An investigation for the appearance of long range nuclear potential on the ultra low energy nuclear synthesis

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Motivation & Starting Point

Phys. Rev. C86 , 044001 (2012),
Few−Body Syst.59, 51 (2018).

The author proposed a Long Range (GPT) Potential which appears,
not only at the 3-body Break-up Threshold (3BT),
but also at the Quasi 2-body Threshold (Q2T).

The GPT Potential represents
1) a Yukawa-type potential for shorter range,
2) but a $1=r^n$ -type potential for longer range.

Energy dependent quasi 2-body AGS-Born (E2Q)

$$Z_{\alpha\beta}(q,q';E) = \frac{g_\alpha(p)g_\beta(p')(1-\delta_{\alpha\beta})}{(E+\varepsilon_B)-q^2/2\mu} \rightarrow \infty$$

at (Q2T): $p = p' = 0,\ \ E_{cm} = (E+\varepsilon_B) = 0,\ \ q = 0$

For $E_{cm} < 0$: with $\sigma^2 = 2\mu|E_{cm}|,\ \ 0 < C_{\alpha\beta}$

$$Z_{\alpha\beta} \rightarrow \frac{g_\alpha(0)g_\beta(0)(1-\delta_{\alpha\beta})}{-|E_{cm}|-q^2/2\mu} = -\frac{C_{\alpha\beta}}{q^2+\sigma^2}$$

Therefore, the Fourier transformation becomes

$$\mathrm{F}\left[\{Z_{\alpha\beta}(q,q';E)\ \right] = V_0 \frac{e^{-\sigma r}}{r} : \qquad V_0 < 0$$

To avoid the divergence at Q2T ,

Adopt a statistical average with a weight : P

$$P = \frac{\sigma^{2\gamma+1} e^{-a\sigma}}{\rho}$$

$$\rho = \int_0^\infty \sigma^{2\gamma+1} e^{-a\sigma} d\sigma = \frac{\Gamma(2\gamma+2)}{a^{2\gamma+2}}$$

$$L\left\{U^{(0)}(\Delta,\sigma;r)\right\} \equiv V_0 \frac{1}{\rho} \int_0^\infty \sigma^{2\gamma+1} e^{-a\sigma} \frac{e^{-\sigma r/2}}{r} d\sigma$$

$$= V_0 \frac{a^{2\gamma+2}}{r(r/2+a)^{2\gamma+2}}$$

GPT-potential: below the Q2T, or 3BT (E_{cm}<0) with parametes a and γ, where $V_0(<0)$ is a potential depth.

$$V_0 \frac{a^{2\gamma+2}}{r(r/2+a)^{2\gamma+2}}$$

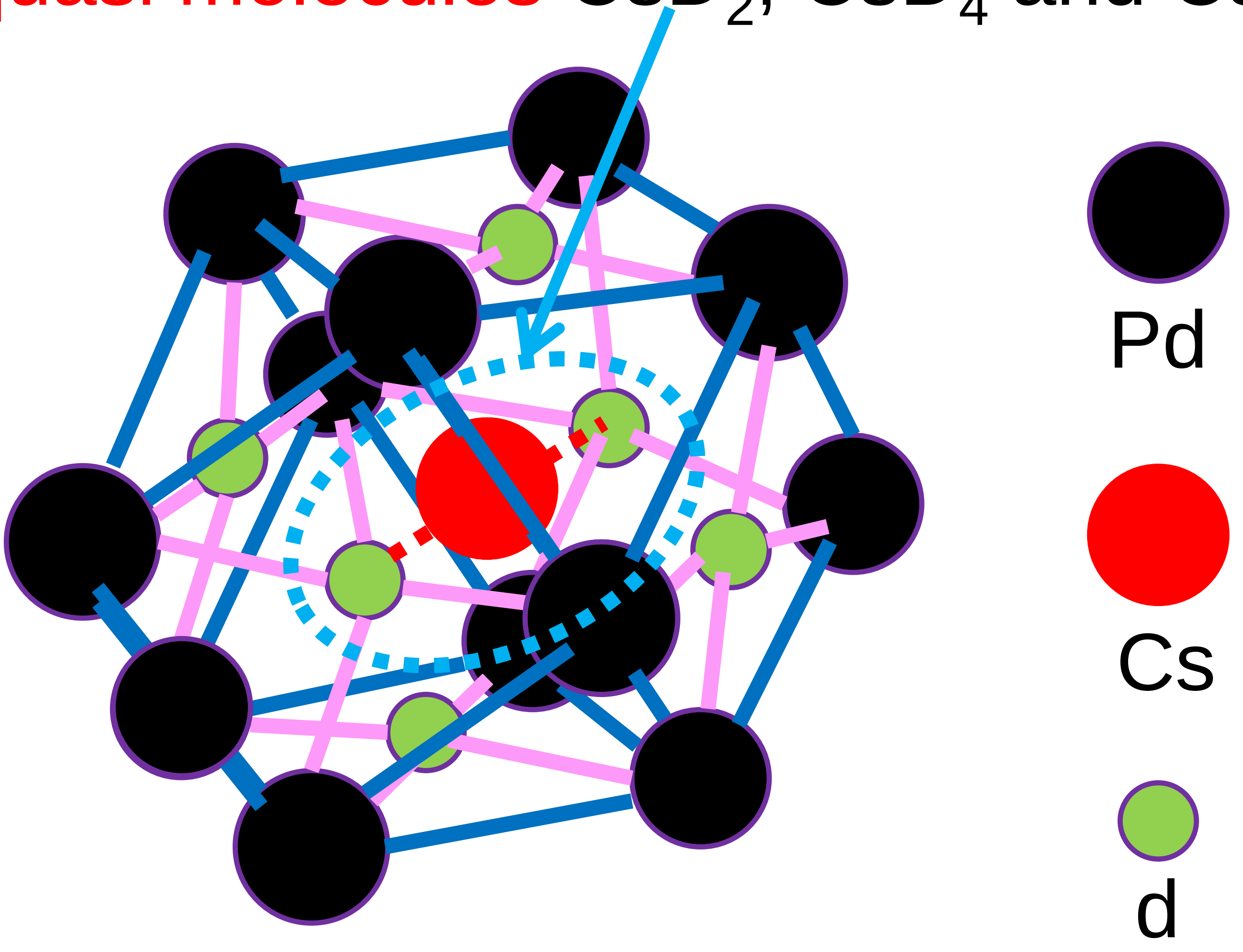
γ	$r \ll a$	GPT-potential	$a \ll r$
-1	V_0/r	V_0/r	V_0/r
$-1/2$	$V_0 e^{-r/2a}/r$	$V_0(2a)/\left[r(r+2a)\right]$	$V_0(2a)/r^2$
0	$V_0 e^{-2r/2a}/r$	$V_0(2a)^2/\left[r(r+2a)^2\right]$	$V_0(2a)^2/r^3$
$1/2$	$V_0 e^{-3r/2a}/r$	$V_0(2a)^3/\left[r(r+2a)^3\right]$	$V_0(2a)^3/r^4$
1	$V_0 e^{-4r/2a}/r$	$V_0(2a)^4/\left[r(r+2a)^4\right]$	$V_0(2a)^4/r^5$
$3/2$	$V_0 e^{-5r/2a}/r$	$V_0(2a)^5/\left[r(r+2a)^5\right]$	$V_0(2a)^5/r^6$
2	$V_0 e^{-6r/2a}/r$	$V_0(2a)^6/\left[r(r+2a)^6\right]$	$V_0(2a)^6/r^7$

The GPT-potential includes automatically
the Efimov-like potential in any systems.

In order to confirm the GPT potential,
we investigate ultra low energy reactions
$^{135}Cs(2d,\gamma)^{139}La$; $^{137}Cs(2d,\gamma)^{141}La$

by the three-ion quasi-molecule CsD_2
in the Pd-cluster with a form: CsD_2Pd_{12}.

Define quasi molecules CsD_2, CsD_4 and CsD_6



$Cs\text{-}d_6\text{-}Pd_{12}$

Cub-octahedron

Our calculation:

$^{137}_{55}Cs(7/2^+:30.07y)+2d \rightarrow \, ^{141}_{57}La(7/2^+:3.2h)+\beta^-$
$\rightarrow \, ^{141}_{58}Ce(7/2^-:32.5d)+\beta^- \rightarrow \, ^{141}_{59}Pr(5/2^+:stable)$

cerium

$^{135}_{55}Cs(7/2^+:2.3x10^6y)+2d \rightarrow \, ^{139}_{57}La(7/2^+:stable)$

$^{135}_{55}Cs(7/2^+: 2.3x10^6y)+4d \rightarrow \, ^{143}_{59}Pr(5/2^+:13.57d)+\beta^-$
$\rightarrow \, ^{143}_{60}Nd(7/2^-:stable)$

neodymium

Calculate D-Cs-D (d-Cs-d) three-body bound states and wave functions.

$Cs\text{-}D_6\text{-}Pd_{12}$ Cub-octahedron

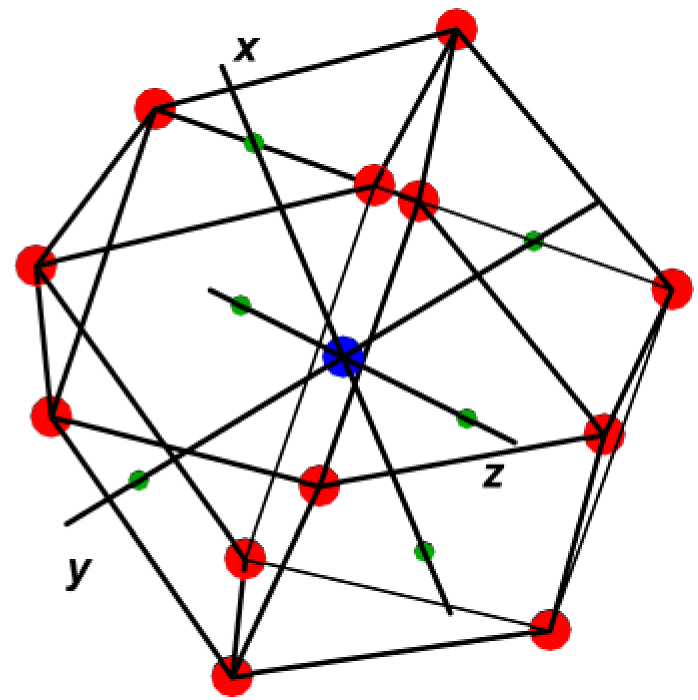
Number of surface :
8-regular triangles + 6-regular squares
=14 surfaces

24-sides
12-apexes

Surface area $S = (6+2\sqrt{3})a^2$

Volume $V = \frac{5\sqrt{2}}{3}a^3$

Radius of the circumscribed sphere a



Pd ● Cs ● D ●

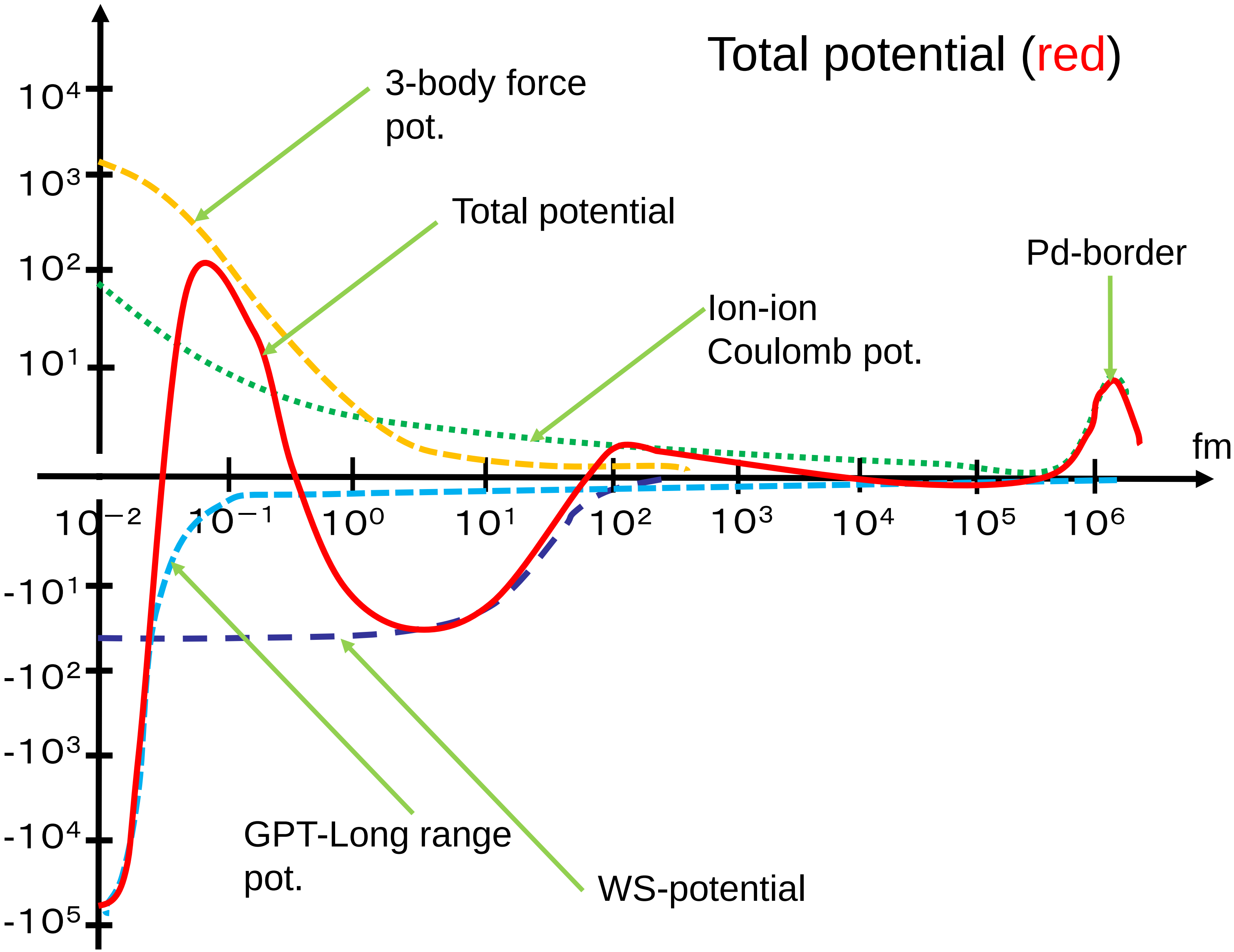
D exists inside of the surface

Pd − Pd distance $R_{Pd} = a = 5.2\,\mathrm{au} \approx 2.75172144 \times 10^5\ \mathrm{fm}$

D − Cs distance $R_D = 3.1\,\mathrm{au} \approx 1.64044932 \times 10^5\ \mathrm{fm}$

$1\,\mathrm{au} = 0.591772\,\mathrm{\mathring{A}}$

Total potential (red)



Potentials in the three-body system

1) Nuclear potential:

$$V_W^{N_iN_j}(r_{ij}) = \frac{V_{W0}^{N_iN_j}}{1+\exp\left(\frac{r_{ij}-R_W^{N_iN_j}}{a_W^{N_iN_j}}\right)}:$$

WS – potential

$$V_{W0}^{Csd} = -79.30\,\mathrm{MeV},\ \ V_{W0}^{dd} = -27.57\,\mathrm{MeV},\ R_W^{Csd} = 10.21\,\mathrm{fm},$$

$$R_W^{dd} = 1.49\,\mathrm{fm},\quad a_W^{Csd} = 0.4\,\mathrm{fm},\quad a_W^{dd} = 0.3\,\mathrm{fm},$$

2) Coulomb potential:

$$V_c^{N_iN_j}(r_{ij}) = \begin{cases} \frac{Z_iZ_je^2}{8\pi R}\left[3-\left(\frac{r_{ij}}{R_c^{N_iN_j}}\right)^2\right] & \text{for}\quad r \le R \\ \frac{Z_iZ_je^2}{8\pi r_{ij}} & \text{for}\quad R \le r \end{cases}$$

$$R_c^{CsH} = 10.21\,\mathrm{fm},\ \ R_c^{H_1H_2} = 10.21\,\mathrm{fm}.$$

3) Pd-N$_i$ potential: one-body potential

$$V_c^{\mathrm{PdN}_i}(r_i) = V_{c0}^{\mathrm{Pd}} \left(\frac{r_i}{a_c^{\mathrm{Pd}}} \right)^{10} \exp\left\{ -\left(\frac{r_i - a_c^{\mathrm{Pd}}}{b_c^{\mathrm{Pd}}} \right)^2 \right\}$$

$$V_{c0}^{\mathrm{Pd}} = 1.0 \times 10^{-4}\ \mathrm{MeV},\ \ a_{c0}^{\mathrm{Pd}} = 5.0 \times 10^5\ \mathrm{fm},$$

$$b_{c0}^{\mathrm{Pd}} = 3.1623 \times 10^5\ \mathrm{fm}.$$

Pd position 1.57×10^6 fm, 2.73 MeV height.

