

Three-body correlations in mesonic-atom-like systems

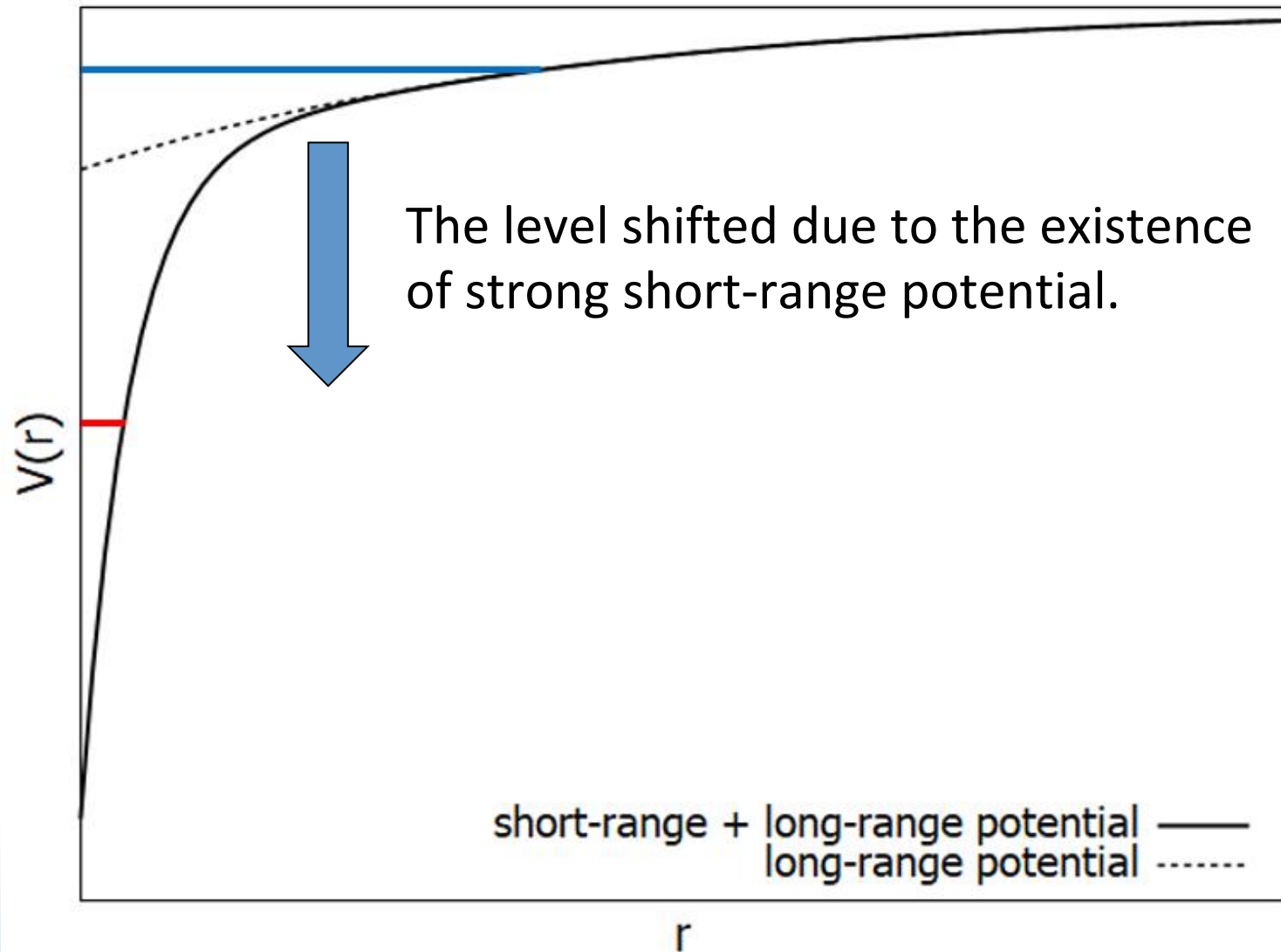
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- Particles are interacting with long- and short-range potential in mesonic atom systems.
 - $K^- p, K^- pp, K^- pn$
- Due to the existence of the short-range potential, the levels are shifted from these with only long-range potential.
 - $\bar{K}N$ interaction is strong short-range attraction[1].

