Four-body Faddeev-type calculation of the $\bar{K}NNN$ system

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K^-pp quasi-bound state – interest to antikaonic nuclei

Theory

Prediction of the existence of deep and narrow $K^- pp$ bound state

T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70

$E_B = -48$ MeV, $\Gamma = 61$ MeV

Many theoretical calculations, different models and inputs (Faddeev, variational calculations, FCA):

$E_B \sim -14 - 80$ MeV, $\Gamma \sim 40 - 110$ MeV

- agree only on the fact that the quasi-bound state in $K^- pp$ exists

Experiment

FINUDA collaboration: $E_B = -115$ MeV, $\Gamma = 67$ MeV

M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303

DISTO collaboration: $E_B = -103$ MeV, $\Gamma = 118$ MeV

T. Yamazaki et al. Phys. Rev. Lett. 104 (2010) 132502

J-PARC E15 experiment: a broad peak in ${}^3He(K^-,\Lambda p)n$ spectrum

A series of 3-body Faddeev calculations with coupled $\bar{K}NN$ - $\pi\Sigma N$ channels

• Three-body pole positions and widths of the quasi-bound states in the $K^- pp$ and $K^- K^- p$ systems were evaluated

N.V.S, A. Gal, J. Mareš, J. Rèvai, Phys. Rev. C 76, 044004 (2007)
J. Rèvai, N.V.S., Phys. Rev. C 90, 034004 (2014)
N.V.S., J. Haidenbauer, Phys. Rev. C 92, 044001 (2015)

◦ No quasi-bound state found in the $K^- d$ system (caused by strong interaction)

• Near-threshold elastic $K^- d$ amplitudes were calculated (including the $K^- d$ scattering length)

• and then used for an approximate calculation of the *1s* level shift and width of (anti)kaonic deuterium

N.V.S., Nucl. Phys. A 890-891, 50-61 (2012)
N.V.S., J. Rèvai, Phys. Rev. C 90, 034003 (2014)

• Faddeev-type equations for strong plus Coulomb potentials were solved for the *1s* level shift and width of kaonic deuterium using

- simple complex *P. Doleschall, J. Rèvai, N.V.S., Phys. Lett. B 744, 105-108 (2015)*
- energy dependent antikaon-nucleon potentials *J. Rèvai, Few Body Syst. 59, 49 (2018)*

→ 4-body $\bar{K}NNN$ system (Faddeev-type equations)

Three-body Faddeev equations in Alt-Grassberger-Sandhas form

E.O. Alt, P. Grassberger, W. Sandhas, Nucl. Phys. B2 (1967) 167

$$U_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})(G_0(z))^{-1} + \sum_{\gamma=1}^{3}(1-\delta_{\alpha\gamma})T_\gamma(z)\,G_0(z)\,U_{\gamma\beta}(z), \qquad \alpha,\beta = 1,2,3$$

$U_{\alpha\beta}(z)$ - 3-body transition operators, $\beta + (\alpha\gamma) \to \alpha + (\beta\gamma)$

$G_0(z)$ - free Green function

$T_\alpha(z)$ - 2-body T-matrix

A separable potential leading to a separable T-matrix

$$V_\alpha = \lambda_\alpha |g_\alpha\rangle\langle g_\alpha| \Rightarrow T_\alpha(z) = |g_\alpha\rangle \tau_\alpha(z) \langle g_\alpha|$$

allows to write the three-body equations in the form

$$X_{\alpha\beta}(z) = Z_{\alpha\beta}(z) + \sum_{\gamma=1}^{3} Z_{\alpha\gamma}(z)\tau_\gamma(z)\,X_{\gamma\beta}(z)$$

with $X_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U_{\alpha\beta}(z)\, G_0(z) | g_\beta \rangle$, $Z_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})\langle g_\alpha | G_0(z) | g_\beta \rangle$

Four-body Alt-Grassberger-Sandhas equations

P. Grassberger, W. Sandhas, Nucl. Phys. B2 (1967) 181

$$U^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})\,\delta_{\alpha\beta}G_0^{-1}(z)\,T_\alpha^{-1}(z)G_0^{-1}(z) +$$
$$+\sum_{\tau,\gamma}(1-\delta_{\sigma\tau})\,U^{\tau}_{\alpha\gamma}(z)G_0(z)\,T_\gamma(z)G_0(z)U^{\tau\rho}_{\gamma\beta}(z)$$

