Hadronic molecules of heavy hadrons with tensor force

Yasuhiro Yamaguchi (RIKEN, Japan)

in collaboration with

Hugo García-Tecocoatzi (UNAM), Alessandro Giachino (INFN Genoa, Genoa Univ.), Atsushi Hosaka (RCNP, Osaka Univ.), Elena Santopinto (INFN Genoa), Sachiko Takeuchi (Japan Coll. Social Work), Makoto Takizawa (Showa Pharmaceutical Univ.).

The 24th European conference on few-body problems in physics (EFB24)

University of Surrey, UK 1-6 September 2019
Outline

1 Introduction
   - Exotic hadrons - Hadronic molecules
   - Hidden-charm pentaquark $Pc$

2 Model setup
   - One Pion Exchange Potential
   - Compact 5-quark potential

3 Numerical results
   - Hidden-charm molecules

4 Summary
Hadron structure: Constituent quark model

Introduction

- Ordinary Hadrons: Baryon \((qqq)\) and Meson \((q\bar{q})\)

\[
\begin{array}{cc}
q & q \\
q & \bar{q} \\
q & q
\end{array}
\]

Baryon (proton, nucleon, ...) Meson \((\pi, K, ...)\)

\(q\): "Constituent quark"

- Exotic Hadrons \((\neq qqq, q\bar{q})\): Multiquark? Multihadron?

\[
\begin{array}{cc}
q & q \\
q & q \\
q & q \\
q & \bar{q} \\
q & \bar{q}
\end{array}
\]

Pentaquark (Compact) Hadronic molecule
Constituent quark picture and beyond

Introduction

▷ e.g. $c\bar{c}$ mesons (Charmonium)

\[
V(r) = -\frac{\alpha_s}{r} + cr + \ldots
\]


S. Godfrey and N. Isgur, PRD\textbf{32}(1985)189

one-g exchange

Confinement

\[
|J/\psi(1S)| = 1^+ +...
\]

\[
|\psi(4155)| = 0^{++}
\]

\[
|\psi(4160)| = 1^{--}
\]

\[
|\psi(4040)| = 0^{++}
\]

\[
|\psi(3770)| = 1^{--}
\]

\[
|\chi_c(2P)| = 1^{++}
\]

\[
|\chi_c(1P)| = 2^{++}
\]

\[
|\chi(1S)| = 2^{--}
\]

\[
|\eta_c(2S)| = 1^{--}
\]

\[
|\eta_c(1S)| = 0^{--}
\]

\[
|\psi(2S)| = 0^{++}
\]
## Constituent quark picture and beyond

### Introduction

- e.g. $c\bar{c}$ mesons (Charmonium) and **Unexpected X, Y, Z**

\[ V(r) = -\frac{\alpha_s}{r} + cr + ... \]

Exotics $\neq c\bar{c}$ have been observed in the Experiments (BaBar, Belle, BESIII, LHCb,...) $\Rightarrow$ **Q. Structure? Physics?**


- S. Godfrey and N. Isgur, PRD32(1985)189
Exotics as Hadronic molecule = Hadron composite system

Expected near the thresholds

\[ X(3872) = D\bar{D}^* \text{ molecule?} \]

\[ D^+D^* - 3879.84 \text{ MeV} \]

\[ D^0\bar{D}^{*0} \text{ } 3871.69 \text{ MeV} \]

\[ D^0\bar{D}^{*0} \text{ } 3871.68 \text{ MeV} \]

⇒ Analogous to Atomic Nuclei

Deuteron \sim pn \text{ bound state } (B = 2.2 \text{ MeV})
Driving force of Nuclei $\Rightarrow$ long range force: $\pi$ exchange
$\Rightarrow$ generating the loosely bound state
Hadronic molecules and $\pi$ exchange potential

**Introduction**

- **Driving force of Nuclei** $\Rightarrow$ **long range force**: $\pi$ exchange
  $\Rightarrow$ generating **the loosely bound state**

- **Strong attraction from** Tensor term ($S \rightarrow D$ mixing)

---

Hadronic molecules and $\pi$ exchange potential

Introduction

- Driving force of Nuclei $\Rightarrow$ long range force: $\pi$ exchange
  $\Rightarrow$ generating the loosely bound state

- Strong attraction from Tensor term ($S - D$ mixing)
  $\Rightarrow$ Important role in the heavy hadronic molecules?