Level rearrangement

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- The shift of the spectrum of the long-range potential is expressed by Deser-Trueman formula (DT) in the case of the two-body systems. [2,3]

$$\delta E^{(2)} = \frac{2\pi}{\mu} |\Psi_{LR}^{(2)}(0)| a_{SR}$$

- DT is the product of the contribution of long-range potential and short-range potential.
 - $\Psi_{LR}^{(2)}(0)$: wave function of the long-range potential
 - a_{SR} : scattering length by the short-range potential

[2] S. Deser et al., Phys. Rev. 96, 774-776 (1954)

[3] T. Trueman, Nucl. Phys. A 26, 57-67 (1961)

Deser-Trueman formula

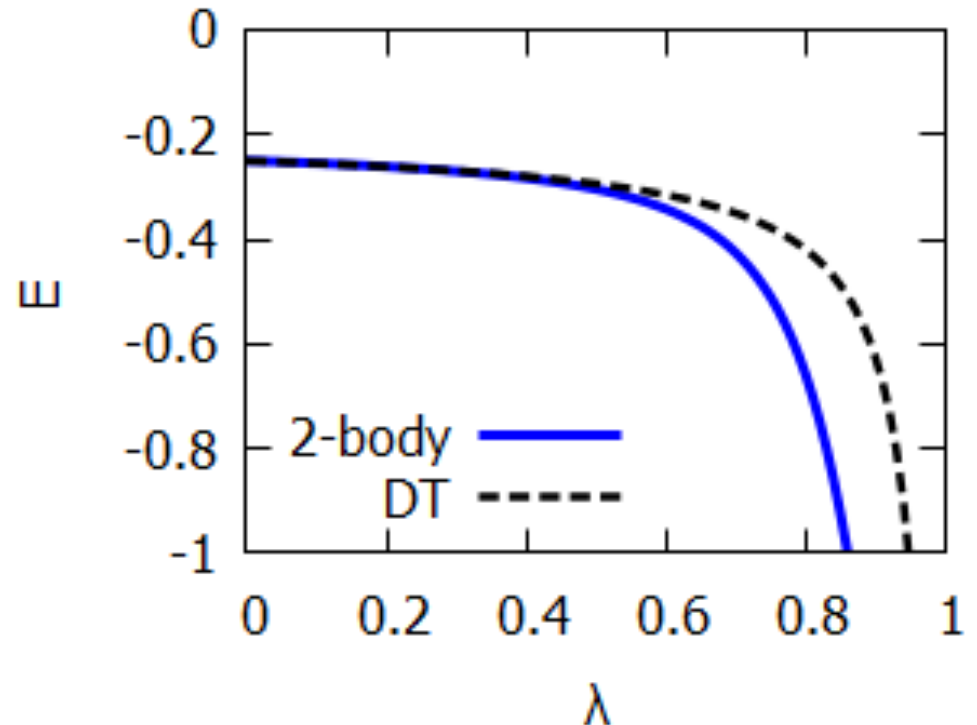
- When the short-range potential is strong enough, the DT does not reproduce the level shift.