$U^{\sigma\rho}_{\alpha\beta}(z)$ - 4-body transition operators, $\beta\subset\rho, \alpha\subset\sigma$

$U^{\tau}_{\alpha\beta}(z)$ - 3-body transition operators, $\alpha,\beta\subset\tau$

$G_0(z)$ - free Green function,

$T_\alpha(z)$ - 2-body *T*-matrix

A separable potential → separable *T*-matrix → four-body equations:

$$\overline{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})\left(\overline{G_0}(z)\right)^{-1}_{\alpha\beta} + \sum_{\tau,\gamma,\delta}(1-\delta_{\sigma\tau})\,\overline{T}^{\tau}_{\alpha\gamma}(z)\left(\overline{G_0}(z)\right)_{\gamma\delta}\overline{U}^{\tau\rho}_{\delta\beta}(z),$$

with $\overline{U}^{\sigma\rho}_{\alpha\beta}(z) = \left\langle g_\alpha \middle| G_0(z)U^{\sigma\rho}_{\alpha\beta}(z)G_0(z) \middle| g_\beta \right\rangle$

$\overline{T}^{\tau}_{\alpha\beta}(z) = \left\langle g_\alpha \middle| G_0(z)U^{\tau}_{\alpha\beta}(z)G_0(z) \middle| g_\beta \right\rangle$

and $\overline{(G_0)}_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z)$

Four-body AGS equations for separable "potentials"

E.O. Alt, P. Grassberger, W. Sandhas, Phys.Rev. C1 (1970) 85
A.Casel, H.Haberzettl, W. Sandhas, Phys. Rev. C25 (1982) 1738

$$\overline{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})\,\delta_{\alpha\beta}\left(\overline{G_0}(z)\right)^{-1}_{\alpha} + \sum_{\tau,\gamma}(1-\delta_{\sigma\tau})\,\overline{T}^{\tau}_{\alpha\gamma}(z)\left(\overline{G_0}(z)\right)_{\gamma}\overline{U}^{\tau\rho}_{\gamma\beta}(z),$$

look similar to the three-body AGS equations in the general form.
Separable form of the "effective potentials" → separable "*T*"-matrix:

$$\overline{T}^{\tau}_{\alpha\beta}(z) = \left|\overline{g}^{\tau}_{\alpha}\right\rangle \overline{\tau}^{\tau}_{\alpha\beta}(z)\left\langle \overline{g}^{\tau}_{\beta}\right|$$

allows to write the four-body equations in the form

$$\overline{X}^{\sigma\rho}_{\alpha\beta}(z) = \overline{Z}^{\sigma\rho}_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta}\overline{Z}^{\sigma\tau}_{\alpha\gamma}(z)\,\overline{\tau}^{\tau}_{\gamma\delta}(z)\,\overline{X}^{\tau\rho}_{\delta\beta}(z)$$

with $\overline{X}^{\sigma\rho}_{\alpha\beta}(z) = \left\langle \overline{g}^{\sigma}_{\alpha}\right|\left(\overline{G_0}(z)\right)_{\alpha}\overline{U}^{\sigma\rho}_{\alpha\beta}(z)\left(\overline{G_0}(z)\right)_{\beta}\left|\overline{g}^{\rho}_{\beta}\right\rangle$

$$\overline{Z}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})\left\langle \overline{g}^{\sigma}_{\alpha}\right|\left(\overline{G_0}(z)\right)_{\alpha\beta}\left|\overline{g}^{\rho}_{\beta}\right\rangle$$

Separabelization of potentials - Hilbert-Schmidt expansion

Lippmann-Schwinger equation:

$$T(p,p';z)=V(p,p')+4\pi\int_0^\infty \frac{V(p,p'')T(p'',p';z)}{z-p''^2/(2\mu)}p''^2dp''$$

