Admittedly the exponential factor is introduced to deal with infrared divergences with the Coulomb gauge (CG), where the photon field $A_\mu$ becomes transverse. It is proved, that in the CG-the interaction part $K_{\alpha\beta}(p)$ is determined as

$$K_{\alpha\beta}(p) = K_{\alpha\beta}^0(p) / \exp\left(\frac{ip\cdot k_{\text{ren}}}{\pi}\right),$$

where separate contributions in the CG are responsible for the different physical processes in the system of interacting photons and leptons.

2 Analytical expressions

A distinctive feature of our approach is that all expressions in the r.h.s. (5) are obtained simultaneously with mass and vortex renormalizations from the commutator of $\gamma_5$ (3) with the generator of the first unitary clothing operator (1). In particular, we present the interaction operator between clothes and photons

$$K_{\alpha\beta}(p) = \sum_{\gamma} \int d\Gamma \left(\gamma_5(p\gamma_\alpha p_\beta) + \gamma_5(p\gamma_\beta p_\alpha)\right)\Psi_\gamma(p,\sigma),$$

with

$$\Psi_\gamma(p,\sigma) = \frac{e^{iK\cdot p}}{p^2 + m^2 + i\epsilon} \left(\gamma_5(p\gamma_\alpha p_\beta) + \gamma_5(p\gamma_\beta p_\alpha)\right)\Psi(\gamma_\alpha\beta).$$

4 The partial eigenvalue equation for para-photons

In this context, we define the partial eigenvalue equation for the para-photons $\Psi^\dagger(\gamma_\alpha\beta)$ which belong to the total angular momentum $J$, viz.,

$$\gamma_5(p^\dagger |A_{\text{fermion-like}} + \bar{U}(\text{off-energy-shell})|^p\sigma),$$

Such a separation implies that only the Fermi-like part survives on the energy-shell, i.e., $p^\dagger = p^\dagger_{\text{fermion-like}}$. For the off-energy-shell part, the task of solving the eigenvalue equation and obtaining the corresponding eigenstates in the CPR is written (see Appendix C in [9]).

5 Decay rate for $\gamma^\dagger p^\dagger \to \gamma^\dagger p$ 

The postion-1 decay in two photons has considered. The corresponding decay rate is given by (see formula (3.197) in [9])

$$\Gamma = \sum_{\gamma} \int_d\frac{d^4k}{k^2} \left|\gamma_5(p_m\gamma_\alpha p_{\text{eigen}} - \gamma_5(p_m\gamma_\beta p_{\text{eigen}})\right|^2, $$

where $\gamma$ is the creation operator for the photon. Similarly to (8), we separate off-energy-shell part which goes to zero if energy conservation law satisfied.

$$\gamma_5(p_m\gamma_\alpha p_{\text{eigen}} - \gamma_5(p_m\gamma_\beta p_{\text{eigen}}) = \frac{e^{iK\cdot p}}{p^2 + m^2 + i\epsilon} \left(\gamma_5(p\gamma_\alpha p_\beta) + \gamma_5(p\gamma_\beta p_\alpha)\right)\Psi(\gamma_\alpha\beta).$$

The postion-1 decay in two photons. The fermion-like part is considered to be the dominant contribution.

$$\gamma_5(p_m\gamma_\alpha p_{\text{eigen}} - \gamma_5(p_m\gamma_\beta p_{\text{eigen}}) = \frac{e^{iK\cdot p}}{p^2 + m^2 + i\epsilon} \left(\gamma_5(p\gamma_\alpha p_\beta) + \gamma_5(p\gamma_\beta p_\alpha)\right)\Psi(\gamma_\alpha\beta).$$