4) Three-cluster potential:

$$V_t(r_1, r_2, r_3) = V_{t0} \exp\left[-\left(\frac{r_{12}}{a_t} \right)^2 - \left(\frac{r_{23}}{a_t} \right)^2 - \left(\frac{r_{31}}{a_t} \right)^2 \right]$$

$$V_{t0} = 1800\ \mathrm{MeV},\ \ a_t = 3.0\ \mathrm{fm}.$$

5) Long range potential:

$$V_e(r_1, r_2, r_3) = \frac{V_{e0}}{\left(\frac{r_{12}}{a_e}\right)^l + \left(\frac{r_{23}}{a_e}\right)^m + \left(\frac{r_{31}}{a_e}\right)^n + 1}$$

Our 3-body Long Range Potential

take $l = m = n = 2$ $(Efimov\ case)$.

$$V_e(r_1, r_2, r_3) = \frac{V_{e0}}{\left(\frac{r_{12}}{a_e}\right)^2 + \left(\frac{r_{23}}{a_e}\right)^2 + \left(\frac{r_{31}}{a_e}\right)^2 + 1} = \frac{V_{e0} a_e^{\,2}}{r_{12}^{\,2} + r_{23}^{\,2} + r_{31}^{\,2} + a_e^{\,2}}$$

$$V_{e0} = -80000\ \text{MeV}, \quad a_e = 500\,\text{fm}.$$

$$V_e(r_1, r_2, r_3) = \frac{V_{e0} a_e^{\,2}}{r_{12}^{\,2} + r_{23}^{\,2} + r_{31}^{\,2} + a_e^{\,2}}$$

Lorentz-type

$$V_{e0} = -80000\ \text{MeV}, \quad a_e = 5000\,\text{fm}.$$

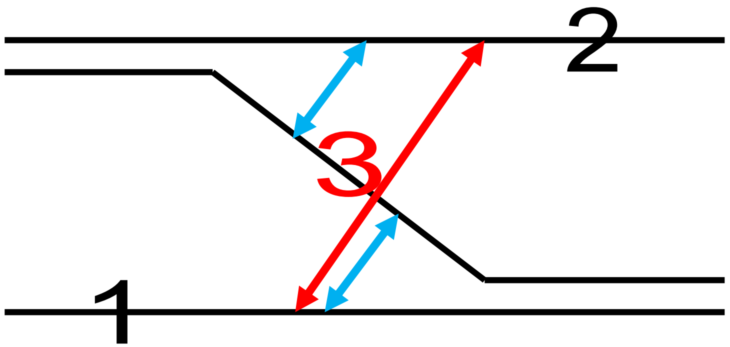
Efimov-potential as a phenomenological form of GPT-potential in 3-body system

For the 3rd particle transfer:

A) $r_{23} = r_2 - r_3 = 0$ or $r_{31} = r_3 - r_1 = 0$

$$V_e(r_1, r_2, r_3) = \frac{V_{e0} a_e^{\,2}}{r_{12}^{\,2} + r_{23}^{\,2} + r_{31}^{\,2} + a_e^{\,2}} \Rightarrow \frac{V_{e0} a_e^{\,2}}{r_{12}^{\,2} + a_e^{\,2}}$$

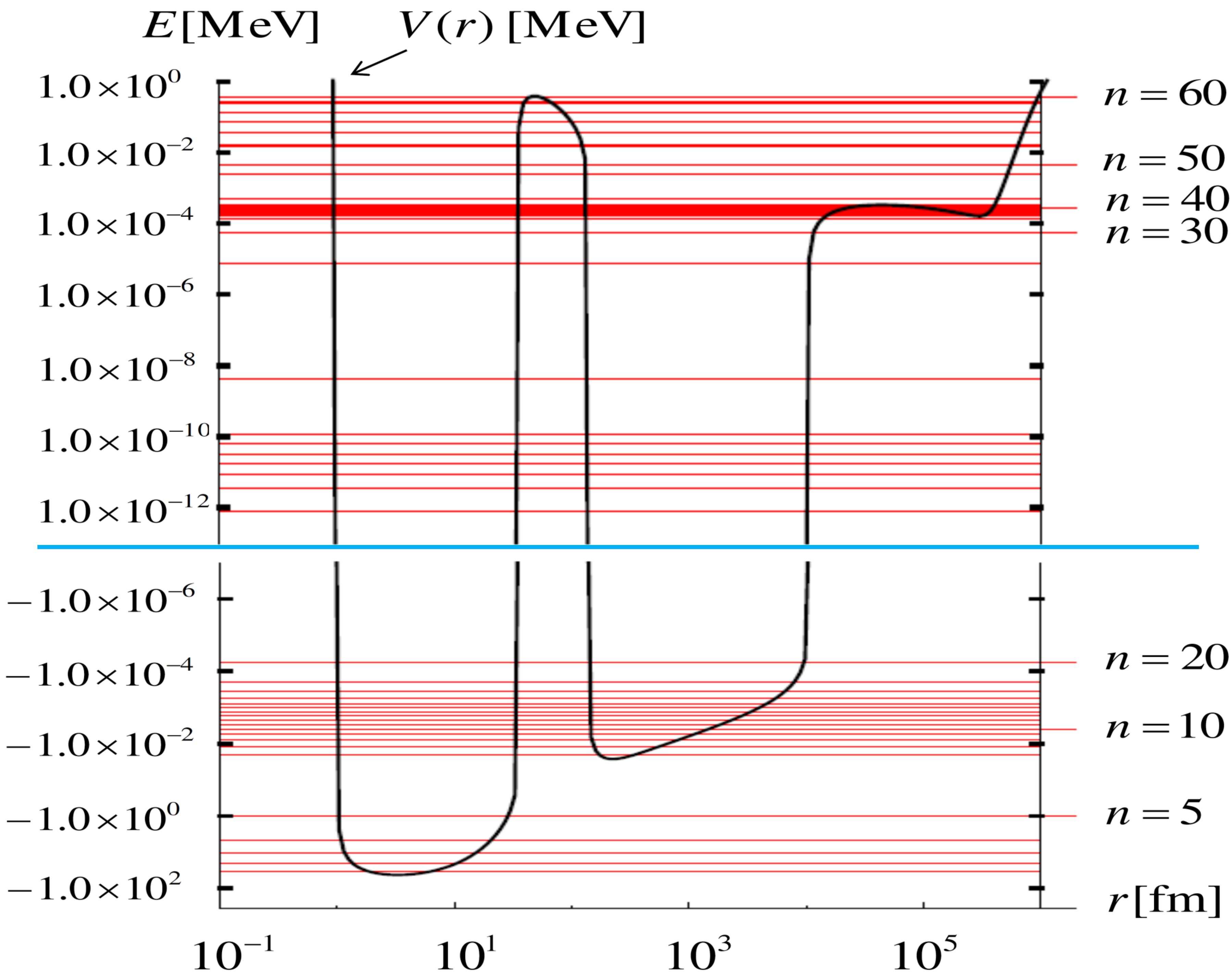
three-body long range (two-body long range pot. = 0)



B) $r_{12} = r_1 - r_2 = 0$

$$V_e(r_1, r_2, r_3) = \frac{V_{e0} a_e^{\,2}}{r_{12}^{\,2} + r_{23}^{\,2} + r_{31}^{\,2} + a_e^{\,2}} \Rightarrow \frac{V_{e0} a_e^{\,2}}{2r_{23}^{\,2} + a_e^{\,2}} = \frac{V_{e0} a_e^{\,2}}{2r_{31}^{\,2} + a_e^{\,2}}$$

two-body long range (three-body long range pot. = 0)



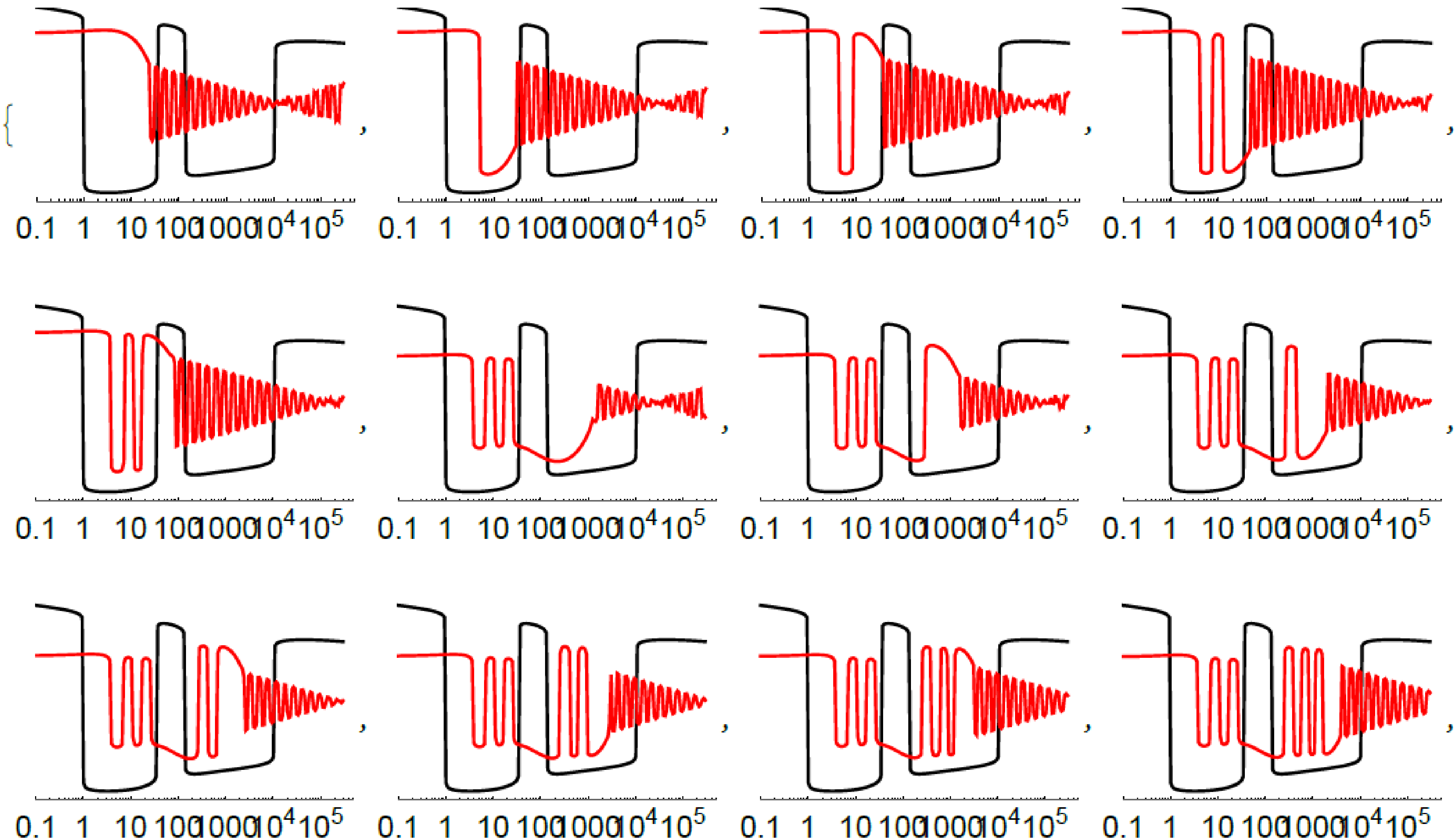
E [MeV]
V(r) [MeV]
1.0×10^0
1.0×10^−2
1.0×10^−4
1.0×10^−6
1.0×10^−8
1.0×10^−10
1.0×10^−12
−1.0×10^−6
−1.0×10^−4
−1.0×10^−2
−1.0×10^0
−1.0×10^2
n = 60
n = 50
n = 40
n = 30
n = 20
n = 10
n = 5
r [fm]
10^−1
10^1
10^3
10^5

