AB INITIO CALCULATIONS OF LIGHT HYPERNUCLEI

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• MOTIVATION
  • Strangeness nuclear physics

• FIRST-PRINCIPLES MODELING OF HYPERNUCLEI
  • Ab initio no-core shell model

• APPLICATIONS
  • Structure of s- and p-shell hypernuclei
  • Charge symmetry breaking puzzle in light mirror hypernuclei
  • Nuclear structure uncertainties
  • Hypertriton decay
  • Few-body resonances

• SUMMARY
MOTIVATION
Strangeness nuclear physics
### Strangeness nuclear physics

Interdisciplinary subject connecting particle physics, nuclear physics and astrophysics.

#### Related topical questions include:

- interaction of (anti)kaons with the nuclear medium
  - possible existence of deeply-bound $K^-$-nuclear states?
  - antikaons in dense matter?
- interaction of hyperons with the nuclear medium
  - $S=-1 \Lambda$ hypernuclei, $\Sigma$-hypernuclei?
  - $S=-2 \Lambda\Lambda$-hypernuclei, $\Xi$ hypernuclei
  - hyperons in dense nuclear matter and neutron stars?
Study of hypernuclei

- Improve understanding of NY interaction
  - strict constraints on NY interaction
  - precise experimental data on hypernuclear spectroscopy
  - supplement (very sparse) hyperon–nucleon scattering data base
- New precision experiments at J-PARC, J-Lab, FAIR, ...
- New constraints from heavy ion collisions: production of light hypernuclei, baryon–baryon interactions (femtoscopy)
- Lattice QCD can be a game changer for strangeness nuclear physics
- Modern developments of NY interactions based on SU(3) chiral EFT / \( \pi \)EFT
- Advanced many-body computational methods are required
FIRST-PRINCIPLES MODELING OF HYPERNUCLEI:
Ab initio no-core shell model
Given a Hamiltonian operator solve the A-body eigenvalue problem:

\[
\left[ \sum_{i \leq A} \frac{\hat{p}_i^2}{2m_i} + \sum_{i < j \leq A-1} \hat{V}_{NN;ij} + \sum_{i < j < k \leq A-1} \hat{V}_{NNN;ijk} + \sum_{i < j = A} \hat{V}_{NY;ij} \right] \Psi = E \Psi
\]

**Ab initio**

- all particles are active (no rigid core)
- exact Pauli principle
- realistic internucleon interactions
- controllable approximations

- Hamiltonian is diagonalized in a finite A-particle harmonic oscillator basis

\[
\Psi(r_1, \ldots, r_A) = \sum_{n \leq N_{\text{max}}} \Phi_n^{\text{HO}}(r_1, \ldots, r_A)
\]

(matrix dimensions up to \(\sim 10^{10}\) with \(\sim 10^{14}\) nonzero elements)

- Systematically improvable: converges to exact results for \(N_{\text{max}} \to \infty\)
The curse of dimensionality

- Basis dimensions for s- and p-shell (hyper)nuclei:

![Graph showing the modelspace dimension for different nuclei](image)

- Strategies:
  - **effective interactions**: Lee–Suzuki, similarity RG
  - **basis reduction**: importance truncation (limit to relevant states), Symmetry-Adapted NCSM (exploit dynamical symmetries)
  - **robust extrapolation technique for “$N_{\text{max}} \to \infty$”**
Convergence in finite HO spaces

- What is the equivalent of Lüscher formula?
- \((N_{\text{max}}, \hbar \omega)\) imposes cutoffs in momentum space (UV) and in position space (IR)
- In a regime with negligible UV corrections, IR corrections are universal for short-range interactions

\[ E(L_{\text{eff}}) = E_\infty + e^{-k_\infty L_{\text{eff}}} + \ldots \]

- \(L_{\text{eff}}\) identified as the size of the hyperspherical cavity associated with \((N_{\text{max}}, \hbar \omega)\) [Wendt et al., PRC 91, 061391 (2015)]
INPUT \( V_{\text{NN}}, V_{\text{NNN}} \) and \( V_{\text{NY}} \) POTENTIALS

**Potentials derived from chiral EFT**

- **long-range part** (\( \pi, K, \eta \)-exchange) predicted by \( \chi \)PT
- **short-range part** parametrized by contact interactions, LECs fitted to experimental data

\[
\begin{align*}
\text{At LO:} & \\
B & \rightarrow B' \quad \text{\( \pi, K, \eta \)-exchange}
\end{align*}
\]

**NN+NNN interaction**

- chiral \( N^3 \)LO NN potential [Entem, Machleidt, PRC 68, 041001 (2003)]
- chiral \( N^2 \)LO NNN potential [Navrátil, FBS 41, 14 (2007)]
- \( NNLO_{\text{sim}} \) NN + NNN potential family [Carlsson et al., PRX 6, 011019 (2016)]

