Measuring the skew-sextupolar component of a crab-cavity from turn-by-turn observations

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With the help of the SPS OP group

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Non-linear effects in a crab-cavity

LONG TERM DYNAMICS OF THE HIGH ... PHYS. REV. ACCEL. BEAMS 19,

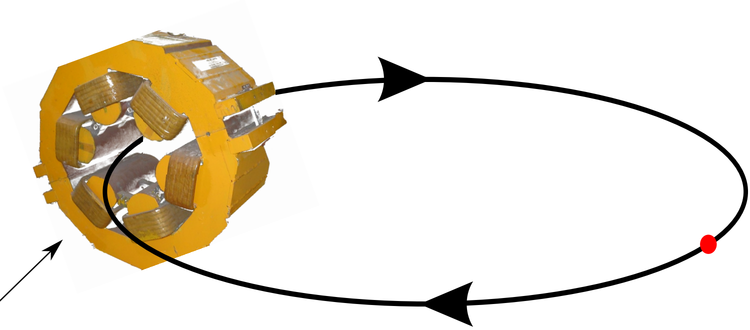
TABLE II. Values of the multipolar rf multipoles for the crab cavity prototypes at nominal deflecting voltage: V_{cc} ¼ 10 MV in units of mTm=m^{n-1}.

		Lorentz method		Panofsky-Wenzel		Helmholtz decom-
		@10 mm	@20 mm	@10 mm	@20 mm	position @20 mm
4RCAV	b_2	–0.06	–0.05	–0.06	–0.06	–0.10
2012	b_3	1159	1159	1161	1161	1156
	b_4	–4	100	63	27	57
RWCAV	b_2	0.01	0.00	0.00	0.01	0.02
2012	b_3	4511	4511	4495	4495	4518
	b_4	–4	–7	–21	7	10
QWCAV	b_2	111.42	111.40	111.43	111.48	113.06
2011	b_3	1266	1267	1257	1260	1279
	b_4	1776	1776	1401	1836	2102
QWCAV	b_2	0.29	0.29	0.29	0.29	0.24
2012	b_3	1074	1073	1078	1078	1073
	b_4	50	67	6	64	22

- b3 is the multipole (sextupole) with the strongest impact on the non-linear beam-dynamic
- In the SPS crab cavities have been installed rotated by 90°. Therefore: b3 $\rightarrow$ a3 (sextupole $\rightarrow$ skew sextupole)

Can we measure A_3?

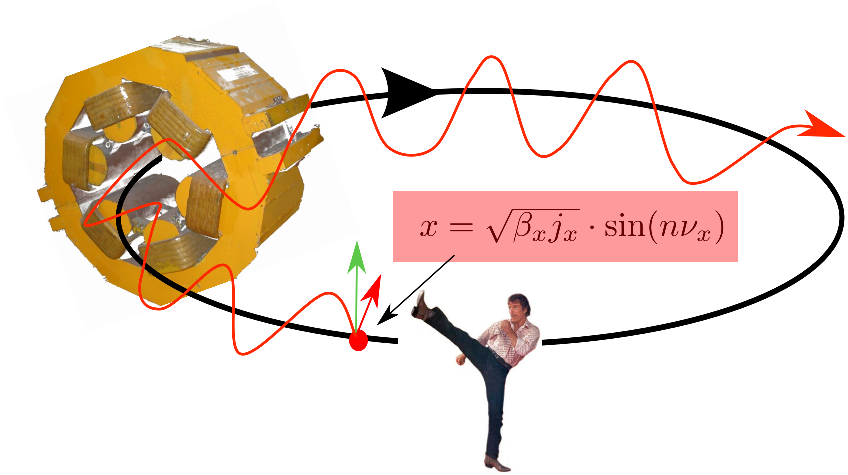
Transverse beam dynamics in presence of a skew-sextupole