The wave function overlap value (WFO: $W_{n;m}$)
between

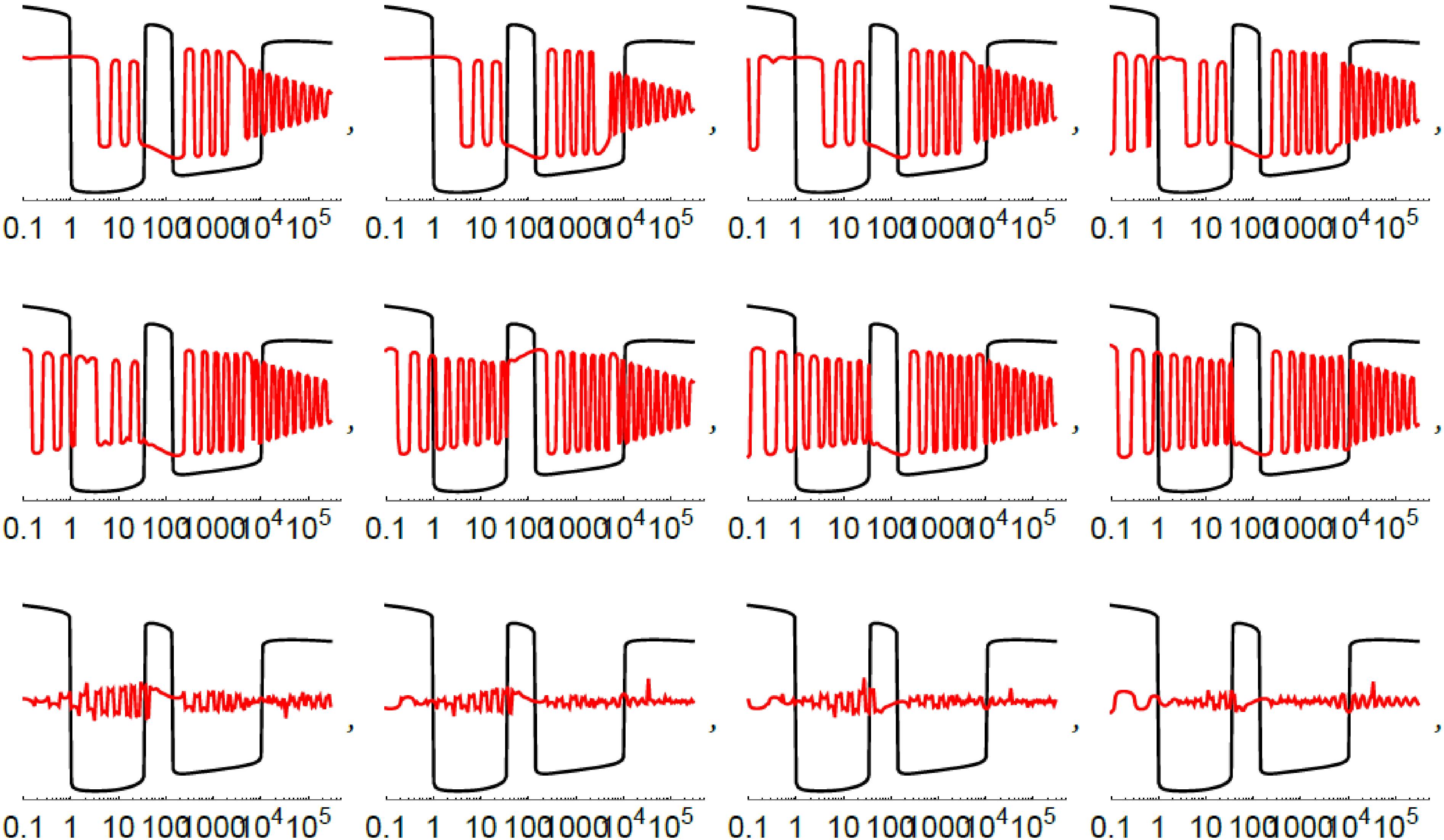
1） the La highest nuclear excited state with
the quantum number n = 5,

2） and the lowest CsD_2 quasi molecular states
is of critical importance for the existence of
the electro-magnetic (EM) transition
in the Cs(2d,γ)La reaction.

$n = 1 \sim 12$



$n = 13 \sim 24$


0.1 1 10 100 1000 10^4 10^5
0.1 1 10 100 1000 10^4 10^5
0.1 1 10 100 1000 10^4 10^5
0.1 1 10 100 1000 10^4 10^5
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0.1 1 10 100 1000 10^4 10^5
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0.1 1 10 100 1000 10^4 10^5
0.1 1 10 100 1000 10^4 10^5
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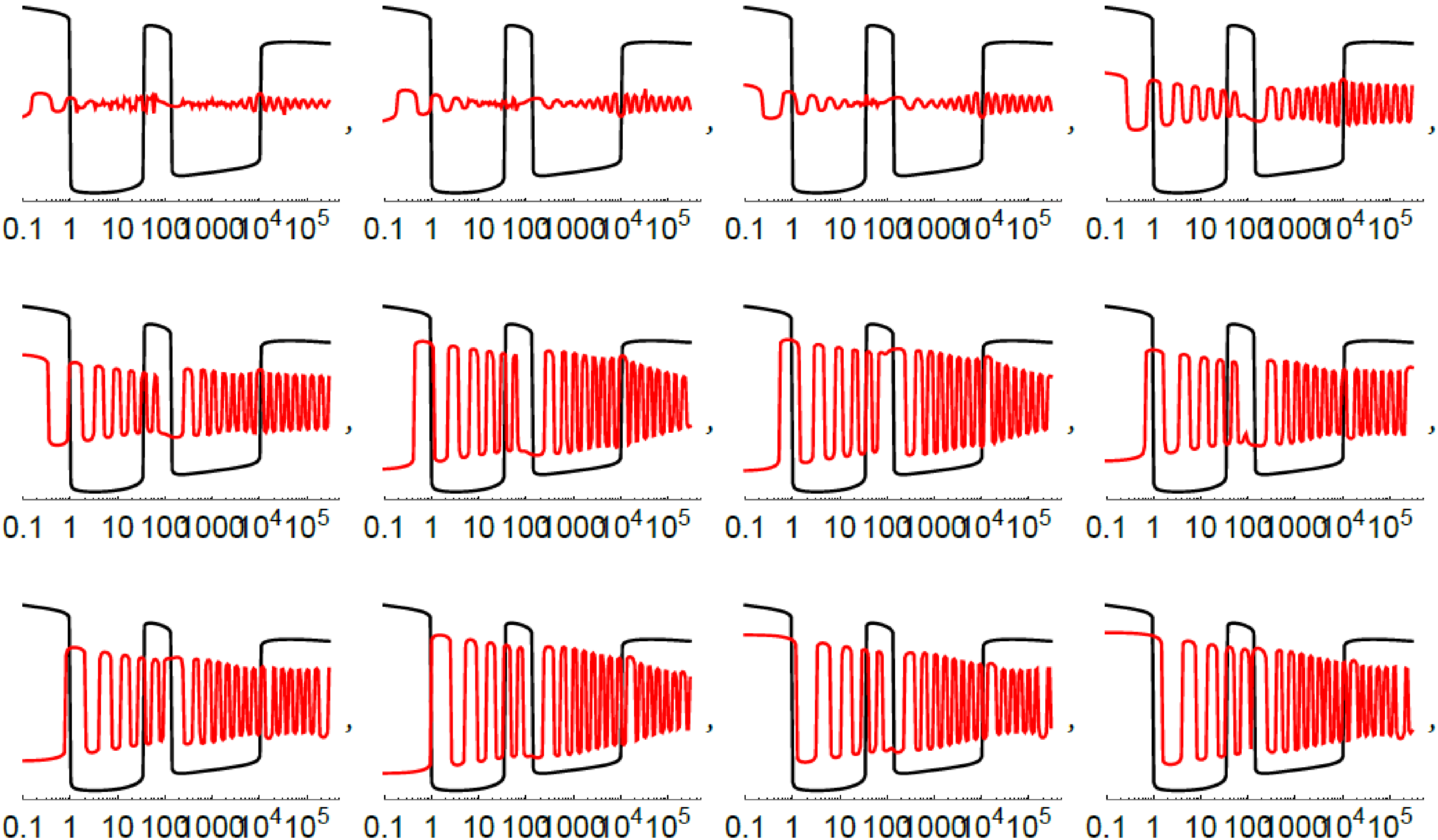
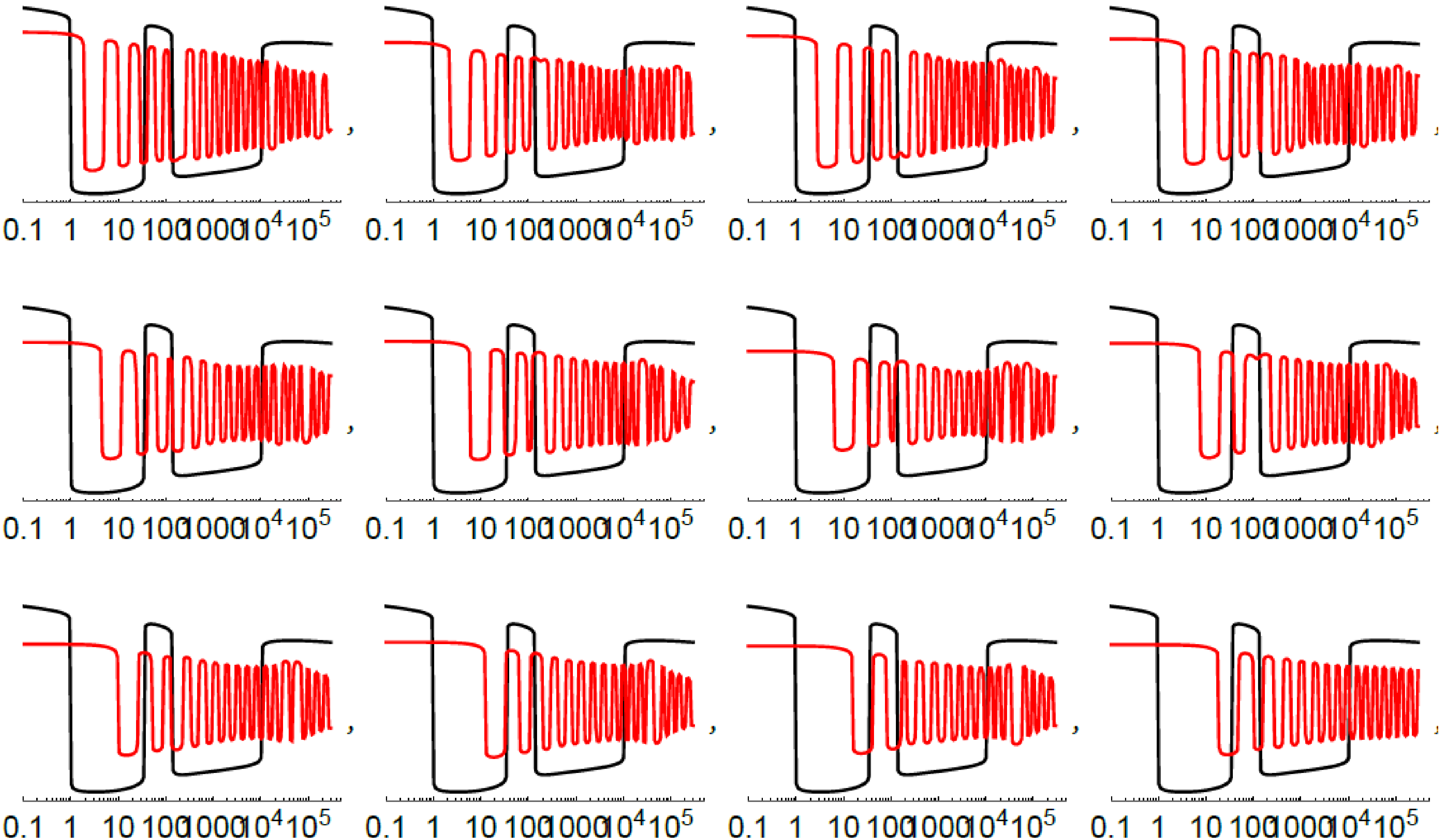
0.1 1 10 100 1000 10^4 10^5