---


N. A. Tornqvist, Z. Phys. **C61** (1994) 525
Hidden-charm pentaquarks

Hadronic molecule

Pentaquark (Compact)

Observation of the Hidden-charm Pentaquark \((c\bar{c}uud)\) in \(\Lambda_b^0 \rightarrow J/\psi K^- p\) Decay? R.Aaij, et al. (LHCb collaboration) PRL 115 (2015) 072001

\(P_c(4380): M = 4380\text{ MeV} \quad P_c(4450): M = 4449.8\text{ MeV}\)
\[\Gamma = 205\text{ MeV} \quad \Gamma = 39\text{ MeV}\]

- \(J^P\)? (3/2\(^-\), 5/2\(^+\)), (3/2\(^+\), 5/2\(^-\)), or (5/2\(^+\), 3/2\(^-\))
- There have been a lot of articles investigating the \(P_c\) states...
  - Hadronic molecule? Compact state? Kinematical effect?


- **$P_c(4450)$ in 2015 → $P_c(4440)$ and $P_c(4457)$**
  - $P_c(4440)$ \( M = 4440.3 \text{ MeV} \)
  - $P_c(4457)$ \( M = 4457.3 \text{ MeV} \)
  - $\Gamma = 20.6 \text{ MeV}$
  - $\Gamma = 6.4 \text{ MeV}$

- **Observation of New state!**
  - $P_c(4312)$ \( M = 4311.9 \text{ MeV} \)
  - $\Gamma = 9.8 \text{ MeV}$

- **$P_c(4380)$ in 2015? “these fits can neither confirm nor contradict the existence of the $P_c(4380)$”**
Introduction: pentaquark

- $P_c$ states reported close to the $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds
- $\pi$ exchange in the heavy hadron system
Hidden-charm meson-baryon molecule...?
Introduction: pentaquark

- $P_c$ states reported close to the $\bar{D}(\star)\Sigma_c(\star)$ thresholds
- $\pi$ exchange in the heavy hadron system enhanced by the heavy quark spin symmetry!

N. Isgur, M. B. Wise, PLB 232 (1989) 113

$\Rightarrow \bar{D}(0^-) - \bar{D}^*(1^-), \Sigma_c(1/2^+) - \Sigma_c^*(3/2^-)$ mixing
Hidden-charm meson-baryon molecule...?
Introduction: pentaquark

▷ $P_c$ states reported close to the $\bar{D}(\ast)\Sigma_c(\ast)$ thresholds

▷ $\pi$ exchange in the heavy hadron system enhanced by the heavy quark spin symmetry!

$\Rightarrow \bar{D}(0^-) - \bar{D}^*(1^-), \Sigma_c(1/2^+) - \Sigma_c^*(3/2^-)$ mixing

$\Rightarrow P_c$ states: $\bar{D}(\ast)\Sigma_c(\ast)$ molecules?

N. Isgur, M. B. Wise, PLB 232 (1989) 113

Hadronic molecule
Compact state: 5-quark configuration
Introduction: pentaquark

- $P_c$ states by the quark cluster model
- 5-quark configurations

\[ S_{q^3} = 1/2, 3/2, \quad S_{c\bar{c}} = 0, 1 \]

Couplings to ($qqc$) baryon-($q\bar{c}$) meson, e.g. $D_c$, are allowed!

