2 A_0(M_H^2)\right.$$
$$\left. - 8\sum_f N_c^f m_f^2 A_0(m_f^2)\right\}$$

$$\boxed{A_0(M^2) = M^2\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right) + 1\right)}$$

$$T^{(6)} = \frac{\hat{v}_T}{32\pi^2}\left\{\left(-6C_H\hat{v}_T^2 + 4C_{H,\text{kin}}\frac{m_H^2}{\hat{v}_T^2}\right)A_0(m_H^2) + (24-16\epsilon)C_{HW}M_W^2 A_0(M_W^2)\right.$$
$$+ (3-2\epsilon)\left[C_{HD} + 4(C_{HW}\hat{c}_w^2 + C_{HB}\hat{s}_w^2 + \hat{c}_w\hat{s}_w C_{HWB})\right] M_Z^2 A_0(M_Z^2)$$
$$\left. + \sum_f N_c^f 2\sqrt{2}\hat{v}_T m_f (C_{fH} + C_{fH}^*) A_0(m_f^2)\right\}$$
$$+ \left[C_{H,\text{kin}} + \hat{v}_T^2\frac{\hat{c}_w}{\hat{s}_w}\left(C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w}C_{HD}\right)\right] T^{(4)}$$

Tadpoles

Tadpole function (unitary gauge):

$$
\begin{aligned}
T^{(4)} = \frac{1}{32\pi^2 \hat{v}_T} \Bigg\{ & 6\left(1-\frac{2\epsilon}{3}\right)\left[2M_W^2 A_0(M_W^2) + M_Z^2 A_0(M_Z^2)\right] + 3M_H^2 A_0(M_H^2) \\
& - 8\sum_f N_c^f m_f^2 A_0(m_f^2) \Bigg\}
\end{aligned}
$$

$$\boxed{A_0(M^2) = M^2\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right) + 1\right)}$$

$$
\begin{aligned}
T^{(6)} = \frac{\hat{v}_T}{32\pi^2} \Bigg\{ & \left(-6C_H \hat{v}_T^2 + 4C_{H,\text{kin}}\frac{m_H^2}{\hat{v}_T^2}\right) A_0(m_H^2) + (24-16\epsilon)C_{HW} M_W^2 A_0(M_W^2) \\
& + (3-2\epsilon)\left[C_{HD} + 4(C_{HW}\hat{c}_w^2 + C_{HB}\hat{s}_w^2 + \hat{c}_w \hat{s}_w C_{HWB})\right] M_Z^2 A_0(M_Z^2) \\
& + \sum_f N_c^f 2\sqrt{2}\hat{v}_T m_f (C_{fH} + C_{fH}^*) A_0(m_f^2) \Bigg\} \\
& + \left[C_{H,\text{kin}} + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w}\left(C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD}\right)\right] T^{(4)}
\end{aligned}
$$

$$\underline{T \sim \frac{m_t^4}{\hat{v}_T^2 m_H^2}}$$

Tadpoles

Tadpole function (unitary gauge):

$$T^{(4)} = \frac{1}{32\pi^2 \hat{v}_T} \left\{ 6\left(1 - \frac{2\epsilon}{3}\right)\left[2M_W^2 A_0(M_W^2) + M_Z^2 A_0(M_Z^2)\right] + 3M_H^2 A_0(M_H^2) \right.$$

$$\left. - 8 \sum N_c^f m_f^2 A_0(m_f^2) \right\}$$

$$A_0(M^2) = M^2 \left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{M^2}\right) + 1\right)$$

Prefer to renormalize in the on-shell scheme to avoid these large corrections

$$T^{(}$$

$$+ (3 - 2\epsilon)\left[C_{HD} + 4(C_{HW}\hat{c}_w^2 + C_{HB}\hat{s}_w^2 + \hat{c}_w\hat{s}_w C_{HWB})\right] M_Z^2 A_0(M_Z^2)$$

$$\left. + \sum_f N_c^f 2\sqrt{2}\hat{v}_T m_f (C_{fH} + C_{fH}^*) A_0(m_f^2) \right\}$$

$$+ \left[C_{H,\mathrm{kin}} + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w}\left(C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD}\right)\right] T^{(4)}$$

$$T \sim \frac{m_t^4}{\hat{v}_T^2 m_H^2}$$

Tadpoles

Impact on decay rate? Use SM as an example.
Examine leading terms in m_t in each scheme.

