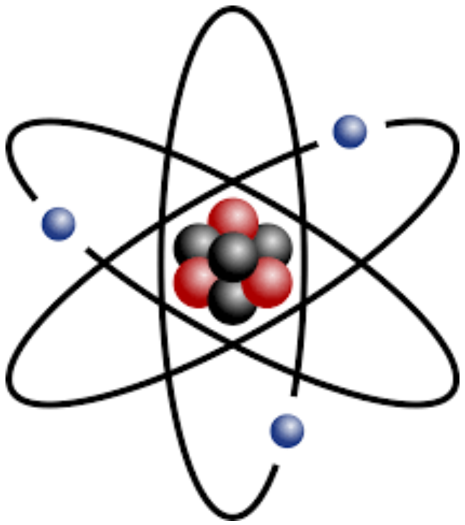
Nuclear structure corrections in light muonic atoms

Sonia Bacca

Johannes Gutenberg University, Mainz

What are muonic atoms?

Exotic atoms



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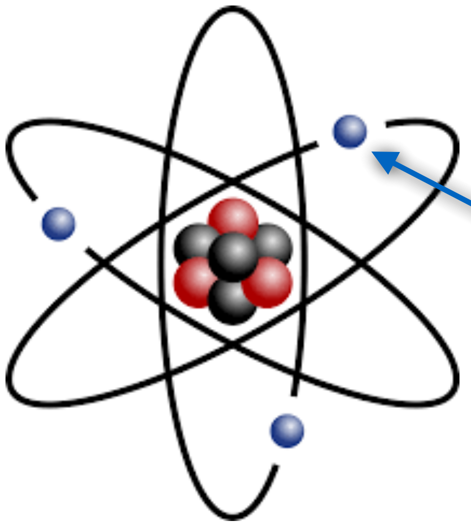

Electron replaced with a muon

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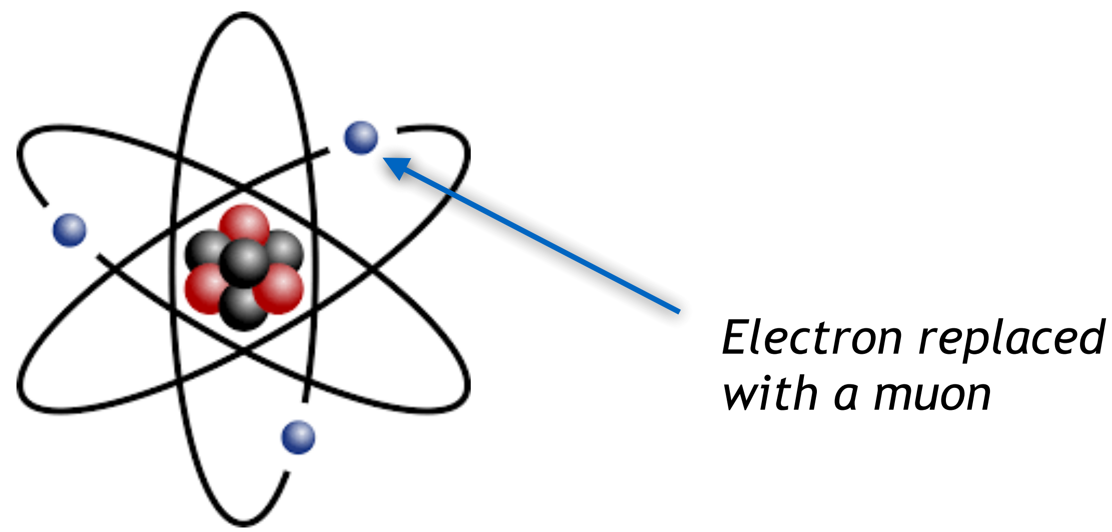
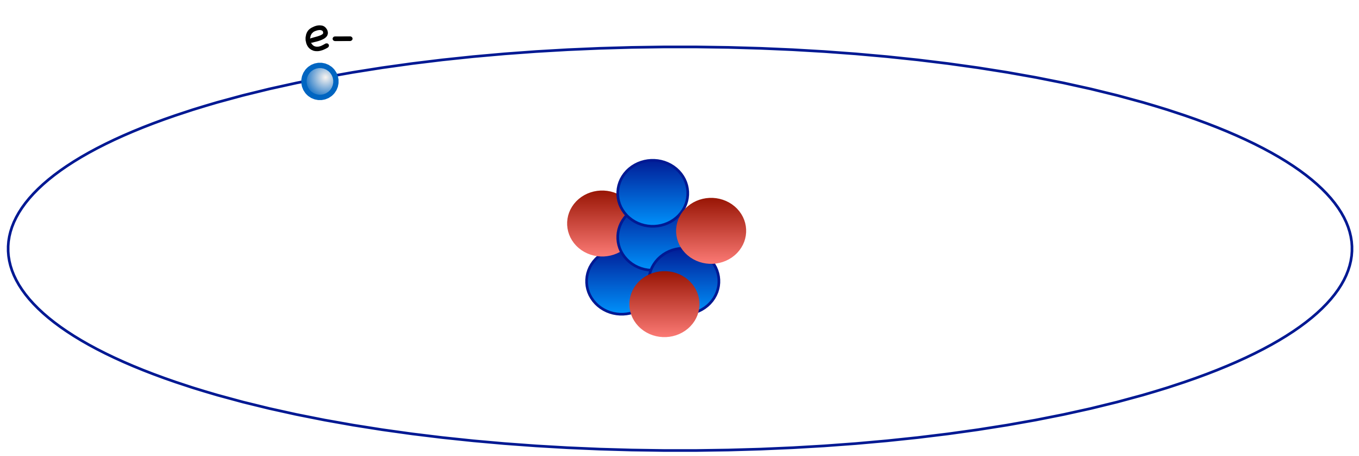
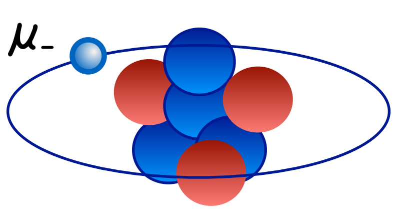


Hydrogen-like systems

Ordinary atoms



Muonic atoms



muon more sensitive to the nucleus

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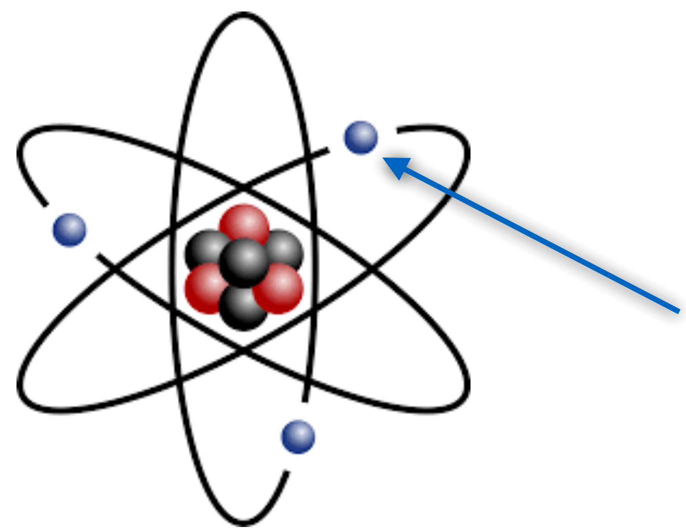

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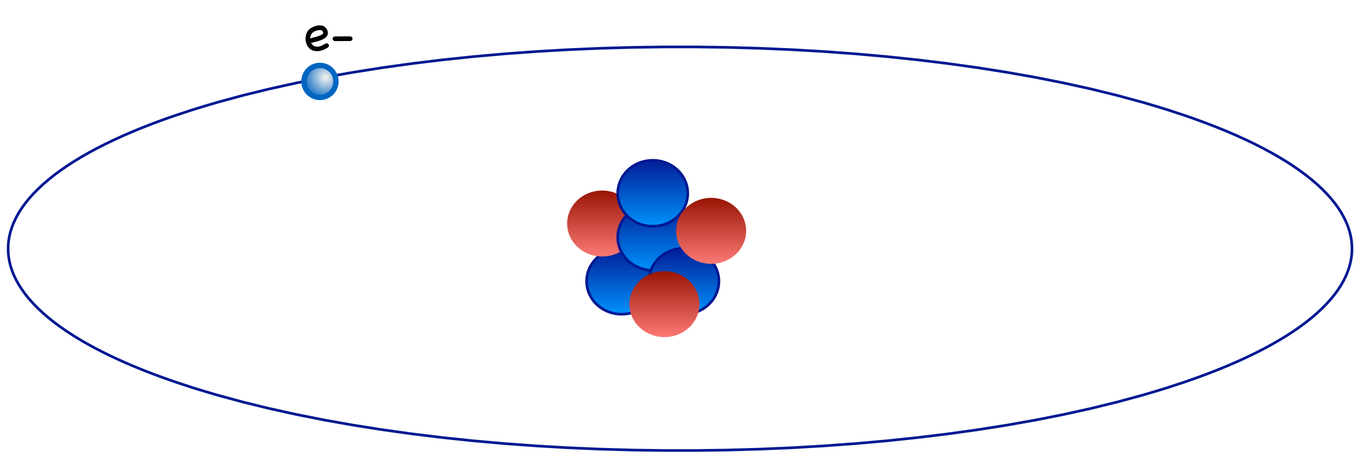
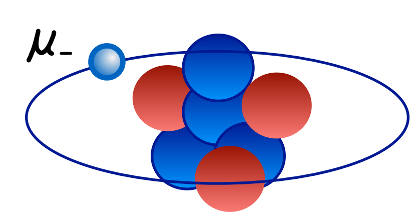


Muonic atoms



muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

Production and measurements

Lamb Shift:
2S-2P splitting in atomic spectrum

$a.$ prompt X-ray $(t \sim 0)$

- μ^- stopped in H_2 gases
- 99% $\rightarrow$ 1S
- 1% $\rightarrow$ 2S $(\tau_{2S} \approx 1\mu s)$

$b.$ delayed X-ray $(t \sim 1\mu s)$

- laser induced 2S$\rightarrow$2P
- measure $K_\alpha^{\text{delayed}}/K_\alpha^{\text{prompt}}$
- $f_{res} = \Delta E_{LS}$

