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THE HEBREW UNIVERSITY OF JERUSALEM

Effective Field Theory for Lattice Nuclei

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The Racah institute for Physics
The Hebrew University, Jerusalem, Israel

Collaboration

Jerusalem, Israel

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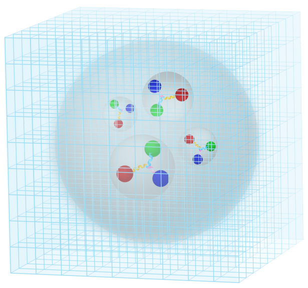
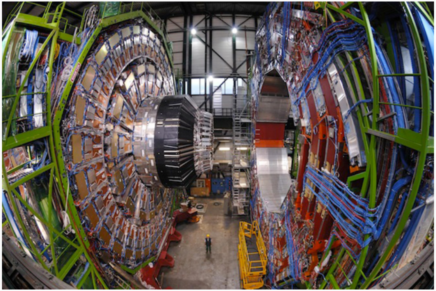
Manchester, UK

J. Kirscher

Lattice QCD

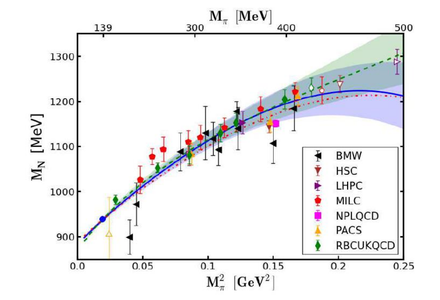
LQCD - a new type of experiment!

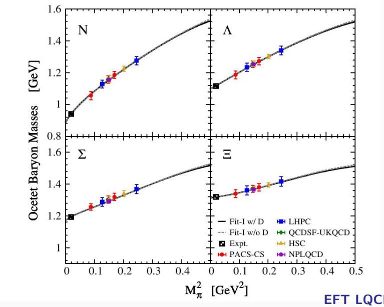




Lattice QCD

- LQCD calculations are usually done at m_u, m_d larger than natural
- parametrized as m_π
- Baryon masses $M_B(m_\pi)$ are predictions
- The limit $m_\pi \longrightarrow m_\pi(\text{nature})$ should be taken
- For the **single baryon** case, we are already there (talk of Constantia Alexandrou)





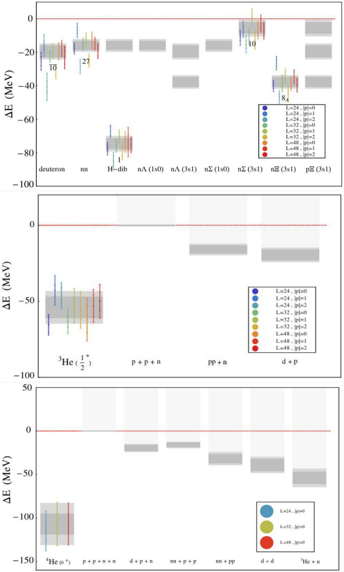
Xui-Lei Ren *et* al., PRD **87** 074001 (2013)

L. Alvarez-Ruso *et* al., ArXiv hep-ph: 1304.0483 (2013)

2-body system - Deutron, dineutron,...

3-body system - ^{3}He, triton

4-body system - ^{4}He



NPLQCD Collaboration, PRD **87** 034506 (2013)

LQCD

- LQCD calculation are done in finite volume
- For 2-body systems Luscher's approach is used to extract **free-space** information
- The 3,4,...-body case is more complicated
- Natural quark masses demand large volumes

Our aim

To extract the free-space data by fitting an appropriate EFT to the finite volume LQCD results.

