Electric-dipole transitions in $^6$Li with a fully microscopic six-body calculation

The 24th European Conference on Few-Body Problems in Physics
University of Surrey, Guildford, UK
2019.9.2-6

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S. Satsuka and WH, Phys. Rev. C 100, 024334 (2019), published on 29 August
Nuclear clustering and E1 transition

• Nuclear clustering play an important role in light N=Z nuclei (e.g. Ikeda diagram, Hoyle state in $^{12}$C)

• Electric-dipole (E1) transition
  – Leading order of electric-multipoles
  – Giant dipole resonance (GDR) phenomena for all nuclei
  – A probe of the structure information
  – Exploring new exotic excitation mode
    E.g. neutron rich unstable nuclei
    Vibration of valence nucleons against the core
    A variant of the macroscopic picture of giant dipole resonance (GDR)
    – Goldhaber-Teller, Steinwedel-Jensen models

K. Ikeda, N. Takigawa, H. Horiuchi, PTPS52 (1972)

T. Yamazaki et al., JHP Report (1986)
Recent photoabsorption measurement of $^6$Li


- Low-lying photoabsorption mainly occurs through E1 transition
- **A two peak structure is found**
  - Their interpretation
    - First peak: GDR of $^6$Li?
    - Second peak: GDR of alpha cluster in $^6$Li?
  - Is the interpretation is correct?
  - What is the role of the nuclear clustering in $^6$Li
    - possible to find new modes in other nuclei

Microscopic six-body calculation
Variational calculation for many-body quantum system

- Many-body wave function $\Psi$ has all information of the nucleon dynamics.
- Solve many-body Schrödinger equation
  \[ H\Psi = E\Psi \]
- Variational principle $\langle \Psi | H | \Psi \rangle = E \geq E_0$ ("Exact" energy)
  (Equal holds if $\Psi$ is the "exact" solution)

- Expand the wave function with the explicitly correlated Gaussian functions
  \[ \Psi = \sum_k c_k \exp\left\{- \sum_{i,j} \beta^{k}_{ij} (r_i - r_j)^2 \right\} \]

  - Optimal parameters $\beta^{k}_{ij}$ are selected stochastically


1. Randomly generate candidates
2. Calculate energy for each candidate
3. Select the basis which gives the lowest energy among them
4. Increase the basis size
5. Return to 1. and repeat the procedure until energy is converged

→ accurate solution can be obtained with a small basis size
Ground-state properties of $^{6}\text{Li}$

\[ H = \sum_{i=1}^{N} T_i - T_{\text{cm}} + \sum_{i<j} v_{ij} \]

Free of spurious c.m. motion

$\nu_{ij}$: Minnesota potential

Reasonable descriptions of s-shell nuclei

"$u$” ⇔ odd-wave strength

Converged only with 600 basis states

Note: 15 parameters for each basis

\[ \begin{array}{cccccccc}
 u & E_0(^6\text{Li}) & E_0(\alpha) & S_{pm} & r_m & r_p & r_n & r_{pp} & S_{ad}^2 \\
 1.00 & -34.63 & -29.94 & 4.7 & 2.20 & 2.20 & 2.20 & 3.62 & 0.856 \\
 0.93 & -33.63 & -29.90 & 3.7 & 2.33 & 2.34 & 2.33 & 3.86 & 0.869 \\
 0.87 & -32.94 & -29.87 & 3.1 & 2.45 & 2.46 & 2.45 & 4.07 & 0.882 \\
 \text{Expt.} & -31.99 & -28.30 & 3.70 & 2.452 \\
\end{array} \]
Cluster degrees of freedom

Six-body calculation for $^6\text{He}$


Distance between two protons
A measure of alpha clustering

- The ratio is unity up to 20 MeV
  Three-body model for the excited state is valid up to 20 MeV
- Large core distortion in the GDR region >30 MeV

Extension to $^6\text{Li}$ case

- Proton in valence or cluster cannot be distinguished
- Spectroscopic factors $S_{ab}^2 = |\langle \Psi^{(a)}(b)|\Psi_{JM_j}^{(6)}(E)\rangle|^2$
  A more direct measure of the nuclear clustering

Ratio of $r_{pp}$ of $^6\text{He}$ and $^4\text{He}$

Excitation energy (MeV)