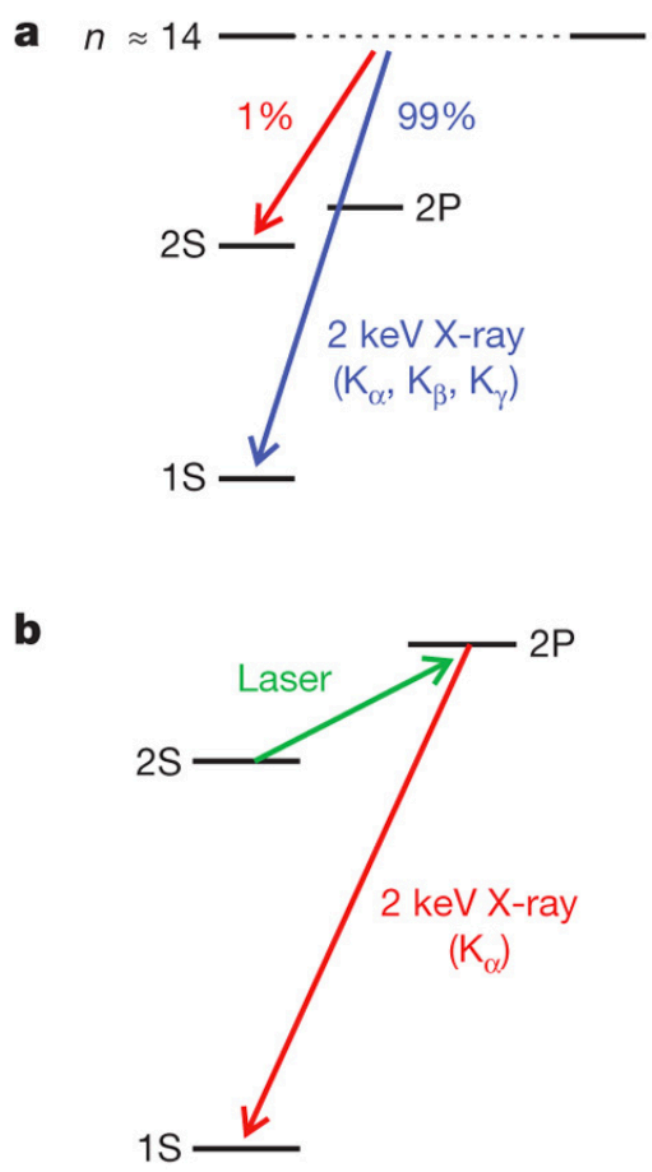
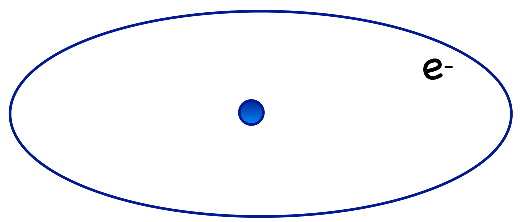


Figure from Pohl *et al.* Nature (2010)

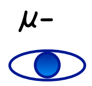
Proton Radius Puzzle

The proton charge radius is measured from:

- electron-proton interactions: 0.8770 ± 0.0045 fm
  - eH spectroscopy
  - e-p scattering



- muonic -proton interactions: 0.8409 ± 0.0004 fm
  - μH Lamb-shift



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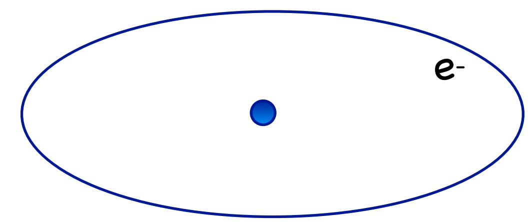

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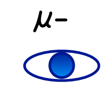

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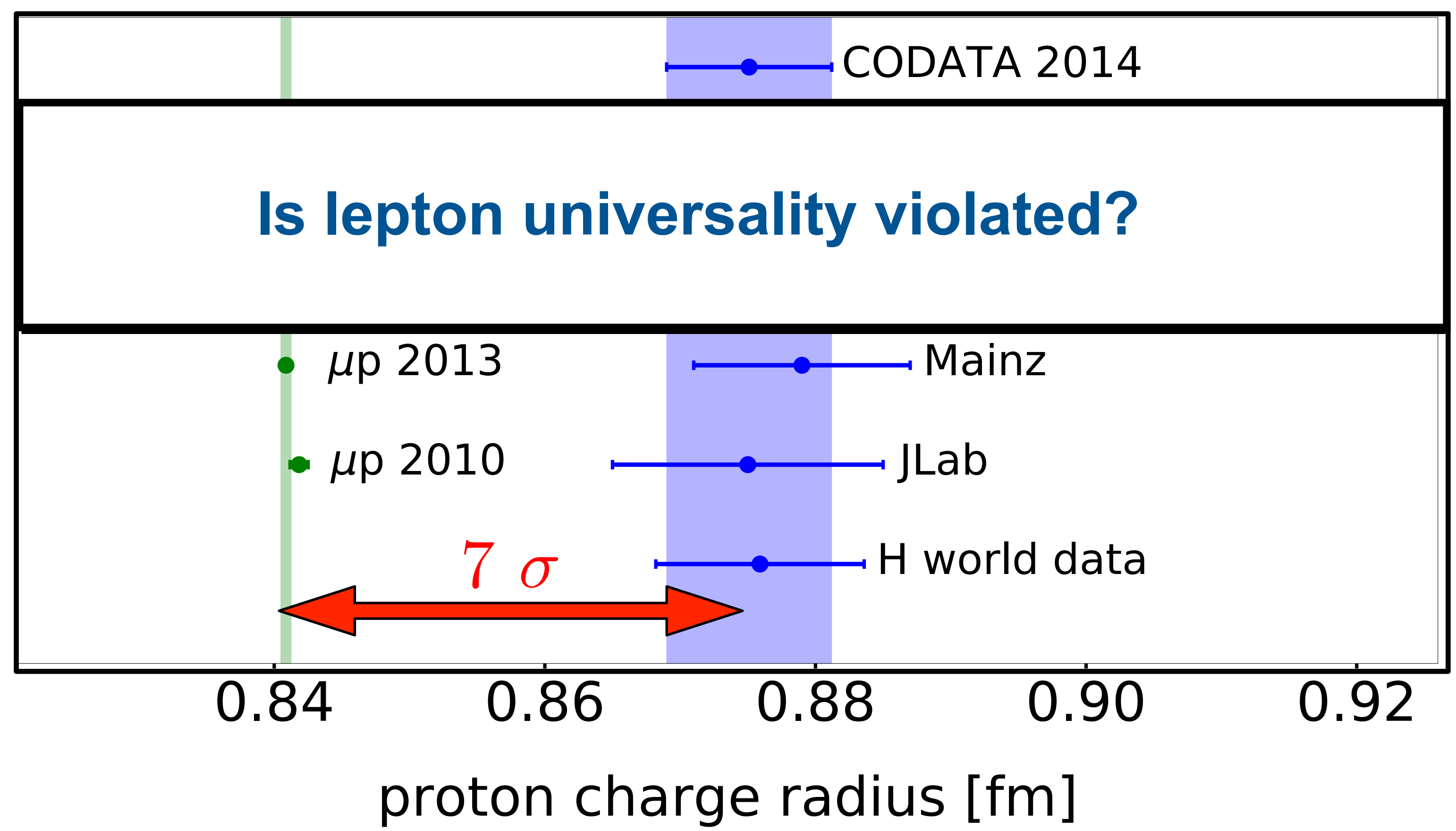

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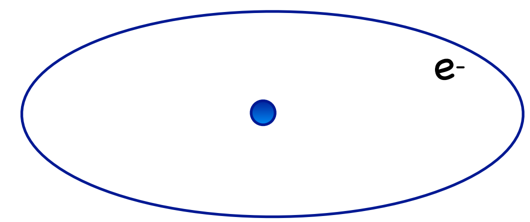

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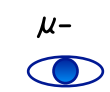

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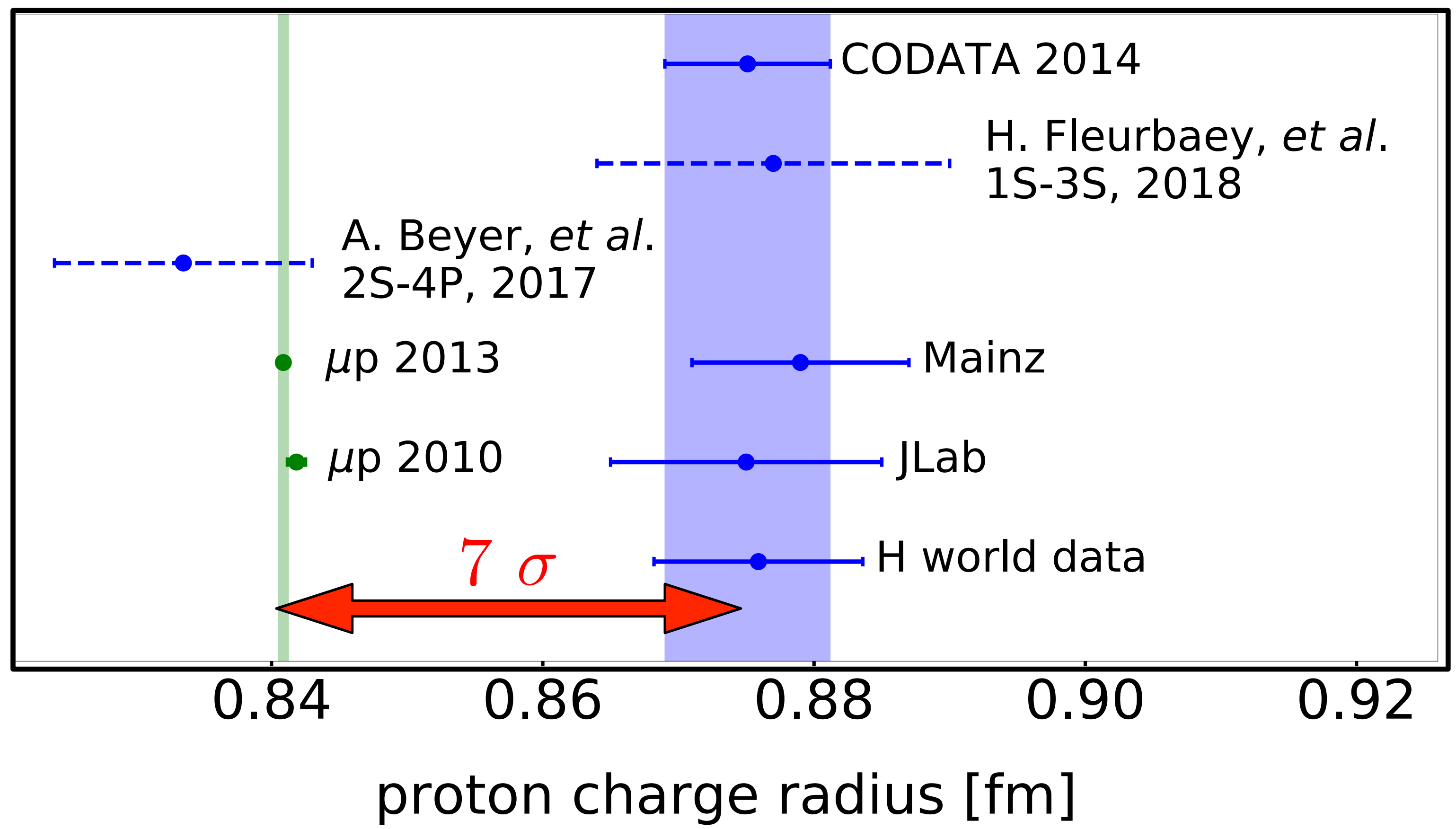

