Recent results from proton intermittency analysis in nucleus-nucleus collisions from NA61 at CERN SPS

Nikolaos Davis
for the NA61/SHINE collaboration

In collaboration with the NA61/SHINE CP Task Force:
Andrzej Rybicki (chair)
Marek Gazdzicki (co-chair)
Nikolaos Antoniou, Fotios Diakonos (NKUA, Athens)
Joanna Stepaniak, Damian Pszczel (NCNR, Warsaw)
Katarzyna Grebieszkow, Tobiasz Czopowicz (WUT, Warsaw)
Olena Linnyk et al (FIAS, Frankfurt)
Adam Bzdak (AGH, Kraków)

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1. QCD Phase Diagram and Critical Phenomena

2. Method of intermittency analysis

3. Previously released results at 150/158A GeV/c

4. New results on Ar+Sc at 150A GeV/c

5. Summary and outlook
Objective: Detection / existence of the QCD Critical Point (CP)

Phase diagram of QCD

- Quark-Gluon Plasma
- Hadronic Phase
- Color Superconductors

- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

Critical Observables; the Order Parameter (OP)

- Event-by-event (global) fluctuations:
  - Variance, skewness, kurtosis – sensitive to experimental acceptance

- Local:
  - density fluctuations of OP in transverse space (stochastic fractal)

Order parameter

Chiral condensate
\[ \sigma(x) = \langle \bar{q}(x)q(x) \rangle \]

Net baryon density
\[ n_B(x) \]

CP observables

Induced critical fluctuations*

* [Y. Hatta and M. A. Stephanov, PRL91, 102003 (2003)]
Self-similar density fluctuations near the CP

Critical Point \( \xrightarrow{\text{divergent correlation}} \) Universality Class & space dimensionality

Critical exponents (power-law) \( \xrightarrow{\text{dictate}} \) Correlations in configuration space

Correlations in momentum space

\( \sigma \)-field: 
\[ \langle n_\sigma(k) n_\sigma(k') \rangle \sim |k - k'|^{-4/3}, \]
\[ n_\sigma(k) = \sigma^2(k) \]

3D-Ising, infinite size system

Baryons: 
\[ \langle n_B(k) n_B(k') \rangle \sim |k - k'|^{-5/3}, \]
\[ n_B = \text{net baryon density at midrapidity} \]
Observing power-law fluctuations: Factorial moments

Experimental observation of local, power-law distributed fluctuations ⇒ Intermittency\(^1\)-\(^3\) in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions\(^3\))

- Net protons used as proxy for net baryons (same critical fluctuations\(^4\)); finally, protons can be used (dominant contribution) & anti-protons dropped.
- Transverse momentum space is partitioned into \(M^2\) cells
- Calculate second factorial moments \(F_2(M)\) as a function of cell size ⇔ number of cells \(M\):

\[
F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i (n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},
\]

where \(\langle \ldots \rangle\) denotes averaging over events.

\(^3\)F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence
\(^4\)Y. Hatta and M. A. Stephanov, PRL91, 102003 (2003)
Factorial moments – removal of noncritical background

- Non-critical pairs & experimental noise must be subtracted from $F_2(M)$.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.
- Crucial parameter: Ratio $\lambda$ of background to total proton multiplicity.
- For $\lambda \lesssim 1$ (background domination), non-critical background is approximated by (uncorrelated) mixed event moments; Critical Monte Carlo (CMC) then shows we can write:

$$\Delta F_2(M) \simeq \Delta F_2^{(e)}(M) \equiv F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

- For a critical system, $\Delta F_2$ scales with cell size (number of cells, $M$) as:

$$\Delta F_2(M) \sim (M^2)^{\varphi_2} ; \quad \varphi_2 : \text{intermittency index}$$

Theoretical prediction\(^1\) for $\varphi_2$

\[
\left\{ \begin{array}{l}
\varphi_{2,cr}^{(p)} = \frac{5}{6} \left(0.833 \ldots\right) \\
\text{net baryons (protons)}
\end{array} \right.
\]

\(^1\)[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]
Bootstrap method used to calculate statistical uncertainties
Bootstrap samples of events created by sampling of events with replacement
\( \Delta F_2(M) \) calculated for each bootstrap sample; variance of sample values provides statistical error of \( \Delta F_2(M) \)


Systematic uncertainties arise from:
- Misidentification of protons & detector effects (e.g. acceptance)
- The fact that \( F_2(M) \) are correlated for different bin sizes \( M \)
- Selection of \( M \)-range to fit for power-law

Bin correlations are partially handled by the bootstrap \( \varphi_2 \) distribution\(^1\), but that is insufficient! The effect of bin correlation has to be investigated through Critical and background Monte Carlo simulation.