Separable potential leads to the separable *T*-matrix

$$V(p,p')=-\sum_{n=1}^{\infty}\lambda_n g_n(p)g_n(p') \quad \Rightarrow \quad T(p,p';z)=-\sum_{n=1}^{\infty}\frac{\lambda_n}{1-\lambda_n}g_n(p)g_n(p')$$

were the eigenvalues λ_n and eigenfunctions $g_n(p)$ are found from

$$g_n(p)=\frac{1}{\lambda_n}4\pi\int_0^\infty \frac{V(p,p'')g_n(p'')}{z-p''^2/(2\mu)}p''^2dp''$$

with normalization condition

$$4\pi\int_0^\infty \frac{g_n(p'')g_{n'}(p'')}{z-p''^2/(2\mu)}p''^2dp''=-\delta_{nn'}$$

Separabelization of "potentials" -
Energy Depenent Pole Expansion/Approximation (EDPE/EDPA)

S. Sofianos, N.J.McGurk, H. Fiedeldey, Nucl. Phys. A318 (1979) 295

Three-body AGS equations:

$$X_{\alpha\beta}(p,p';z)=Z_{\alpha\beta}(p,p';z)+\sum_{\gamma=1}^{3}4\pi\int_0^\infty Z_{\alpha\gamma}(p,p'';z)\tau_\gamma(p'';z)X_{\gamma\beta}(p'',p';z)p''^2dp''$$

1. Evaluate eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p;z)$ of the AGS system from

$$g_{n\alpha}(p;z)=\frac{1}{\lambda_n}\sum_{\gamma=1}^{3}4\pi\int_0^\infty Z_{\alpha\gamma}(p,p';z)\tau_\gamma(p';z)g_{n\gamma}(p';z)p'^2dp'$$

with normalization condition

$$\sum_{\gamma=1}^{3}4\pi\int_0^\infty g_{n\gamma}(p';z)\tau_\gamma(p';z)g_{n'\gamma}(p';z)p'^2dp'=-\delta_{nn'}$$

at fixed (binding) energy $z=E_B$

2. Calculate energy dependent form-factors

$$g_{n\alpha}(p;z)=\sum_{\gamma=1}^{3}4\pi\int_0^\infty Z_{\alpha\gamma}(p,p';z)\,\tau_\gamma(p';E_B)\,g_{n\gamma}(p';E_B)\,p'^2dp'$$

and propagators

$$\left(\Theta(z)\right)^{-1}_{mn}=\sum_{\gamma=1}^{3}4\pi\int_0^\infty g_{m\gamma}(p';z)\,\tau_\gamma(p';E_B)\,g_{n\gamma}(p';E_B)\,p'^2dp'-$$
$$\sum_{\gamma=1}^{3}4\pi\int_0^\infty g_{m\gamma}(p';z)\,\tau_\gamma(p';z)\,g_{n\gamma}(p';z)\,p'^2dp'$$

so the separable form of the three-body amplitudes is

$$\boxed{X_{\alpha\beta}(p,p';z)=\sum_{m,n=1}^{\infty}g_{m\alpha}(p;z)\,\Theta_{mn}(z)\,g_{n\beta}(p';z)}$$

EDPE/EDPA:

- Needs only one solution of the eigenvalue equation
- Accurate already with one term (EDPA)
- EDPE converges faster than Hilbert-Schmidt expansion

Four-body equations for the $\bar{K}NNN$ system

Two types of partitions: 3+1 and 2+2:

$$|\bar{K}+(NNN)\rangle$$
$$|N+(\bar{K}NN)\rangle$$
$$|(\bar{K}N)+(NN)\rangle$$

The channels σ_α :

$$1_{NN}\;:\;|\bar{K}+(N_1+N_2N_3)\rangle,\;|\bar{K}+(N_2+N_3N_1)\rangle,\;|\bar{K}+(N_3+N_1N_2)\rangle$$

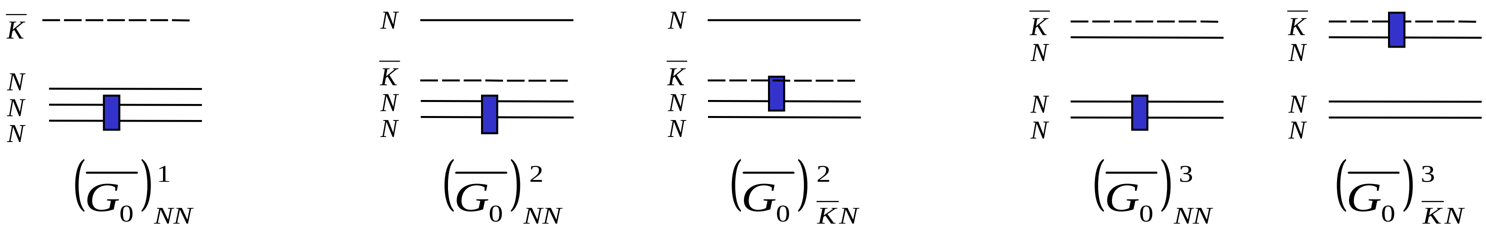
$$2_{NN}\;:\;|N_1+(\bar{K}+N_2N_3)\rangle,\;|N_2+(\bar{K}+N_3N_1)\rangle,\;|N_3+(\bar{K}+N_1N_2)\rangle$$