Experimental efforts

- **Higher precision electron scattering experiments**

  Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2 Jefferson Lab

  ISR measurement, not competitive, Phys.Lett. B **771** (2017) 194-198

  Repeat traditional measurement with windowless gas jet target (2019/20)

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  PAUL SCHERRER INSTITUT PSI

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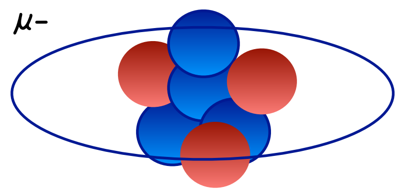
Understanding the Proton Radius Puzzle

Strong experimental program at PSI (Switzerland) from the CREMA collaboration to unravel the mystery by studying other muonic atoms:

- μD (results released)
- μ^4He^+ (analyzing data)
- μ^3He^+ (analyzing data)
- μ^3H (impossible?)
- μ^6Li^{2+}, μ^7Li^{2+} (future)



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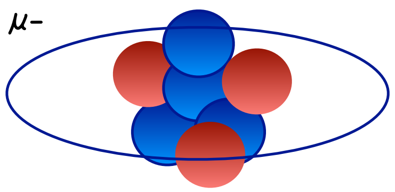

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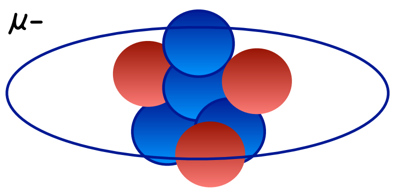

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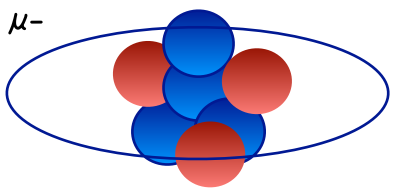

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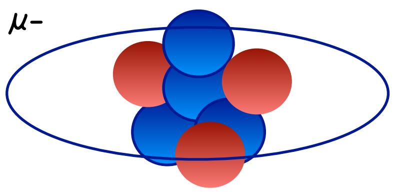

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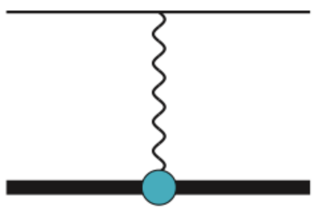

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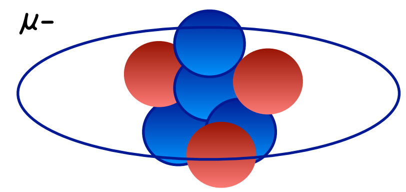

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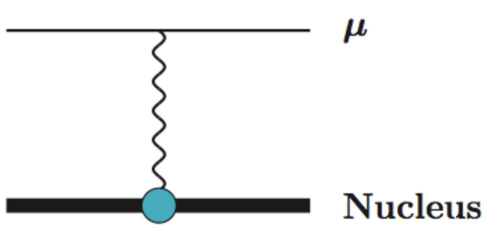
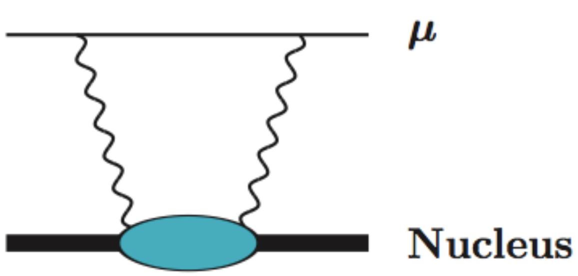


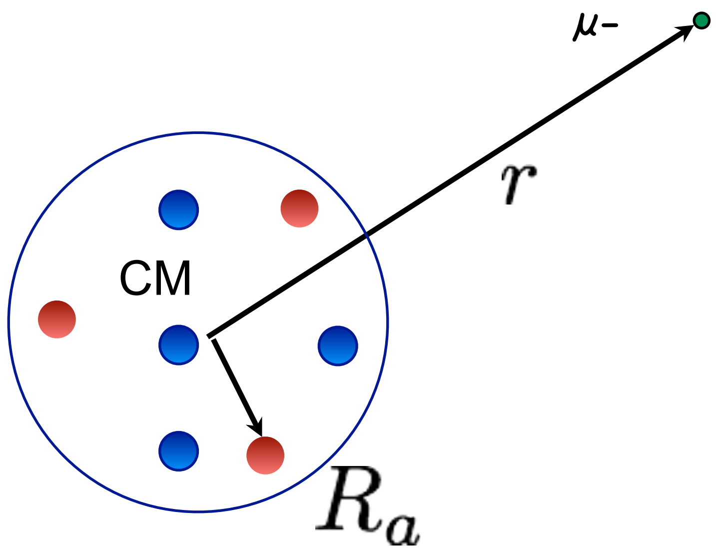
unknown
NS corrections



Theoretical derivation of TPE

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



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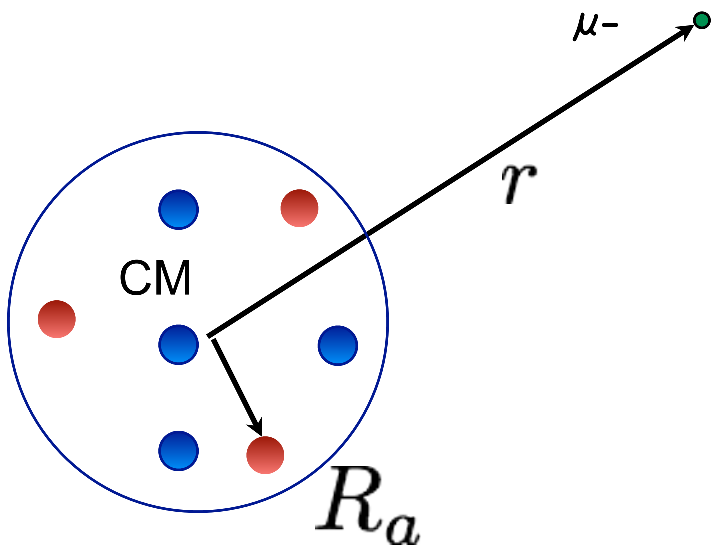

Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

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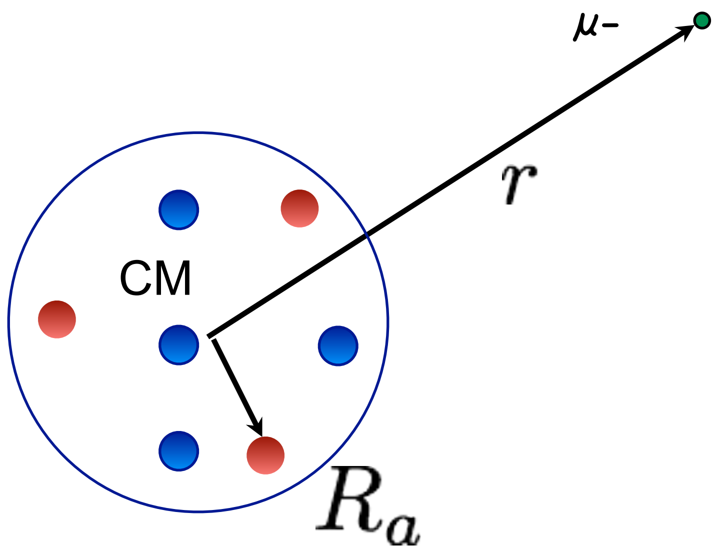

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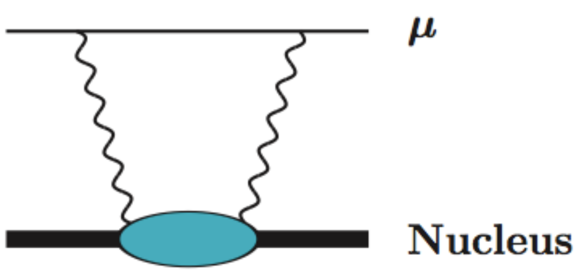
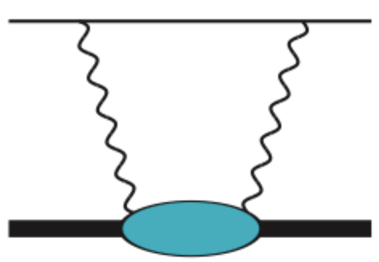
Using perturbation theory at second order
one obtains the expression for TPE
up to order $(Z\alpha)^5$



Theoretical derivation of TPE

- Non relativistic term



Take non-relativistic kinetic energy in muon propagator
Neglect Coulomb force in the intermediate state
Expand the muon matrix elements in powers of η

η= $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

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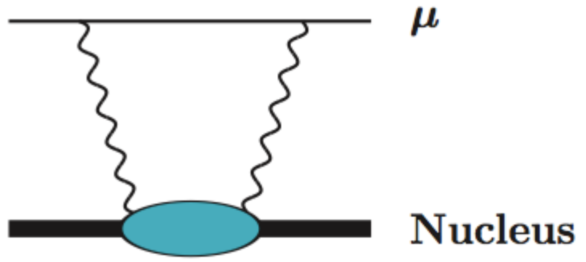

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$\delta^{(0)}$ $\delta^{(1)}$ $\delta^{(2)}$

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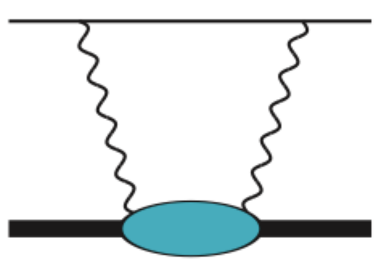

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$\delta^{(0)}$ $\delta^{(1)}$ $\delta^{(2)}$

- ★ $|\boldsymbol{R}-\boldsymbol{R}'|$ "virtual" distance traveled by the proton between the two-photon exchange
- ★ Uncertainty principle $|\boldsymbol{R}-\boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$
- ★ $\eta = \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} = 0.17$

Theoretical derivation of TPE

- Non relativistic term

★ $\delta^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

dominant term, related to the energy-weighted integral of the dipole response function

$$\delta^{(0)}_{D1} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

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★ $\delta^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$ Related to Zemach moment elastic contribution

$$\delta^{(1)}_{Z3} = \frac{\pi}{3} m_r (Z\alpha)^2 \phi^2(0) \iint d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0^p(\boldsymbol{R}) \rho_0^p(\boldsymbol{R}')$$

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★ $\delta^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

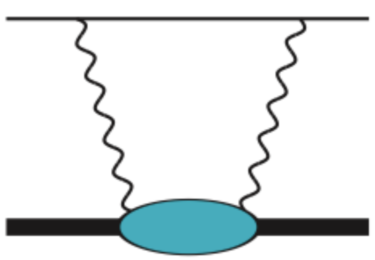
leads to energy-weighted integrals of three different response functions