Mixing of Compact state and Hadronic Molecule!
Compact state: 5-quark configuration

Introduction: pentaquark

- $P_c$ states by the quark cluster model
- 5-quark configurations

\begin{align*}
S_{q^3} &= 1/2, 3/2, \quad S_{c\bar{c}} = 0, 1 \\
S_{q^3} &= 1/2, \quad S_{c\bar{c}} = 0, 1
\end{align*}

- $[q^3 \, 8_c \, 3/2]$: Color magnetic int. is attractive!
Compact state: 5-quark configuration

Introduction: pentaquark

  $P_c$ states by the quark cluster model
- 5-quark configurations

\begin{align*}
S_{q^3} = 1/2, 3/2, \ S_{c \bar{c}} = 0, 1 & \quad S_{q^3} = 1/2, \ S_{c \bar{c}} = 0, 1 \\
[q^3 8_c 3/2]: \text{Color magnetic int. is attractive!} \\
\Rightarrow \text{Couplings to (qqc) baryon-(q\bar{c}) meson, e.g. } \bar{D}\Sigma_c, \\
& \text{are allowed!}
\end{align*}

Mixing of Compact state and Hadronic Molecule!
Model setup in this study

- Hadronic molecule \((MB) + \text{Compact state } (5q)\)

\[ MB + 5q \]

Hadronic molecule

\[ 8_c \]
Model setup in this study

- **Hadronic molecule** ($MB$) + **Compact state** ($5q$)

\[ MB \text{ coupled to } 5q \text{ (Feshbach Projection)} \]

\[ MB + 5q \]

Hadronic molecule

5 Sep 2019 Yasuhiro Yamaguchi (RIKEN) EFB24 (Univ. of Surrey, UK)
Model setup in this study

- Hadronic molecule \((MB) + \) Compact state \((5q)\)
  \[ \Rightarrow MB \text{ coupled to } 5q \text{ (Feshbach Projection)} \]

**Interaction of hadrons \((M \text{ and } B)\)**

- Long range interaction: One pion exchange potential (OPEP)
- Short range interaction: \(5q\) potential

Model setup in this study

- **Hadronic molecule** \((MB) + \text{Compact state} (5q)\)
  \(
  \Rightarrow \ MB \text{ coupled to } 5q \ (\text{Feshbach Projection})
  \)

### Interaction of hadrons \((M \text{ and } B)\)

- **Long range interaction**: One pion exchange potential (OPEP)
- **Short range interaction**: 5q potential (\(\rightarrow\) Local Gaussian)

Spin dependence \(\rightarrow\) Spin structure of 5q


**$\bar{D}^*(-) Y_c$ Interaction: Long range force**

**HQS and OPEP**

- One pion exchange potential

\[ V^\pi_{\bar{D}^*(-) Y_c - \bar{D}^*(-) Y_c} = \frac{g_{D^*} g_{\pi} g_{Y_c} g_{Y_c}}{f_\pi^2} \left[ \vec{S}_1 \cdot \vec{S}_2 C(r) + S_{S_1 S_2} T(r) \right] \]

- (Contact term is removed)

- Form factor with Cutoff $\Lambda$ (determined by the hadron size)

\[ F(q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}, \quad \Lambda_{\bar{D}} \sim 1130 \text{ MeV}, \quad \Lambda_{Y_c} \sim 840 \text{ MeV} \]

2. Short range force: 5-quark potential

\[ M \quad M \]

\[ B \quad B \]
Model: 5-quark potential

- 5-quark potential $\Rightarrow$ **Local Gaussian potential** is employed.
- Massive $M_{5q}$ (few hundred MeV above $\bar{D}^*\Sigma^*_c$) $\Rightarrow$ **Attractive**

Channel $i, j = \bar{D}^{(*)}\Lambda_c, \bar{D}^{(*)}\Sigma^*_c$ with $S$–wave

$$M \quad M$$

$i$ \quad \quad \quad \quad \quad \quad j \quad \Rightarrow -f S_i S_j e^{-\alpha r^2}$
Model: 5-quark potential

- 5-quark potential ⇒ **Local Gaussian potential** is employed.
  - Massive $M_{5q}$ (few hundred MeV above $\bar{D}^*\Sigma_c^*$) → Attractive

\[
M \quad M \quad i \quad j \quad \Rightarrow -f S_i S_j e^{-\alpha r^2}
\]