$\overline{\text{MS}}$ scheme for b-quark mass and electric charge:

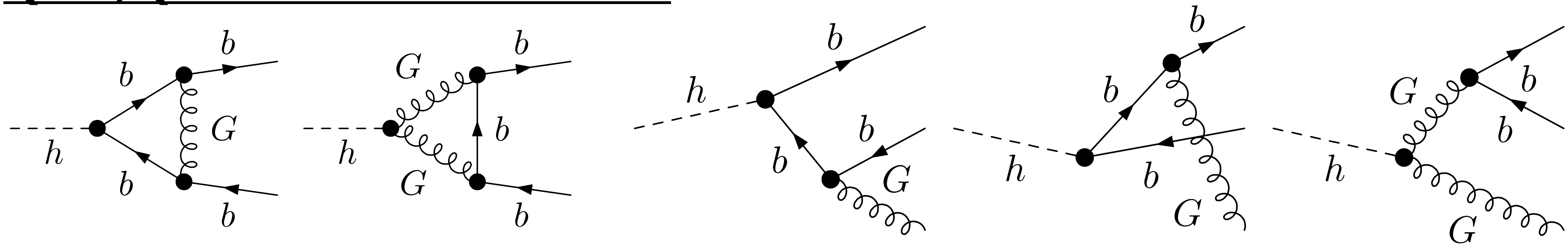
$$\frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2}\frac{m_t^4}{\hat{v}_T^2 m_H^2} \approx -15\%$$

On-shell scheme

$$\frac{[\Gamma_t]^{\text{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2\hat{v}_T^2}\left(-6 + N_c\frac{7-10\hat{c}_w^2}{3\hat{s}_w^2}\right) \approx -3\%$$

Large NLO corrections

QCD/QED-like corrections



How large are these corrections?
Keeping only logarithmic corrections and setting $\mu = m_H$

$$\frac{\Gamma^{(1)}_{g,\gamma}}{\Gamma^{(4,0)}} \approx \ln^2\left(\frac{m_b^2}{m_H^2}\right)\frac{\hat{v}_T^2}{\pi}\left(C_F\alpha_s C_{HG} + Q_b^2\alpha c_{h\gamma\gamma}\right)$$

$$+ \; c_{m_b}\ln\left(\frac{m_b^2}{m_H^2}\right)\frac{3}{2}\left(\frac{C_F\alpha_s + Q_b^2\alpha}{\pi}\right)\left[1 + 2\hat{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right)\right.\right.$$

$$\left.\left. + \frac{\hat{c}_w}{\hat{s}_w}C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}}{2\sqrt{2}}\right)\right]$$

$$c_{h\gamma\gamma} = C_{HB}\hat{c}_w^2 + C_{HW}\hat{s}_w^2 - C_{HWB}\hat{c}_w\hat{s}_w$$

Large NLO corrections

QCD/QED-like corrections

Numerically:

$$\frac{\Gamma^{(1)}_{g,\gamma}}{\Gamma^{(4,0)}} \approx \hat{v}_T^2 \left(2.4 C_{HG} + 0.02 c_{h\gamma\gamma}\right) - 0.5 c_{m_b}\left[1 + 2\hat{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4}\left(1-\frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}}{2\sqrt{2}}\right)\right]$$

🟪 Large coefficient of C_{HG} from double log

→ IR log, could be dealt with via resummation

🟨 Can set $c_{m_b} = 0$

→ Preference to use $\overline{\text{MS}}$ scheme for QCD/QED type corrections

On-shell or $\overline{\text{MS}}$?

We would like to renormalize the b-quark mass in the $\overline{\text{MS}}$ scheme (and to allow the resummation of mass logarithms).

This leads to large tadpole contributions!

Can use decoupling relations to define $\overline{m}_b^{(\ell)}$and $\bar{e}^{(\ell)}$ in a low energy theory where tadpole contributions from the top quark are included in decoupling constants.

[Bednyakov, Kniehl, Pikelner, Veretin: Nucl.Phys. B916 (2017) 463-483]

Decoupling

Considering the low energy part of the theory we can write:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z)\overline{m}_b^{(\ell)}(\mu)$$

$$\overline{e}(\mu) = \zeta_e(\mu, m_t, m_H, M_W, M_Z)\overline{e}^{(\ell)}(\mu)$$

Only $d = 4$ terms

$$\zeta_i = 1 + \zeta_i^{(4,1)} + \zeta_i^{(6,1)}$$

We make this replacement in the $\overline{\text{MS}}$ renormalized decay rate

$$\overline{\Gamma}_\ell^{(4,1)} = \overline{\Gamma}^{(4,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right)$$

$$\overline{\Gamma}_\ell^{(6,1)} = \overline{\Gamma}^{(6,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(6,1)} + \zeta_e^{(6,1)}\right) + 2\overline{\Gamma}^{(6,0)}\zeta_b^{(4,1)}$$

$$+ \sqrt{2}C_{bH}\frac{\left(\overline{v}^{(\ell)}\right)^3}{\overline{m}_b^{(\ell)}}\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right)$$