$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$

Theoretical derivation of TPE

- Coulomb term

Consider the Coulomb force in the intermediate states
Naively $\delta_C^{(0)} \sim (Z\alpha)^6$, actually logarithmically enhanced
$\delta_C^{(0)} \sim (Z\alpha)^5 \log(Z\alpha)$ Friar (1977), Pachucki (2011)
Related to the dipole response function



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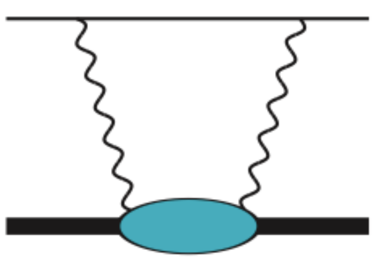

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Related to the dipole response function

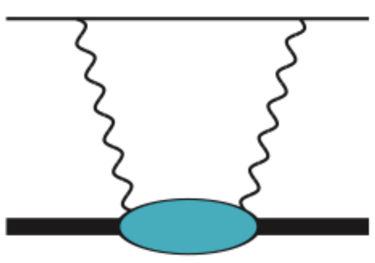
Relativistic terms

Take the relativistic kinetic energy in muon propagator
Related to the dipole response function

$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right)\, S_{D_1}(\omega)$$

Theoretical derivation of TPE

Coulomb term



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Finite nucleon-size corrections

Consider finite nucleon-size by including their charge distributions and obtain terms, e.g.,

$$\delta_{R1}^{(1)} = -8\pi m_r (Z\alpha)^2 \phi^2(0) \int\int d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'| \left[\frac{2}{\beta^2}\rho_0^{pp}(\boldsymbol{R},\boldsymbol{R}') - \lambda\rho_0^{np}(\boldsymbol{R},\boldsymbol{R}')\right]$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^N_{\rm Zem} + \delta^A_{\rm pol} + \delta^N_{\rm pol}$$

Theoretical derivation of TPE

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$$\delta^{A}_{\rm pol} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \delta^{(1)}_{Z3} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C}$$
$$+\delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \delta^{(1)}_{Z1} + \delta^{(2)}_{NS}$$

$$\delta^{A}_{\rm Zem} = -\delta^{(1)}_{Z3} - \delta^{(1)}_{Z1}$$

Friar an Payne ('97)

Need to calculate $\delta_{\rm TPE}$ and related uncertainties.

A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

Roughly: 95% 4% 1%

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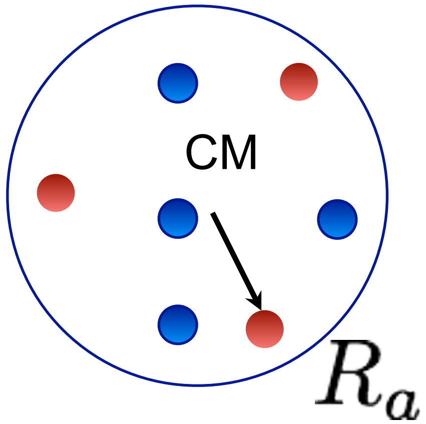
*Leidemann, Rosenfelder ’95 using few-body methods

Ab Initio Nuclear Theory

- Solve the Schrödinger equation for few-nucleons

$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$



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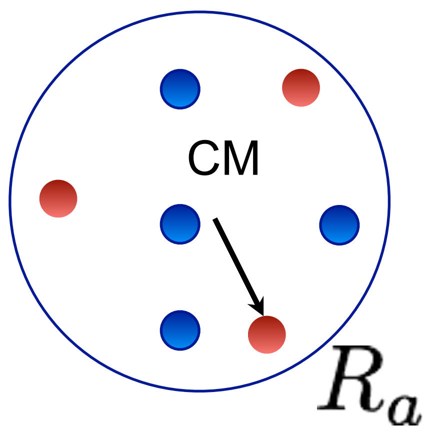

Hyper-spherical harmonics expansions for A=3,4,6,7 → Barnea, Leidemann, Orlandini PRC **61** (2000) 054001

For A=2 we use an harmonic oscillator expansion

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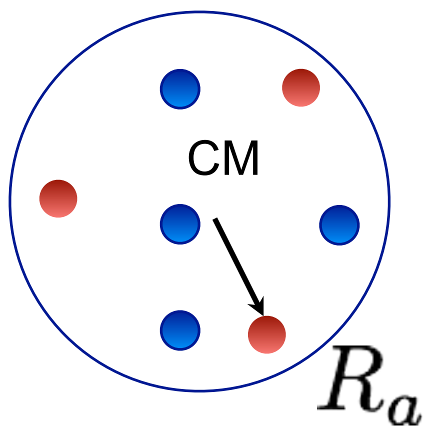

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Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

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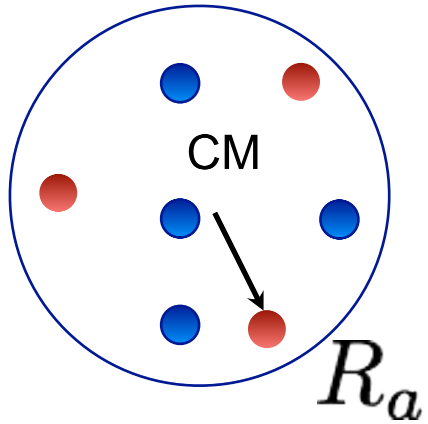

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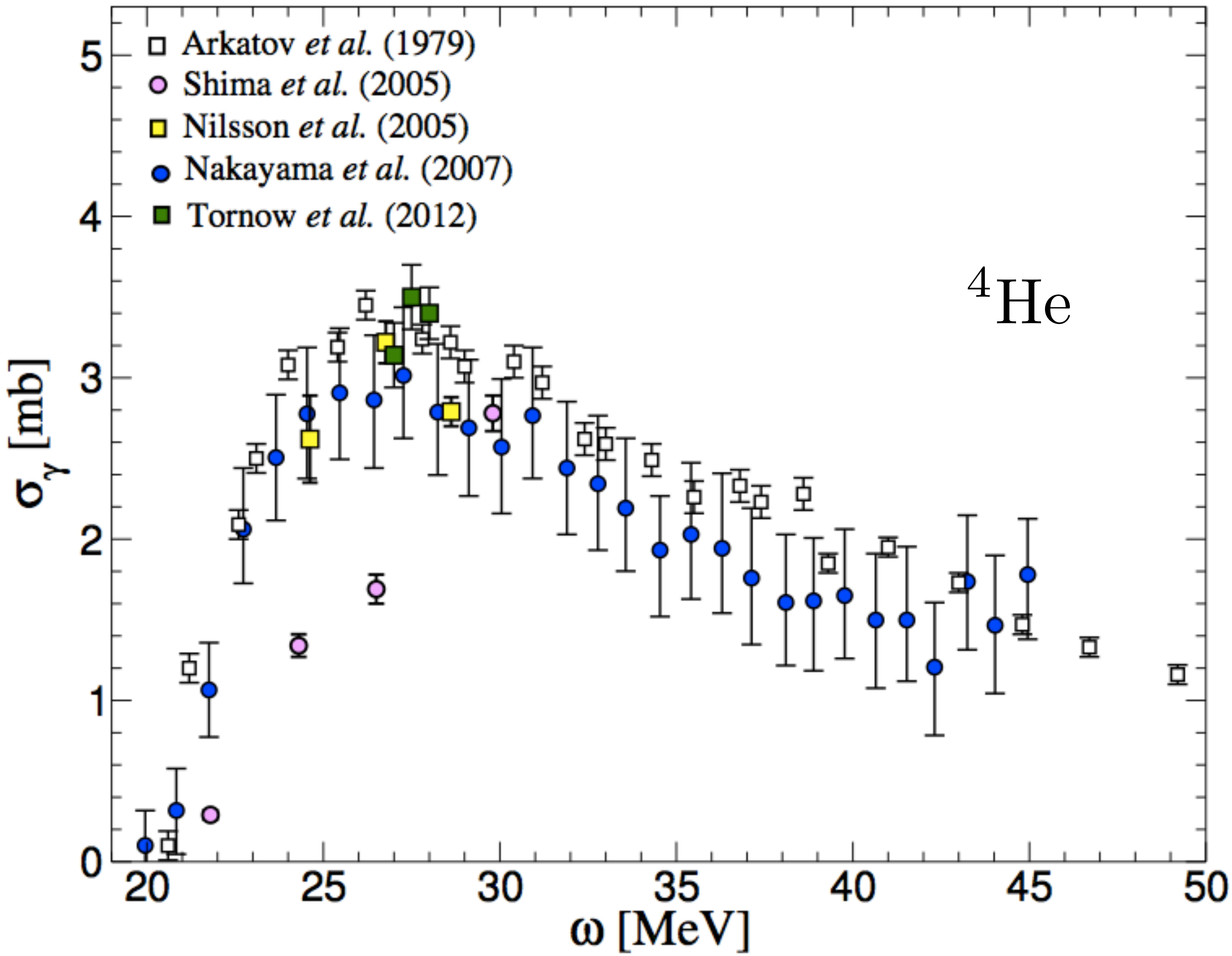
  Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

- We will use nuclear interactions derived from chiral effective filed theory (at various orders) and traditional potentials (AV18+UIX)