Few nucleons in a BOX

Box with periodic boundary conditions

the free space Schroedinger equation $\boldsymbol{x} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_A\}$

$$\Big[-\sum_i^A \frac{\nabla_i^2}{2m_i} + V(\boldsymbol{x}) \Big] \Psi(\boldsymbol{x}) = E\Psi(\boldsymbol{x})$$

Is replaced by the box equation

$$\Big[-\sum_i^A \frac{\nabla_i^2}{2m_i} + V_L(\boldsymbol{x}) \Big] \Psi_L = E_L \Psi_L$$

where the potential is replaced by a sum over all mirror images

$$V_L(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_A) = \sum_{\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots \boldsymbol{n}_A} V(\boldsymbol{x}_1 + \boldsymbol{n}_1 L, \boldsymbol{x}_2 + \boldsymbol{n}_2 L, \ldots, \boldsymbol{x}_A + \boldsymbol{n}_A L)$$

Solution

Asymptotic solution - Luscher's formula [Luscher (1986)]

Exact solution - Box SVM (Gaussians) [Yin & Blume (2013)]

The free space Hamiltonian with $V(\boldsymbol{z})$ a short range potential

$$H_F = T + V(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

The box Hamiltonian

$$H_L = T + V_L(\boldsymbol{x}_1 - \boldsymbol{x}_2) \qquad V_L(\boldsymbol{z}) = \sum_{\boldsymbol{n}} V(\boldsymbol{z} + \boldsymbol{n}L)$$

Let Ψ_F be an eigenstate of H_F

$$H_F \Psi_F = E_F \Psi_F$$

For large L we expect $\Psi_L \approx \Psi_F$.

The trial function

$$\Psi_0(\boldsymbol{z}) = \sum_{\boldsymbol{n}} \Psi_F(\boldsymbol{z} + \boldsymbol{n}L) \qquad \boldsymbol{z} = \boldsymbol{x}_1 - \boldsymbol{x}_2$$

is an periodic function, therefore a legitimate solution.

The 2-body Luscher formula (continue)

The free space Hamiltonian with $V(\boldsymbol{z})$ a short range potential

$$H_L \Psi_0 = \left(T + \sum_{\boldsymbol{n}'} V(\boldsymbol{z} + \boldsymbol{n}'L) \right) \sum_{\boldsymbol{n}} \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

$$= \sum_{\boldsymbol{n}} \underbrace{(T + V(\boldsymbol{z} + \boldsymbol{n}L))}_{H_F} \Psi_F(\boldsymbol{z} + \boldsymbol{n}L) + \sum_{\boldsymbol{n}' \neq \boldsymbol{n}} V(\boldsymbol{z} + \boldsymbol{n}'L) \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

$$= \sum_{\boldsymbol{n}} E_F \Psi_F(\boldsymbol{z} + \boldsymbol{n}L) + \sum_{\boldsymbol{n}' \neq \boldsymbol{n}} V(\boldsymbol{z} + \boldsymbol{n}'L) \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

$$= E_F \Psi_0(\boldsymbol{z}) + \eta(\boldsymbol{z})$$

where

$$\eta(\boldsymbol{z}) = \sum_{\boldsymbol{n}' \neq \boldsymbol{n}} V(\boldsymbol{z} + \boldsymbol{n}'L) \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

Now

$$\Delta E_L = \frac{\langle \Psi_0 | H_L | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle} - E_F = \frac{\langle \Psi_0 | \eta \rangle}{\langle \Psi_0 | \Psi_0 \rangle}$$

The 2-body Luscher formula (continue)

The free space Hamiltonian with $V(\boldsymbol{z})$ a short range potential

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The 2-body Luscher formula (continue)

For a short range interaction, asymptotically $\boxed{\Psi_F = Ae^{-\kappa r}/r}$ with $\kappa L \gg 1$.

It follows that $\langle \Psi_0 | \Psi_0 \rangle \approx N_{boxes}$

$$\Delta E_L = \frac{1}{N_{boxes}} \sum_{\boldsymbol{n}''} \sum_{\boldsymbol{n}' \neq \boldsymbol{n}} \int d\boldsymbol{z} \Psi_0^\dagger(\boldsymbol{z} + \boldsymbol{n}''L) V(\boldsymbol{z} + \boldsymbol{n}'L) \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

$$\approx \sum_{|\boldsymbol{n}|=1} \int d\boldsymbol{z} \Psi_0^\dagger(\boldsymbol{z}) V(\boldsymbol{z}) \Psi_F(\boldsymbol{z} + \boldsymbol{n}L)$$

$$\approx \sum_{|\boldsymbol{n}|=1} \int d\boldsymbol{z} \Psi_0^\dagger(\boldsymbol{z}) V(\boldsymbol{z}) A \frac{e^{-\kappa|\boldsymbol{z}+\boldsymbol{n}L|}}{|\boldsymbol{z}+\boldsymbol{n}L|}$$

