Four-Body Scale
in Universal Few-Boson Systems.

B. Bazak 1  J. Kirscher 2  S. König 3  M. Pavón Valderrama 4  
N. Barnea 1  U. van Kolck 5,6

1 The Racah Institute of Physics, The Hebrew University
2 Department of Physics and Astronomy, The University of Manchester
3 Institut fr Kernphysik, Technische Universität Darmstadt
4 Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University
5 Institut de Physique Nucléaire, Université Paris-Saclay
6 Department of Physics, University of Arizona

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The Problem
Identification of universal properties of $A$ bosons which are correlated with characteristics of $A' < A$ bosons.

“Can we understand complexity (nuclear chart, molecules) from few-body dynamics?”
The Problem
Identification of universal properties of A bosons which are correlated with characteristics of A′ < A bosons.

"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

\[ \lim_{\Lambda \rightarrow \infty} (\aleph^M_{-1}) = 0 \Rightarrow \text{scale-invariant A-boson bound states} \]

Conjecture
As soon as the \( A-1 \) boson system is constrained by more than \( A \) parameters, the A-boson system is sensitive to \( (A-1) \)-unobservable interaction details.

Leading Order:
\[ P(A + n)^{2/3} \text{-body collisions} = P(A)^{2/3} \text{-body collisions} \forall |n| \leq A \]

Next-to-leading Order (3 bodies, 3 constraints):
Two long-range constraints, one short-range constraint. (3 bodies, 3 constraints): \[ \Rightarrow \text{four-body details are resolved!} \]
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"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

\[ \lim_{\Lambda \to \infty} (\aleph M) - 1 = 0 \]
\[ \Rightarrow \text{scale-invariant } A \text{-boson bound states} \]

Conjecture
As soon as the A - 1 boson system is constrained by more than A parameters, the A-boson system is sensitive to \((A - 1)^{-}\text{unobservable interaction details}.

Leading Order:
\[ P(A + n)_{2\text{-}/3\text{-body collisions}} = P(A^2)_{2\text{-}/3\text{-body collisions}} \quad \forall|n| \leq A \]

Next-to-leading Order (3 bodies, 3 constraints):
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\[ \lim_{\Lambda \to \infty} (\aleph^M) - 1 = 0 \implies \text{scale-invariant A-boson bound states} \]

\[ \lim |r_{nm}| \to 0 \]

\[ \begin{cases} \text{Conjecture} \
\text{As soon as the A}_{A-1} \text{boson system is constrained by more than A parameters, the A-boson system is sensitive to } (A-1) \text{-unobservable interaction details.} \end{cases} \]

Leading Order:

\[ P(A+n/2-3/2 \text{-body collisions}) = P(A_2-3 \text{-body collisions}) \forall |n| \leq A \]

Next-to-leading Order (3 bodies, 3 constraints):

Two long-range constraints, one short-range constraint.

(3 bodies, 3 constraints) \implies four-body details are resolved!
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"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

\[ \lim_{\Lambda \to \infty} (\aleph^M) - 1 = 0 \] implies scale-invariant A-boson bound states

\[ \lim_{|r_{nm}| \to 0} \begin{cases} u(r_1, \ldots, r_{A-1}, A) \prod_{i < j} |r_{ij}| \forall n, m & \text{and with } 0 < |u| < \infty \text{ for any } |r_{nm}| \to 0 \text{ (no finite polynomial)} \end{cases} \]

Conjecture
As soon as the A−1 boson system is constrained by more than A parameters, the A-boson system is sensitive to \((A - 1)\)-unobservable interaction details.


Leading Order:
\[ P \left( A + n \text{ 2-/3-body collisions} \right) = P \left( A \text{ 2-/3-body collisions} \right) \forall |n| \leq A \]

\[ \mathcal{L} = \psi^+ \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^+ \psi)^2 - D(\psi^+ \psi)^3 \]
\[ \lim_{\Lambda \to \infty} (\mathcal{N} M)^{-1} = 0 \]
\[ \Rightarrow \text{ scale-invariant } A\text{-boson bound states} \]

**Leading Order:**
\[ P \left( A + n \text{ 2-/3-body collisions} \right) = P \left( A \text{ 2-/3-body collisions} \right) \quad \forall \lvert n \rvert \leq A^{2.5} \]

\[ \mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 \]

\[ \Psi = 12 \cdot Y + 6 \cdot H \xrightarrow{\text{id}} Y + H \quad (\text{S. König, accurate calibration}); \quad \Psi = \sum_n c_n \cdot e^{-n^\dagger A_n n} \quad (B. Bazak, A > 4). \]
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"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

$$\lim_{\Lambda \to \infty} (\aleph M)^{-1} = 0$$
$$\Rightarrow \text{scale-invariant } A\text{-boson bound states}$$

