UNIVERSITY OF TRENTO - Italy

Trento Institute for Fundamental Physics and Applications

Beryllium-9 in Cluster Effective Field Theory

E. Filandri[1,2]

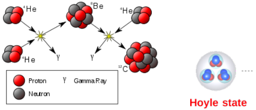
P. Andreatta[1,2], C.A. Manzata[1], C. Ji[3], W. Leidemann[1,2], G. Orlandini[1,2]

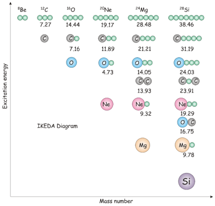
[1]University of Trento, 38123 Trento, Italy
[2]INFN-TIFPA Trento Institute of Fundamental Physics and Applications, Via Sommarive, 14, 38123 Povo-Trento, Italy
[3]Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China

Summary

Motivations

- Experimental evidence for the cluster structure of light nuclei is well documented [Freer, 2007 and references therein]
  - α decay in ^{8}Be
  - the Hoyle state in ^{12}C
  - Observations of many systems predicted in the Ikeda diagram
  - α-cluster structure in ^{56}Ni [H Akimune et al, 2013]
  - α-cluster structure in the ground state of ^{40}Ca displayed in a (p,pα) knockout reaction [A.A Cowley, 2013]





- Effective field theories (EFTs) provides a controlled framework to exploit the separation of scales in nuclei, by now mainly few-body system have been studied within the EFTs and much success have been achieved [P.F Bedaque, U. van Kolck, 2002][E.Braaten, H-W Hammer, 2006]

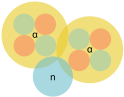
Our purpose is to describe halo or cluster nuclei (e.g.^{6}He, ^{12}C,^{9}Be, . . .) and some reactions of astrophysical interest, specializing in very low energies where clusters of nucleon behave coherently

- To describe such cluster nuclei we use an EFT formulated with contact interactions among nucleon and alpha-particles

Cluster Effective Field Theory

Many nuclei have the probability distribution of the valence neutrons that extend beyond the core (halo nuclei), others some parts of the system which can be seen as separated subsystems (Borromean nuclei).

These cluster nuclei can be well described by an EFT. We focus on the Borromean system provided by the nucleus of ^{9}Be



The energy needed in order to separate the system into the three effective degrees of freedom is:

$$B(^9\text{Be}) = BE(^9\text{Be}) - 2BE(\alpha) = 1.572\text{MeV}$$

and the break-up threshold of ^{4}He into $^3\text{H} + p$:

$$S_p(^4\text{He}) = 19.813\text{MeV}$$

$\Rightarrow$ Separation of scales, needed for an EFT approach.
The two types of subsystems are the $\alpha\alpha$ pair and the αn one. The $\alpha\alpha$ interaction is dominated by the ^1S virtual state, while the αn interaction has a resonance in the $^2P_{\frac{3}{2}}$ one at low energies.

Power Counting

αn

From a physical interpretation one would expect that the two scales are given by

$$M_{lo} = \sqrt{2\mu_{\alpha n} Q_{\alpha decay}(^5\text{He})} \approx 30\text{MeV} \qquad M_{hi} = \sqrt{2\mu_{\alpha n} S_p(^4\text{He})} \approx 140\text{MeV}$$

We adopt the following power counting [Bedaque et. al, 2003]

$$\frac{1}{a_1} \sim M_{lo}^2 M_{hi} \quad and \quad r_1 \sim M_{hi}$$

a_1 being the scattering length and r_1 the effective range.
Using experimental values for a_1 and r_1:

$$M_{lo} \approx 50\text{MeV} \qquad M_{hi} \approx 170\text{MeV}$$

$\alpha\alpha$

Here we have three different scales of interest:

$$M_{lo} = \sqrt{2\mu_{\alpha\alpha} Q_{\alpha decay}(^8\text{Be})} \approx 20\text{MeV} \qquad M_{hi} = \sqrt{2\mu_{\alpha\alpha} S_p(^4\text{He})} \approx 260\text{MeV} \quad \text{and} \quad k_C = 4\alpha\mu_{\alpha\alpha}.$$