Consequently

$$\boxed{\Delta E_L = C\frac{e^{-\kappa L}}{L} + O(e^{-\sqrt{2}\kappa L})}$$

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\end{aligned}$$

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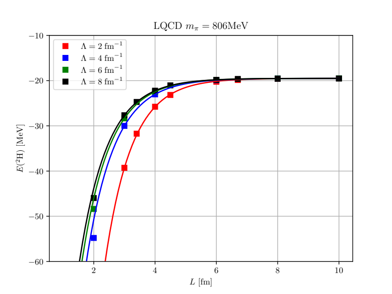
Exact solution - Box SVM

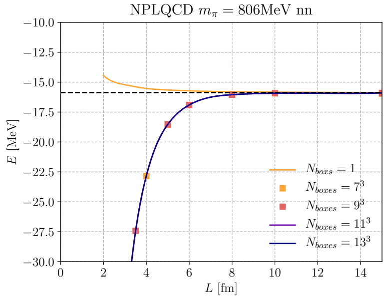
- Gaussian basis
- Variational
- Allows for analytical evaluation of matrix elements - even in a box
- The price is summation over all the particle box configurations
- Usually converges with 5-9 boxes in each dimension.

The 2-body case

Pion mass $m_\pi = 806$MeV

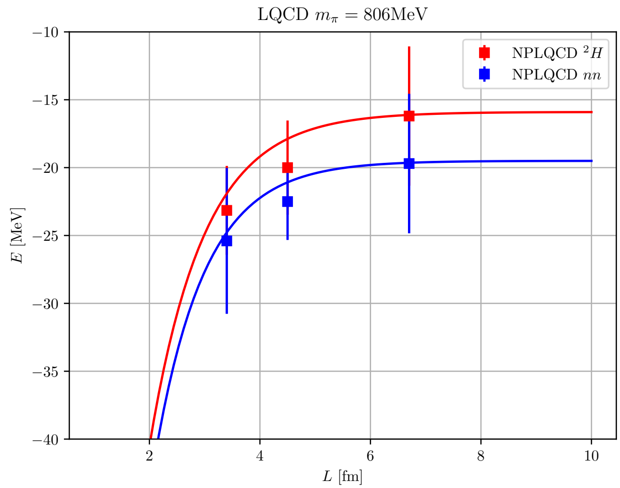
We use $\not\pi$EFT at leading order



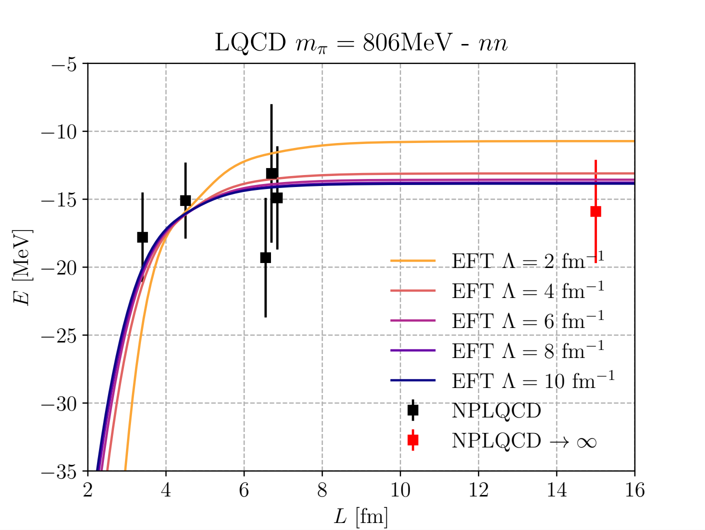


Comparing to the “experimental” data