Channel $i, j = \bar{D}(\ast)\Lambda_c, \bar{D}(\ast)\Sigma_c(\ast)$ with $S$–wave

**Free Parameters**

Strength $f$ and Gaussian para. $\alpha$ (→ may be fixed in the future)
($f$ vs $E$ will be shown latter. $\alpha = 1$ fm$^{-2}$ is fixed.)
Model: 5-quark potential

- 5-quark potential ⇒ **Local Gaussian potential** is employed.
  - Massive $M_{5q}$ (few hundred MeV above $\bar{D}^*\Sigma_c^*$) → **Attractive**

\[
\begin{align*}
M & \quad M \\
\downarrow & \quad \downarrow \\
B & \quad B
\end{align*}
\]
\[i \quad j \quad \Rightarrow -f S_i S_j e^{-\alpha r^2}
\]

Channel $i, j = \bar{D}^*(\Lambda_c), \bar{D}^*(\Sigma_c^*)$ with $S-$wave

**Free Parameters**

- Strength $f$ and Gaussian para. $\alpha$ (⇒ may be fixed in the future)
  - ($f$ vs $E$ will be shown latter. $\alpha = 1$ fm$^{-2}$ is fixed.)

**Relative strength $S_i$**

- Spectroscopic factors ⇒ determined by the spin structure of $5q$
Model: 5-quark potential

- 5-quark potential ⇒ **Local Gaussian potential** is employed.
- Massive $M_{5q}$ (few hundred MeV above $D^{*} \Sigma_c^{*}$) ⇒ Attractive

\[
\begin{align*}
M & \quad M \\
i & \quad j \\
B & \quad B
\end{align*}
\]

Channel $i, j = D^{(*)} \Lambda_c, \bar{D}^{(*)} \Sigma_c^{(*)}$ with $S-$wave

**Free Parameters**

Strength $f$ and Gaussian para. $\alpha$ (→ may be fixed in the future)
($f$ vs $E$ will be shown latter. $\alpha = 1 \text{ fm}^{-2}$ is fixed.)

**Relative strength $S_i$**

Spectroscopic factors ⇒ determined by the spin structure of $5q$
Spectroscopic factor $S_i$ (Spin structure)

$5q$ potential

- S-factor: $S_i = \langle (\bar{D} Y_c)_i | 5q \rangle$

**Table:** Spectroscopic factors $S_i$ for each meson-baryon channel.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$S_{c\bar{c}}$</th>
<th>$S_{3q}$</th>
<th>$\bar{D} \Lambda_c$</th>
<th>$\bar{D}^* \Lambda_c$</th>
<th>$\bar{D} \Sigma_c$</th>
<th>$\bar{D}^* \Sigma_c$</th>
<th>$\bar{D}^* \Sigma_{c'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>(i) 0</td>
<td>1/2</td>
<td>0.4</td>
<td>0.6</td>
<td>-0.4</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>(ii) 1</td>
<td>1/2</td>
<td>0.6</td>
<td>-0.4</td>
<td>0.2</td>
<td>-</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>(iii) 1</td>
<td>3/2</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.8</td>
<td>-</td>
<td>-0.5</td>
</tr>
<tr>
<td>3/2</td>
<td>(i) 0</td>
<td>3/2</td>
<td>-</td>
<td>0.0</td>
<td>-</td>
<td>-0.5</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>(ii) 1</td>
<td>1/2</td>
<td>-</td>
<td>0.7</td>
<td>-</td>
<td>0.4</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>(iii) 1</td>
<td>3/2</td>
<td>-</td>
<td>0.0</td>
<td>-</td>
<td>-0.7</td>
<td>-0.8</td>
</tr>
<tr>
<td>5/2</td>
<td>(i) 1</td>
<td>3/2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Spectroscopic factor $S_i$ (Spin structure)