With the following power counting

$$a_0 \sim \frac{M_{hi}^2}{M_{lo}^3} \quad and \quad r_0 \sim \frac{1}{3k_C} \sim \frac{1}{M_{hi}}$$

the known energy resonance position and width are reproduced and using again the experimental values we get:

$$M_{lo} \approx 20\text{MeV} \qquad M_{hi} \approx 170\text{MeV}$$

EFT expansion validity

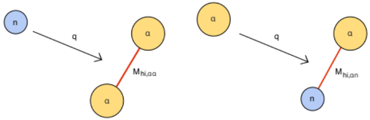
In the αn case the $\frac{M_{lo}}{M_{hi}}$ expansion is up to NLO with an error of order:

$$O\left(\frac{M_{lo,\alpha n}}{M_{hi,\alpha n}}\right) \sim 0.3$$

In the $\alpha\alpha$ expansion we consider the terms up to NLO:

$$O\left(\frac{M_{lo,\alpha\alpha}}{M_{hi,\alpha\alpha}}\right) \sim 0.1$$

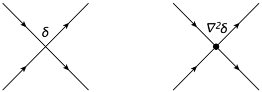
- In order to evaluate the range of validity of our EFT, we should also take a look to the breakdown scale of our system
  - Since we have two two-body subsystems, we have also two different high momentum scales
    $\Rightarrow$ For the three-body problem we have to take the strictest constraint



$$M_{hi} = Min\{M_{hi,\alpha n}, M_{hi,\alpha\alpha}\}$$

The Potentials

The short-range interaction between two particles can be expanded in a series of a contact term and its derivatives



In our particular case we describe the interaction of the two couples, αn and $\alpha\alpha$, with potentials of the form [P.Recchia, 2015]:

$$\langle \boldsymbol{x}|V|\boldsymbol{x}'\rangle = \left(\lambda_0 + \lambda_1(\nabla^2 + \nabla'^2)\right)\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(\boldsymbol{x})$$

In momentum space:

$$V(\boldsymbol{p},\boldsymbol{p}') = \lambda_0 + \lambda_1(p^2 + p'^2) = \sum_{i,j}^{1} p^{2i}\lambda_{ij}p'^{2j}$$

where $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the relative momenta and:

$$\lambda = \begin{pmatrix} \lambda_0 & \lambda_1 \\ \lambda_1 & 0 \end{pmatrix} \qquad \lambda_0, \lambda_1 = \text{coefficients to be determined}$$

In general we can expand a potential in partial-wave components

$$V(\boldsymbol{p},\boldsymbol{p}') = p^l p'^l g(p) g(p') \sum_{ij=0}^{1} p^{2i} \lambda_{ij} p'^{2j} (2l+1) P_l(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{p}'})$$

P_l= Legendre polynomial and $g(p)$ regulates the short-distance dependence of the interaction:

$$g(p=0) = 1 \quad and \quad g(p \to \infty) = 0$$

- i and j could be larger than 1$\Rightarrow$ we limit them in order to get a phase shift expansion up to the effective range order
- Partial wave expansion $\Rightarrow$ in the ^{9}Be problem the two interactions have both a dominant wave ($l=0$ for $\alpha\alpha$, $l=1$ for αn)
- In the $\alpha\alpha$ case we have also a long range Coulomb potential V_C
- Our aim is to find an explicit expression for the coefficients λ_0 and λ_1 in terms of the scattering length, on the effective range and with a dependence on a cutoff Λ
- Λ is necessary in our model to take care of the ultraviolet divergences
- We choose the cutoff regularization$\Rightarrow$ it reproduce known features (e.g the negative sign of the coefficients in the effective range expansion beyond the scattering length)
- We take

$$g(p) = \exp^{-(\frac{p}{\Lambda})^2}$$

λ_0, λ_1 coefficient & Wigner Bound

The coefficients for the potential are found from the Lippman-Schwinger equation:

αn

$$T(\boldsymbol{p},\boldsymbol{p}') = V(\boldsymbol{p},\boldsymbol{p}') + \int \frac{d\boldsymbol{q}}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{q}) \frac{1}{E - \frac{q^2}{2\mu_{\alpha n}} + i\epsilon} T(\boldsymbol{q},\boldsymbol{p}')$$