Decoupling

Decomposing the decay rate as before:

$$\overline{\Gamma}_{\ell}^{(1)} = \overline{\Gamma}_{\ell,g,\gamma}^{(1)} + \overline{\Gamma}_{\ell,t}^{(1)} + \overline{\Gamma}_{\ell,\mathrm{rem}}^{(1)}$$

We now find

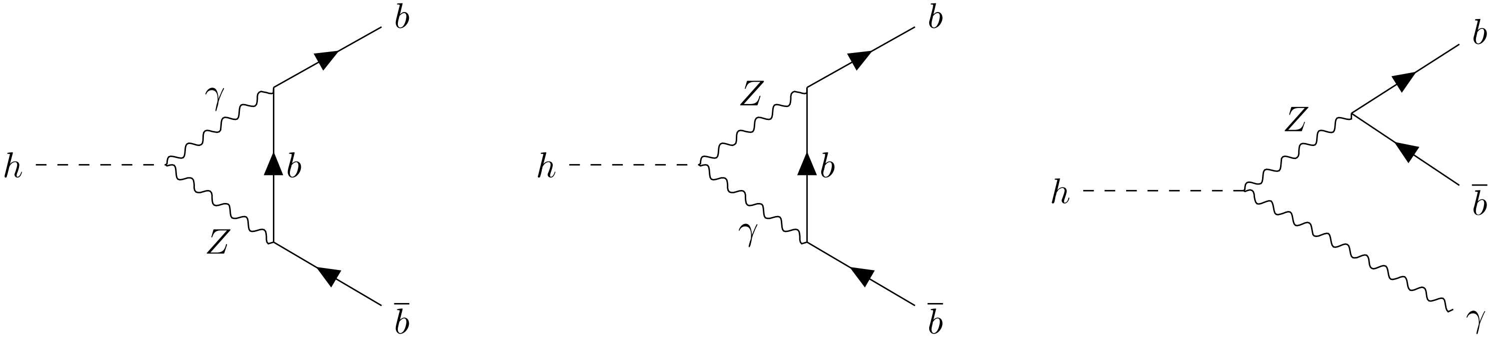
$$\overline{\Gamma}_{\ell,g,\gamma} = \overline{\Gamma}_{g,\gamma}\,, \qquad \overline{\Gamma}_{\ell,t} = [\Gamma_t]^{\mathrm{O.S.}}$$

Can view this as:

- QCD/QED corrections calculated in $\overline{\text{MS}}$ scheme
- Contributions from top loops calculated on-shell (same for heavy gauge bosons) – no large tadpoles!

hZɣ - vertex

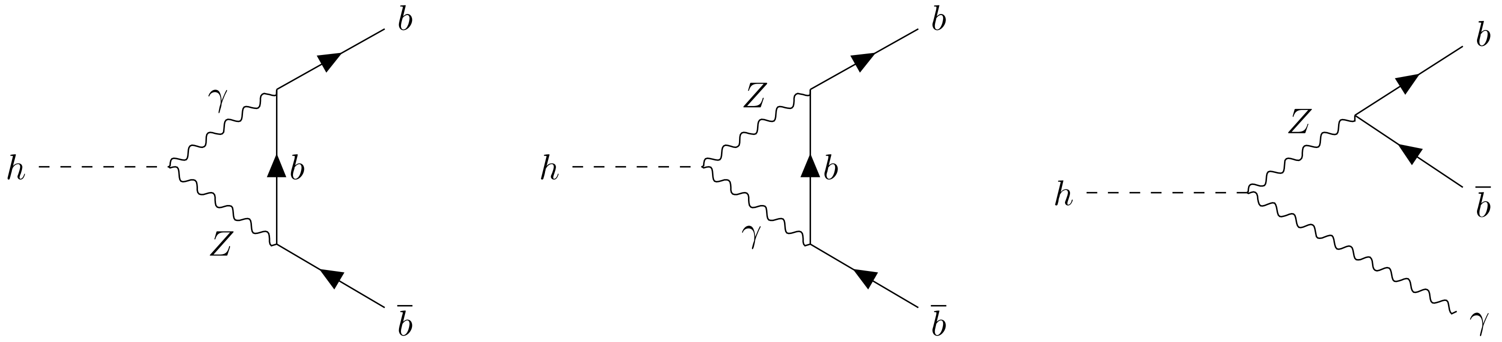
Most QED-type corrections can be obtained from the QCD ones. The hZγ- vertex however introduces new contributions.