$5q$ potential

- **S-factor:** $S_i = \langle (\bar{D}Y_c)_i | 5q \rangle$

**Table:** Spectroscopic factors $S_i$ for each meson-baryon channel.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$S_{c\bar{c}}$</th>
<th>$S_{3q}$</th>
<th>$\bar{D}\Lambda_c$</th>
<th>$\bar{D}^*\Lambda_c$</th>
<th>$\bar{D}\Sigma_c$</th>
<th>$\bar{D}\Sigma^*_c$</th>
<th>$\bar{D}^*\Sigma_c$</th>
<th>$\bar{D}^<em>\Sigma^</em>_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>(i) 0</td>
<td>1/2</td>
<td>0.4</td>
<td>0.6</td>
<td>-0.4</td>
<td>-</td>
<td>0.2</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>(ii) 1</td>
<td>1/2</td>
<td>0.6</td>
<td>-0.4</td>
<td>0.2</td>
<td>-</td>
<td>-0.6</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>(iii) 1</td>
<td>3/2</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.8</td>
<td>-</td>
<td>-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>3/2</td>
<td>(i) 0</td>
<td>3/2</td>
<td>-</td>
<td>0.0</td>
<td>-</td>
<td>-0.5</td>
<td>0.6</td>
<td>-0.7</td>
</tr>
<tr>
<td></td>
<td>(ii) 1</td>
<td>1/2</td>
<td>-</td>
<td>0.7</td>
<td>-</td>
<td>0.4</td>
<td>-0.2</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>(iii) 1</td>
<td>3/2</td>
<td>-</td>
<td>0.0</td>
<td>-</td>
<td>-0.7</td>
<td>-0.8</td>
<td>-0.2</td>
</tr>
<tr>
<td>5/2</td>
<td>(i) 1</td>
<td>3/2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

- $\bar{D}Y_c$ with **Large $S_i$** will play an important role.
Numerical Results for Hidden-charm sector

Bound state and Resonance
- Coupled-channel Schrödinger equation for $\bar{D}\Lambda_c$, $\bar{D}^*\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}\Sigma^*_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma^*_c$ (6 $MB$ components).
- For $J^P = 1/2^-, 3/2^-, 5/2^-$ (Negative parity)
Results ($f^5q$ vs $E$) of charm $\bar{D}Y_c$ for $J^P = 1/2^-$

- Energy with $V_\pi + V^{5q}(f^5q)$. (Y. Yamaguchi et al, PRD 96 (2017), 114031)

$$J^P = 1/2^-$$

Dashed line: Thresholds, Red line: Energy obtained

- For small $f^5q$, **No bound state**
  ⇒ The OPEP attraction is not enough to generate a state
- $5q$ potential helps to generate the states near the thresholds
  ⇔ **Large S-factor** (Spin structure)
Results ($f^{5q} \text{ vs } E$) for $J^P = 3/2^-, 5/2^-$

- Energy with $V_\pi + V^{5q}(f^{5q})$. (Y. Yamaguchi et al, PRD96 (2017), 114031)

$$J^P = 3/2^-$$

$$J^P = 5/2^-$$

- For small $f^{5q}$, **No bound state**
  - The OPEP attraction is not enough to generate a state
- $5q$ potential helps to generate the states **near the thresholds**
  - **Large S-factor** (Spin structure)
In 2019, New $P_c$ states by LHCb

- $f^{5q} = 45$ is fixed to reproduce new $P_c$'s

In 2019, New $P_c$ states by LHCb

- $f^{5q} = 45$ is fixed to reproduce new $P_c$'s


- $(E, \Gamma)$ in our pred. consistent with Exp for $P_c(4312), P_c(4440), P_c(4457)$

- Missing three $(1/2^-, 3/2^-, 5/2^-)$ states below $\bar{D}^*\Sigma_c^*$

- Broad $P_c(4380)$ consistency with Exp for $P_c(4312), P_c(4440), P_c(4457)$

- Further theoretical and experimental studies are necessary...
In 2019, New $P_c$ states by LHCb