Other systematic uncertainties are estimated by varying proton and \( M \)-range selection

\(^1\) [B. Efron, *The Annals of Statistics* 7,1 (1979)]

- 3 sets of NA49 collision systems were analysed\(^1\), at 158A GeV/c: “C”+C, “Si”+Si, Pb+Pb (“C”=C,N ; “Si”=Si,Al,P)
- Factorial moments of proton transverse momenta analyzed at mid-rapidity
- Fit with \( \Delta F_2^{(e)}(M ; C, \phi_2) = e^C \cdot (M^2)^{\phi_2} \), for \( M^2 \geq 6000 \)
- No intermittency detected in the “C”+C, Pb+Pb datasets.

![Graphs showing data, mixed, and power-law fit for NA49 “Si”+Si @ 158A GeV/c]

- Evidence for intermittency in “Si”+Si – but large statistical errors.
- Based on CMC simulation, we estimate a fraction of \(~ 1\%\) critical protons are present in the sample.
- Estimated intermittency index\(^1\): \( \phi_{2,B} = 0.96^{+0.38}_{-0.25} \) (stat.) \( \pm 0.16 \) (syst.)

$F_2(M)$ of data and mixed events overlap $\Rightarrow$

Subtracted moments $\Delta F_2(M)$ fluctuate around zero $\Rightarrow$

No intermittency effect is observed.

Preliminary analysis with CMC simulation indicates an upper limit of $\sim 0.3\%$ critical protons

[PoS(CPOD2017) 054]

NA61/SHINE CP task force created to verify and extend these results. Task force is spearheaded by IFJ Krakow group, with important contributions from Athens (NKUA), Warsaw (WUT, NCNR) and Frankfurt (FIAS).

Intermittency analysis process:

- Proton selection via particle energy loss $dE/dx$
- Removal of split tracks – $q_{\text{inv}}$ distribution & cut of proton pairs
- Probe $\Delta p_T$ distribution of proton pairs for power-law like behaviour in the limit of small $p_T$ differences
- Calculate factorial moments $F_2(M), \Delta F_2(M)$ for selected protons
- Calculate intermittency index $\phi_2$ (when possible) & estimate its statistical uncertainty

Results were obtained for:

- 0-5%, 5-10% and 10-15% centrality bins
- 80%, 85% and 90% minimum proton purity selections
Split tracks & the $q_{\text{inv}}$ cut

- **Split tracks** (sections of the same track erroneously identified as two close tracks) must be removed.

- **Three cuts** to root them out:
  1. Ratio of points / potential points in a track (removes most)
  2. Minimum track distance in the detector (pair cut)
  3. $q_{\text{inv}}$ cut (pair cut, physics-significant) in the ratio of $P(q_{\text{inv}}^{\text{data}})/P(q_{\text{inv}}^{\text{mixed}})$

$$q_{\text{inv}}(p_i, p_j) \equiv \frac{1}{2} \sqrt{-(p_i - p_j)^2}, \; p_i : 4\text{-momentum of } i^{\text{th}} \text{ track.}$$

- Remove possible split tracks ⇒

- **Universal cutoff** of $q_{\text{inv}} > 7 \text{ MeV/c}$ applied to all sets before analysis.

![Graphs showing ratio data/mix vs $q_{\text{inv}}$ for different centrality and purity settings](attachment:graphs.png)
**$\Delta p_T$ distributions: NA61 data vs EPOS**

- **Ar+Sc at 150A GeV/c:**
  
  $\Delta p_T = \frac{1}{2} \sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$

  distributions of protons selected for intermittency analysis

- In NA61 data, we see strong correlations in $\Delta p_T \to 0 \Rightarrow$ indication of intermittent behaviour

Δp_T distributions & \(F_2(M)\): NA61 data vs EPOS

Ar+Sc NA61, cent.10-15%, pur.90%, Δp_T Ratio

NA61 data

Ar+Sc EPOS, cent.10-15%, pur.90%, Δp_T Ratio

EPOS

Ar+Sc NA61, cent.10-15%, pur > 90%

\[F_2(M)\]