$$2_{\bar{K}N}\;:\;|N_1+(N_2+\bar{K}N_3)\rangle,\;|N_2+(N_3+\bar{K}N_1)\rangle,\;|N_3+(N_1+\bar{K}N_2)\rangle,$$
$$|N_1+(N_3+\bar{K}N_2)\rangle,\;|N_2+(N_1+\bar{K}N_3)\rangle,\;|N_3+(N_2+\bar{K}N_1)\rangle$$

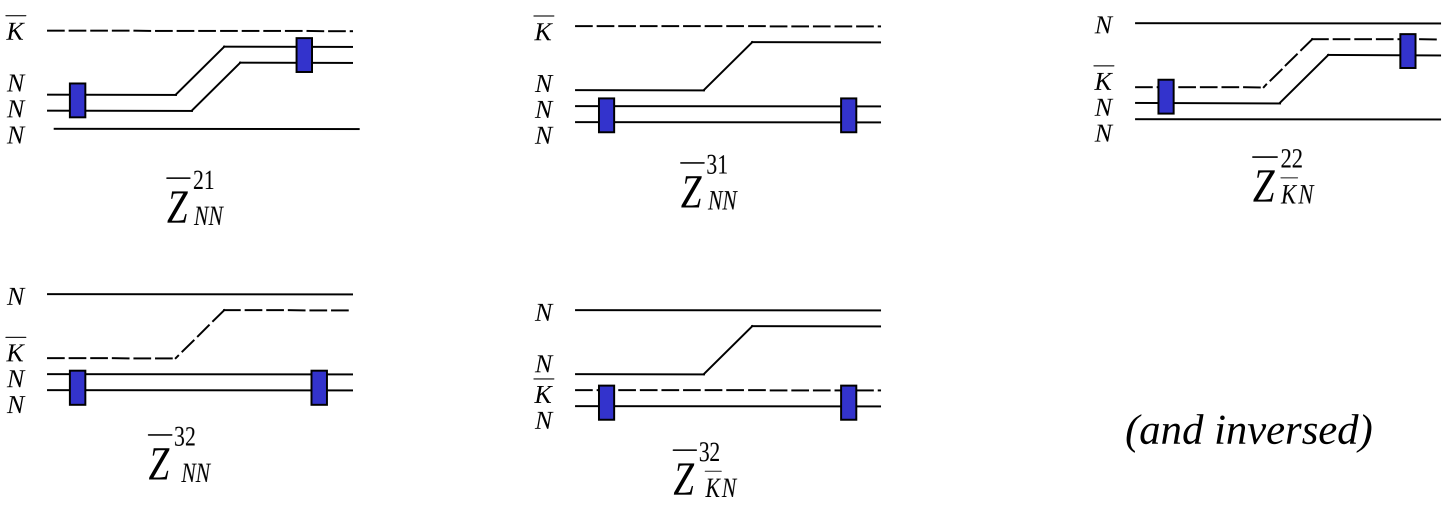
$$3_{NN}\;:\;|(N_2N_3)+(\bar{K}+N_1)\rangle,\;|(N_3N_1)+(\bar{K}+N_2)\rangle,\;|(N_1N_2)+(\bar{K}+N_3)\rangle$$

$$3_{\bar{K}N}\;:\;|(\bar{K}N_1)+(N_2+N_3)\rangle,\;|(\bar{K}N_2)+(N_3+N_1)\rangle,\;|(\bar{K}N_3)+(N_1+N_2)\rangle$$

Propagators



Z operators



(and inversed)