Leading Order:
$$\mathcal{P} \left( A + n \text{ 2-/3-body collisions} \right) = \mathcal{P} \left( A \text{ 2-/3-body collisions} \right) \quad \forall |n| \leq A$$

$$\mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3$$

Next-to-leading Order (3 bodies, 3 constraints):

*Two long-range constraints, one short-range constraint.*

\[
\mathcal{L} = \psi^\dagger \left( i \partial_0 + \frac{\nabla^2}{2m} \right) \psi - C (\psi^\dagger \psi)^2 - D (\psi^\dagger \psi)^3
\]
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\[ \lim_{\Lambda \to \infty} (\mathbb{L}^M)_{-1} = 0 \Rightarrow \text{scale-invariant } A\text{-boson bound states} \]

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As soon as the \( A-1 \) boson system is constrained by more than \( A \) parameters, the A-boson system is sensitive to \((A-1)\)-unobservable interaction details.

Leading Order:
\[ \mathcal{L}(A+n/2/3\text{-body collisions}) = \mathcal{L}(A/2/3\text{-body collisions}) \quad \forall |n| \leq A \]

Next-to-leading Order (3 bodies, 3 constraints):
Two long-range constraints, one short-range constraint.

\[ \mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 \]
\[
\mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger \psi)^2 - D(\psi^\dagger \psi)^3 - F(\psi^\dagger \psi)^4
\]
\[ \mathcal{L} = \psi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^\dagger\psi)^2 - D(\psi^\dagger\psi)^3 - F(\psi^\dagger\psi)^4 \]
The Problem
Identification of universal properties of A bosons which are correlated with characteristics of $A'$ bosons.

"Can we understand complexity (nuclear chart, molecules) from few-body dynamics?"

$$\lim_{\Lambda \to \infty} (\mathbb{A} M^{-1}) = 0 \Rightarrow$$ scale-invariant $A$-boson bound states

$$\lim_{|r_{nm}| \to 0} = \left\{ \begin{array}{ll}
\frac{1}{u(r_{12}, \ldots, r_{A-1, A})} \prod_{i < j} |r_{ij}| & \forall n, m \text{ and with } 0 < |u| < \infty \text{ for any } |r_{nm}| \to 0 \text{ (no finite polynomial)}
\end{array} \right.$$

Conjecture
As soon as the $A-1$ boson system is constrained by more than $A$ parameters, the $A$-boson system is sensitive to $(A-1)$-unobservable interaction details.

**Leading Order:**
$$P(A+n/2-/3-body\ collisions) = P(A+2-/3-body\ collisions) \forall |n| \leq A$$

Next-to-leading Order (3 bodies, 3 constraints):
Two long-range constraints, one short-range constraint.

⇒ four-body details are resolved!

$$\mathcal{L} = \psi^+ \left( i\partial_0 + \frac{\nabla^2}{2m} \right) \psi - C(\psi^+ \psi)^2 - D(\psi^+ \psi)^3 - F(\psi^+ \psi)^4$$
Conjecture

As soon as the $A - 1$ boson system is constrained by more than $A$ parameters, the $A$-boson system is sensitive to $(A - 1)$-unobservable interaction details.

\[
\lim_{|r_{nm}| \to 0} \left\{ \frac{u(r_{12}, \ldots, r_{A-1,A})}{\prod_{i<j} |r_{ij}|} \right\} = \forall n, m
\]

and with $0 < |u| < \infty$ for any $|r_{nm}| \to 0$ (no finite polynomial)
The Problem

Identification of universal properties of A bosons which are correlated with characteristics of A′ < A bosons.

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\[ \lim_{\Lambda \to \infty} (\mathbb{M} - 1) = 0 \Rightarrow \text{scale-invariant A-boson bound states} \]

\[ \lim_{|r_{nm}| \to 0} = \mu \left( r_{12}, \ldots, r_{A-1,A} \right) \prod_{i < j} |r_{ij}| \forall n, m \text{ and with } 0 < |\mu| < \infty \text{ for any } |r_{nm}| \to 0 \text{ (no finite polynomial)} \]

Conjecture

As soon as the A-1 boson system is constrained by more than A parameters, the A-boson system is sensitive to \((A-1)^{-}\text{unobservable interaction details.}\)

Leading Order:

\[ P(A + \frac{n}{2}) = P(A - \frac{n}{2}) \forall |n| \leq A \]

Next-to-leading Order (3 bodies, 3 constraints):

Two long-range constraints, one short-range constraint. (3 bodies, 3 constraints): \( \Rightarrow \) four-body details are resolved!