An example



S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

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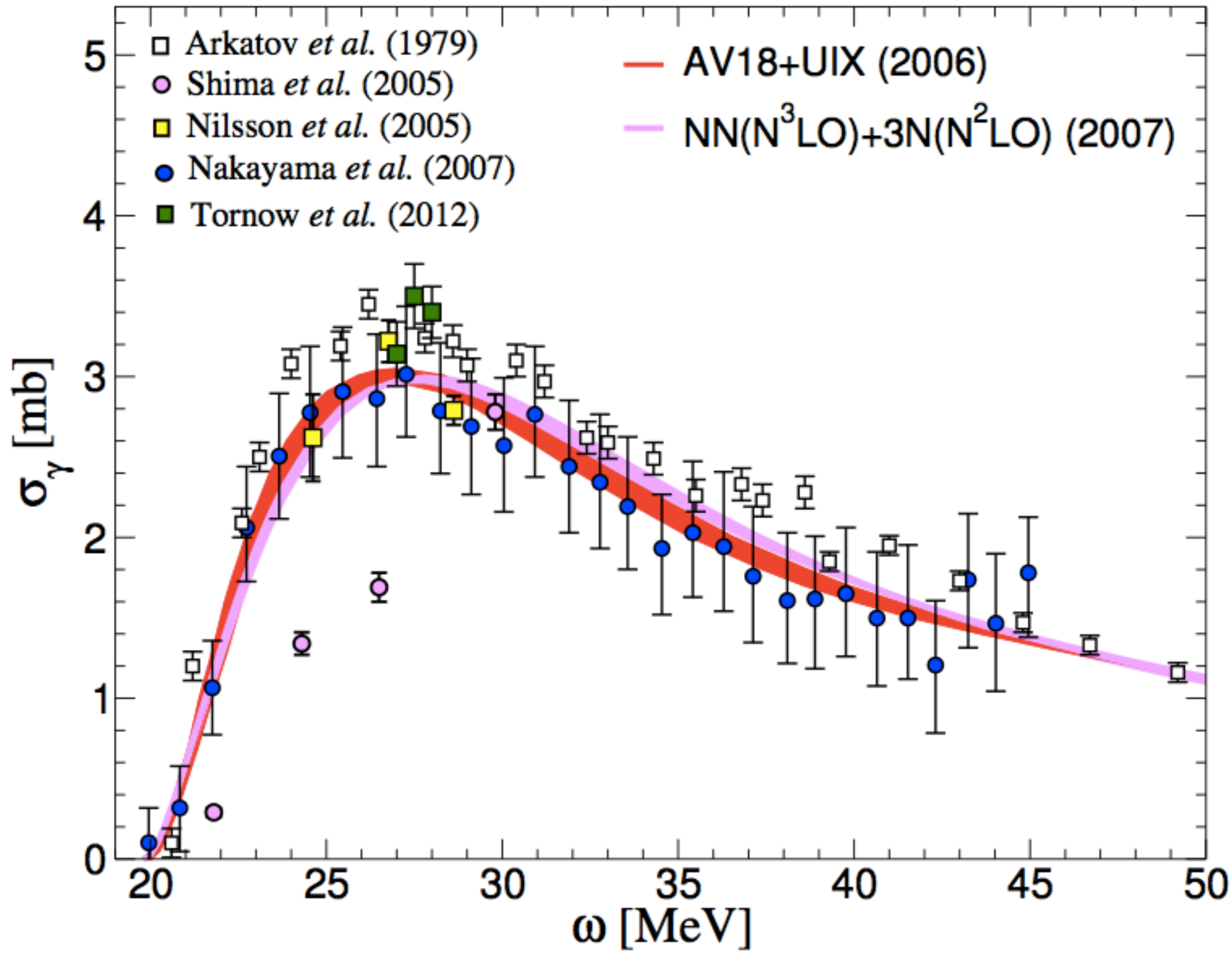
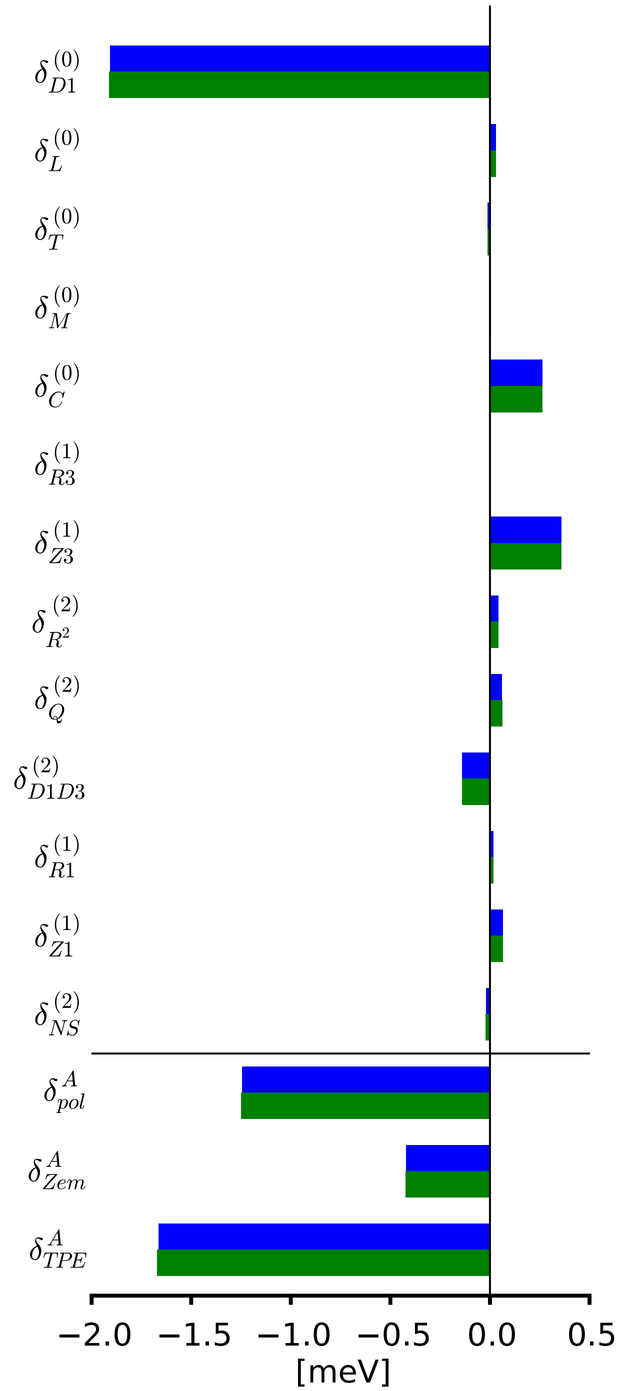


S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Muonic Deuterium



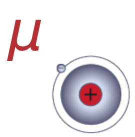
	AV18	χEFT
$\delta^{(0)}_{D1}$	-1.907	-1.912
$\delta^{(0)}_{L}$	0.029	0.029
$\delta^{(0)}_{T}$	-0.012	-0.012
$\delta^{(0)}_{M}$	0.003	0.003
$\delta^{(0)}_{C}$	0.262	0.262
$\delta^{(1)}_{R3}$	-	-
$\delta^{(1)}_{Z3}$	0.357	0.359
$\delta^{(2)}_{R^2}$	0.042	0.041
$\delta^{(2)}_{Q}$	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.139	-0.139
$\delta^{(1)}_{R1}$	0.017	0.017
$\delta^{(1)}_{Z1}$	0.064	0.064
$\delta^{(2)}_{NS}$	-0.020	-0.021
δ^{A}_{pol}	-1.243	-1.248
δ^{A}_{Zem}	-0.421	-0.423
δ^{A}_{TPE}	-1.664	-1.671

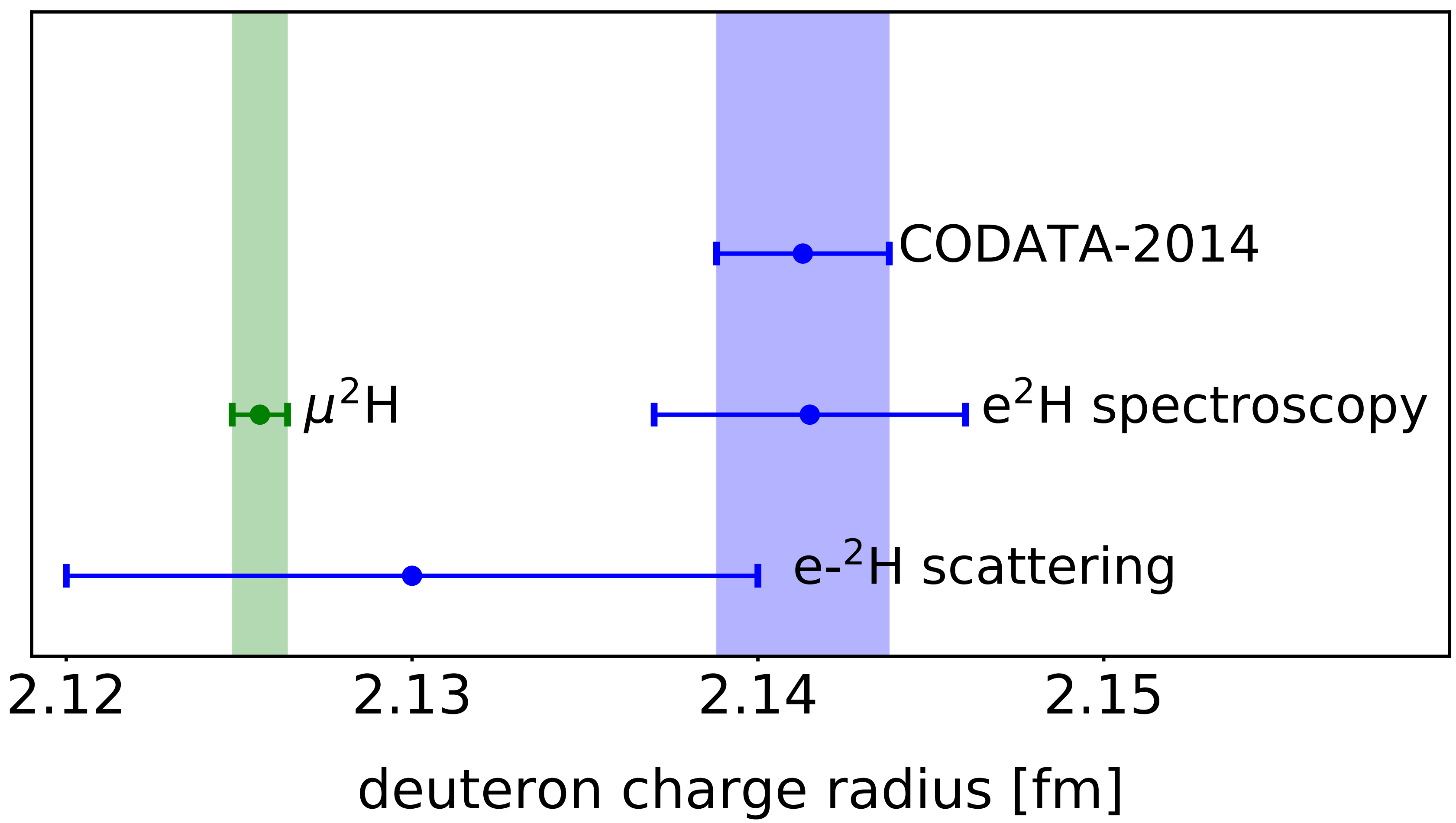
J. Hernandez et al, Phys. Lett. B **736**, 344 (2014)

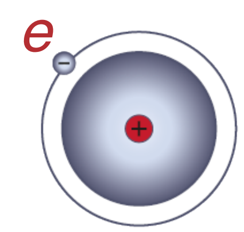
AV18 in agreement with Pachucki (2011)+ Pachucki, Wienczek (2015)

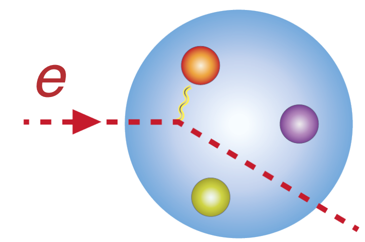
Deuteron radius puzzle

Pohl et al, Science **353**, 669 (2016)









$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2 \rangle + \delta_{\rm TPE}$$

Hernandez et al., PLB **736**, 334 (2014)