$\alpha\alpha$

$$T_{SC}(\boldsymbol{p},\boldsymbol{p}') = \langle \Psi_{\boldsymbol{p}}^{(-)} | V_S(\boldsymbol{p},\boldsymbol{p}') | \Psi_{\boldsymbol{p}}^{(+)} \rangle - \int \frac{dp''}{(2\pi)^3} \langle \Psi_{\boldsymbol{p}'}^{(-)} | V_S G_C^{(+)} | \Psi_{\boldsymbol{p}''}^{(+)} \rangle \frac{1}{E - \frac{q^2}{2\mu_{\alpha n}} + i\epsilon} T(\boldsymbol{p},\boldsymbol{p}'')$$

$$\Psi_{\boldsymbol{p}}^{(\pm)} \rangle = 1 + G_C^{\pm} |\boldsymbol{p}\rangle \,, \quad G_C^{\pm} = \text{the retarded/advanced Coulomb Green's function}$$

- Putting the partial wave decomposition into the Lippman-Schwinger equations we can extract the coefficient comparing the on shell T-matrix with the effective range expansion up to k^2 order
  - We get two different solutions: one with positive λ_0, the other with negative

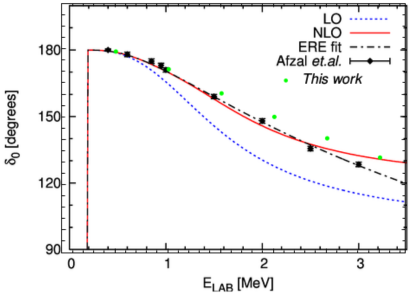
The presence of a Wigner bound [E.P. Wigner,1955]

$$r \leq 2[R - \frac{R^2}{a} + \frac{R^3}{3a^2}]$$

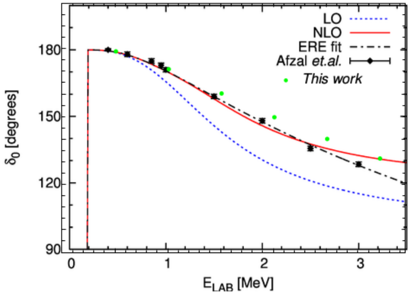
r = effective range parameter , a = scattering lenght , R = interaction rate

$$\text{limit the cutoff } \Lambda \Rightarrow \Lambda^{\text{MAX}}_{\alpha n} = 340\text{MeV} \qquad \Lambda^{\text{MAX}}_{\alpha\alpha} = 230\text{MeV}$$

Negative λ_0



Positive λ_0



[C. Ji]

- Choosing the regulator $g(p) = \exp^{-(\frac{p}{\Lambda})^2}$ we have a good agreement with experimental data

Nonsymmetrized Hyperspherical Harmonics

- The aim is to calculate the ground-state energy of the ^{9}Be by diagonalizing the Hamiltonian on a proper basis
  - We work in the momentum space and we use a Nonsymmetrized Hyperspherical Harmonics basis (NSHH)$\Rightarrow$ similar to NSHH in coordinate space[M.Gattobigio, et. al, 2011] [S.Deflorian, et. al., 2013]
  - We extract the states with a chosen symmetry using the Casimir operator for a set of N_s elements

$$\hat{C}(N_1, \cdots, N_n) = \sum_{s=1}^{n} b_{\Lambda_s} \hat{C}_s(N_s) \; ; \qquad \hat{C}_s = \sum_{j>i}^{N_s} \hat{P}_{ij}$$

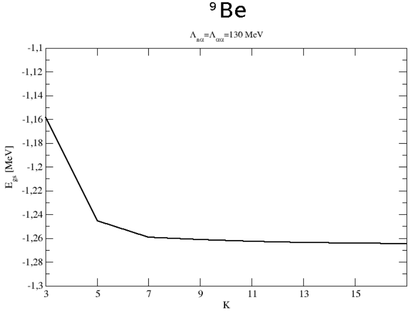
$$b_{\Lambda_s} = \begin{cases} 1 & \Lambda_s = A, M \\ -1 & \Lambda_s = S \end{cases} \qquad \hat{P}_{ij} = \text{permutation operator}$$

The $\hat{C}(N_1, \cdots, N_n)$ operator commutes with the Hamiltonian, diagonalizing the matrix $\tilde{H} = \hat{H} + \gamma \hat{C}_s$:

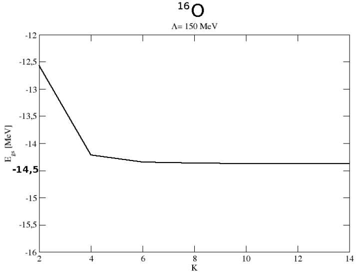
$$\tilde{E}_{k,\Lambda} = E_{k,\Lambda} + \gamma \sum_{s=1}^{n} b_s \lambda_{\Lambda_s} \quad (k = 0, 1, 2, \cdots, N_{max}(\Lambda)) \qquad \gamma > \frac{|E_{min}|}{\sum_{s=1}^{n} N_s}$$