- $f^{5q} = 45$ is fixed to reproduce new $P_c$'s


- $(E, \Gamma)$ in our pred. consistent with Exp for $P_c(4312), P_c(4440), P_c(4457)$

- Missing three $(1/2^-, 3/2^-, 5/2^-)$ states below $\bar{D}^* \Sigma_c^*$

- Broad $P_c(4380) \leftrightarrow$ Our prediction with 4376 MeV
  $\rightarrow$ Further theoretical and experimental studies are necessary...
\( J^P \) assignment for \( P_c(4440) \) and \( P_c(4457) \)

\[
\begin{array}{ccc}
1/2- & 3/2- \\
\text{4457} & \text{4462} & \Sigma_c D^* \\
\text{4440} & \text{4442}
\end{array}
\]

- \( J^P \) assignment
  - \( P_c(4440) : 3/2^- \)
  - \( P_c(4457) : 1/2^- \)

\[ \Rightarrow E(1/2^-) > E(3/2^-) \]

- OPEP of the \( \bar{D}^* \Sigma_c \) channel
  - \( 1/2^- : \begin{align*} 2S, & \quad 4D \\ 4C & \quad 2\sqrt{2}T \\ 2\sqrt{2}T & \quad -2C + 4T \end{align*} \)
  - \( 3/2^- : \begin{align*} 4S, & \quad 2D, & \quad 4D \\ -2C & \quad -2T & \quad -4T \\ -2T & \quad 4C & \quad 2T \\ -4T & \quad 2T & \quad -2C \end{align*} \)

\* \( C \): Central force, \( T \): Tensor force
**J^P assignment for** $P_c(4440)$ and $P_c(4457)$

- **J^P assignment**
  - $P_c(4440): 3/2^-$
  - $P_c(4457): 1/2^-$

  $\Rightarrow E(1/2^-) > E(3/2^-)$

- **OPEP of the $\bar{D}^*\Sigma_c$ channel**

  1/2\(^-\): \(^2\)S, \(^4\)D
  
  $\begin{pmatrix}
  4C & 2\sqrt{2}T \\
  2\sqrt{2}T & -2C + 4T
  \end{pmatrix}$

  3/2\(^-\): \(^4\)S, \(^2\)D, \(^4\)D
  
  $\begin{pmatrix}
  -2C & -2T & -4T \\
  -2T & 4C & 2T \\
  -4T & 2T & -2C
  \end{pmatrix}$

  * C: Central force, T: Tensor force

- **S — D, D — D couplings producing the attraction from the tensor force**

  $\Rightarrow 1/2^-: \(^2\)S — \(^4\)D$

  $3/2^-: \(^4\)S — \(^2\)D, \(^4\)S — \(^4\)D, \(^2\)D — \(^4\)D$

  $\uparrow 14$ MeV

  **More attractive!**
Summary

- Hidden-charm pentaquarks as Hadronic molecule + Compact multiquark

- $\bar{D}(\ast) Y_c$ Interaction
  - Long range force: OPEP with the tensor force, enhanced by the heavy quark symmetry
  - Short range force: Coupling to Compact 5$q$ states

- The OPEP is not enough to generate the bound state. $\rightarrow$ OPEP + 5$q$ potential generates the states

- Applying this model to New hidden-charm pentaquarks by LHCb in 2019
  $\Rightarrow$ our prediction is consistent with EXP

- The $J^P$ assignment $P_c(4440)$: $3/2^-$ and $P_c(4457)$: $1/2^-$ understood by the tensor force of the OPEP