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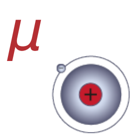

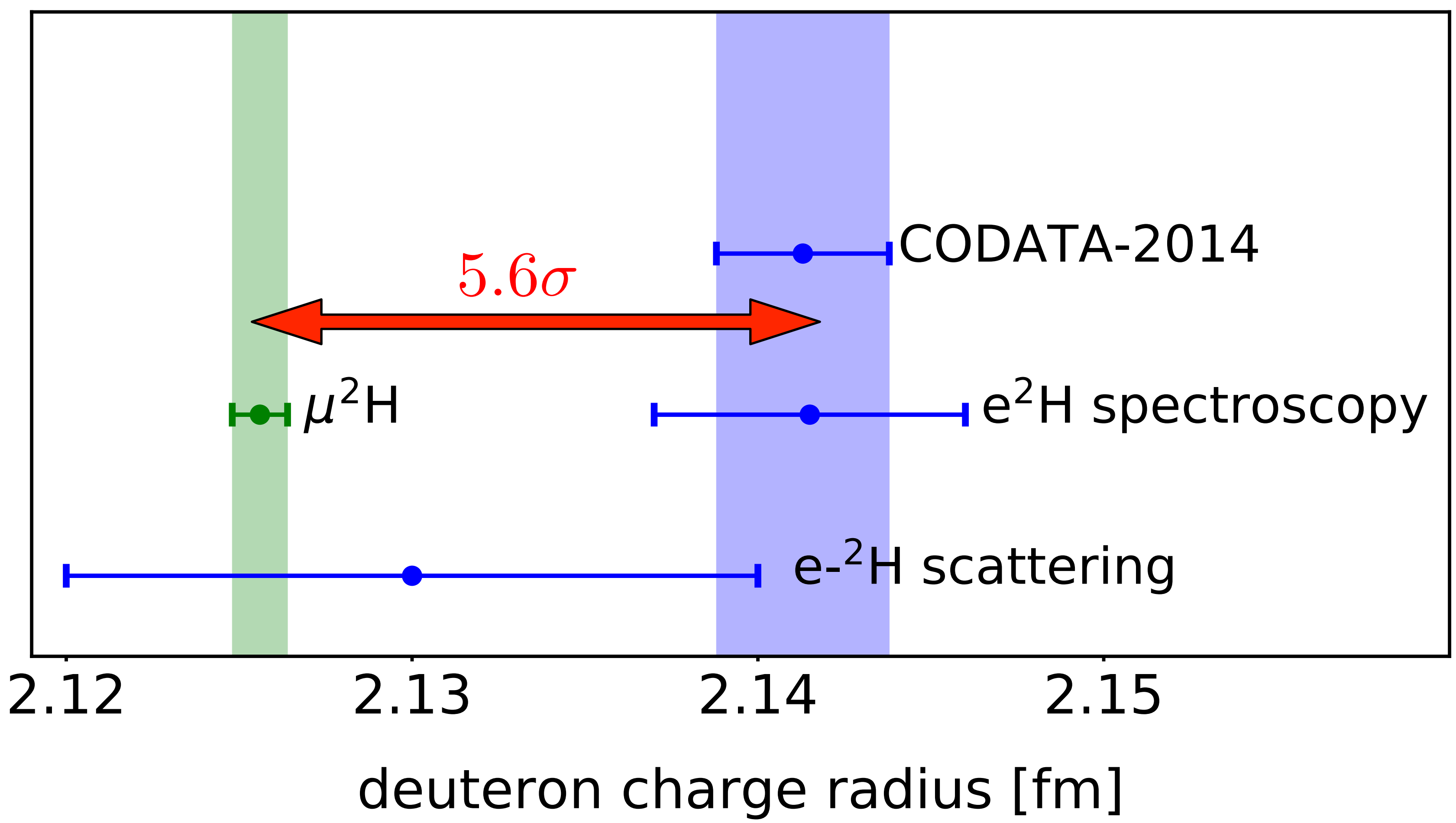

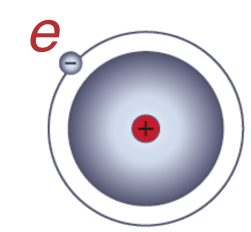

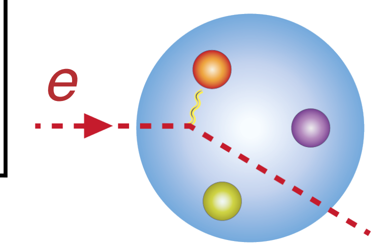

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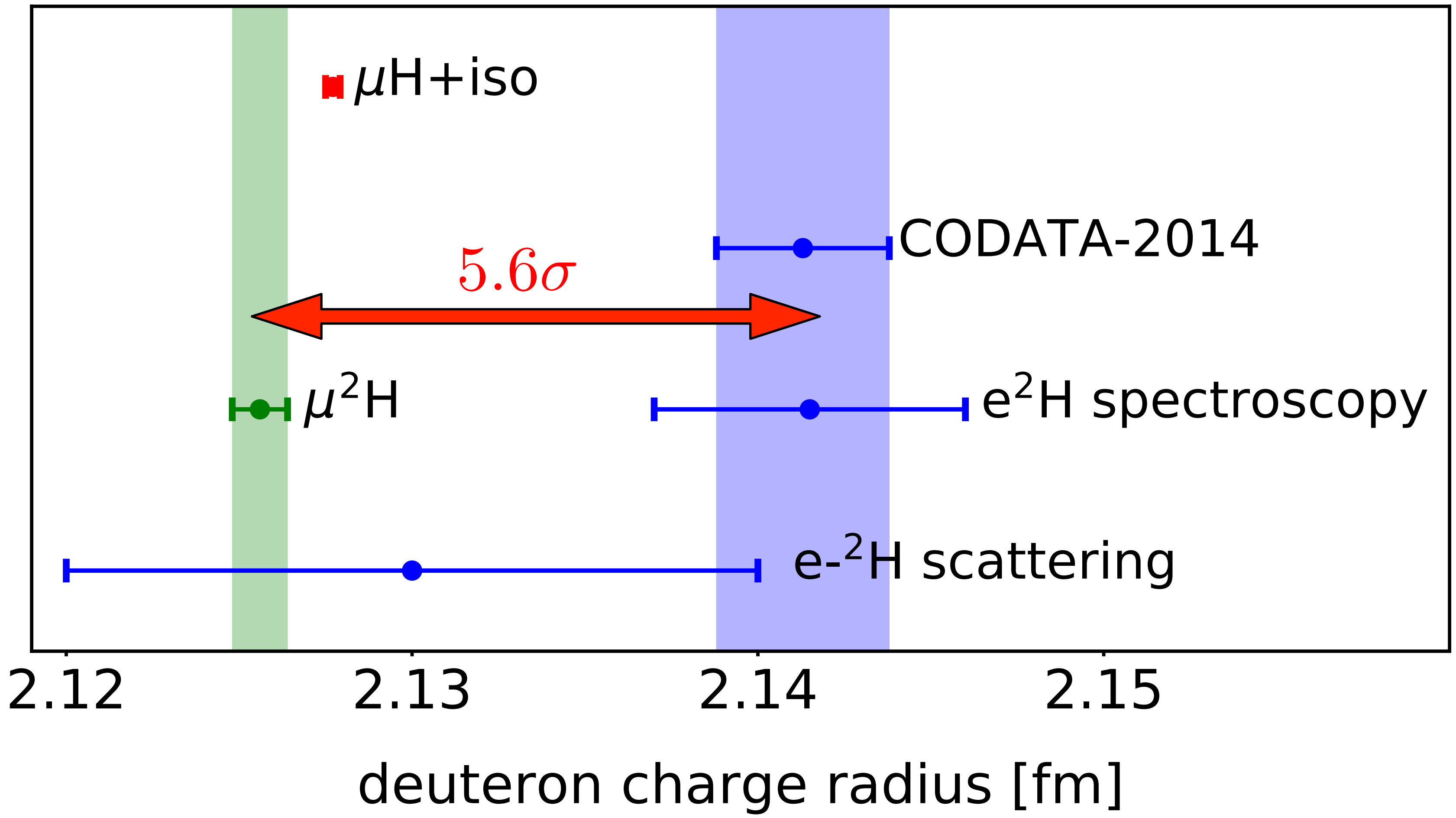

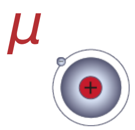

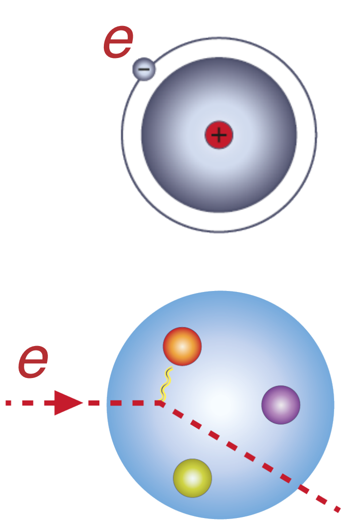

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μH+iso: r_p from μH and deuterium isotopic shift $r^2_d - r^2_p$: Parthey et al., PRL **104** 233001 (2010)

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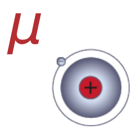