Four-body equations in operator form:

$$\hat{X} = \hat{Z}\,\hat{\tau}\,\hat{X}$$

Three nucleons – antisymmetrization,
after antisymmetrization
(without spin-isospin variables)

$$\bar{X}^{\sigma}_{\alpha} = \begin{pmatrix} \bar{X}^{1}_{NN} \\ \bar{X}^{2}_{NN} \\ \bar{X}^{2}_{\bar{K}N} \\ \bar{X}^{3}_{NN} \\ \bar{X}^{3}_{\bar{K}N} \end{pmatrix}$$

$$\bar{Z}^{\sigma\rho}_{\alpha} = \begin{pmatrix} 0 & \bar{Z}^{12}_{NN} & 0 & \bar{Z}^{13}_{NN} & 0 \\ \bar{Z}^{21}_{NN} & 0 & 0 & \bar{Z}^{23}_{NN} & 0 \\ 0 & 0 & \bar{Z}^{22}_{\bar{K}N} & 0 & \bar{Z}^{23}_{\bar{K}N} \\ \bar{Z}^{31}_{NN} & \bar{Z}^{32}_{NN} & 0 & 0 & 0 \\ 0 & 0 & \bar{Z}^{32}_{\bar{K}N} & 0 & 0 \end{pmatrix}$$

$$\bar{\tau}^{\rho}_{\alpha\beta} = \begin{pmatrix} \bar{\tau}^{1}_{NN,NN} & 0 & 0 & 0 & 0 \\ 0 & \bar{\tau}^{2}_{NN,NN} & \bar{\tau}^{2}_{NN,\bar{K}N} & 0 & 0 \\ 0 & \bar{\tau}^{2}_{\bar{K}N,NN} & \bar{\tau}^{2}_{\bar{K}N,\bar{K}N} & 0 & 0 \\ 0 & 0 & 0 & \bar{\tau}^{3}_{NN,NN} & \bar{\tau}^{3}_{NN,\bar{K}N} \\ 0 & 0 & 0 & \bar{\tau}^{3}_{\bar{K}N,NN} & \bar{\tau}^{3}_{\bar{K}N,\bar{K}N} \end{pmatrix}$$

Full system of equations with:

- 1-term separable $\bar{K}N$ potential $\Rightarrow$
- 2-term separable NN potential
- 1-term separabelized 3-body $NNN, \bar{K}NN$ ‘‘T-matrices’’ and ‘‘3-body’’ $\bar{K}N + NN$ ‘‘T-matrices’’

$$\bar{Z}^{\sigma g(m_3 n_3)}_{\alpha(m_\alpha n_\alpha);(i,ss')}$$

σ, g – 4-частичные канала
m_3, n_3 – 3-частичн [illegible] /здесь=(1,1)/
α = NN или K̄N
m_α, n_α = 1,2 для NN или = 1 для K̄N
i – изоспин α в σ (= i' α в g);
s, s' – спин α в σ, α в g