$$H = \sum_i^2 T_i - T_{cm} + V_{12}^{LR} + \lambda V_{12}^{SR}$$

$$V_{ij}^{SR} = -C_{SR} \mu_{SR}^3 \exp(-\mu_{SR}^2 r_{ij}^2)$$

$$V_{ij}^{LR} = -\frac{\text{erf}(\mu_{LR} r_{ij})}{r_{ij}}$$

$$\mu_{SR} = \mu_{LR} = 30$$



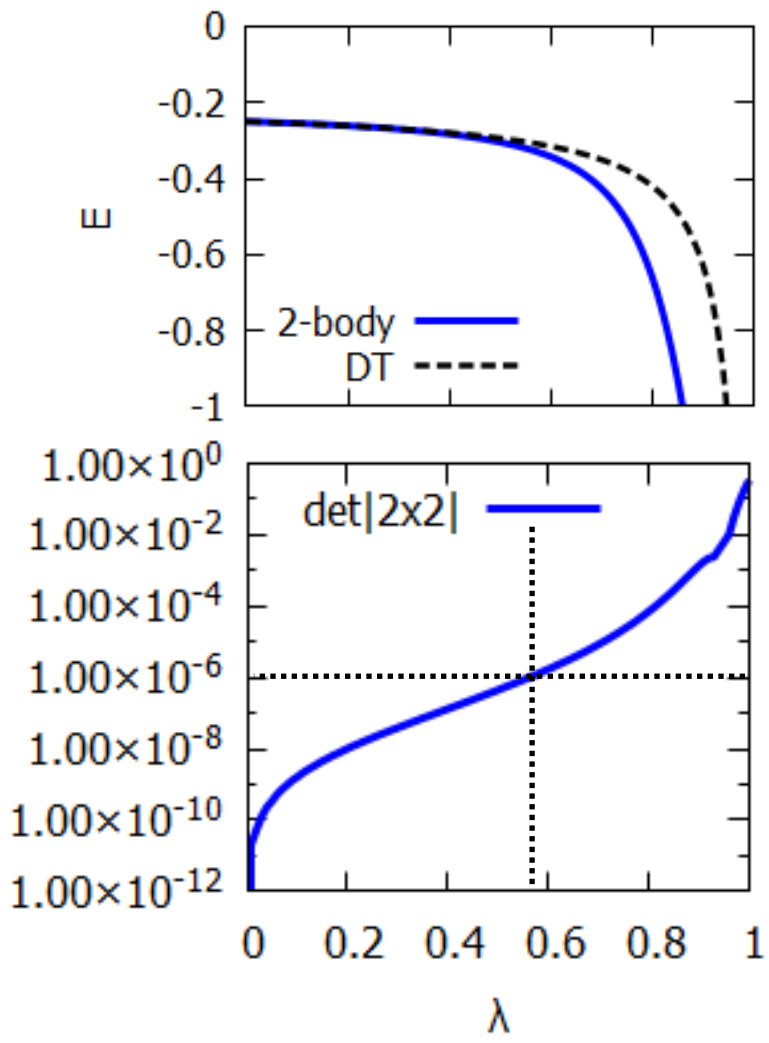
- The determinant method [4] is adopted to evaluate whether the DT works well quantitatively.

	λ_1	λ_2	λ_3	\dots
LR_I	$\delta E(LR_I, \lambda_1)$	$\delta E(LR_I, \lambda_2)$	$\delta E(LR_I, \lambda_3)$	\dots
LR_{II}	$\delta E(LR_{II}, \lambda_1)$	$\delta E(LR_{II}, \lambda_2)$	$\delta E(LR_{II}, \lambda_3)$	\dots
LR_{III}	$\delta E(LR_{III}, \lambda_1)$	$\delta E(LR_{III}, \lambda_2)$	$\delta E(LR_{III}, \lambda_3)$	\dots
\vdots	\vdots	\vdots	\vdots	

- If the energy shift is expressed by the DT, the determinant of the blued sub matrix is always 0

$$S_2 = \begin{vmatrix} \delta E(LR_I, \lambda_1) & \delta E(LR_I, \lambda_2) \\ \delta E(LR_{II}, \lambda_1) & \delta E(LR_{II}, \lambda_2) \end{vmatrix} = \begin{vmatrix} A_{LR_I} B_{SR}(\lambda_1) & A_{LR_I} B_{SR}(\lambda_2) \\ A_{LR_{II}} B_{SR}(\lambda_1) & A_{LR_{II}} B_{SR}(\lambda_2) \end{vmatrix} = 0$$

- Determinant method analysis on the two-body system.



*The DT represents the two-body correlation

*The criteria whether the determinant is zero or not depends on the calculation accuracy

$$\lambda_c \sim 0.6$$

- Extended the DT for the three-body systems[3].

$$\sum_{i>j=1}^3 \frac{2\pi}{\mu} |\Psi_{LR,ij}^{(3)}(0)|^2 a_{SR,ij}$$

- Summing up the DT for all pair in a three-body system.
 - $\Psi_{LR,ij}^{(3)}(0)$: three-body wave function of long-range potential when the two particles labeled i and j contact.
 - $a_{SR,ij}$: scattering length of i and j particles by short-range potential.
 - Including only two-body correlations
- The determinant method is still useful under the extension.

- To overview the behavior we firstly tackle to these two simple models consisting of bosons.