- The employment of the NSHH basis avoids the explicit symmetrization procedure
- Its extra flexibility allows to pass from one physical model to another

Convergence of ground state energy



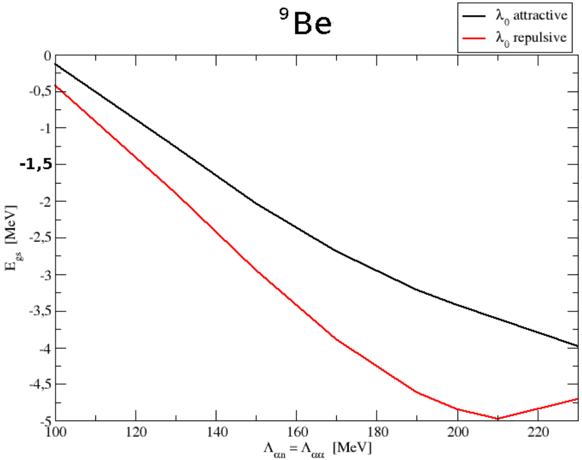
K	E_{gs}[MeV]
3	-1.1582708
5	-1.2453006
7	-1.2593364
9	-1.2611099
11	-1.2624669
13	-1.2634732
15	-1.2641629
17	-1.2646203



- Rapid convergence increasing the number of basis radial functions

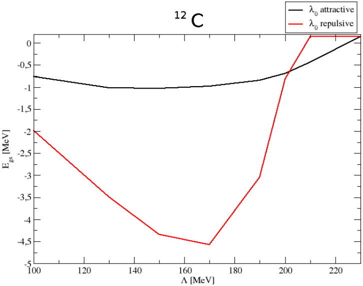
K	E_{gs}[MeV]
2	-12.569698
4	-14.215357
6	-14.345383
8	-14.364673
10	-14.370449
12	-14.375405
14	-14.377645

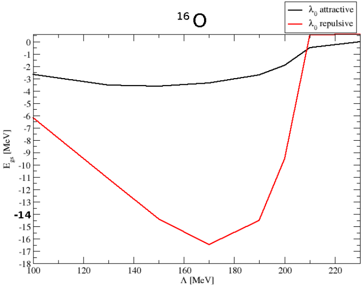
Dependence on the cutoff

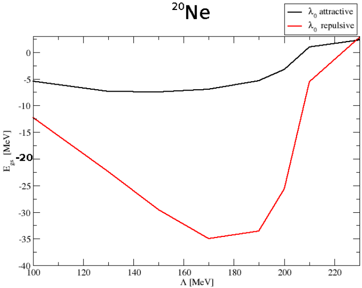


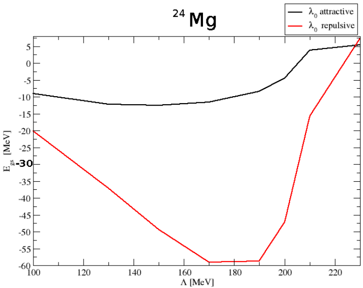
- To eliminate cutoff dependence a 3-body force is required

Other examples









Conclusion & Future perspectives

- In our work we have studied ^{9}Be ground state with non-local αn and $\alpha\alpha$ potentials derived from Cluster EFT
  - The potentials are regularized by a Gaussian cutoff and the potential parameters are fitted to reproduce the scattering parameters in the calculated T-matrix
  - We implemented such non-local potential models in a NSHH code in momentum space
- For a selected cutoff value and a solution of λ_0, λ_1 we are able to reproduce the experimental value of ground state energy for most of the studied nuclei
  - The strong cutoff dependence and the case of ^{12}C indicate something missed in the description

Work is in progress for $\cdots$

- Inclusion of three body force
- Calculation of the cross-section of ^{9}Be photodisintegration:

$$^9\text{Be} + \gamma \rightarrow \alpha + \alpha + n$$

Inverse reaction is important for the formation of ^{12}C in supernovae events as an alternative to triple alpha process

  - The prediction of the photodisintegration will be realized with the LIT method [V.D Efros, W. Leidemann and G. Orlandini, 1994]

Thank you for your attention