	1(1) NN(1);Φ1	1(1) NN(1);1Φ	1(1) NN(2),Φ1	1(1) NN(2);1Φ	2(1) NN(1);Φ1	2(1) NN(1);1Φ	2(1) NN(2);Φ1	2(1) NN(2);1Φ	2(1) K̄N(1);ΦΦ	2(1) K̄N(1);1Φ	2(1) K̄N(1);Φ1	2(1) K̄N(1);11	3(1) NN(1);Φ1	3(1) NN(1);1Φ	3(1) NN(2);Φ1	3(1) NN(2);1Φ	3(1) K̄N(1);Φ1	3(1) K̄N;1Φ
1(1) NN(1);Φ1					$\bar{Z}^{12(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{12(1,1)}_{NN(1,2)};(\Phi,11)$	∅					$\bar{Z}^{13(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{13(1,1)}_{NN(1,2)};(\Phi,11)$	∅		
1(1) NN(1);1Φ					∅	$\bar{Z}^{12(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{12(1,1)}_{NN(1,2)};(1,\Phi\Phi)$					∅	$\bar{Z}^{13(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{13(1,1)}_{NN(1,2)};(1,\Phi\Phi)$		
1(1) NN(2);Φ1					$\bar{Z}^{12(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{12(1,1)}_{NN(2,2)};(\Phi,11)$	∅					$\bar{Z}^{13(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{13(1,1)}_{NN(2,2)};(\Phi,11)$	∅		
1(1) NN(2);1Φ					∅	$\bar{Z}^{12(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{12(1,1)}_{NN(2,2)};(1,\Phi\Phi)$					∅	$\bar{Z}^{13(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{13(1,1)}_{NN(2,2)};(1,\Phi\Phi)$		
2(1) NN(1);Φ1	$\bar{Z}^{21(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{21(1,1)}_{NN(1,2)};(\Phi,11)$	∅									$\bar{Z}^{23(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{23(1,1)}_{NN(1,2)};(\Phi,11)$	∅		
2(1) NN(1);1Φ	∅	$\bar{Z}^{21(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{21(1,1)}_{NN(1,2)};(1,\Phi\Phi)$									∅	$\bar{Z}^{23(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{23(1,1)}_{NN(1,2)};(1,\Phi\Phi)$		
2(1) NN(2);Φ1	$\bar{Z}^{21(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{21(1,1)}_{NN(2,2)};(\Phi,11)$	∅									$\bar{Z}^{23(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{23(1,1)}_{NN(2,2)};(\Phi,11)$	∅		
2(1) NN(2);1Φ	∅	$\bar{Z}^{21(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{21(1,1)}_{NN(2,2)};(1,\Phi\Phi)$									∅	$\bar{Z}^{23(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{23(1,1)}_{NN(2,2)};(1,\Phi\Phi)$		
2(1) K̄N(1);ΦΦ									$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(\Phi,\Phi\Phi)$	∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(\Phi,\Phi1)$	∅					$\bar{Z}^{23(1,1)}_{\bar{K}N(1,1)};(\Phi,\Phi1)$	∅
2(1) K̄N(1);1Φ									∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(1,\Phi1)$					∅	$\bar{Z}^{23(1,1)}_{\bar{K}N(1,1)};(1,\Phi\Phi)$
2(1) K̄N(1);Φ1									$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(\Phi,1\Phi)$	∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(\Phi,11)$	∅					$\bar{Z}^{23(1,1)}_{\bar{K}N(1,1)};(\Phi,11)$	∅
2(1) K̄N(1);11									∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(1,1\Phi)$	∅	$\bar{Z}^{22(1,1)}_{\bar{K}N(1,1)};(1,11)$					∅	$\bar{Z}^{23(1,1)}_{\bar{K}N(1,1)};(1,1\Phi)$
3(1) NN(1);Φ1	$\bar{Z}^{31(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{31(1,1)}_{NN(1,2)};(\Phi,11)$	∅	$\bar{Z}^{32(1,1)}_{NN(1,1)};(\Phi,11)$	∅	$\bar{Z}^{32(1,1)}_{NN(1,2)};(\Phi,11)$	∅										
3(1) NN(1);1Φ	∅	$\bar{Z}^{31(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{31(1,1)}_{NN(1,2)};(1,\Phi\Phi)$	∅	$\bar{Z}^{32(1,1)}_{NN(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{32(1,1)}_{NN(1,2)};(1,\Phi\Phi)$										
3(1) NN(2);Φ1	$\bar{Z}^{31(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{31(1,1)}_{NN(2,2)};(\Phi,11)$	∅	$\bar{Z}^{32(1,1)}_{NN(2,1)};(\Phi,11)$	∅	$\bar{Z}^{32(1,1)}_{NN(2,2)};(\Phi,11)$	∅										
3(1) NN(2);1Φ	∅	$\bar{Z}^{31(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{31(1,1)}_{NN(2,2)};(1,\Phi\Phi)$	∅	$\bar{Z}^{32(1,1)}_{NN(2,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{32(1,1)}_{NN(2,2)};(1,\Phi\Phi)$										
3(1) K̄N(1);Φ1									$\bar{Z}^{32(1,1)}_{\bar{K}N(1,1)};(\Phi,1\Phi)$	∅	$\bar{Z}^{32(1,1)}_{\bar{K}N(1,1)};(\Phi,11)$	∅						
3(1) K̄N(1);1Φ									∅	$\bar{Z}^{32(1,1)}_{\bar{K}N(1,1)};(1,\Phi\Phi)$	∅	$\bar{Z}^{32(1,1)}_{\bar{K}N(1,1)};(1,\Phi1)$						