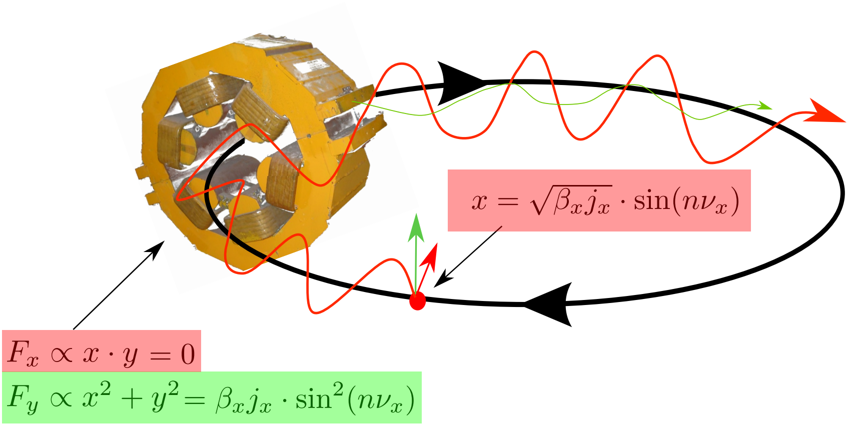
$$F_x \propto x \cdot y$$

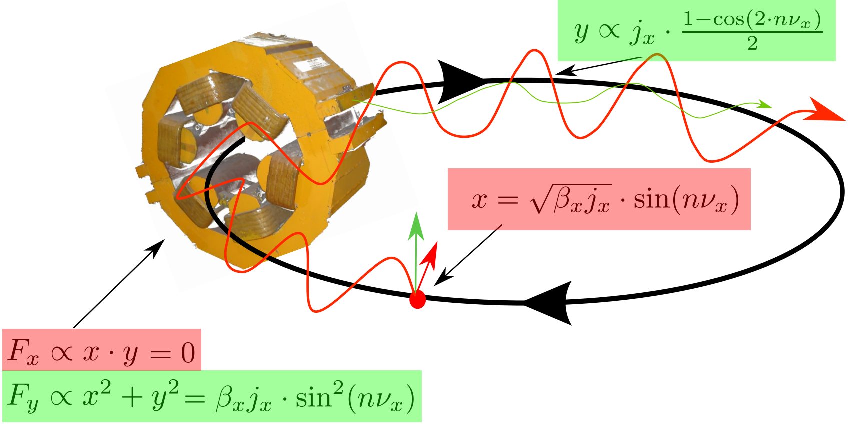
$$F_y \propto x^2 + y^2$$

If we excite the horizontal betatron motion with a kick...



The skew sextupole couples horizontal and vertical motion

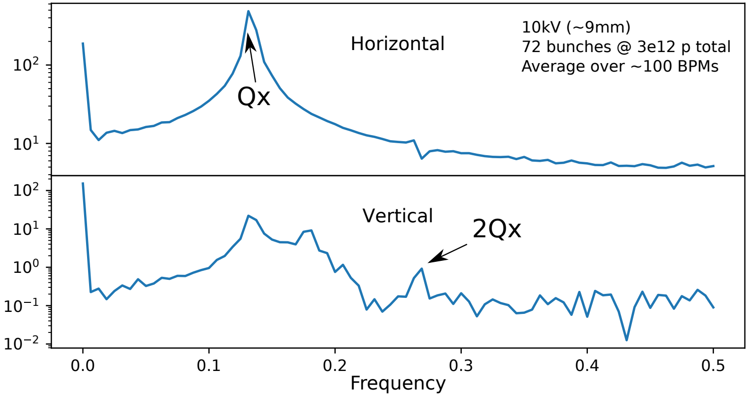




- V_{20}: Spectral line with frequency $2Q_x$ $\rightarrow$ Damps following the horizontal decoherence
- V_{00}: Static offset of the orbit $\rightarrow$ No damping (?)

BPMs can measure the beam motion as long as the beam move rigidly.
With multiple bunches decoherence plays an important role!