Distance between two protons
A measure of alpha clustering

Extension to $^6\text{Li}$ case

- Proton in valence or cluster cannot be distinguished
- Spectroscopic factors $S_{ab}^2 = |\langle \Psi^{(a)}(b)|\Psi_{JM_j}^{(6)}(E)\rangle|^2$
  A more direct measure of the nuclear clustering
Configurations for the final state

(i) Single particle excitations

\[ M_{1\mu}(E1)\psi_i(6\text{Li}) \]

"Coherent E1 state"

Well account for the E1 sumrule

\[ 3 \times 600 \text{ basis states} \]

(ii) \( \alpha+n+n \) disintegration

\[ \psi_i(4\text{He})\chi(R, r) \]

Valence nucleon excitation

\[ 2 \times 8100 \text{ basis states} \]

(iii) \( h+t \) disintegration

\[ \psi_i(3\text{He})\psi_j(3\text{H})\chi(R) \]

GDR configuration

490 basis states

Basis functions for all subsystems are obtained by SVM

**E1 operator**

\[ M_{1\mu} = e \sum_{i \in p} (r_i - x_6)_\mu = \sqrt{\frac{4\pi}{3}} e \sum_{i \in p} \gamma_{1\mu}(r_i - x_6) \]

**Correlated Gaussian + global vector**

\[ F_{LM_L}(v, A, x) = \exp \left( -\frac{1}{2} \tilde{x} A x \right) \gamma_{LM_L}(\tilde{v} x) \]

\[ \tilde{x} A x = \sum_{i, j=1}^{N-1} A_{ij} x_i \cdot x_j \quad \tilde{v} x = (\sum_{i=1}^{N-1} v_i x_i) \]

Diagonalization with 18490 bases

- Distortion of the clusters are taken into account through their pseudo states
- \( \sim 2000 \) states found below 100 MeV
E1 transition strengths

- E1 strength distribution
  - Some prominent strengths around 12, 23, 33 MeV
- Categorize them with respect to the spectroscopic factors: α+p+n, h+t, and the others
- Various excitations appear after opening the h+t threshold

Low-lying states are dominated by the α+p+n configurations
E1 transition densities

\[ \rho_{p/n}^{tr}(r) = \sum_{i=p/n} \langle \Psi_{f}^{(6)} \| Y_{i}(r_{i} - x_{6}) \delta(|r_{i} - x_{6}| - r) \| \Psi_{0}^{(6)} \rangle, \]

\[ \langle \Psi_{f}^{(6)} \| \mathcal{M}(E1) \| \Psi_{0}^{(6)} \rangle = \epsilon \sqrt{\frac{4\pi}{3}} \int_{0}^{\infty} dr \rho_{p}^{tr}(r). \]

Goldhaber-Teller (GT) mode: out-of-phase transition between protons and neutrons
Photoabsorption of $^6$Li

First peak: GDR of $^6$Li?
Second peak: GDR of alpha cluster in $^6$Li?


Note: Only a one-peak structure is found in S. Bacca et al., PRL89, 052502 (2002)
**E1 excitations of $^6$He**


**Transition density ($x_6$: c.m of $^6$He)**

$$\rho_{p/n}^{tr}(E_v,r) = \langle \Psi_1(E_v) \| \sum_{i=p/n} \mathcal{Y}_1(r_i - x_6) \times \delta(|r_i - x_6| - r) \| \Psi_0 \rangle.$$  

**E1 matrix element**

$$\langle \Psi_1(E_v) \| \mathcal{M}_1 \| \Psi_0 \rangle = e \sqrt{\frac{4\pi}{3}} \int_0^\infty \rho_{p/n}^{tr}(E_v,r)dr.$$  

**Graphs**

- (a) 3.0 MeV
- (b) 7.2 MeV
- (c) 17.7 MeV
- (d) 32.9 MeV

**Figures**

- "Soft" dipole mode
- Giant dipole mode
Summary

- Electric-dipole transitions in $^6$Li with a fully microscopic six-body calculations
  - Explicitly correlated basis approach
    - Distortion of clusters are taken into account
    - Emergence of nuclear clustering

- Nuclear clustering plays an important role in the excitation of light nuclei
  - Various excitation modes appear with increasing the excitation energy following the threshold rule
    - Soft GT dipole mode (4+1+1 cluster), 3+3 cluster, giant dipole excitation modes
    - Exploring soft GT dipole and other cluster excitations (e.g. $^7$Li, $^9$Be, $^{18}$F, $^{20}$Ne) are interesting