	1(1) NN(1);Φ1	1(1) NN(1);1Φ	1(1) NN(2);Φ1	1(1) NN(2);1Φ	2(1) NN(1);Φ1	2(1) NN(1);1Φ	2(1) NN(2);Φ1	2(1) NN(2);1Φ	2(1) K̄N(1);ΦΦ	2(1) K̄N(1);1Φ	2(1) K̄N(1);Φ1	2(1) K̄N(1);11	3(1) NN(1);Φ1	3(1) NN(1);1Φ	3(1) NN(2);Φ1	3(1) NN(2);1Φ	3(1) K̄N(1);Φ1	3(1) K̄N;1Φ
1(1) NN(1);Φ1	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,1);(ΦΦ,11)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,1);(Φ1,1Φ)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,2);(ΦΦ,11)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,2);(Φ1,1Φ)														
1(1) NN(1);1Φ	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,1);(1Φ,Φ1)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,1);(11,ΦΦ)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,2);(1Φ,Φ1)	$\bar{\tau}^{1(1)}_{NN,NN}$ (1,2);(11,ΦΦ)														
1(1) NN(2);Φ1	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,1);(ΦΦ,11)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,1);(Φ1,1Φ)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,2);(ΦΦ,11)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,2);(Φ1,1Φ)														
1(1) NN(2);1Φ	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,1);(1Φ,Φ1)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,1);(11,ΦΦ)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,2);(1Φ,Φ1)	$\bar{\tau}^{1(1)}_{NN,NN}$ (2,2);(11,ΦΦ)														
2(1) NN(1);Φ1					$\bar{\tau}^{2(1)}_{NN,NN}$ (1,1);(ΦΦ,11)	∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (1,2);(ΦΦ,11)	∅	∅	∅	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (1,1);(ΦΦ,11)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (1,1);(Φ1,11)						
2(1) NN(1);1Φ					∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (1,1);(11,ΦΦ)	∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (1,2);(11,ΦΦ)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (1,1);(1Φ,ΦΦ)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (1,1);(11,ΦΦ)	∅	∅						
2(1) NN(2);Φ1					$\bar{\tau}^{2(1)}_{NN,NN}$ (2,1);(ΦΦ,11)	∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (2,2);(ΦΦ,11)	∅	∅	∅	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (2,1);(ΦΦ,11)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (2,1);(Φ1,11)						
2(1) NN(2);1Φ					∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (2,1);(11,ΦΦ)	∅	$\bar{\tau}^{2(1)}_{NN,NN}$ (2,2);(11,ΦΦ)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (2,1);(1Φ,ΦΦ)	$\bar{\tau}^{2(1)}_{NN,\bar{K}N}$ (2,1);(11,ΦΦ)	∅	∅						
2(1) K̄N(1);ΦΦ					∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,1);(Φ1,ΦΦ)	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,2);(Φ1,ΦΦ)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(ΦΦ,ΦΦ)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(Φ1,ΦΦ)	∅	∅						
2(1) K̄N(1);1Φ					∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,1);(11,ΦΦ)	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,2);(11,ΦΦ)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(1Φ,ΦΦ)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(11,ΦΦ)	∅	∅						
2(1) K̄N(1);Φ1					$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,1);(ΦΦ,11)	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,2);(ΦΦ,11)	∅	∅	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(ΦΦ,11)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(Φ1,11)						
2(1) K̄N(1);11					$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,1);(1Φ,11)	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,NN}$ (1,2);(1Φ,11)	∅	∅	∅	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(1Φ,11)	$\bar{\tau}^{2(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(11,11)						
3(1) NN(1);Φ1													$\bar{\tau}^{3(1)}_{NN,NN}$ (1,1);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (1,2);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{NN,\bar{K}N}$ (1,1);(ΦΦ,11)	∅
3(1) NN(1);1Φ													∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (1,1);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (1,2);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{NN,\bar{K}N}$ (1,1);(11,ΦΦ)
3(1) NN(2);Φ1													$\bar{\tau}^{3(1)}_{NN,NN}$ (2,1);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (2,2);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{NN,\bar{K}N}$ (2,1);(ΦΦ,11)	∅
3(1) NN(2);1Φ													∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (2,1);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{NN,NN}$ (2,2);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{NN,\bar{K}N}$ (2,1);(11,ΦΦ)
3(1) K̄N(1);Φ1													$\bar{\tau}^{3(1)}_{\bar{K}N,NN}$ (1,1);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{\bar{K}N,NN}$ (1,2);(ΦΦ,11)	∅	$\bar{\tau}^{3(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(ΦΦ,11)	∅
3(1) K̄N(1);1Φ													∅	$\bar{\tau}^{3(1)}_{\bar{K}N,NN}$ (1,1);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{\bar{K}N,NN}$ (1,2);(11,ΦΦ)	∅	$\bar{\tau}^{3(1)}_{\bar{K}N,\bar{K}N}$ (1,1);(11,ΦΦ)