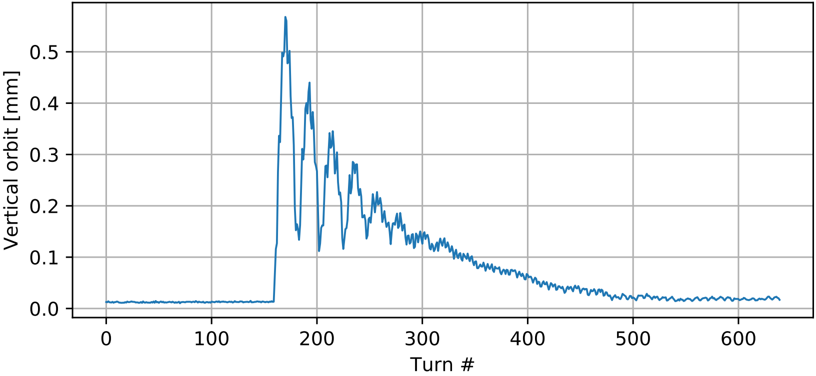
Spectral analysis of V_{20}



For each acquisition:

1. $\mathbf{Q_x}$: average over each horizontal BPM (à la Laskar)
2. $\mathbf{H_{10}}$ **amplitude, phase and damping** (**damping**: average over each horizontal BPM)
3. **Undamp** the **vertical** signal
4. Evaluate **amplitude and phase** of $\mathbf{V_{20}}$ for each vertical BPM

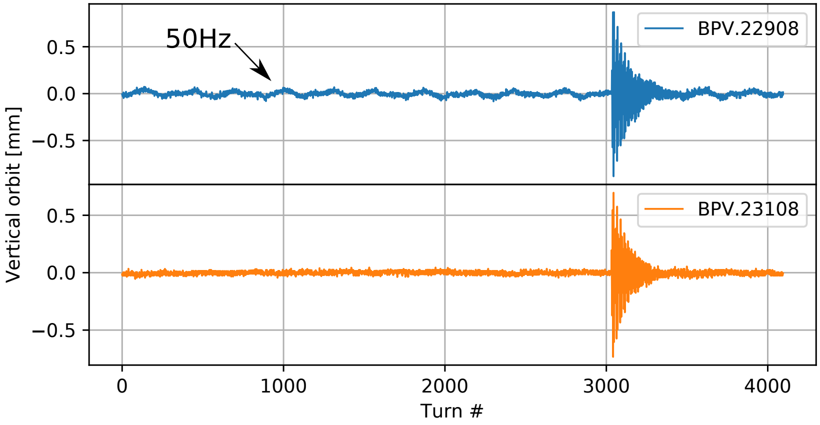
Analysis of V_{00}



For each acquisition:

1. **Orbit** is obtained from the average of **1000 turns before the kick**
2. $\mathbf{V_{00}}$ is the difference of the orbit and the average of **100 turns after the kick**
3. **...Damping?**

Typical BPM signal... no averaging, no filtering

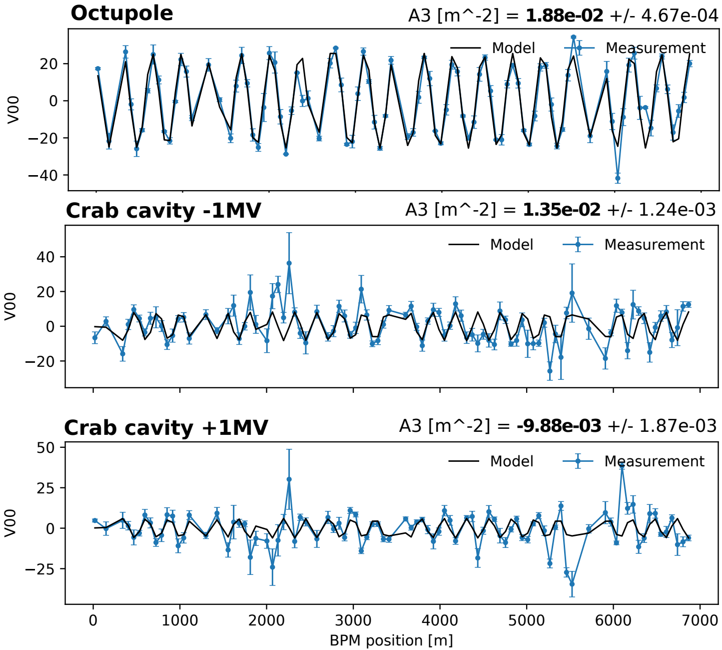


- **Amplitude and phase of 50Hz** is evaluated using **3000 turns** before the kick
- **50Hz is purged** from the signal