$$\overline{\Gamma}^{(6,1)}_{h\gamma Z} = \frac{\hat{v}_T^2}{\pi}\Gamma^{(4,0)}\left\{ c_{h\gamma Z}\, v_b Q_b \alpha\, F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)\right\}$$

hZɣ - vertex

Most QED-type corrections can be obtained from the QCD ones.
The hZγ- vertex however introduces new contributions.



$$\overline{\Gamma}^{(6,1)}_{h\gamma Z} = \frac{\hat{v}_T^2}{\pi}\Gamma^{(4,0)}\left\{ c_{h\gamma Z}\, v_b Q_b \alpha\, F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)\right\}$$

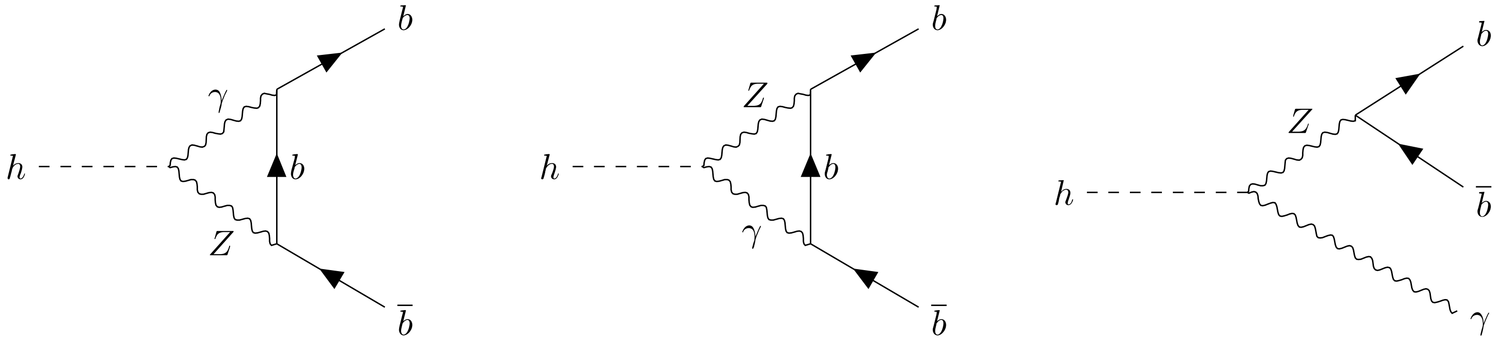
$$c_{h\gamma Z} = 2(C_{HB} - C_{HW})\hat{c}_w\hat{s}_w \\ +C_{HWB}(\hat{c}_w^2 - \hat{s}_w^2)$$

Vector coupling: SM Zbb vertex

$$v_b = -(\frac{1}{2} + 2Q_b\hat{s}_w^2)/(2\hat{c}_w\hat{s}_w)$$

hZɣ - vertex

Most QED-type corrections can be obtained from the QCD ones. The hZγ- vertex however introduces new contributions.



$$\overline{\Gamma}^{(6,1)}_{h\gamma Z} = \frac{\hat{v}_T^2}{\pi}\Gamma^{(4,0)}\left\{ c_{h\gamma Z}\, v_b Q_b \alpha\, F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)\right\}$$

Kinematic function. Simplifies in $m_b \to 0$ limit to

$$F_{h\gamma Z}\left(z, \hat{\mu}^2, 0\right) = -12 + 4z - \frac{4}{3}\pi^2(1-z)^2 + \left(3 + 2z + 2(1-z)^2\ln(1-z)\right)\ln(z)$$
$$+4(1-z)^2\mathrm{Li}_2(z) - 6\ln(\hat{\mu}^2)$$

Numerical Results

Look at ratios:

$$\Delta^{\mathrm{LO}}(\mu_R, \mu_C) \equiv \frac{\overline{\Gamma}_\ell^{(4,0)}(\mu_R, \mu_C) + \overline{\Gamma}_\ell^{(6,0)}(\mu_R, \mu_C)}{\overline{\Gamma}_\ell^{(4,0)}(m_H, m_H)}$$

$$\Delta^{\mathrm{NLO}}(\mu_R, \mu_C) \equiv \Delta^{\mathrm{LO}}(\mu_R, \mu_C) + \frac{\overline{\Gamma}_\ell^{(4,1)}(\mu_R, \mu_C) + \overline{\Gamma}_\ell^{(6,1)}(\mu_R, \mu_C)}{\overline{\Gamma}_\ell^{(4,0)}(m_H, m_H)}$$

Use separate scales for Wilson coefficients $C_i(\mu_C)$ and the $\overline{\mathrm{MS}}$ parameters $\overline{m}_b^{(\ell)}(\mu_R)$, $\overline{e}^{(\ell)}(\mu_R)$

Uncertainties from varying each independently by factors of 2 and combining in quadrature.