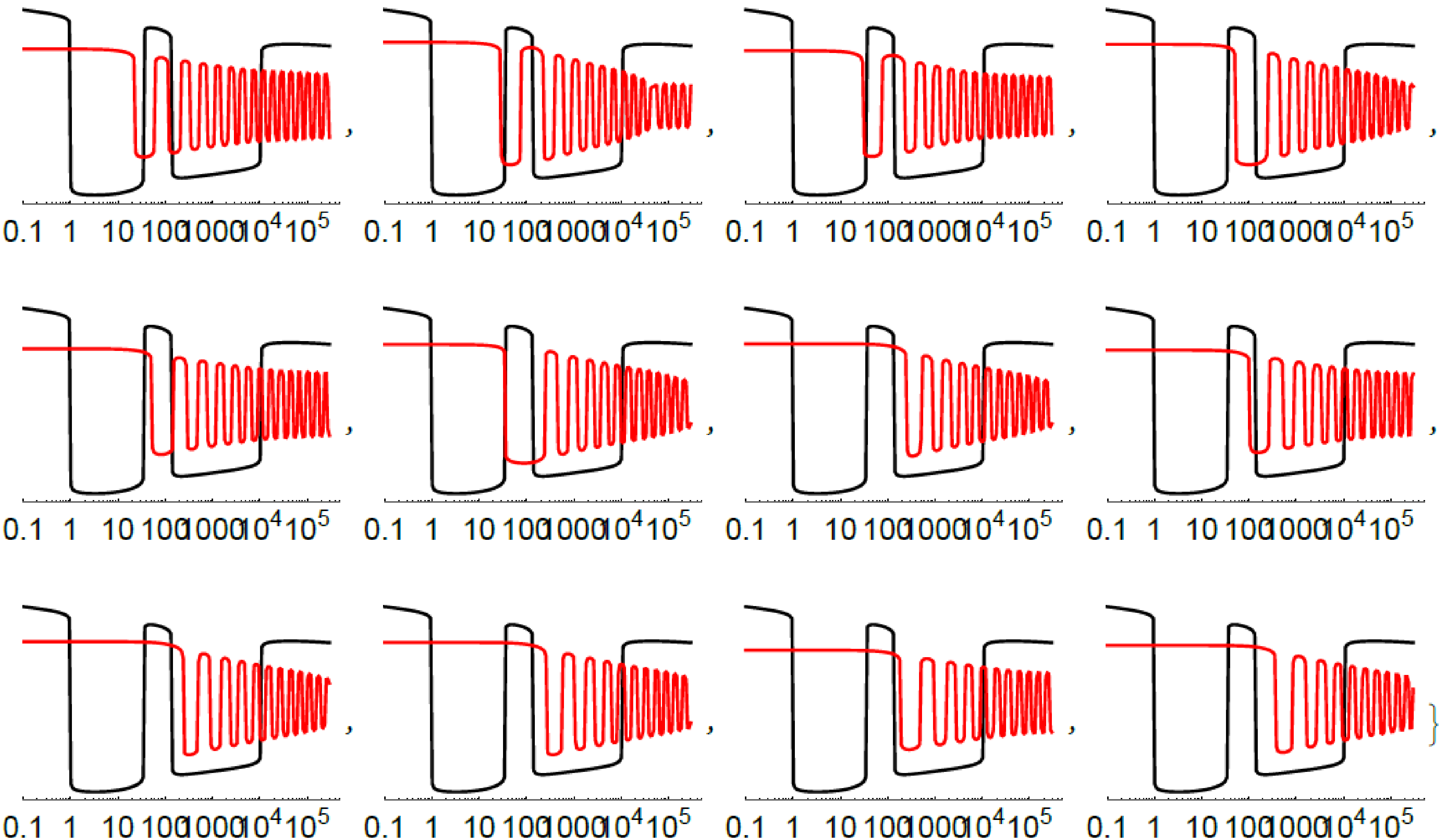
$n = 25 \sim 36$

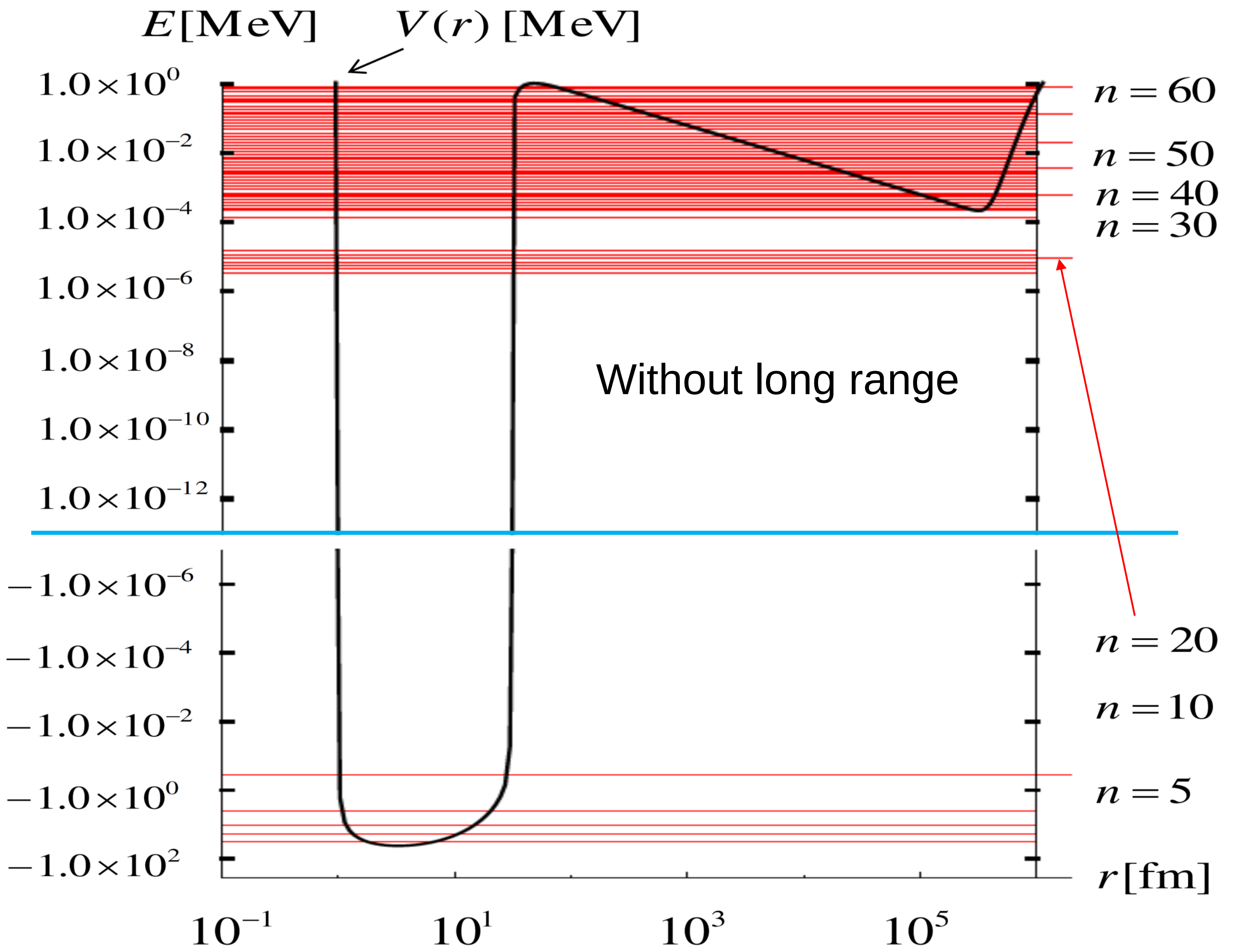


$n = 37 \sim 48$



$n = 49 \sim 60$

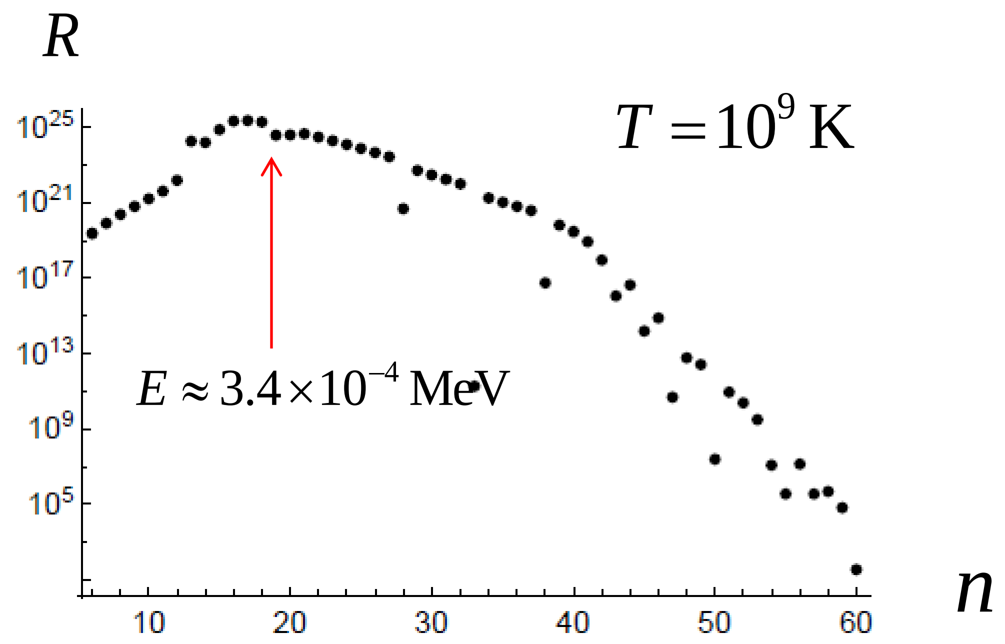


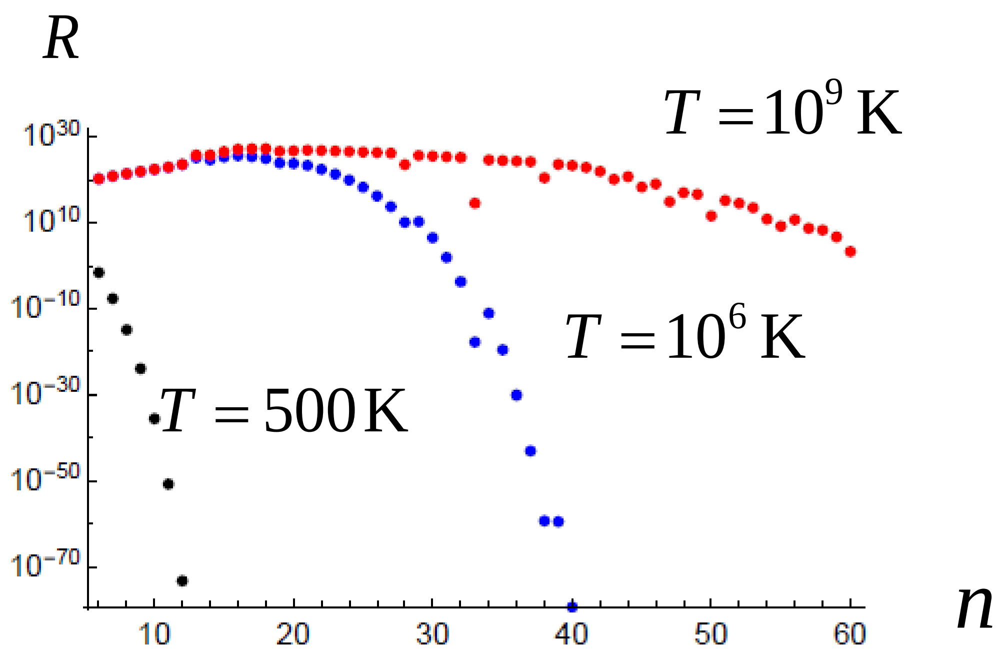


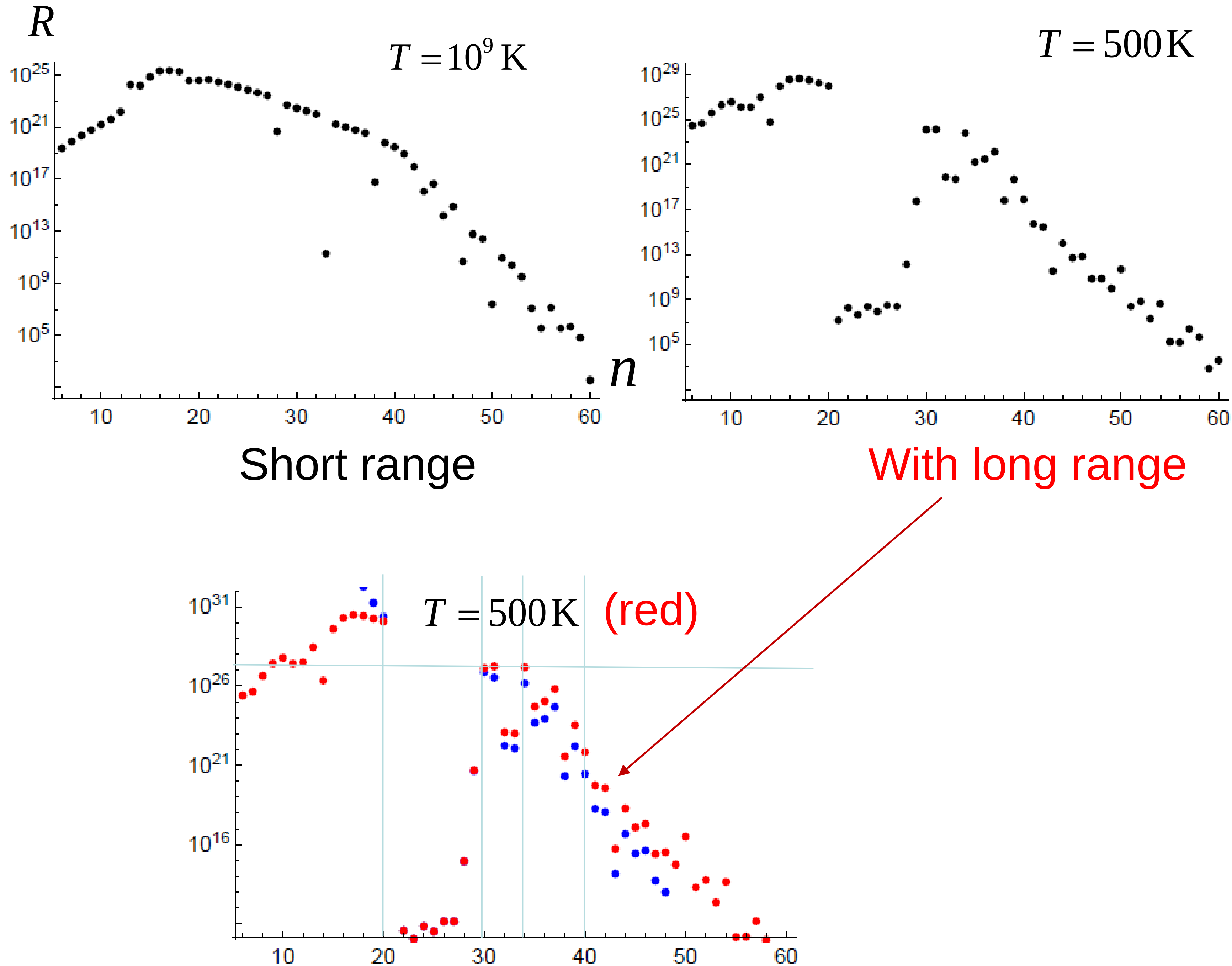
E[MeV]
V(r) [MeV]
1.0×10^0
1.0×10^−2
1.0×10^−4
1.0×10^−6
1.0×10^−8
1.0×10^−10
1.0×10^−12
−1.0×10^−6
−1.0×10^−4
−1.0×10^−2
−1.0×10^0
−1.0×10^2
n = 60
n = 50
n = 40
n = 30
n = 20
n = 10
n = 5
Without long range
r[fm]
10^−1
10^1
10^3
10^5

Short range
Nuclear potential

R: transition rate







R
T = 10^9 K
n
Short range
T = 500 K
With long range
T = 500 K
(red)

・Transition probability from $|\psi_i\rangle$ to $|\psi_f\rangle$ by the spontaneous emission in the vacume

E1- transition

$$W^{\mathrm{E1}}_{i\to f}(E_i,E_f)=\frac{(E_i-E_f)^3}{3\pi\varepsilon_0 h_b^4 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2=\frac{4\,(E_i-E_f)^3}{3h_b^3c^2}\left(\frac{1}{4\pi\varepsilon_0 h_b c}\right)\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2$$

$$=\begin{cases}\dfrac{4\,(E_i-E_f)^3}{3h_b^3c^2}\alpha\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2 & \text{cgs unit}\\[2ex] \dfrac{4\,(E_i-E_f)^3}{3}\alpha\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2 & \text{Natural unit}\end{cases}$$

E2-transition

$$W^{\mathrm{E2}}_{i\to f}(E_i,E_f)=\frac{1}{20}\frac{4(E_i-E_f)^5}{3\pi\varepsilon_0 h_b^6 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|\frac{1}{2}\left(3z_k^2-x_k^2-y_k^2\right)Z_k e\right|\Psi_i\right\rangle\right|^2$$

$$\rightarrow W^{\mathrm{E2'}}_{i\to f}=\frac{1}{20}\frac{4(E_i-E_f)^5}{3\pi\varepsilon_0 h_b^6 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|\frac{1}{2}r_k^2 Z_k e\right|\Psi_i\right\rangle\right|^2$$

M1-transition

$$W^{\mathrm{M1}}_{i\to f}(E_i,E_f)=\frac{4(E_i-E_f)^3}{3\pi\varepsilon_0 h_b^4 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|\frac{Z_k e\,g_k}{2m_k}(\vec{L}_k+\vec{S}_k)\right|\Psi_i\right\rangle\right|^2$$

g_k : gyromagnetic ration of nucleus

・transition time $\tau_{i\to f}=1/W_{i\to f}$

・Let us obtain the transition probability from $|\psi_i\rangle$ to $|\psi_f\rangle$ by photon emission from CsD_2Pd_{12} in the thermal equilibrium of temperature *T*. The average La number at the energy E_i and with the temperature *T* is given by the Maxwell-Boltzmann distribution;

$$f_{\mathrm{MB}}(E_i,T)=\exp\left[-\frac{E_i}{k_B T}\right]/Z \qquad Z=\sum_{j=1}\exp\left[-\frac{E_j}{k_B T}\right] \qquad k_B=1.380649\times10^{-23}\ \mathrm{J/K} \approx 8.6171\times10^{-8}\ \mathrm{MeV/K}$$

In the radiation field of the thermal equilibrium with the temperature *T* and the energy E_i , the average photon number is give by the Bose-Einstein statistics,

$$f_{\mathrm{BE}}(E_i-E_f)=\frac{1}{\exp\left[\frac{E_i-E_f}{k_B T}\right]-1}$$

for Black body radiation

Therefore, the transition probability for the unit time, and the unit number of La is given by

$$\frac{dN^{\mathrm{E1}}_{i\to f}}{dt}=f_{\mathrm{MB}}(E_i,T)\left[\underbrace{\frac{4(E_i-E_f)^3}{3\pi\varepsilon_0 h_b^4 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2}+\underbrace{f_{\mathrm{BE}}(E_i-E_f,T)\frac{4(E_i-E_j)^3}{3\pi\varepsilon_0 h_b^4 c^3}\sum_{k=1}^{3}\left|\left\langle\Psi_f\left|Z_k e\vec{r}_k\right|\Psi_i\right\rangle\right|^2}\right]$$