Ar+Sc EPOS, cent.10-15%, pur > 90%

\[F_2(M)\]
NA61/SHINE: Ar+Sc at 150A GeV/c: $F_2(M)$

NA61/SHINE preliminary

N. Davis (IFJ PAN)
NA61/SHINE: Ar+Sc at 150A GeV/c: $\Delta F_2(M)$

NA61/SHINE preliminary

Ar+Sc NA61, cent.0-5%, pur > 80%

Ar+Sc NA61, cent.5-10%, pur > 80%

Ar+Sc NA61, cent.10-15%, pur > 80%

Ar+Sc NA61, cent.0-5%, pur > 85%

Ar+Sc NA61, cent.5-10%, pur > 85%

Ar+Sc NA61, cent.10-15%, pur > 85%

Ar+Sc NA61, cent.0-5%, pur > 90%

Ar+Sc NA61, cent.5-10%, pur > 90%

Ar+Sc NA61, cent.10-15%, pur > 90%
Ar+Sc EPOS: $F_2(M)$, $\Delta F_2(M)$

NA61/SHINE preliminary
Ar+Sc 150 $\Delta F_2(M)$ – statistical significance of signal

1. $\Delta F_2(M)$, NA61 Ar+Sc @ 150 GeV/c bootstrap distributions
2. $\Delta F_2(M)$, random background bootstrap distributions

- Contour map of sigmas from the median

$\sim 2 - 3\sigma$ separation of $\Delta F_2(M)$ from zero in Ar+Sc 150

Average ArSc150 $\Delta F_2(M) \sim 2 - 3\sigma$ away from random background $\Delta F_2(M)$

Based on CMC simulation*, we estimate a fraction of 0.7% critical protons are present in the sample.

Indication of intermittency effect in middle-central NA61/SHINE Ar+Sc collisions

First possible evidence of CP signal in NA61/SHINE

Effect quality increases with increased proton purity selection, up to 90% proton purity; EPOS does not reproduce observed effect.
Expanding the analysis to other NA61/SHINE systems (Xe+La, Pb+Pb) and SPS energies (Ar+Sc) will hopefully lead to a more reliable interpretation of the observed intermittency signal in terms of the critical point.
The NA61/SHINE CP task force, led by the IFJ Krakow group, is already working on extending the Ar+Sc scan to lower energies, as well as scrutinizing intermittency methodology in order to reduce detector dependence and improve result robustness.
Thank you!
Acknowledgements

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Back Up Slides
Subtracting the background from factorial moments

- Experimental data is noisy ⇒ a background of non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.

Partitioning of pairs into critical/background

$$\langle n(n-1) \rangle = \langle n_c(n_c-1) \rangle + \langle n_b(n_b-1) \rangle + 2\langle n_b n_c \rangle$$

critical background cross term

$$\Delta F_2(M) = F_2^{(d)}(M) - \lambda(M)^2 \cdot F_2^{(b)}(M) - 2 \cdot \lambda(M) \cdot (1 - \lambda(M)) f_{bc}$$

correlator data background ratio $\frac{\langle n \rangle_b}{\langle n \rangle_d}$

- The cross term can be neglected under certain conditions (non-trivial! Justified by Critical Monte Carlo* simulations)

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk:
    \[ \tilde{d}^{(B,2)}_F = 1/3 \Rightarrow \phi_2 = 5/6 \]
  - Lower / upper bounds of Lévy walks \( p_{\text{min},\text{max}} \) plugged in.
  - Cluster center exponential in \( p_T \), slope adjusted by \( T_c \) parameter.
  - Poissonian proton multiplicity distribution.

### Input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( p_{\text{min}} ) (MeV)</th>
<th>( p_{\text{max}} ) (MeV)</th>
<th>( \lambda_{\text{Poisson}} )</th>
<th>( T_c ) (MeV)</th>
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<tbody>
<tr>
<td>Value</td>
<td>0.1 → 1</td>
<td>800 → 1200</td>
<td>( \langle p \rangle_{\text{non-empty}} )</td>
<td>163</td>
</tr>
</tbody>
</table>