**NY interaction**

- chiral LO potential [Polinder et al., NPA 779, 244 (2006)], NLO developed
- \( \Lambda N - \Sigma N \) mixing explicitly taken into account:

\[
V_{\text{NY}} = \begin{pmatrix} V_{\Lambda N - \Lambda N} & V_{\Lambda N - \Sigma N} \\ V_{\Sigma N - \Lambda N} & V_{\Sigma N - \Sigma N} \end{pmatrix} + \Delta m
\]

Coupled-channel \( \Lambda \)-hypernucleus – \( \Sigma \)-hypernucleus problem!
APPLICATION:
Structure of s- and p-shell hypernuclei

[Gazda, Mareš, Navrátil, Roth, Wirth, FBS 55, 857 (2014)]
[Wirth, Gazda, Navrátil, Calci, Langhammer, Roth, PRL 113, 192502 (2014)]
[Wirth, Gazda, Navrátil, Roth, PRC 97, 064315 (2018)]
Aims

• Develop an ab initio computational technique for $A \geq 5$ hypernuclei
• Test the performance of existing NY interaction models

Validation for $A = 3, 4$ hypernuclei

- two formulations of NCSM developed: in relative Jacobi-coordinate HO basis (squares) and Slater-determinant s.p. HO basis (crosses)
- calculations agree with exact Faddeev results [Nogga et al., NPA 914, 140 (2013)]

First applications

- systematic study from $A = 3 \Lambda^3H$ to $A = 13 \Lambda^{13}C$
  [Gazda et al., FBS 55, 857 (2014); Wirth, Gazda et al., PRL 113, 192502 (2014); Wirth, Gazda, et al., PRC 97, 064315 (2018)]
• calculations with SRG-evolved NN+NNN and bare NY potentials
• surprisingly good performance of chiral LO NY potentials for low-lying states
• reveals deficiencies of the phenomenological potential
STRUCTURE OF s- AND p-SHELL HYPERNUCLEI: $^{13}_\Lambda$C

- calculations with SRG-evolved NN+NNN and bare NY potentials
- surprisingly good performance of chiral LO NY potentials for low-lying states
- reveals deficiencies of the phenomenological potential, as well as deficiencies of the chiral LO potential at higher partial waves
APPLICATION:
Charge symmetry breaking puzzle in light mirror hypernuclei

[Gazda, Gal, PRL 116, 122501 (2016)]
[Gazda, Gal, NPA 954, 161 (2016)]
### Charge symmetry in hadron physics

- Invariance of the strong interaction under the interchange of up and down quarks (protons and neutrons).
- Broken in QCD by the up and down light quark mass differences and their QED interactions, expected to break down at \((m_u - m_d)/M \approx 10^{-3}\).

### Charge symmetry breaking in nuclear physics

- Manifest in pp and nn scattering lengths, well understood.
- \(^3\text{He} - ^3\text{H}: \Delta E_{\text{SI}}^{\text{CSB}} \approx 70\,\text{keV}\)

### Charge symmetry breaking in hypernuclear physics

- Poor p\(^\Lambda\) and no n\(^\Lambda\) scattering data.
- Highly suppressed in \(^3\Lambda\)H.
- \(^4\Lambda\)He – \(^4\Lambda\)H energy level splittings, \(\Delta B_{\Lambda}^{\text{CSB}} \approx 200\,\text{keV}\).
**CSB in $A = 4$ hypernuclei**

<table>
<thead>
<tr>
<th></th>
<th>$^3\text{H} + \Lambda$</th>
<th>0</th>
<th>$^3\text{He} + \Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^+$</td>
<td>1.067 ± 0.08</td>
<td>0.984 ± 0.05</td>
<td>Δ$B_\Lambda = \text{−0.083 ± 0.094}$</td>
</tr>
<tr>
<td>$0^+$</td>
<td>2.157 ± 0.077</td>
<td>1.09 ± 0.02</td>
<td>Δ$B_\Lambda = +0.233 ± 0.092$</td>
</tr>
</tbody>
</table>

- **Recently reaffirmed** by J-PARC E13 observation of $^4\Lambda\text{He}(1^+_{\text{exc.}} \rightarrow 0^+_{\text{g.s.}})$ γ-ray transition [Yamamoto et al., PRL 115, 222501 (2015)] and MAMI-A1 determination of $B_\Lambda(^4\Lambda\text{H})$ [Esser et al., PRL 114, 232501 (2015)]
- Until recently, no calculation was able to reproduce large Δ$B_\Lambda$
- **CSB due to Λ – Σ$^0$ mixing and related to ΛN – ΣN coupling** [Gazda, Gal, PRL 116, 122501 (2016); Gazda, Gal, NPA 954, 161 (2016)]
Electromagnetic $\Lambda - \Sigma^0$ mixing