Spontaneous emission（with the particle system） Stimulated emission（with the photon system）

$$=f_{\mathrm{MB}}(E_i,T)\left[W^{\mathrm{E1}}_{ij}(E_i,E_f)+f_{\mathrm{BE}}(E_i-E_f,T)W^{\mathrm{E1}}_{ij}(E_i,E_f)\right]=\frac{W^{\mathrm{E1}}_{ij}(E_i,E_f)/Z}{\exp\left[\frac{E_i}{k_B T}\right]-\exp\left[\frac{E_f}{k_B T}\right]}$$

E2、M1 transitions as well

Recent Experimental Results:

By Iwamura et al. (MHI), (2002)

$^{133}_{55}Cs$ (7/2^{+}:stable)+4d → $^{141}_{59}Pr$(5/2^{+}:stable)

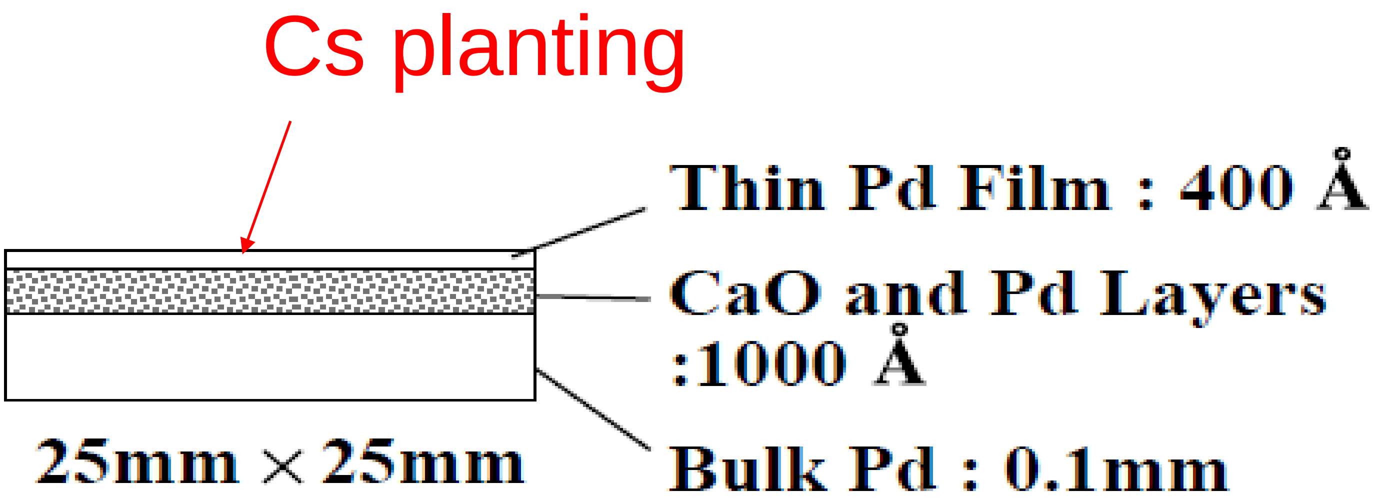
Praseodymium

$^{88}_{38}Sr$(0^{+}:82.58%)+4d → $^{96}_{42}Mo$(0^{+}:16.68%)

strontium molybdenum

$^{88}_{38}Sr$(0^{+}:82.58%)+2d → $^{92}_{40}Zr$(0^{+}:17.15%) zirconium

Hioki et al, Toyota-Nagoya univ. group confirmed (2013)


Cs planting
Thin Pd Film : 400 Å
CaO and Pd Layers
:1000 Å
Bulk Pd : 0.1mm
25mm × 25mm

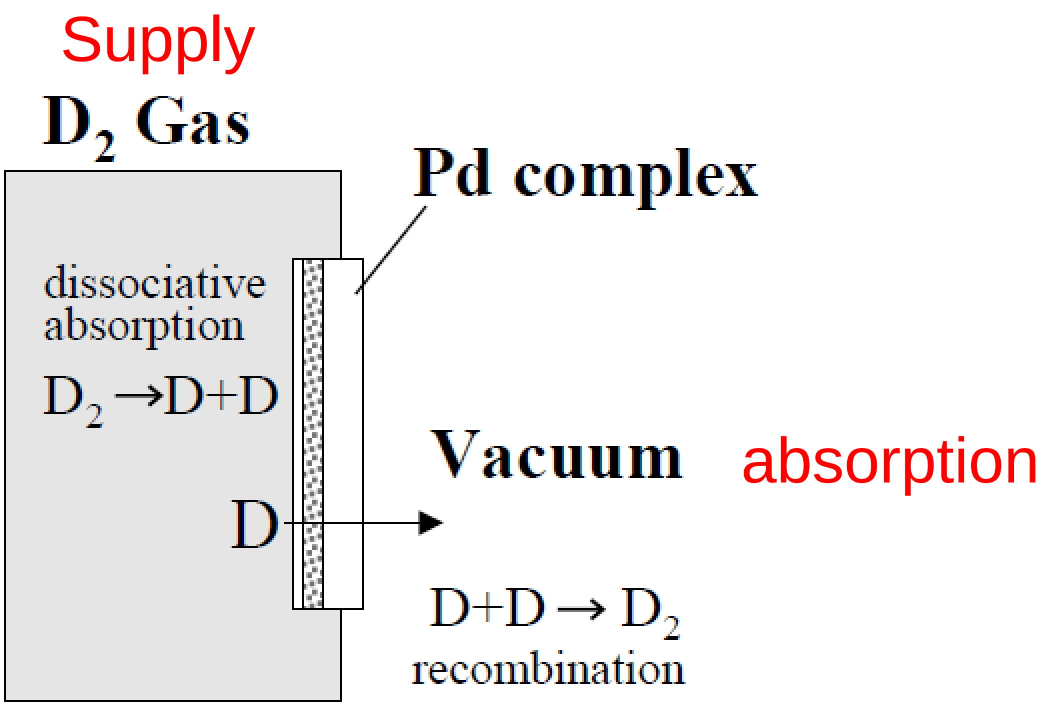
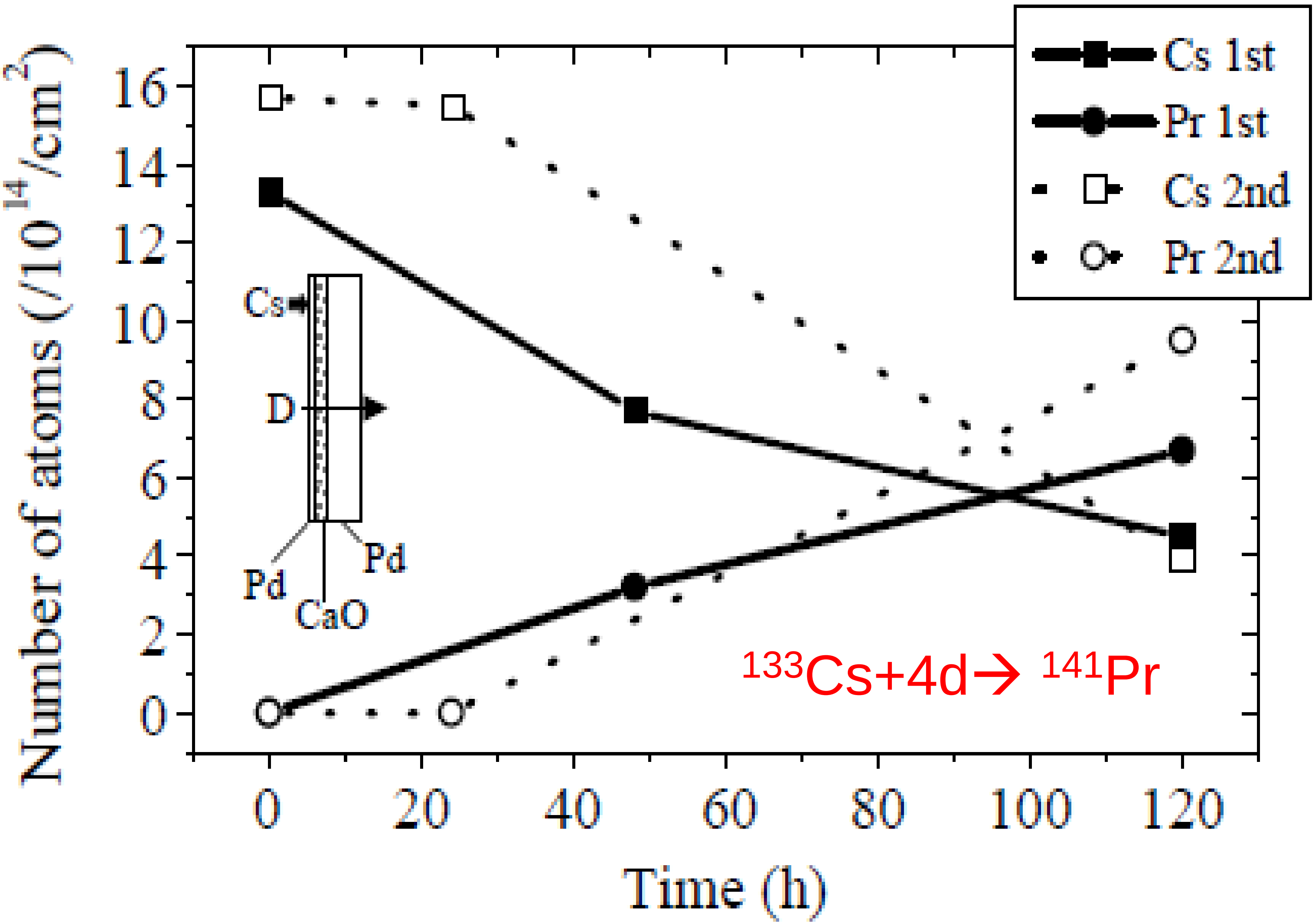
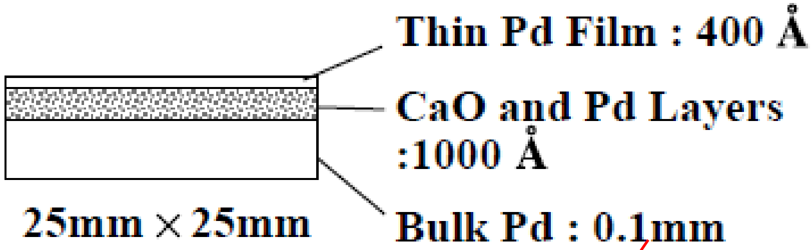
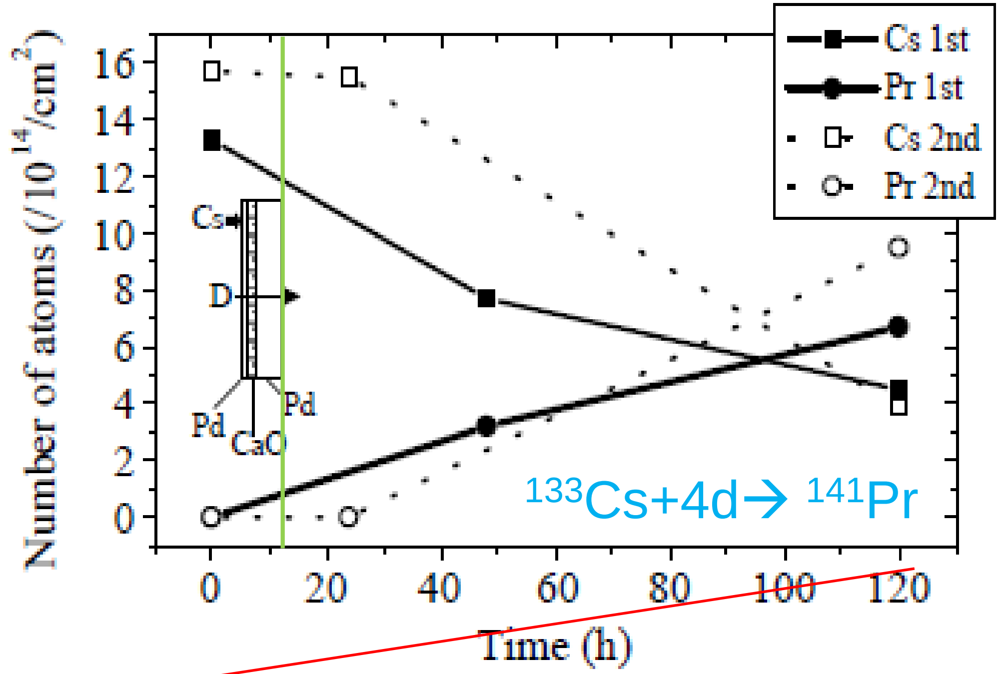


Fig. 1. D_2 gas permeation through the Pd complex.


Number of atoms (/10^14/cm^2)
Time (h)
Cs 1st
Pr 1st
Cs 2nd
Pr 2nd
Cs
D
Pd
CaO
Pd
133Cs+4d→ 141Pr
0
2
4
6
8
10
12
14
16
0
20
40
60
80
100
120

(a)





$$n = 1.4 \times 10^{15} \ /\mathrm{cm}^2$$

$$T = 343\mathrm{K} \approx 70^\circ\mathrm{C}$$

Transition Probability for:

$^{135}Cs+2d \rightarrow {}^{139}La$

1) No long range & electron-ion pot.

$$\sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx 7.1 \times 10^{7} / \mathrm{cm}^2 \sim$$

$$\frac{7 \times 10^{14}}{73 \times 10^{8}} \approx 10^{5}\ \mathrm{y}$$

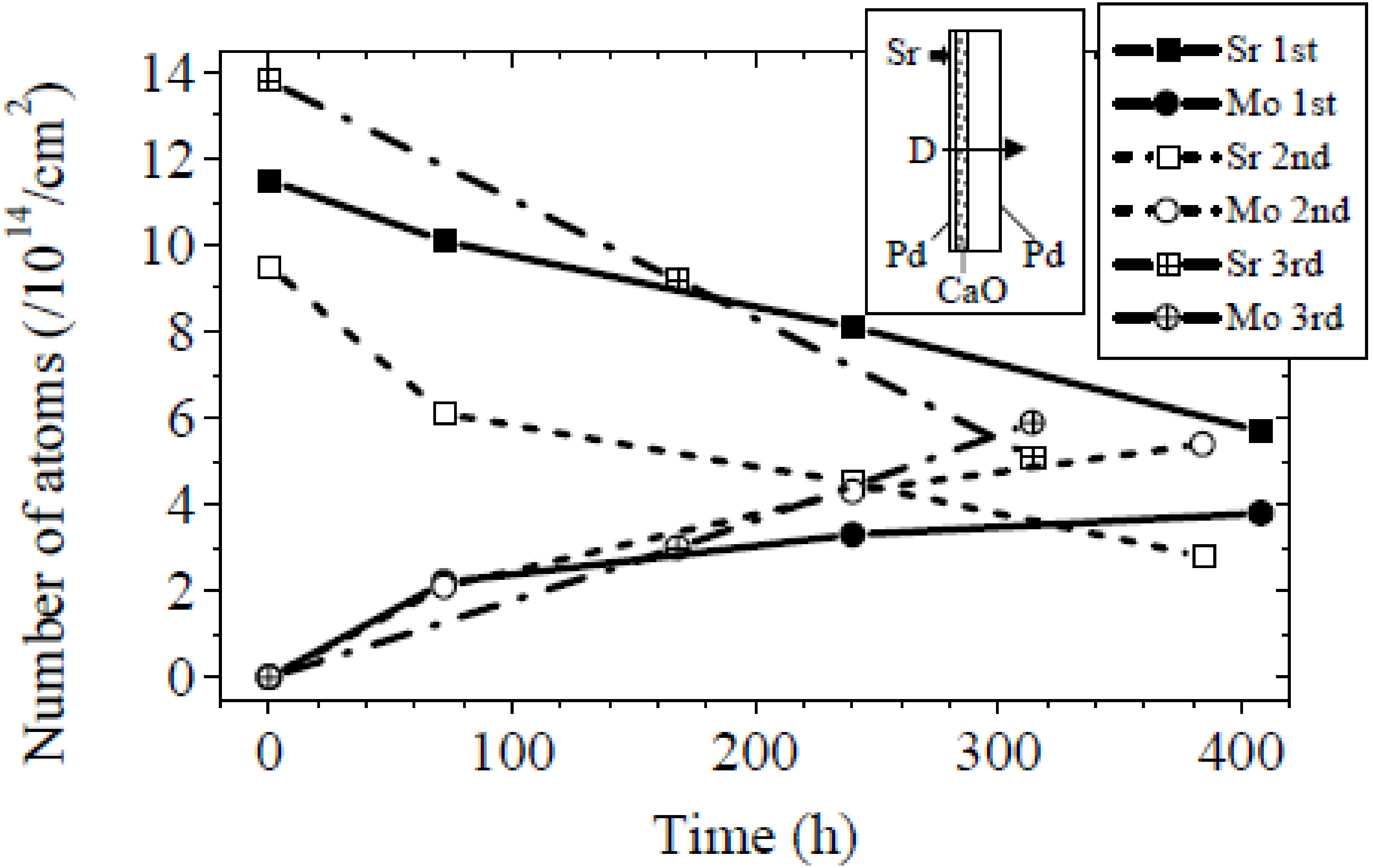
2) With long range, No electron-ion pot.

$$W_{i\to f}^{E2'}(L) = \sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.5 \times 10^{16} / \mathrm{cm}^2$$

Our result is 36 times larger than Experiment.