Model1

$$H_I = \sum_i^3 T_i - T_{cm} + \sum_{i>j=1}^3 V_{ij}^{LR} + \lambda \sum_{i>j=1}^3 V_{ij}^{SR}$$

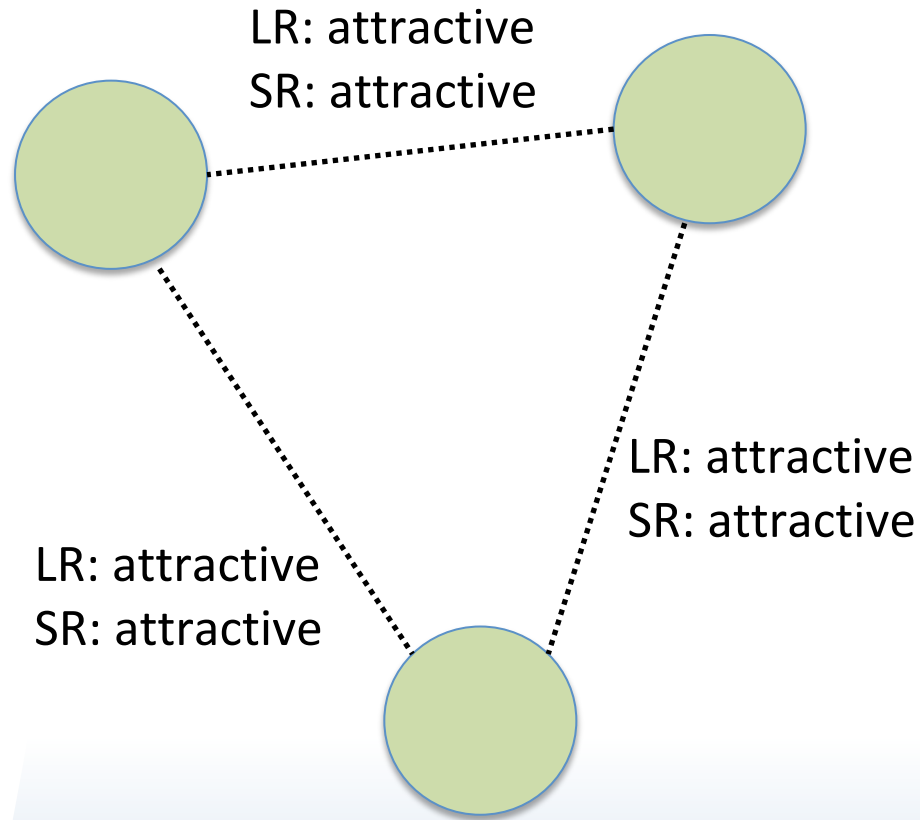
$$V_{ij}^{LR} = -\frac{\text{erf}(\mu_{LR} r_{ij})}{r_{ij}} \quad V_{ij}^{SR} = -C_{SR} \mu_{SR}^3 \exp(-\mu_{SR}^2 r_{ij}^2)$$

Model2

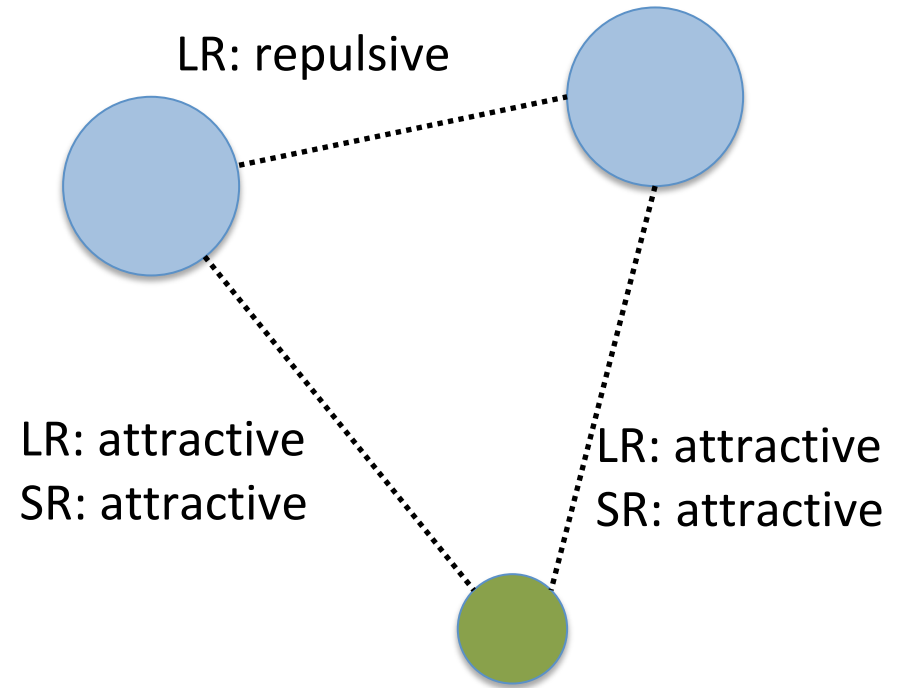
$$H_{II} = \sum_i^3 T_i - T_{cm} + \sum_{i>j=1}^3 V_{ij}^{LR} + \lambda \sum_{i=1}^2 V_{i3}^{SR}$$