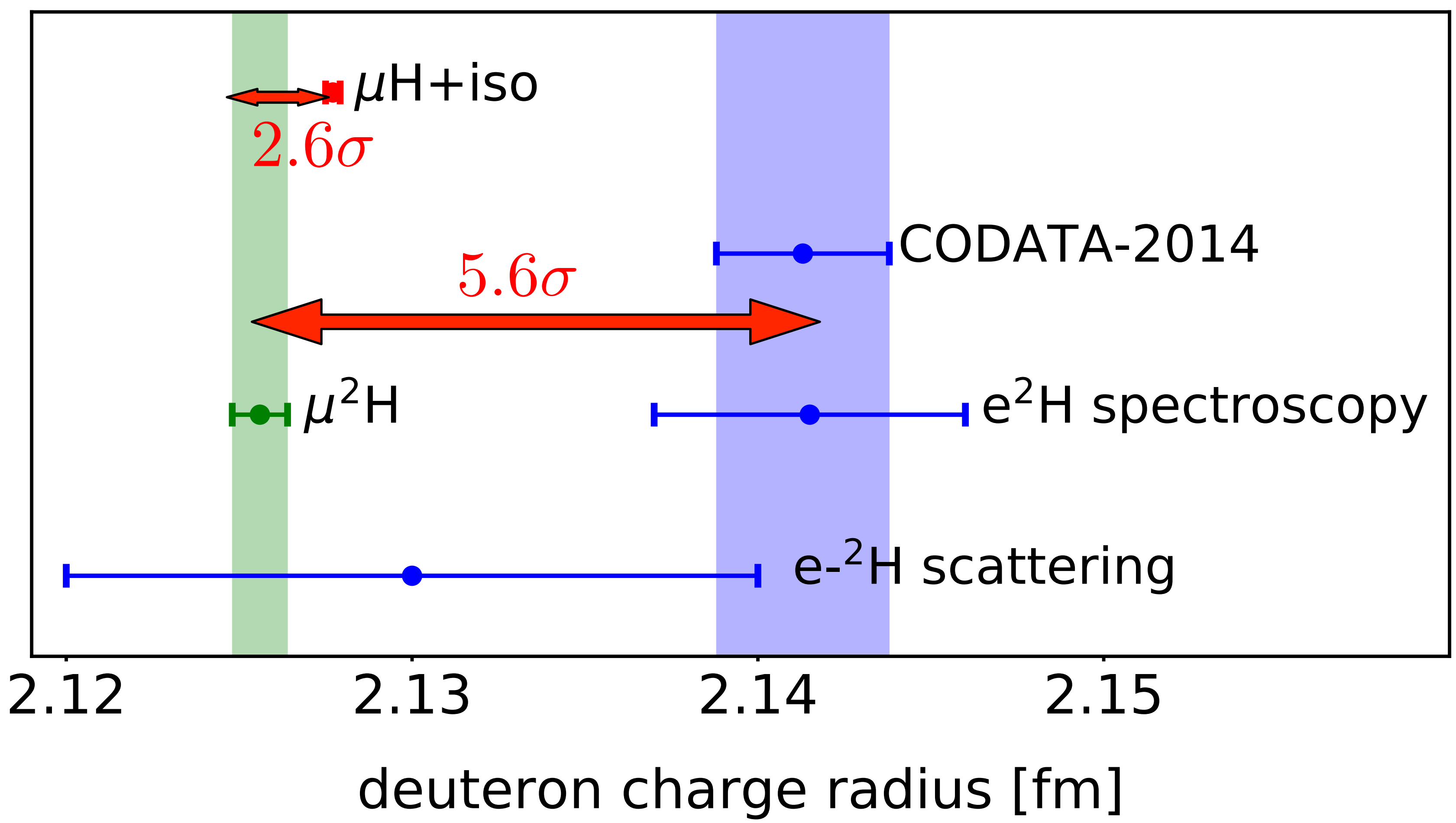

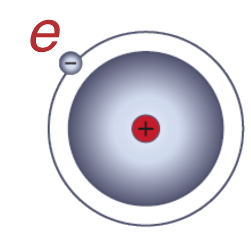

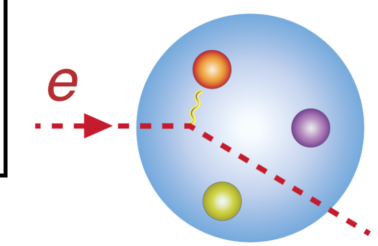

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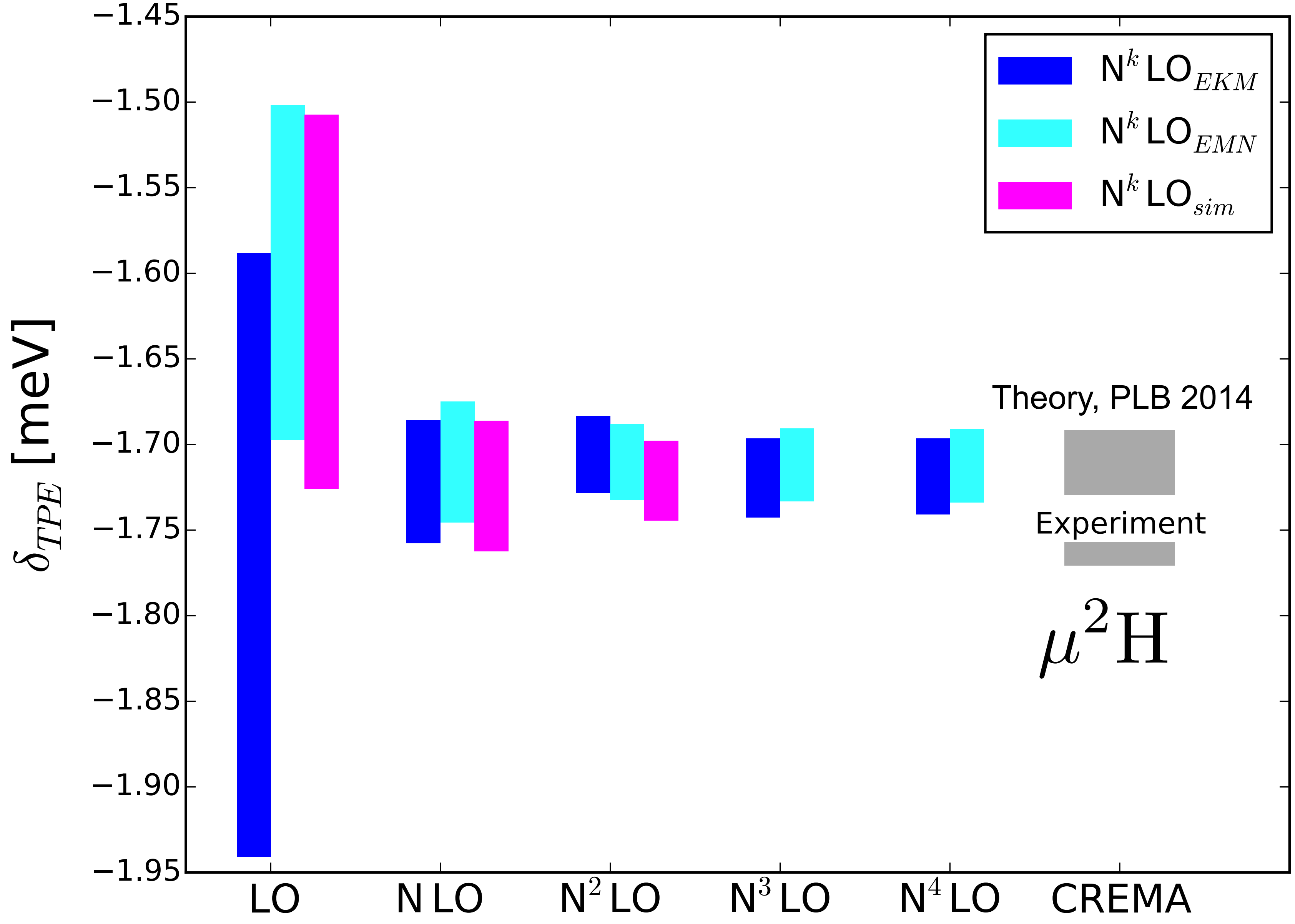
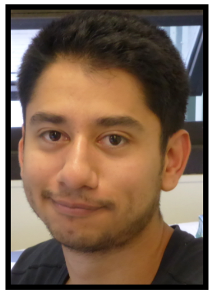
Order-by-order chiral expansion

Statistical and systematic uncertainty analysis

J. Hernandez et al, Phys. Lett. B **778**, 377 (2018)

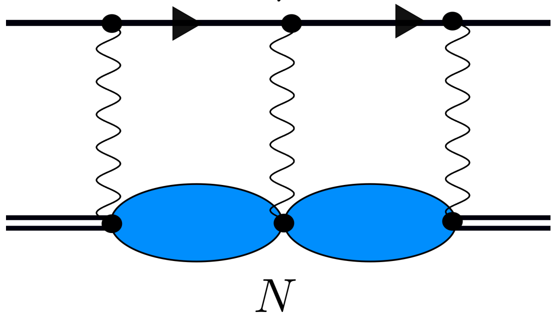




Only sightly mitigate the “small” proton radius puzzle (2.6 to 2 σ)

Higher order corrections in α

Three-photon exchange



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

$(Z\alpha)^6$ correction, negligible

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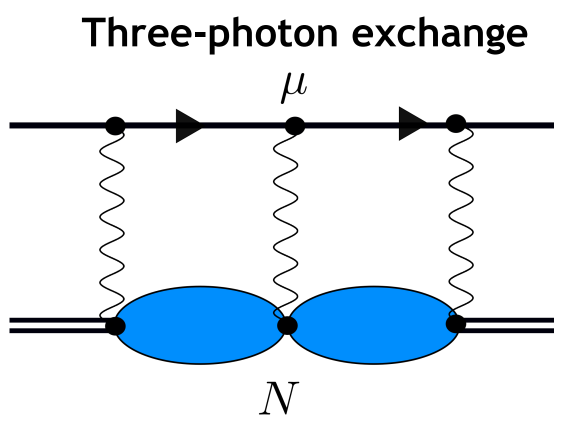

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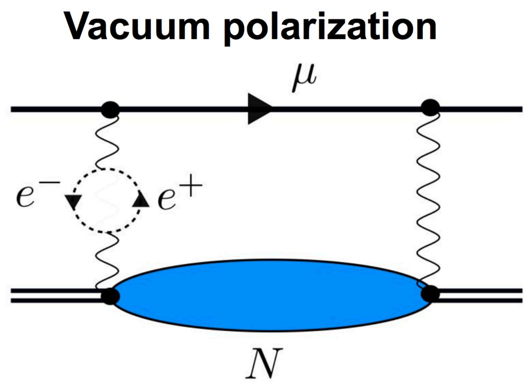

One the many α^6 corrections, supposedly the largest

Kalinowski, Phys. Rev. A **99** 030501 (2019)

$$\delta_{\rm TPE} = -1.750^{+14}_{-16}\ {\rm meV}$$ Theory

$$\delta_{\rm TPE} = -1.7638(68)\ {\rm meV}$$ Exp

Consistent within 1σ
solves the small deuteron-radius puzzle

Large deuteron-radius puzzle still unsolved!
New data on electron scattering expected from MAMI and from the future MESA