Back up
Experimental mass and Our prediction

<table>
<thead>
<tr>
<th>State</th>
<th>Mass</th>
<th>Width</th>
<th>Our pred. ((M, \Gamma, J^P))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_c(4312)^+)</td>
<td>4311.9 ± 0.7^{+6.8}_{-0.6}</td>
<td>9.8 ± 2.7^{+3.7}_{-4.5}</td>
<td>(4312, 5, \frac{1}{2}^-)</td>
</tr>
<tr>
<td>(P_c(4380)^+)</td>
<td>4380 ± 8 ± 29</td>
<td>205 ± 18 ± 86</td>
<td>(4376, 8, \frac{3}{2}^-)</td>
</tr>
<tr>
<td>(P_c(4440)^+)</td>
<td>4440.3 ± 1.3^{+4.1}_{-4.7}</td>
<td>20.6 ± 4.9^{+8.7}_{-10.1}</td>
<td>(4442, 26, \frac{3}{2}^-)</td>
</tr>
<tr>
<td>(P_c(4457)^+)</td>
<td>4457.3 ± 0.6^{+4.1}_{-1.7}</td>
<td>6.4 ± 2.0^{+5.7}_{-1.9}</td>
<td>(4462, 6.6, \frac{1}{2}^-)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4524, 1.5, \frac{1}{2}^-)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4521, 23, \frac{3}{2}^-)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4511, 55, \frac{5}{2}^-)</td>
</tr>
</tbody>
</table>

(in units of MeV)
# Coupled-channels

<table>
<thead>
<tr>
<th>Channels</th>
<th>$\bar{D} Y_c(2S+1L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^-$</td>
<td>$\bar{D}\Lambda_c(2S)$, $\bar{D}^<em>\Lambda_c(2S)$, &lt;br&gt; $\bar{D}\Sigma_c(2S)$, $\bar{D}\Sigma_c^</em>(4D)$, &lt;br&gt; $\bar{D}^<em>\Sigma_c(2S,4D)$, $\bar{D}^</em>\Sigma_c^*(2S,4D,6D)$</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>$\bar{D}\Lambda_c(2D)$, $\bar{D}^<em>\Lambda_c(4S,2D,4D)$, &lt;br&gt; $\bar{D}\Sigma_c(2D)$, $\bar{D}\Sigma_c^</em>(4S,4D)$, &lt;br&gt; $\bar{D}^<em>\Sigma_c(4S,2D,4D)$, $\bar{D}^</em>\Sigma_c^*(4S,2D,4D,6D,6G)$</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>$\bar{D}\Lambda_c(2D)$, $\bar{D}^<em>\Lambda_c(2D,4D,4G)$, &lt;br&gt; $\bar{D}\Sigma_c(2D)$, $\bar{D}\Sigma_c^</em>(4D,4G)$, &lt;br&gt; $\bar{D}^<em>\Sigma_c(2D,4D,4G)$, $\bar{D}^</em>\Sigma_c^*(6S,2D,4D,6D,4G,6G)$</td>
</tr>
</tbody>
</table>

- 6 $\bar{D} Y_c$ channels: $\bar{D}\Lambda_c$, $\bar{D}^*\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$.
- $S - D$ mixing induced by the Tensor force ($S_{12}$)
Heavy hadron-$\pi$ coupling
HQS and OPEP

- Effective Lagrangians: Heavy hadron and $\pi$


  \[
  \mathcal{L}_{\pi HH} = -\frac{g_{\pi}}{2f_\pi} \text{Tr} \left[ H \gamma_\mu \gamma_5 \partial^\mu \bar{\pi} \pi H \right], \quad H = \frac{1+\gamma^\nu}{2} \left[ D^\ast_\mu \gamma^\mu - D \gamma_5 \right]
  \]

- Heavy meson: $\bar{D}^{(*)} \bar{D}^{(*)} \pi$ (\textit{DD}\pi: Parity violation)
Effective Lagrangians: Heavy hadron and $\pi$


$\pi$

$g_\pi$

$\bar{D}(\ast)$ \hspace{1cm} $\bar{D}^\ast$

Heavy meson: $\bar{D}(\ast)\bar{D}(\ast)\pi$ \hspace{1cm} ($DD\pi$: Parity violation)

$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi} \text{Tr} \left[ H\gamma_\mu\gamma_5 \partial^\mu \hat{\pi} \tilde{H} \right], \quad H = \frac{1+\gamma^5}{2} \left[ D^*_\mu \gamma^\mu - D \gamma_5 \right]$
Effective Lagrangians: Heavy hadron and $\pi$