$\bar{\tau}^{\sigma(m_3=1)}_{\alpha\beta(m_2,m_2')}(ii';ss')$

σ – 4-частичный канал

α, β = NN или K̄N

$m_2; m_2'$ = 1,2 (для NN) или =1 (для K̄N)

ii'; ss' – изоспин (i) или спин (s), соответствующие α(i; s) или β(i'; s')

Two-body input

Antikaon-nucleon potentials with coupled $\bar{K}N - \pi\Sigma$ channels, fitted to:

- $1s$ level shift and width of kaonic hydrogen (by SIDDHARTA)

$$\Delta_{1s}^{SIDD} = -283 \pm 36 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22 \text{ eV}$$

- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- ○ $\Lambda(1405)$ with one- or two-pole structure

$$M_{\Lambda(1405)}^{PDG} = 1405.1^{+1.3}_{-1.0} \text{ MeV}, \quad \Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0 \text{ MeV}$$

Four-body calculations: the "exact optical" versions of the

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

constructed for the three-body calculations

Two-term *NN* potential

Reproduce: Argonne V18 *NN* phase shifts (with sign change),
deuteron binding energy, and scattering lengths

Quantum numbers of the 4-body system $K^- ppn: \; I^{(4)} = 0, S^{(4)} = 1/2,$

orbital momentum $L^{(4)} = 0$

Three-body subsystems

$\bar{K}NN \left(I^{(3)} = 1/2, S^{(3)} = 0 \text{ or } 1 \right)$

$K^- pp$ pole positions (MeV):
$-53.3 - i\,32.4$ *(1-pole phen)* , $-47.4 - i\,24.9$ *(2-pole phen)*, $-32.2 - i\,24.3$ *(chiral)*

$NNN \left(I^{(3)} = 1/2, S^{(3)} = 1/2 \right)$

$NNN \left({}^3H \text{ or } {}^3He \right)$ binding energy: 9.95 MeV

2+2 (‘‘Three-body’’) system

$\bar{K}N + NN \left(I^{(4)} = 0, S^{(4)} = 1/2, I_{\bar{K}N} = I_{NN} = 0 \text{ or } 1 \right)$

A special system with two non-interacting pairs of particles;
3-body system of equations has to be solved

Solution of the four-body equations

- Write down the equations for distinguishable nucleons ✓
- Antisymmetrize the equations ✓
- Evaluate momentum and spin-isospin parts of the kernel functions Z ✓
- Calculate separable 3-body $\bar{K}NN, NNN$ and ‘‘3-body’’ $\bar{K}N + NN$ ✓ “T-matrices”:
  - perform additional 3-body and ‘‘3-body’’ calculations,
  - calculate eigenvalues and eigenfunctions at fixed energy
  - evaluate form-factors and propagators for arbitrary energy
- Write the program code for four-body calculations in complex z-plane ✓
- Perform tests of the solutions

Preliminary results

Four-body quasi-bound state

$K^- ppn$ pole positions:

– 87.5 – *i* 38.2 MeV *(1-pole phenomenological $\bar{K}N$ potential)*

– 80.9 – *i* 34.3 MeV *(2-pole phenomenological $\bar{K}N$ potential)*

A system of equations with:

- 1-term separable $\bar{K}N$ potential
- 2-term separable NN potential
- 1-term separabelized 3-body $NNN, \bar{K}NN$ ''T-matrices''
  and ''3-body'' $\bar{K}N + NN$ ''T-matrices'' (EDPA)

was solved

Three-body subsystem (calculated earlier)

$K^- pp$ pole positions (MeV):

– 53.3 – *i* 32.4 *(1-pole phen)* , – 47.4 – *i* 24.9 *(2-pole phen)*