- Physical $\Lambda$ and $\Sigma^0$ hyperons have mixed isospin composition in terms of the SU(3) pure-isospin $\Lambda$ ($I = 0$) and $\Sigma$ ($I = 1$) hyperons
- Mixing angle proportional to EM mass matrix element $\langle \Sigma^0 | \delta m | \Lambda \rangle$

Relating $\Lambda - \Sigma^0$ CSB mixing to $\Lambda N - \Sigma^0 N$ coupling

- For NY interaction models with explicit $N \Sigma - N \Lambda$ coupling, the electromagnetic $\Lambda - \Sigma^0$ mixing relates matrix elements of $V^{\text{CSB}}_{N \Lambda}$ with $V_{N \Sigma - N \Lambda}$:

$$\langle N \Lambda | V^{\text{CSB}}_{N \Lambda} | N \Lambda \rangle = -2 \frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{M_{\Sigma^0} - M_{\Lambda}} \tau_{N3} \frac{1}{\sqrt{3}} \langle N \Sigma | V_{N \Sigma - N \Lambda} | N \Lambda \rangle$$

[Gal, PLB 744, 352 (2015)]

- The first microscopic model which generates large $\Delta B_\Lambda (0^+_{\text{g.s.}}) \approx 200$ keV in $A = 4$ hypernuclei!

[Gazda, Gal, PRL 116, 122501 (2016)]
Figure 1: Cutoff momentum ($\Lambda_{EFT}$) dependence of the difference $\Delta E_{CSB}^x$ of the excitation energies $E_x(0^+_{g.s.} \rightarrow 1^+)$ in $^4_\Lambda$He and $^4_\Lambda$H in ab initio NCSM calculations without $V_{CSB}^{\Lambda N}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral NY interactions.
Figure 1: Cutoff momentum ($\Lambda_{\text{EFT}}$) dependence of the difference $\Delta E_{x}^{\text{CSB}}$ of the excitation energies $E_{x}(0_{g.s.}^{+} \rightarrow 1^{+})$ in $^{4}_{\Lambda}\text{He}$ and $^{4}_{\Lambda}\text{H}$ in ab initio NCSM calculations without and with $V_{\Lambda N}^{\text{CSB}}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral NY interactions.
APPLICATION:
Nuclear Structure uncertainties
[unpublished]
Aim

What are the theoretical uncertainties of hypernuclear properties resulting from the remaining freedom in the constructions of nuclear NN+NNN interactions?

The NNLO$_{\text{sim}}$ family of NN+NNN potentials

- Parameters fitted to reproduce simultaneously $\pi N$, NN, and NNN low-energy observables
- family of 42 Hamiltonians where the experimental uncertainties propagate into LECs

\[
\begin{align*}
T_{\text{NN}}^{\text{lab,max}} &\leq 125, \ldots, 290 \text{ MeV} \\
\Lambda_{\text{EFT}} &\leq 450, \ldots, 600 \text{ MeV}
\end{align*}
\]

\begin{align*}
\left\{ 42 V_{\text{NN}}+V_{\text{NNN}} \right. \text{ potentials} \\
\left. \right. \\
\text{All Hamiltonians give equally good description of the fit data} \\
\text{Note that } \Delta E^{(3}\text{He}/3\text{H}) \approx 0 \text{ (fitted) while } \Delta E_{\text{g.s.}}^{(4}\text{He}) \approx 1.5 \text{ MeV}
\end{align*}

[Carlsson et al., PRX 6, 011019 (2016)]
\( ^3 \Lambda \text{He} J^\pi = 1/2^+ \)

- converged NCSM calculations (\( N_{\text{max}} = 70 \))
- \( \Delta E_{\text{g.s.}} \approx 0.1 \text{ MeV} \approx \Delta B_{\Lambda} \)
  (\( B_{\Lambda} \approx 0.13 \text{ MeV} \))

\( ^4 \Lambda \text{He(H)} J^\pi = 0^+, 1^+ \)

- \( N_{\text{max}} = 20(16) \) for \( J^\pi = 0^+(1^+) \)
- e.g. for \( \Lambda = 500 \text{ MeV}, T_{\text{max}} = 290 \text{ MeV} \)

\( ^4 \Lambda \text{He} J^\pi = 0^+ \text{ g.s.: } \Delta E_{\text{g.s.}} \approx 0.5 \text{ MeV} \)

\( ^4 \Lambda \text{He} J^\pi = 1^+ \text{ exc.: } \Delta E_{\text{exc.}} \approx 0.6 \text{ MeV} \)
\( ^5_\Lambda \text{He} J^\pi = 1/2^+ \)

- \( N_{\text{max}} = 10 \) for \( J^\pi = 1/2^+ \)
- e.g. for \( \Lambda = 500 \text{ MeV}, T_{\text{max}} = 290 \text{ MeV} \):

\[ E_{gs}(\text{MeV}) \]
\[ L_{\text{eff}} (\text{fm}) \]

\( ^5_\Lambda \text{He} \) overbinding problem

- exp. \( B_\Lambda(^5_\Lambda \text{He}) = 3.12(2) \text{ MeV} \), 
  \( B_\Lambda = E(^4\text{He}) - E(^5_\Lambda \text{He}) \)
- Hard to reproduce by any NY interaction model
- Evidence of missing \( \Lambda\text{NN} \) forces?