$\pi$
$\bar{D}^{(*)}$  $\bar{D}^*$  $\Sigma_c^{(*)}$  $\Lambda_c$  $\Sigma_c^{(*)}$
$g_\pi$  $g_1$  $g_4$

$\bar{D}^{(*)}\pi$  $\Sigma_c^{(*)}\pi$  $\Lambda_c\Sigma_c^{(*)}\pi$
(DD$\pi$: Parity violation)

$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi} \text{Tr} \left[ H \gamma_\mu \gamma_5 \partial^\mu \hat{\pi} \hat{H} \right], \quad H = \frac{1+i\gamma_5}{2} \left[ D^{*}_\mu \gamma^\mu - D \gamma_5 \right]$

$\Sigma_c^{(*)}\Sigma_c^{(*)}\pi, \Lambda_c\Sigma_c^{(*)}\pi$  ($\Lambda_c\Lambda_c\pi$: Isospin breaking)

$\mathcal{L}_{\pi BB} = -\frac{3}{4f_\pi} g_1 (i\nu_\kappa) \varepsilon^{\mu\nu\lambda\kappa} \text{tr} \left[ \bar{S}_\mu \partial_\nu \hat{\pi} S_\lambda \right] - \frac{g_4}{2f_\pi} \text{tr} \left[ \bar{\Lambda}_c \partial_\mu \hat{\Lambda}_c \right] + \text{H.c.},$

$S_\mu = \Sigma_{c\mu}^* - \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma_5 \Sigma_c, \quad g_\pi = 0.59, g_1 = 1.00, g_4 = 1.06$
Spectroscopic factors $S_i$ (Spin structure)

5$q$ potential

- Spin of 5$q$ states $\rightarrow S_{c\bar{c}}$ and $S_{3q}$ configuration
  
  e.g. for $J^P = 1/2^-$, (i), (ii), (iii)

\[
\begin{array}{c|cc}
J^P = 1/2^- & S_{c\bar{c}} & S_{3q} \\
\hline
\text{type (i)} & 0 & 1/2 \\
\text{(ii)} & 1 & 1/2 \\
\text{(iii)} & 1 & 3/2 \\
\end{array}
\]
Spectroscopic factors $S_i$ (Spin structure)

5$q$ potential

- Spin of 5$q$ states $→ S_{c\bar{c}}$ and $S_{3q}$ configuration
  e.g. for $J^P = 1/2^-$, (i), (ii), (iii)

- Overlap of the spin wavefunctions of 5-quark state and $\bar{D}Y_c$

  \[
  S_i = \langle (\bar{D}Y_c)_i | 5q \rangle
  \]

  $⇒$ Relative strength of couplings to $\bar{D}Y_c$ channel

\[
\begin{array}{c|cc}
 J^P = 1/2^- & S_{c\bar{c}} & S_{3q} \\
\hline 
 \text{type (i)} & 0 & 1/2 \\
 (ii) & 1 & 1/2 \\
 (iii) & 1 & 3/2 \\
\end{array}
\]
Volume integrals of the potentials

- Bound and Resonant states appears for $f^{5q} \gtrsim 25$
  $\Leftrightarrow$ Large? Small?
Volume integrals of the potentials

- Bound and Resonant states appears for $f^{q}_{5q} \gtrsim 25$
  $\Leftrightarrow$ Large? Small?

▶ Volume integral $V(q = 0) = \int V(r)dr^3$

Comparison with the $NN$ interaction (Bonn potential)


$$\left| V_{f=25}^{5q}(0) \right| = 1.1 \times 10^{-4} \text{ MeV} \sim 0.03 |C_{NN}^\sigma(0)|$$

($C_{NN}^\sigma$ : Central force of $\sigma$ exchange)

- $\left| V_{f=25}^{5q}(0) \right|$ is much smaller than $|C_{NN}^\sigma(0)|$.

However, the bound and resonant states are obtained!