Uncertainties quantifications

Uncertainties sources

- Numerical
- Nuclear model
- Nucleon-size
- Truncation of multiples
- η-expansion
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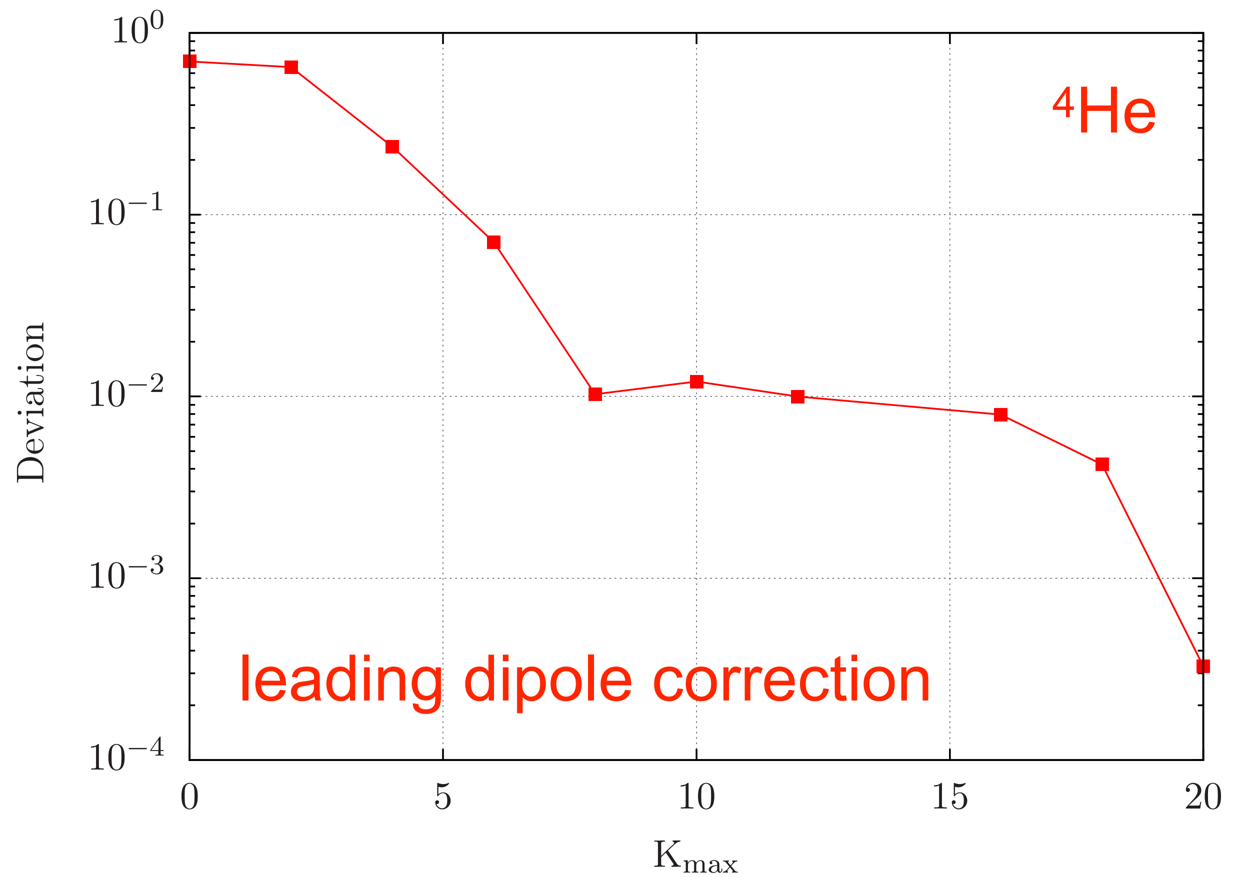
C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)



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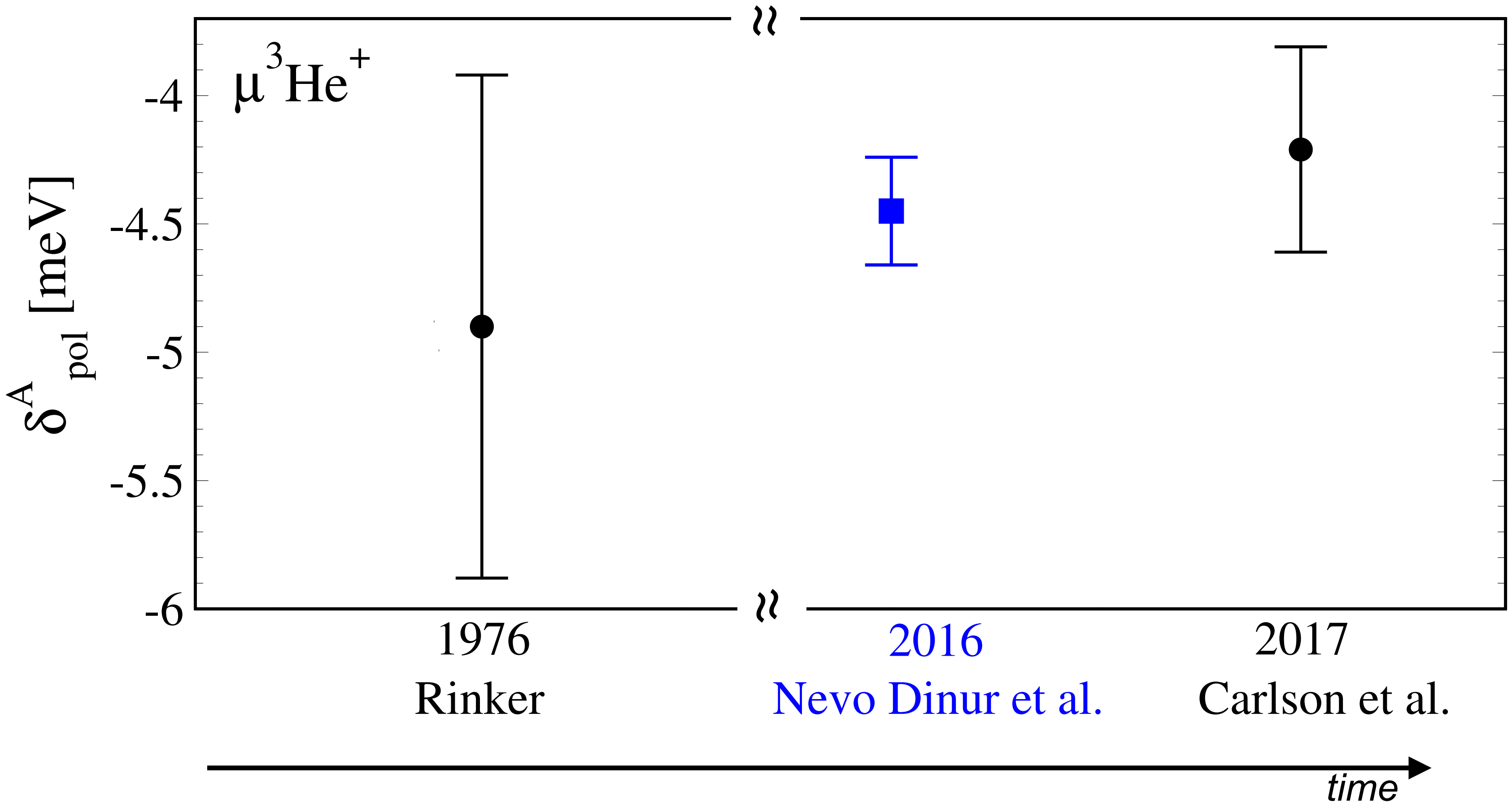

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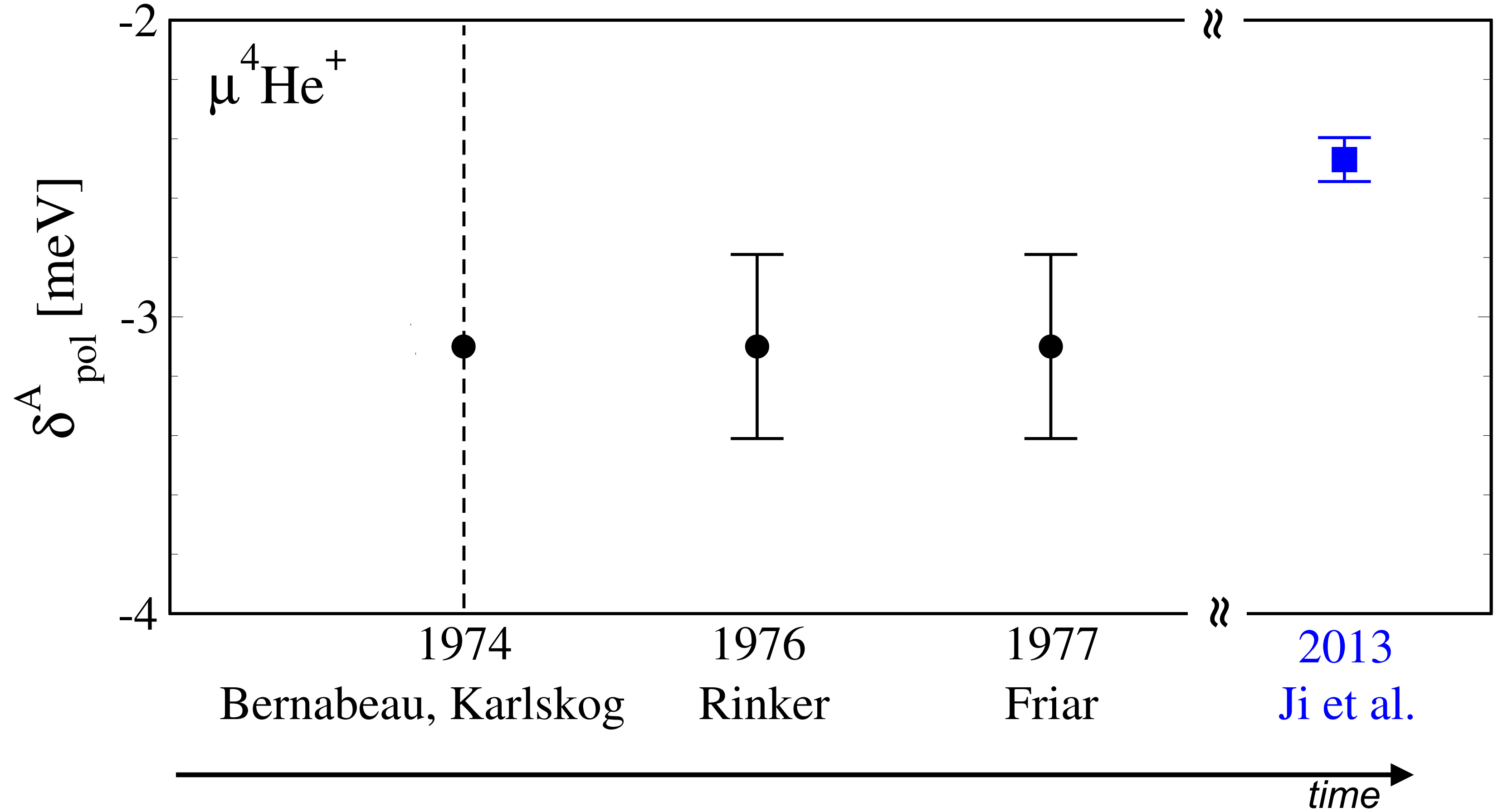

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What about $\mu^{6,7}Li^{++}$?

Go to Poster Session,
Contribution by Simone Li Muli
Thu 18:30

Summary and Outlook

- Ab initio calculations have allowed to substantially reduce uncertainties in TPE
- Independently on the nature of the puzzle, these calculations are needed to support any spectroscopic measurement with muonic atoms
- In the future we will investigate the hyperfine splitting of muonic deuterium and the Lamb shift in muonic lithium atoms

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N.Barnea, **O.J. Hernandez**, C.Ji, **S.Li Muli, N.Nevo Dinur**, **A. Poggialini**

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Thank you for your attention!

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C.Ji et al., JPG: Part. Nucl. **45**, 093002 (2018)

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6