Proton selection

- Employ $p_{tot}$ region where Bethe-Bloch bands do not overlap
  $(3.98 \text{ GeV}/c \leq p_{tot} \leq 126 \text{ GeV}/c)$
- Fit $dE/dx$ distribution with 4-gaussian sum for $\alpha = \pi, K, p, e$ — Bins: $p_{tot}, p_T$
- 30 Bins in $\log_{10}(p_{tot})$: $10^{0.6} \rightarrow 10^{2.1}$ GeV/c
- 20 Bins in $p_T$: $0.0 \rightarrow 2.0$ GeV/c
- Proton purity: probability for a track to be a proton, $P_p = p/(\pi + K + p + e)$
- Additional cut along Bethe-Blochs
  (avoid low-reliability region between $p$ and $K$ curves)
Used $dE/dx$ spectra from Ar+Sc @150 data in the 6% - 18% centrality interval
For each track, assign a $dE/dx$ value based on particle species and phase space bin
Apply $dE/dx$ & purity cuts identical to NA61/SHINE data
### Ar+Sc at 150A GeV/c: NA61 data vs EPOS

#### EPOS – proton $p_T$ statistics

| Centrality | #events   | $\langle p \rangle_{|p_T|\leq1.5 \text{ GeV}, |y_{CM}|\leq0.75}$ | $\Delta p_{x,y}$ |
|------------|-----------|-------------------------------------------------|-----------------|
|            |           | Non-empty | With empty |                      |
| 0- 5%      | 293,412   | $3.06 \pm 1.60$ | $2.89 \pm 1.70$ | $0.35 - 0.43$ |
| 5-10%      | 252,362   | $2.72 \pm 1.45$ | $2.49 \pm 1.58$ | $0.35 - 0.43$ |
| 10-15%     | 274,072   | $2.45 \pm 1.33$ | $2.16 \pm 1.48$ | $0.35 - 0.43$ |

#### $^{40} Ar + ^{45} Sc$ NA61 data – proton $p_T$ statistics

| Centrality | #events   | $\langle p \rangle_{|p_T|\leq1.5 \text{ GeV}, |y_{CM}|\leq0.75}$ | $\Delta p_{x,y}$ |
|------------|-----------|-------------------------------------------------|-----------------|
|            |           | Non-empty | With empty |                      |
| 0- 5%      | 144,362   | $3.44 \pm 1.79$ | $3.30 \pm 1.89$ | $0.46 - 0.58$ |
| 5-10%      | 148,199   | $3.00 \pm 1.61$ | $2.79 \pm 1.73$ | $0.46 - 0.58$ |
| 10-15%     | 142,900   | $2.81 \pm 1.53$ | $2.58 \pm 1.66$ | $0.45 - 0.57$ |
$p_{x,y}$ spectra comparison – NA61 vs EPOS (0 – 15%)
Split tracks; the $q_{inv}$ cut in analysed datasets

- Split tracks can create false positive for intermittency ⇒ must be reduced or removed.

- $q_{inv}$-test – distribution of track pairs: $q_{inv}(p_i, p_j) \equiv \frac{1}{2} \sqrt{(p_i - p_j)^2}$, $p_i$ : 4-momentum of $i^{th}$ track.

- Calculate ratio $q_{inv}^{data}/q_{inv}^{mixed}$ ⇒ peak at low $q_{inv}$ (below 20 MeV/c): possible split track contamination.

- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis ⇒ “dip” in low $q_{inv}$, peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]

- Universal cutoff of $q_{inv} > 25$ MeV/c applied to all sets before analysis.
NA49 analysis – $\Delta p_T$ distributions

- We measure correlations in relative $p_T$ of protons via
  
  $$\Delta p_T = \frac{1}{2}\sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$$

- Strong correlations for $\Delta p_T \to 0$ indicate power-law scaling of the density-density correlation function $\Rightarrow$ intermittency presence

- We find a strong peak in the “Si”+Si dataset

- A similar peak is seen in the $\Delta p_T$ profile of simulated CMC protons with the characteristics of “Si”+Si.
Noisy CMC (baryons) – estimating the level of background

- $F_2(M)$ of noisy CMC approximates “Si”+Si for $\lambda \approx 0.99$
- $\Delta F_2^{(e)}(M)$ reproduces critical behaviour of pure CMC, even though their moments differ by orders of magnitude!

- Noisy CMC results show our approximation is reasonable for dominant background.
$q_{inv}$ proton distributions – NA61/SHINE

Ar+Sc NA61, cent.0-5%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.0-5%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.0-5%, pur.90%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.90%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.90%, $q_{inv}$ Ratio