The LECs are fitted to the **reported** free-space results

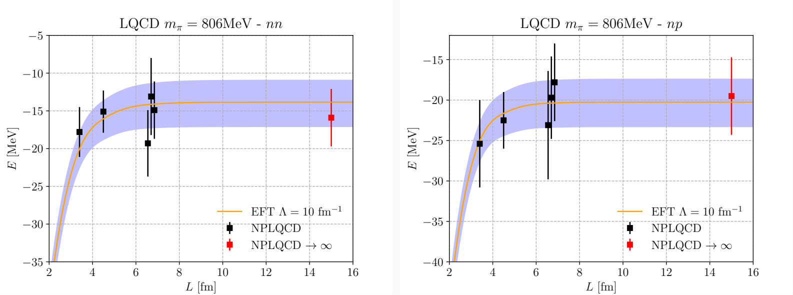


Fit to all reported data

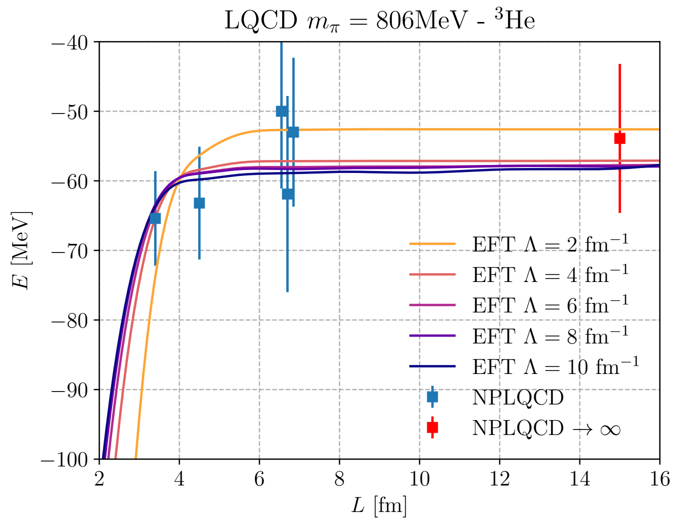


New free-space estimates

LECs are fitted to all available data points



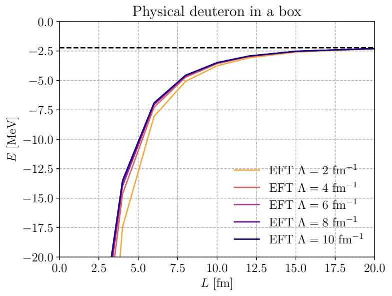
The 3-body case

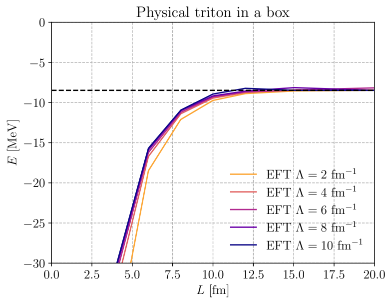


The natural case

In nature light nuclei are weakly bound

The asymptotic energy appears at large volume



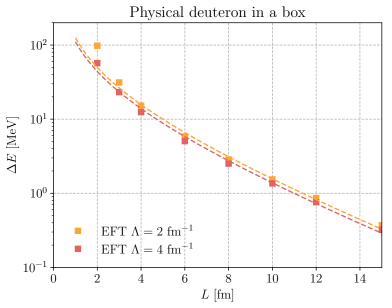


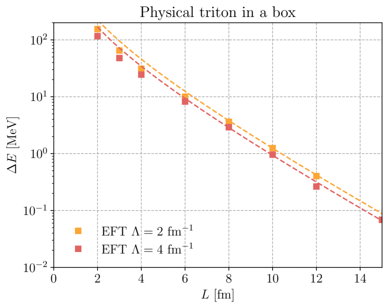
We try fitting to

$$\Delta E_2 = C_2 \frac{e^{-\kappa_2 L}}{\kappa_2 L}$$

$$\Delta E_3 = C_3 \frac{e^{-\kappa_3 L}}{\kappa_3 L}$$

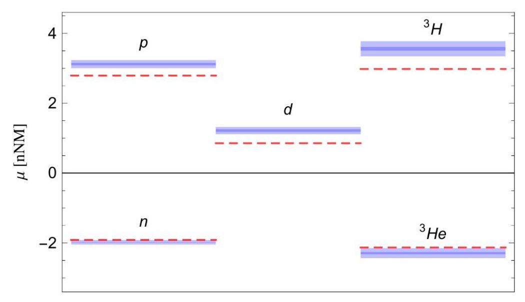
Here $\kappa_2 = \sqrt{m_N B_2}$ and $\kappa_3 = \sqrt{\frac{4}{3} m_N (B_3 - B_2)}$