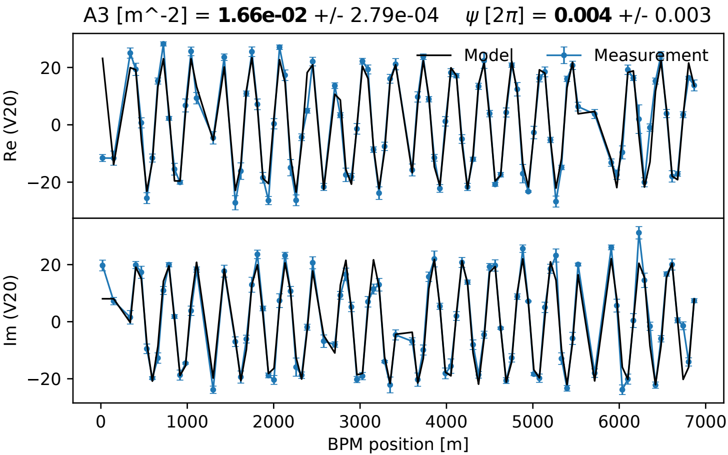
Measurements/Experimental results

- **20/10/2017** test with a **static skew-sextupole**
  - No skew sextupoles is present in SPS, a 5 mm **vertical bump in an octupole** (LOE.33002) was used to produce a **feed-down**.
  - The measurement was repeated for an **octupole strength of K3 =** $\pm$**2,** $\pm$**5** and a **vertical bump of** $\pm$**5mm**
  - **Q20 optics** was used.
- **10/10/2018** measurement was repeated with the **crab cavity**
  - The measurement was repeated for a crab cavity **voltage of: 0.1 and** $\pm$**1 MV**
  - **Q26 optics** was used.

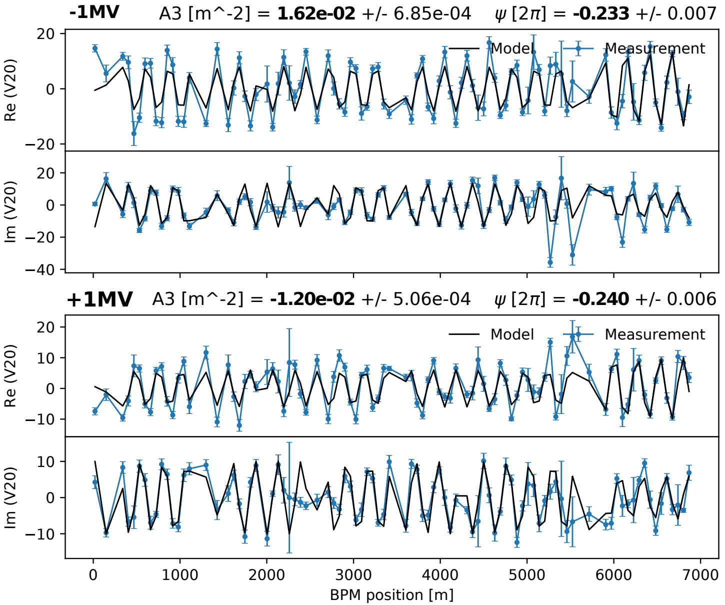
Skew sextupole strength from V_{00}



Skew sextupole strength from V_{20} (Octupole)



Skew sextupole strength from V_{20} (Crab cavity)



Summary

- Measurements of V_{00} and V_{20} with a static skew sextupole (vertical bump in an octupole) shows agreement with theory

V_{00}	V_{20}	Model	
1.88e-2	1.66e-2	1.63e-2	m^{-2}

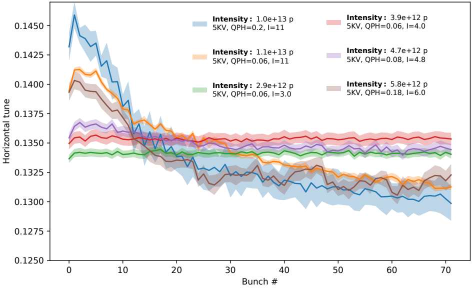
- Measurements of the skew-sextupolar component of the crab cavity are higher than expected (results do not take into account for BPM response):