$$V_{ij}^{LR} = -\frac{\text{erf}(\mu_{LR} r_{ij})}{r_{ij}} \quad V_{ij}^{SR} = -C_{SR} \mu_{SR}^3 \exp(-\mu_{SR}^2 r_{ij}^2)$$

Model1



Model2



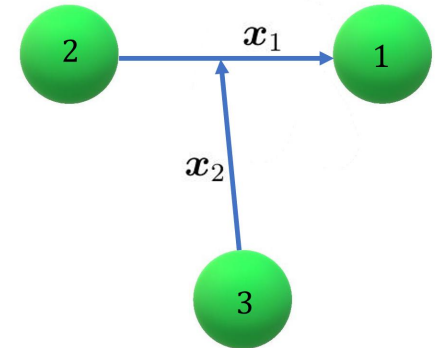
How three-body calc. is done

□ Three-body calculation is done by stochastic variation method with the correlated Gaussian[5].

– The wave function is expanded by the correlated Gaussian

$$|\Psi^{(3)}\rangle = \sum_k c_k \mathcal{S} \exp\left(-\frac{1}{2} \tilde{\mathbf{x}} A_k \mathbf{x}\right)$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} \quad \begin{aligned} \mathbf{x}_1 &= \mathbf{r}_1 - \mathbf{r}_2 \\ \mathbf{x}_2 &= \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3 \end{aligned}$$



– The variation parameters are determined to minimize the binding energy.