Magnetic Moments

Magnetic moments and polarizations



$$\Delta E = \mu B + \frac{1}{2}\beta_M B^2$$

The $\not{\pi}$EFT Lagrangian at NLO

$$\begin{aligned}\mathcal{L} = & N^\dagger \left\{ (i\partial_0 - e\hat{Q}A_0) + \frac{1}{2m}\left(\boldsymbol{\nabla} - ie\hat{Q}\boldsymbol{A}\right)^2 + \hat{g}_\mu \frac{e}{2m}\boldsymbol{\sigma}\cdot\boldsymbol{B} \right\} N \\ & + C_T (N^\dagger P_i N)^2 + C_S (N^\dagger \bar{P}_3 N)^2 + D_1 (N^\dagger N)^3 + \ldots \\ & + L_1 (N^\dagger P_i N)^\dagger (N^\dagger \bar{P}_3 N) B_i + L_2 i\epsilon_{ijk} (N^\dagger P_i N)(N^\dagger P_j N) B_k\end{aligned}$$

The magnetic current

The one-body current

$$\boldsymbol{\mu}^{(1)} = \sum_{i=1}^{A} \frac{|e|}{2m}\left[\frac{g_p + g_n}{2}\boldsymbol{\sigma}_i + \frac{g_p - g_n}{2}\boldsymbol{\sigma}_i \boldsymbol{\tau}_{i,z}\right]$$

The two-body current

$$\boldsymbol{\mu}^{(2)} = \sum_{i<j}^{A} \left[L_1(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_{i,z} - \boldsymbol{\tau}_{j,z}) + L_2(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\right]\delta_\Lambda(\boldsymbol{r}_{ij})$$

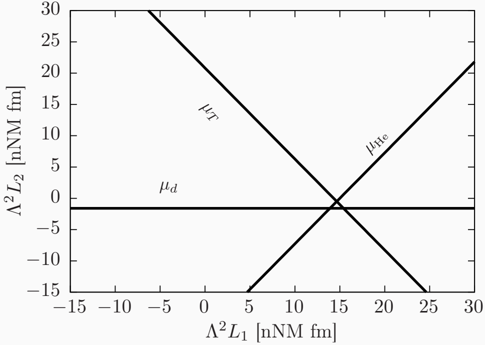
The magnetic LECs

Observables

The deutron magnetic moment μ_d

The $A = 3$ magnetic moments $\mu_T, \mu_{^3\text{He}}$

The transition matrix element $t_{01} = \langle ^1S_0 | \boldsymbol{\mu} | ^3S_1 \rangle$



Observations

$m_\pi = 140$MeV - Consistencey between the different observables

$m_\pi = 806$MeV - Error bars too large, t_{01} a bit off

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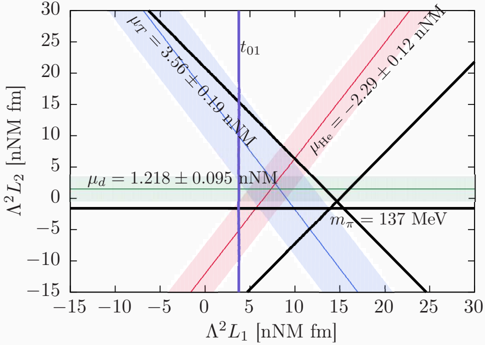

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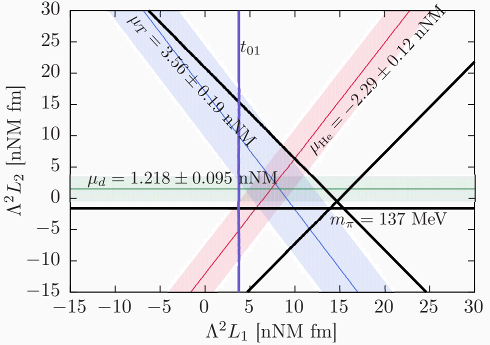

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Conclusions

Thanks