Cavity setting	V_{00}	V_{20}	Model	
+1MV	1.35e-2	1.62e-2	0.27e-2	m^{-2}
-1MV	-0.99e-2	-1.20e-2	-0.27e-2	m^{-2}

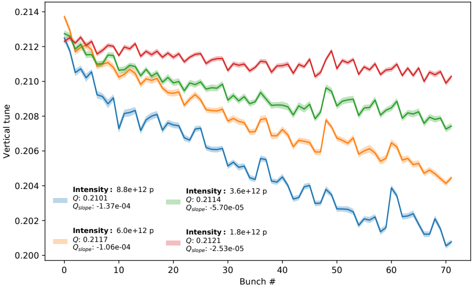
- In the case of the crab cavity a phase not expected, appears in the spectral line V_{20}
- The damping of the V_{00} is also not understood
- A SixTrack multiparticle tracking has been set-up to shed some light on these issues.

SPS multibunch detuning (what lucky coincidence!)

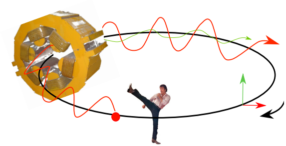
Horizontal plane



Vertical plane

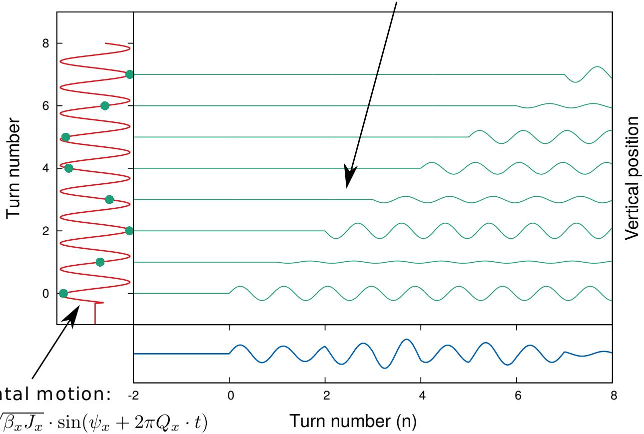


- 72 bunches in the ring
- **Horizontal** plane is **ok** up to $\sim 4 \cdot 10^{12}$
- **Vertical** plane exhibits a strong **tuneshift**
- **No excitation on the vertical plane allowed!** ...but we don't need it



Induced vertical motion at turn 'n' induced by kick 't':

$$A_3 \cdot x^2(t) \cdot \theta(s_{\text{bpm}} - s_{\text{skew}} + (n-t)C) \cdot \sqrt{\beta_y^{\text{bpm}} \beta_y^{\text{skew}}} \sin(\Delta\psi_y + 2\pi(n-t)Q_y)$$



Horizontal motion:

$$x(t) = \sqrt{\beta_x J_x} \cdot \sin(\psi_x + 2\pi Q_x \cdot t)$$

$$y(n) = \sum_{t=0}^{n} A_3 \sqrt{\beta_y^{\text{bpm}} \beta_y^{\text{skew}}} \sin(\Delta\psi_y + 2\pi(n-t)Q_y) \times \left[\sqrt{\beta_x^{\text{skew}} J_x} \sin(\psi_x^{\text{skew}} + 2\pi t Q_x)\right]^2$$

$$y(n) = \sum_{t=0}^{n} A_3 \sqrt{\beta_y^{\mathrm{bpm}} \beta_y^{\mathrm{skew}}} \sin(\Delta\psi_y + (n-t)\nu_y) \times \left[\sqrt{\beta_x^{\mathrm{skew}} J_x} \sin(\psi_x^{\mathrm{skew}} + t\nu_x) \right]^2$$

$$\Downarrow \qquad \sum_{t=0}^{n} e^{it\nu} = \frac{1 - e^{i(n+1)\nu}}{1 - e^{i\nu}}$$