3) No long range, with electron-ion pot.

$$W_{i\to f}^{E2'}(S) = \sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.1 \times 10^{8} / \mathrm{cm}^2$$



(a)

$^{88}_{38}Sr + 4d \rightarrow ^{96}_{42}Mo$

Conclusion

1) Wave function overlapping:

$$W_{56}^{L} / W_{56}^{S} \sim 10^{7}$$

Oryu et al. Few Body Syst. (2019) 60 : 42

2) Transition probability for an approximated E2 gives

$$W_{i\to f}^{L} \equiv W_{i\to f}^{E2'}(L) = \sum_{i=6}^{n_{\max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.5 \times 10^{16} / \mathrm{cm}^2$$

$$W_{i\to f}^{S} \equiv W_{i\to f}^{E2'}(S) = \sum_{i=6}^{n_{\max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i\to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.1 \times 10^{8} / \mathrm{cm}^2$$

$$W_{i\to f}^{L} / W_{i\to f}^{S} \sim 10^{8}$$

3) Long Range Potential is essential to obtain the ultra low energy nuclear synthesis.

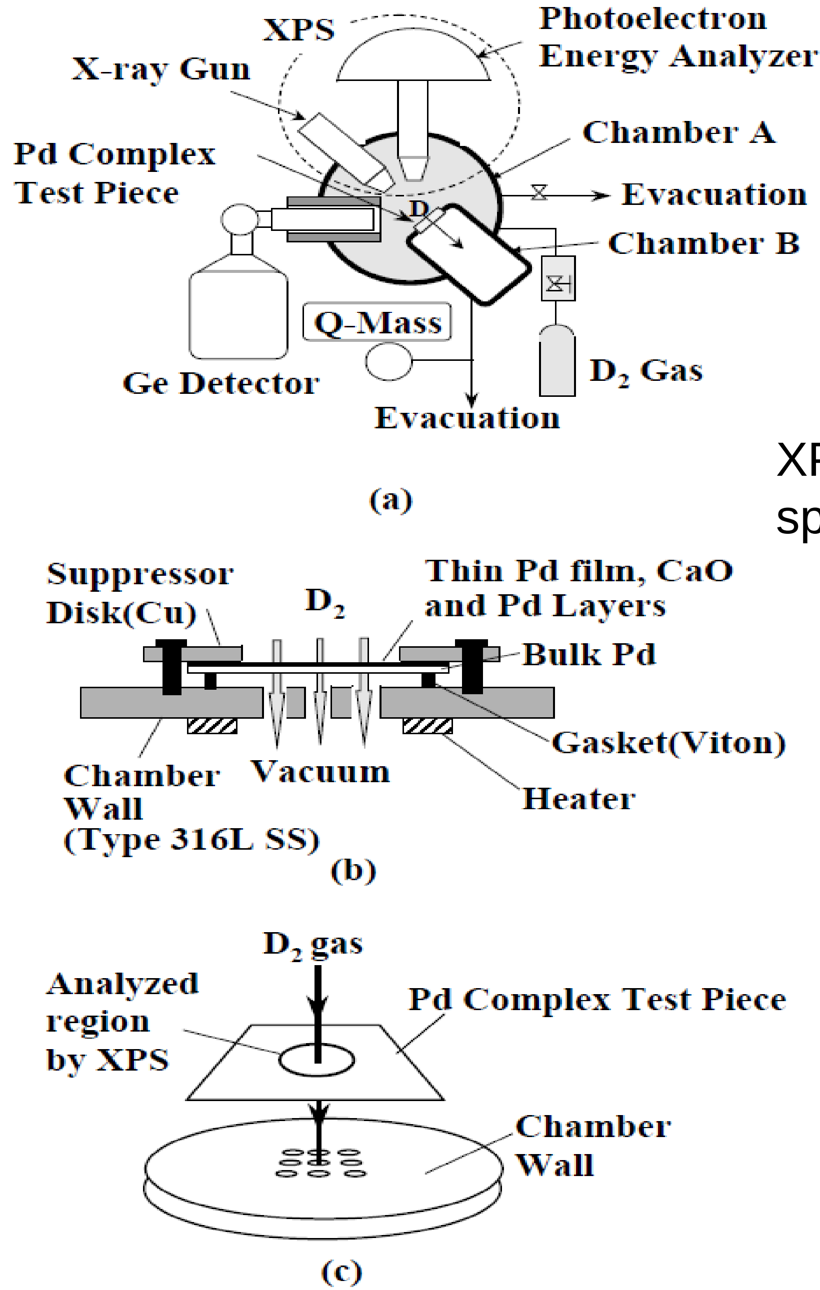
4) The GPT $1/r^n$-type potentials are promising for the D-particle transfer potential in D-Cs-D of $1/r^2 (or\ 1/r^3) - type$, while D_2-transfer in D_2-Cs-D_2 could be $1/r^3 (or\ 1/r^4) - type$ potential etc.

5) Therefore, pure D-absorption into Pd complex
never occur the D+D $\rightarrow$ ^{4}He fusion,
because D-Pd$_n$ is not a three-body system
but a many-body system,
then no GPT potential could be made.

6) Our *theoretical* calculation is the first success for
the description of ultra low energy nuclear synthesis
after the Experimental breakthrough was done.

7) As a conclusion, our Few-Body community could
contribute to ultra low energy nuclear sysnthesis
by the GPT long range potential.

Thank you very much for your attention!



XPS: X-ray photoelectron spectroscopy

Fig. 3. (a) Experimental apparatus, (b) Schematic of test setup in the vicinity of Pd complex test piece, (c) Path of D_2 gas flowing through Pd complex test piece and chamber wall.

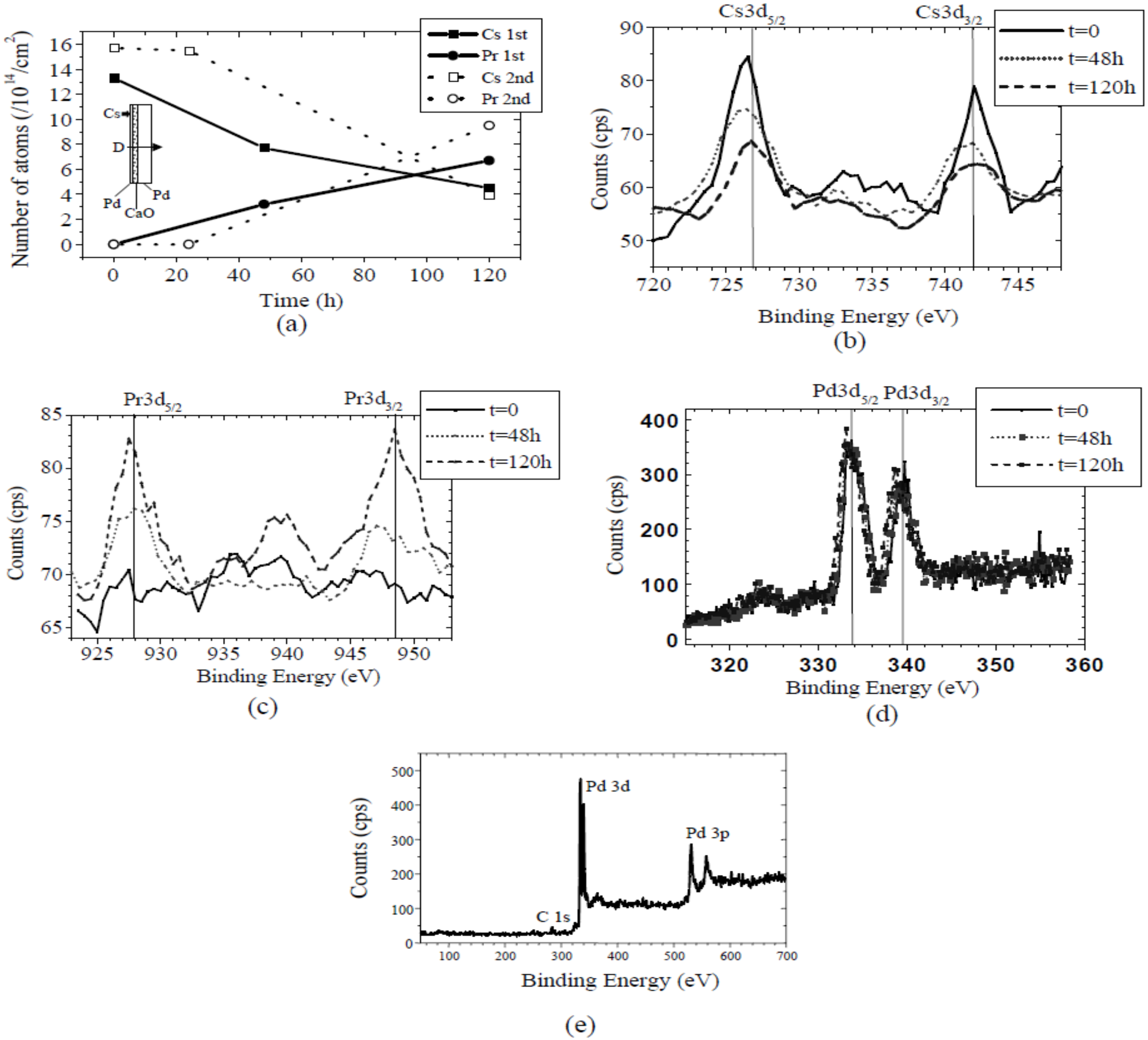


Fig. 4. Experimental results obtained by D_2 gas permeation through Pd complex (Pd/CaO/Pd) deposited with Cs: (a) Time variation in number of Cs and Pr atoms (number of atoms per cm^2), (b) XPS spectrum of Cs for experiment run #1, (c) XPS spectrum of Pr for experiment run #1, (d) XPS spectrum of Pd for experiment run #1, (e) Wide-range XPS spectrum for experiment run #1.

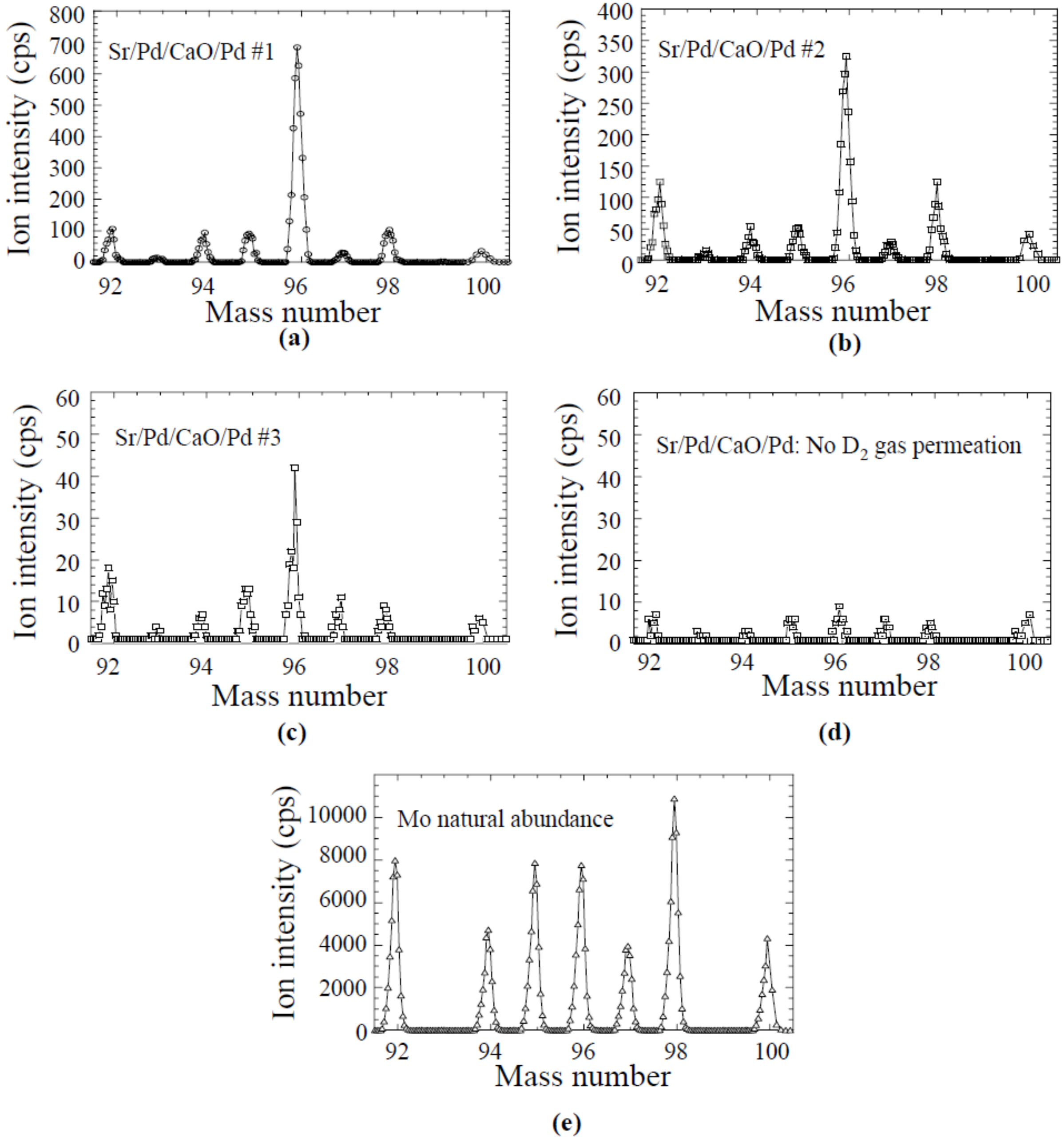


Fig. 9. Anomalous isotopic composition of detected Mo: (a) Isotopic composition of detected Mo for run #1, (b) Isotopic composition of detected Mo for run #2, (c) Isotopic composition of detected Mo for run #3, (d) SIMS analysis for Pd complex test piece with added Sr without D_2 gas permeation, (e) Natural abundance of Mo analyzed by SIMS.