Numerical Results

Leading Order result:

$$\Delta^{\mathrm{LO}}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\mathrm{NP}}^2} \Big\{ (3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} - (1.41 \pm 0.07)\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD} \pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}_{Hq}^{(1)} \pm 0.18\tilde{C}_{Ht} \pm 0.11\tilde{C}_{Hq}^{(3)} \pm 0.08\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{qtqb}^{(1)} \pm 0.03\frac{\tilde{C}_{tW}}{\overline{e}^{(\ell)}} \pm 0.03(\tilde{C}_{HW} + \tilde{C}_{tH}) + \ldots \Big\}$$

Numerical Results

Leading Order result:

$$
\begin{aligned}
\Delta^{\mathrm{LO}}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda^2_{\mathrm{NP}}} \Big\{ \\
& (3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} \\
& - (1.41 \pm 0.07)\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD} \\
& \pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}^{(1)}_{Hq} \pm 0.18\tilde{C}_{Ht} \pm 0.11\tilde{C}^{(3)}_{Hq} \\
& \pm 0.08\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}^{(1)}_{qtqb} \pm 0.03\frac{\tilde{C}_{tW}}{\bar{e}^{(\ell)}} \pm 0.03(\tilde{C}_{HW} + \tilde{C}_{tH}) + \dots \Big\}
\end{aligned}
$$

From running of LO Wilson coefficients

🟪 → Additional scaling for Minimal Flavour Violation (MFV) scenarios $\bar{v}/m_b \sim 80$

Numerical Results

Next-to-Leading Order result:

$$\begin{aligned}\Delta^{\text{NLO}}(m_H, m_H) = 1.13^{+0.01}_{-0.04} + \frac{(\bar{v}^{(\ell)})^2}{\Lambda^2_{\text{NP}}} \Big\{ & \left(4.16^{+0.05}_{-0.14}\right)\, \tilde{C}_{HWB} + \left(2.40^{+0.04}_{-0.09}\right)\, \tilde{C}_{H\Box} \\ & + \left(-1.73^{+0.04}_{-0.03}\right)\, \frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}} \tilde{C}_{bH} + \left(1.33^{+0.01}_{-0.04}\right) \tilde{C}_{HD} + \left(2.75^{+0.49}_{-0.48}\right) \tilde{C}_{HG} \\ & + \left(-0.12^{+0.04}_{-0.01}\right)\, \tilde{C}^{(3)}_{Hq} + \left(-0.08^{+0.05}_{-0.01}\right)\, \tilde{C}_{Ht} + \left(0.06^{+0.00}_{-0.05}\right)\, \tilde{C}^{(1)}_{Hq} \\ & + \left(0.03^{+0.02}_{-0.01}\right)\, \frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}} \tilde{C}^{(1)}_{qtqb} + \left(0.00^{+0.07}_{-0.04}\right)\, \frac{\tilde{C}_{tG}}{g_s} + \left(-0.03^{+0.01}_{-0.01}\right) \tilde{C}_{tH} \\ & + \left(0.03^{+0.01}_{-0.01}\right) \tilde{C}_{HW} + \left(-0.01^{+0.01}_{-0.00}\right) \tilde{C}_{tW} + \ldots \Big\}\end{aligned}$$

Numerical Results

	SM	$\tilde{C}_{HWB}$	$\tilde{C}_{H\Box}$	$\tilde{C}_{bH}$	$\tilde{C}_{HD}$
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO large-m_t	-3.1%	-4.6%	3.2%	3.5%	-9.0%
NLO remainder	-2.2%	-1.9%	-1.2 %	0.6%	-2.0%
NLO correction	12.9%	11.3%	20.2%	22.3%	7.1%

Conclusions

- Computed the decay $h \to b\bar{b}$ at NLO including all operators in the dimension-6 SMEFT
- Result depends on 45 Wilson coefficients
- Several subtleties in SMEFT NLO not encountered in SM
- Large corrections from Tadpoles and/or QED-QCD corrections removed through decoupling relations
- QCD corrections dominant, but large m_t limit EW corrections still significant
- EW corrections not accurately accounted for using a universal K-factor