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

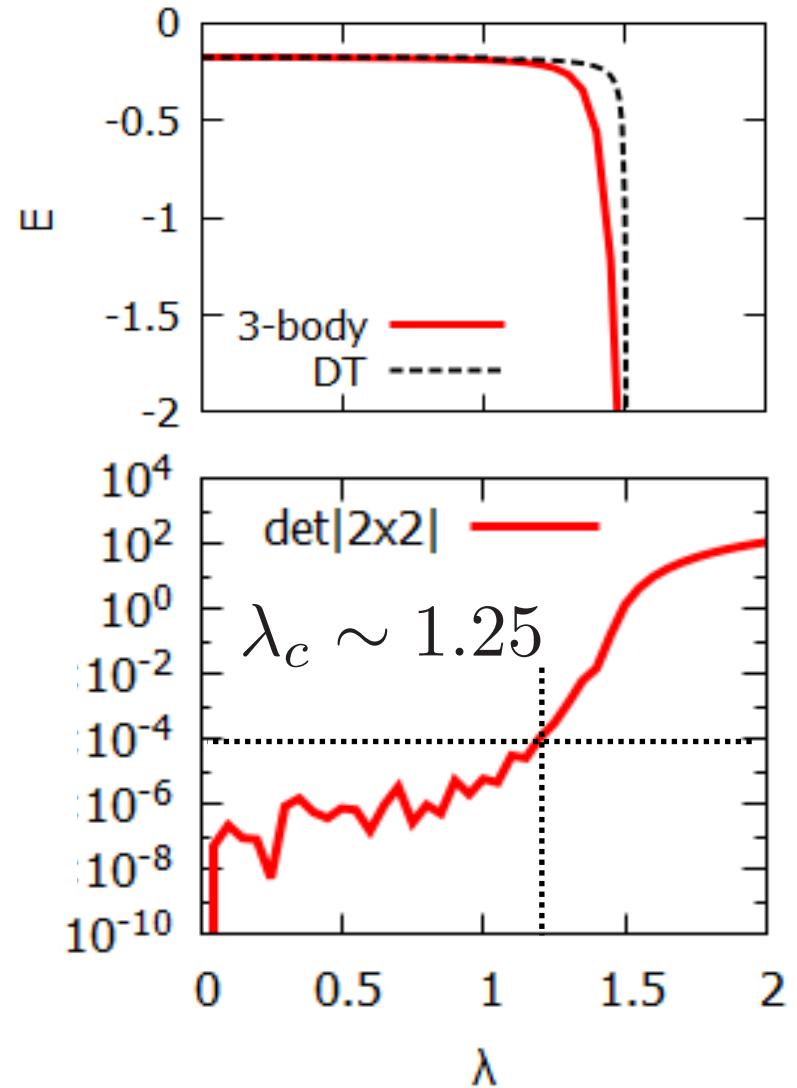
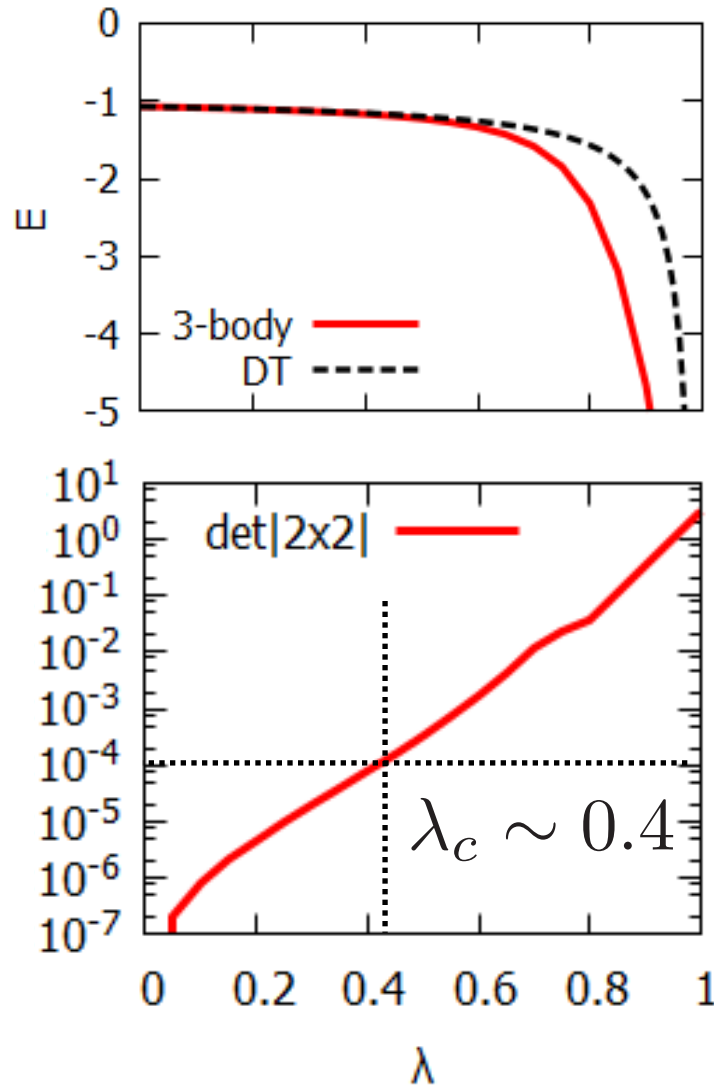
How three-body calc. is done

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- The difficulty of the calculation is that both long-range and short-range model space have to be treated simultaneously.
- The energies are converged until $\sim 10^{-4}$.

Results of three-body systems

- Level shift and determinant method of model 1,2



- The ranges of the strength of the short-range potential where Deser-Trueman formula is available are evaluated by the determinant method.
- 2body: $\lambda_c \sim 0.6$
- 3body
 - Model1 $\lambda_c \sim 0.4$
 - Model2 $\lambda_c \sim 1.25$
- The application to physical system e.g. KNN gives the prediction on to the KN interaction.