LO NY \( \chi \text{EFT} \) cutoff dependence

For \( \Lambda_{\text{NY}} = 550, \ldots, 700 \text{ MeV} \):
\( \Delta B_\Lambda(^5_\Lambda \text{He}) \approx 3.7 \text{ MeV} \)

Altogether
\( B_\Lambda \approx 2.27 - 7.62 \text{ MeV}! \)
APPLICATION:
Mesonic decay of the hypertriton
[unpublished] (preliminary)
### $^3\Lambda$H lifetime puzzle

- The weakly-bound $^3\Lambda$H ($B_\Lambda \approx 0.13$ MeV) is expected to have lifetime within few \% of the free $\Lambda$ hyperon lifetime.
- Faddeev calculation: $\tau = 0.94\tau_\Lambda$ [Kamada et al., PRC 57, 1595 (1998)].
- Recent heavy-ion $^3\Lambda$H production experiments yield lifetimes shorter by $\gtrsim 30$\% (wo. avg.): $\tau = 142^{+24}_{-21} \pm 29$ ps ($0.54^{+0.09}_{-0.08}\tau_\Lambda$) [ALICE collab., PLB 754, 360 (2016); STAR collab. PRC 97, 054909 (2018)].

### $^3\Lambda$H decay

- mesonic modes (not Pauli blocked as in heavier hypernuclei):
  $$^3\Lambda H \rightarrow \pi^- (\pi^0)^+^3\text{He}(^3\text{H}) / \pi^- (\pi^0) + d + p (n) / \pi^- (\pi^0) + p + n + p (n)$$
- rare non-mesonic modes: $^3\Lambda$H $\rightarrow$ n + d / n + n + p

### Our aim

- Pionic FSI in $^3\Lambda H \rightarrow \pi^- + ^3\text{He}$
  (closure calculation yields $\tau = 1.23\tau_\Lambda$ [Gal, Garcilazo, PLB 791, 48 (2018)].)
Mesonic decay of the hypertriton

Decay rate

\[ \Gamma(\Lambda H \rightarrow ^3\text{He} + \pi^-) \propto \int \langle \Psi_{^3\text{He}} \phi_{\pi} | \hat{O} | \Psi_{\Lambda H} \rangle \]

- \( \phi_{\pi} \) plane wave (distorted waves in progress)
- \( \Psi_{^3\text{He}}, \Psi_{\Lambda H} \) from NCSM

- \( \Gamma \) sensitive to IR properties of \( \Psi_{\Lambda H} \)
- good convergence for \( \Lambda_{UV} > 800 \text{ MeV} \)
- \( \Sigma^0,^- \) hyperons decrease \( \Gamma \) by \( \sim 20\% \)
APPLICATION:
Few-body resonances
[unpublished] (preliminary)
Few-body resonances

Harmonic-oscillator representation of the scattering equation

- varying $N_{\text{max}}$, $\hbar \omega \rightarrow$ NCSM eigenenergies $E_i$, eigenfunctions $\Psi_i$
- phase shifts $\delta_L(E_i) = - \arctan \frac{S_N^L(E_i)}{C_N^L(E_i)}$
- $\Lambda n n$, $^3\Lambda H J^\pi = 3/2^+$, ...

[Shirokov et al., PRC 94, 064320 (2016)]

$\Lambda n n$ resonance state

- Signal of particle-stable $\Lambda n n$ [C. Rappold et al. (HypHI collab. PRC 88, 041001 (2013))]
- Three-body $\Lambda + n + n \rightarrow \Lambda + n + n$ scattering in hyperspherical HO basis
- No bound subsystems, “democratic” decay
- Preliminary!
Ab initio calculations of light hypernuclei

- No-core shell model is a powerful and reliable technique to study s- and p-shell hypernuclei
- Allow to test hyperon–nucleon interaction models
- High precision allows to address important questions of hypernuclear physics, such as:
  - charge-symmetry breaking in mirror hypernuclei
  - quantification of systematic theoretical uncertainties of hypernuclear observables
- New applications:
  - hypernuclear decays
  - exotic few-ody resonances
Thank you!