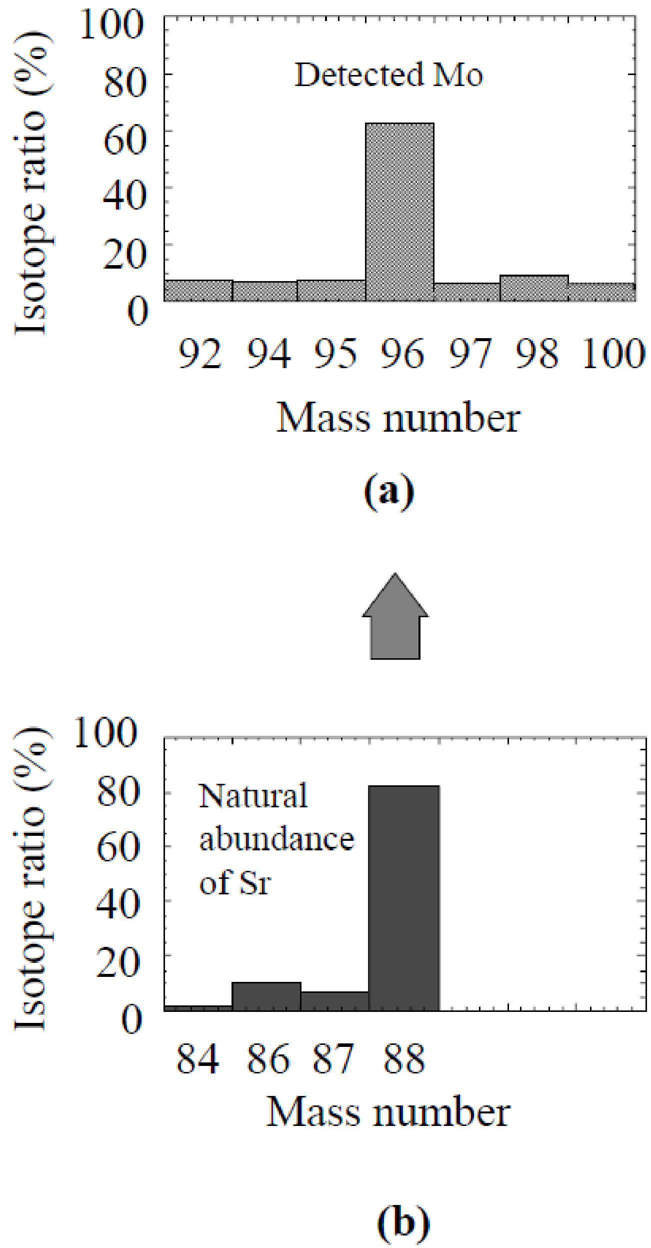


Fig. 10. Relationship of mass numbers between given Sr and detected Mo: (a) Isotopic composition of detected Mo, (b) Isotopic composition of given Sr.

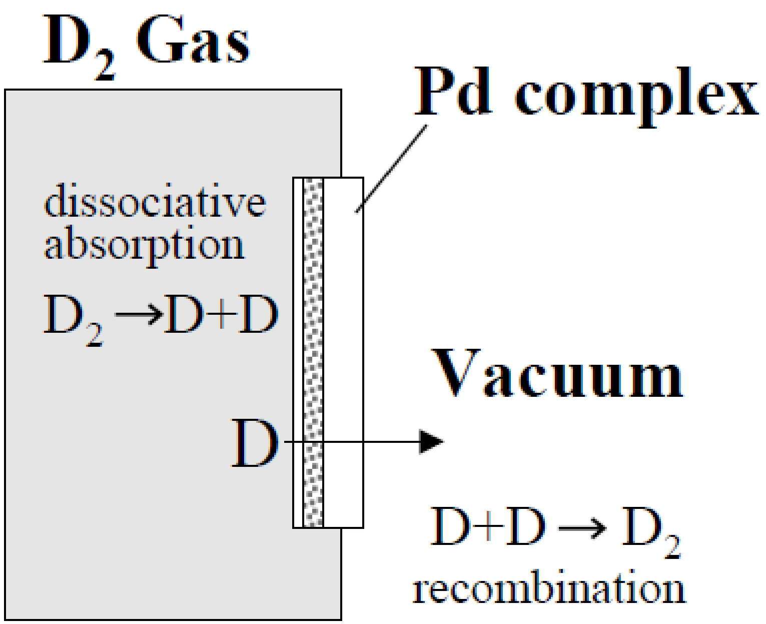
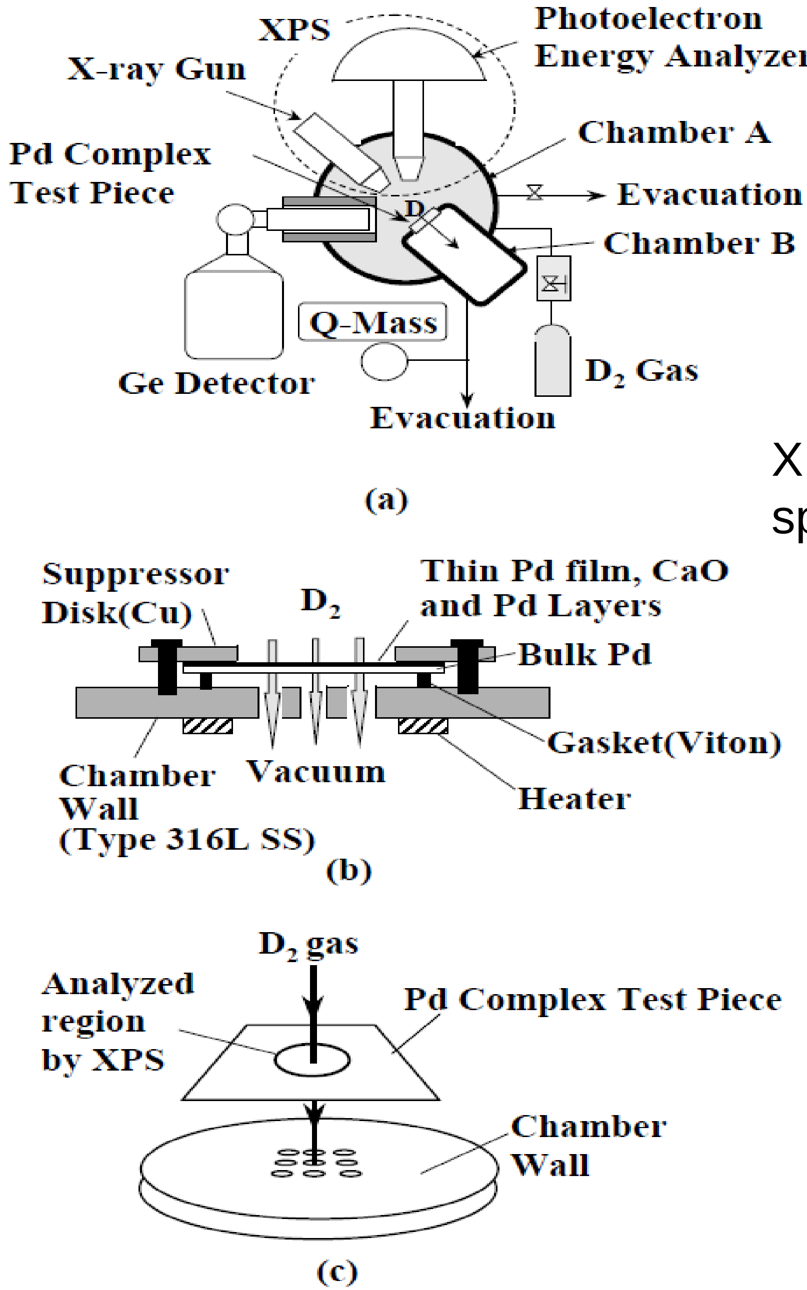


Fig. 1. D_2 gas permeation through the Pd complex.



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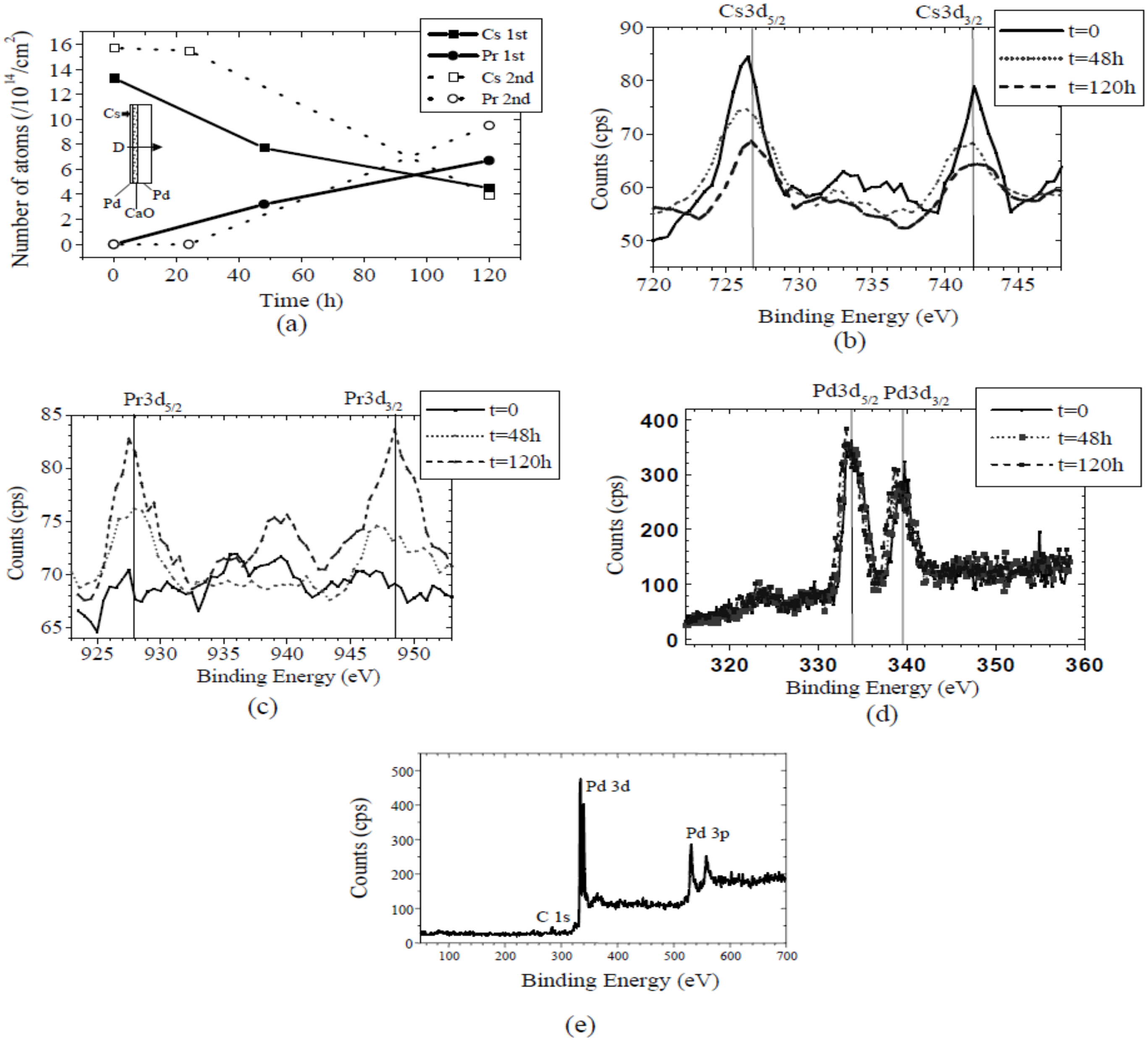
XPS: X-ray photoelectron spectroscopy



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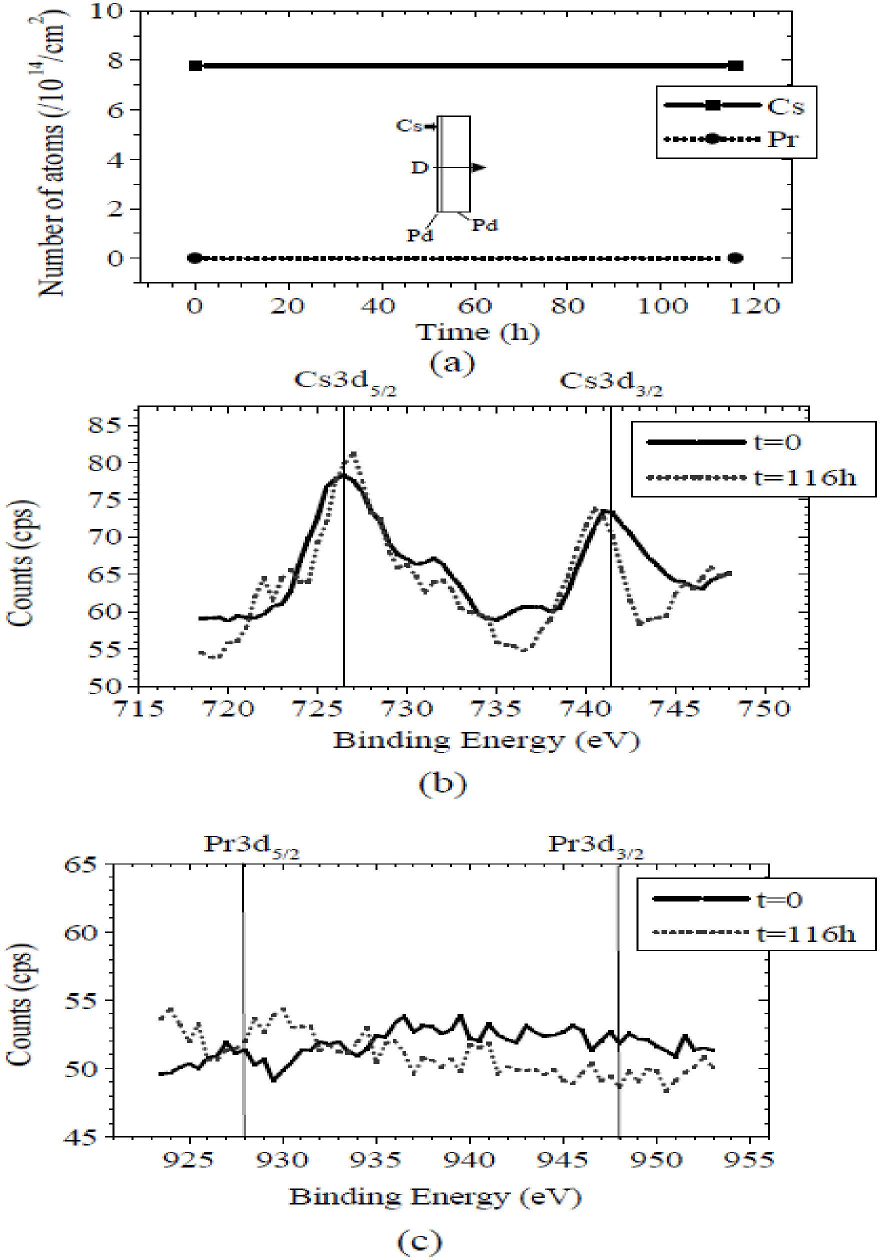


Fig. 5. Experimental results obtained by D_2 gas permeation through thin film and bulk Pd with added Cs: (a)Time variation in number of Cs and Pr atoms, (b) XPS spectrum of Cs, (c)XPS spectrum of Pr.