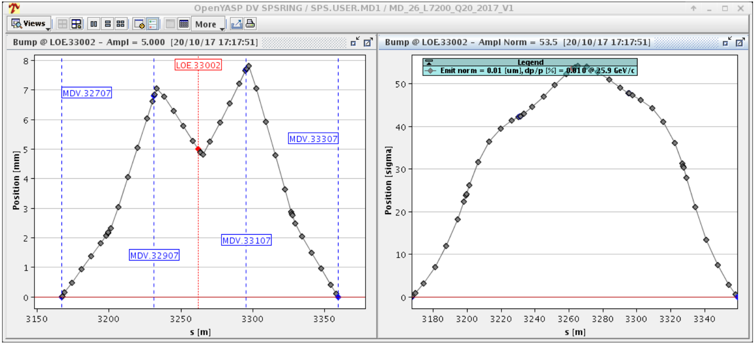
$$y(n) = \mathbf{V_{20}} + V_{02} + V_{01} + V_{00}$$

$$V_{20} = A_3 J_x \frac{\beta_x^{\mathrm{p}} \sqrt{\beta_y^{\mathrm{o}} \beta_y^{\mathrm{p}}}}{8i} \cdot \left[\frac{e^{i(2\nu_x + \nu_y - \Delta\psi_y + 2\psi_x^{\mathrm{p}})}}{e^{i(2\nu_x + \nu_y)} - 1} - \frac{e^{i(2\nu_x - \nu_y + \Delta\psi_y + 2\psi_x^{\mathrm{p}})}}{e^{i(2\nu_x - \nu_y)} - 1} \right] e^{2i\nu_x n} - \mathrm{c.c.}$$

The skew-sextupole **drives** an oscillation with frequency $\mathbf{2}Q_x$ on the **vertical plane** $\propto A_3 \cdot J_x$

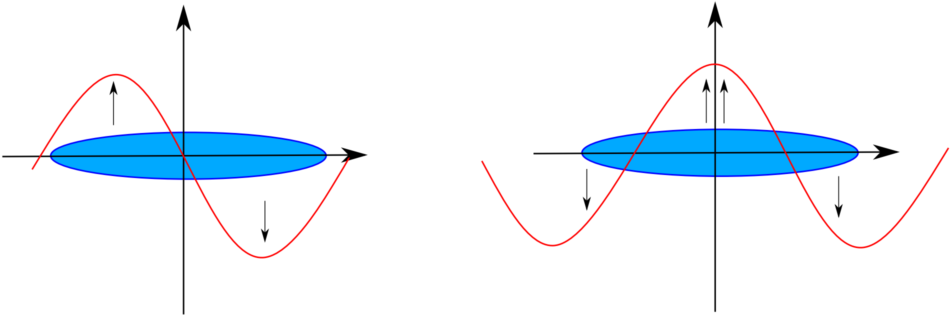
No skew-sextupoles in SPS: Octupole + vertical bump

- LOE.33002 was used
- ±5mm vertical bump
- $K_3 = \pm\ 5\ \&\ \pm 2\ \mathrm{m}^{-4}$ ($K_3 = \pm 2$ produces an A_3 very close to the C.C. one $\simeq 0.013\mathrm{m}^{-3}$)



Time dependent A_3 + longitudinal beam emittance

- Standard operation: head and tail of the bunch see opposite A_3 $\rightarrow$ average to 0
- Running the crab-cavity on-crest $\rightarrow$ A_3 does not average to zero



- Bunch length(4σ): 3ns
- Energy spread(1σ): 1.5‰

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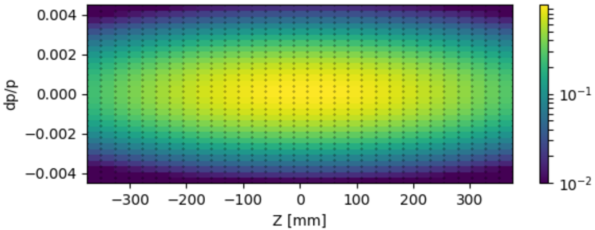

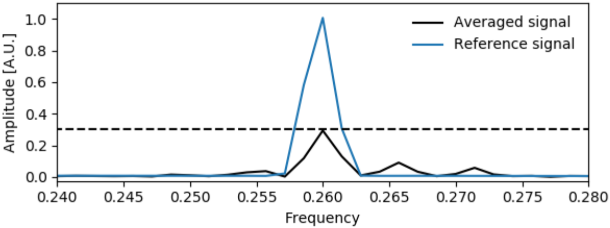

Bunch length(4σ): 3ns Energy spread(1σ): 1.5‰ C.C.: 680kV