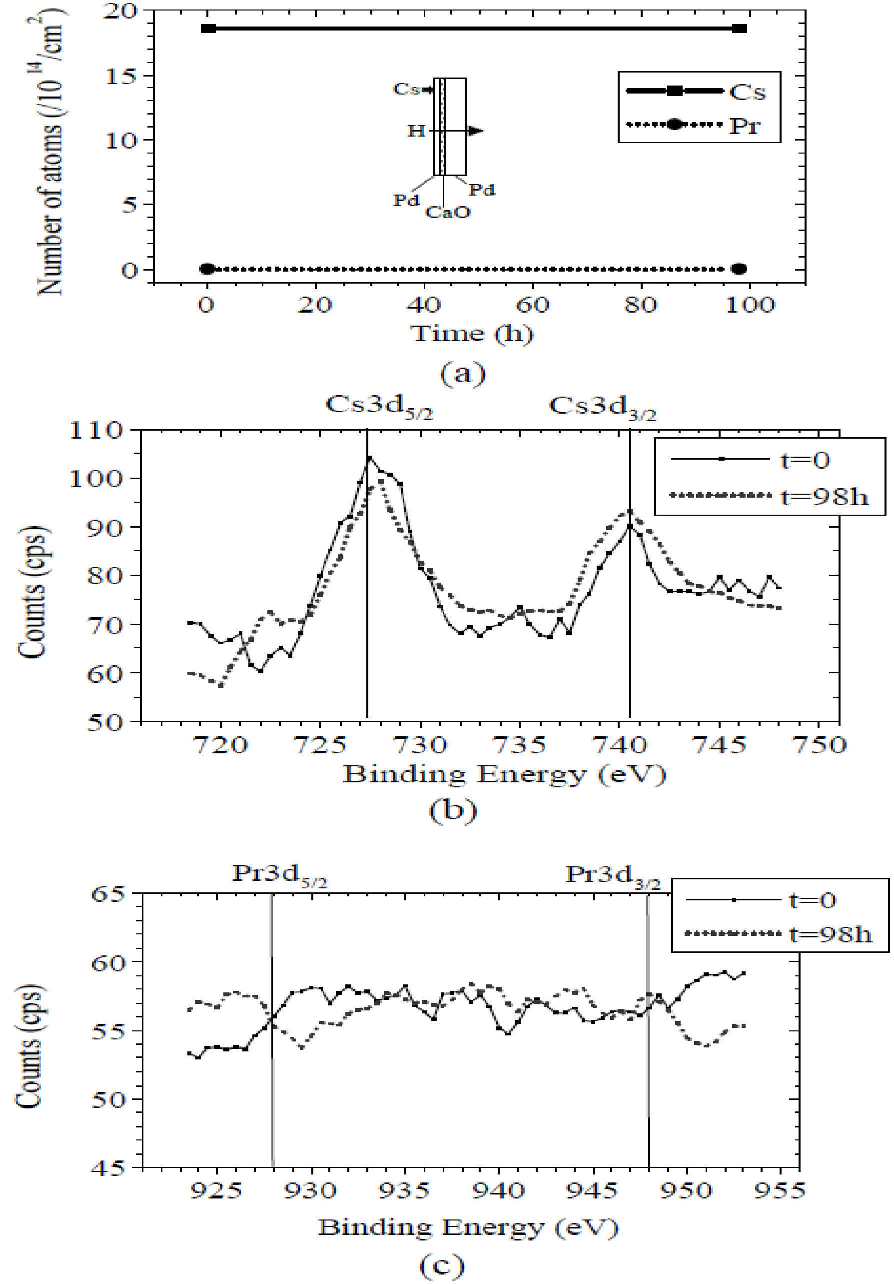


Fig. 6. Experimental results obtained by H_2 gas permeation through Pd complex (Pd/CaO/Pd) with added Cs: (a)Time variation in number of Cs and Pr atoms, (b)XPS spectrum of Cs, (c) XPS spectrum of Pr.

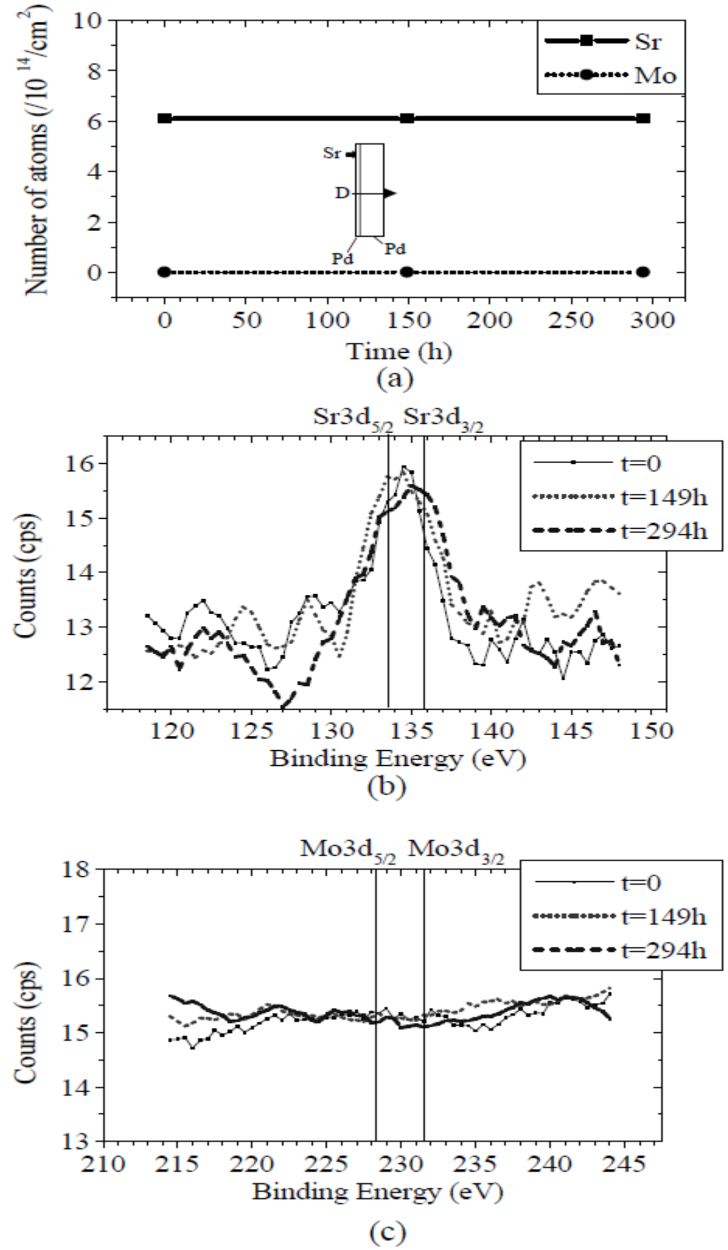
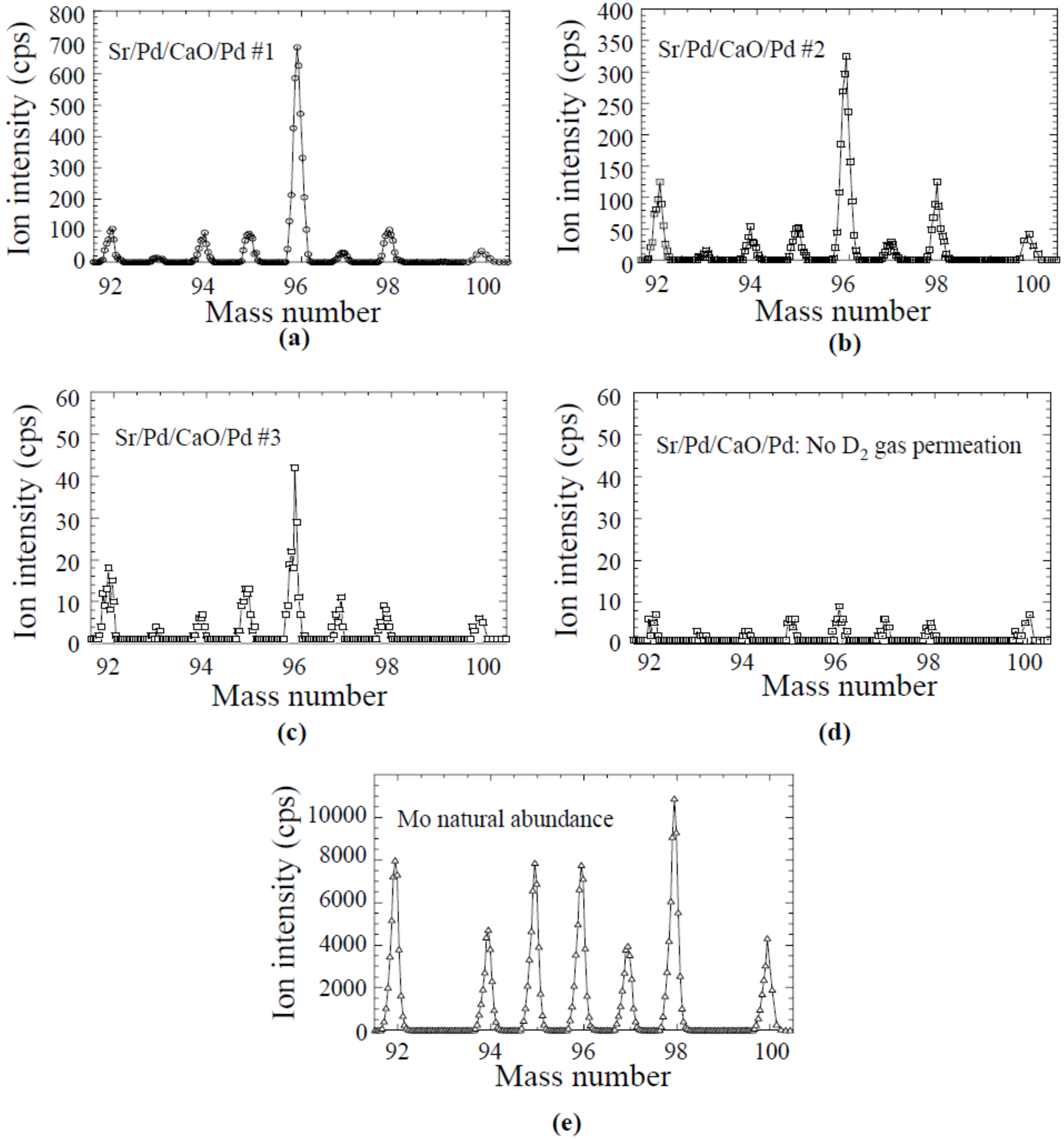


Fig. 8. Experimental results obtained by D_2 gas permeation through thin film and bulk Pd deposited with Sr: (a) Time variation in number of Sr and Mo atoms, (b) XPS spectrum of Sr, (c) XPS spectrum of Mo.



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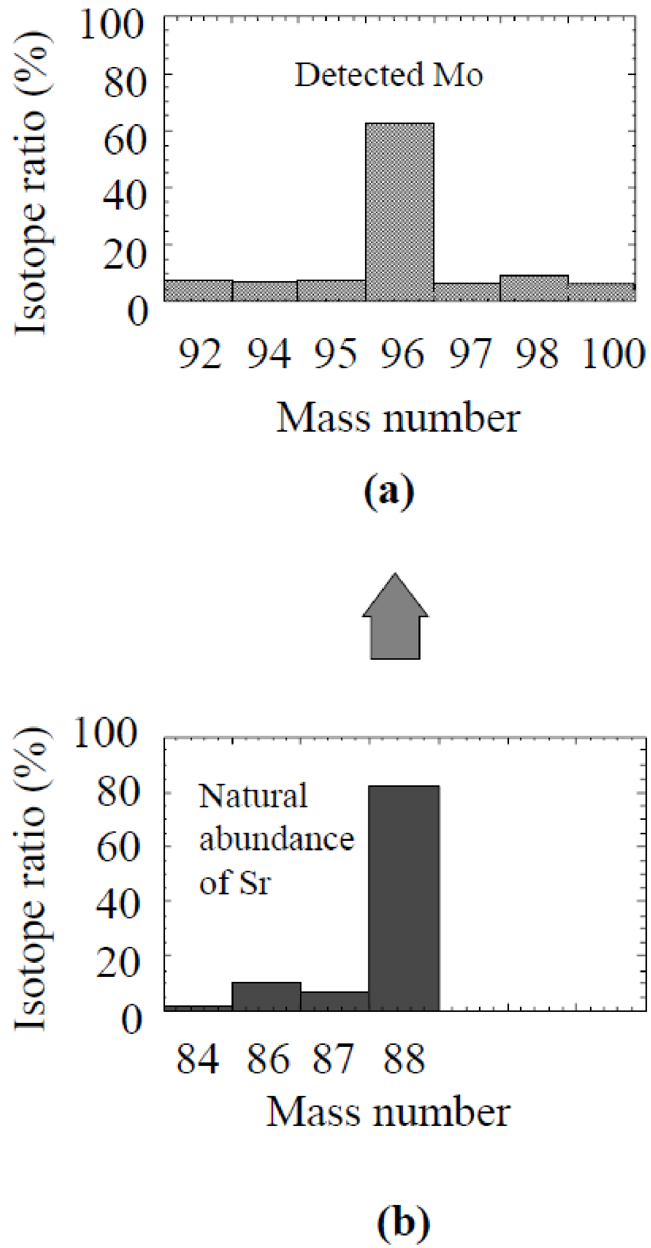

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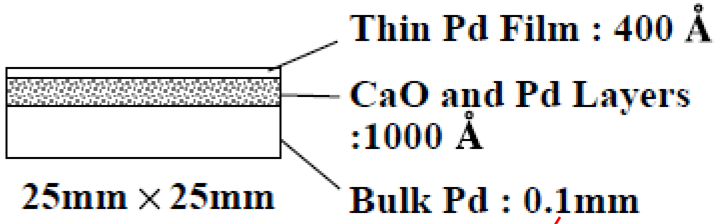
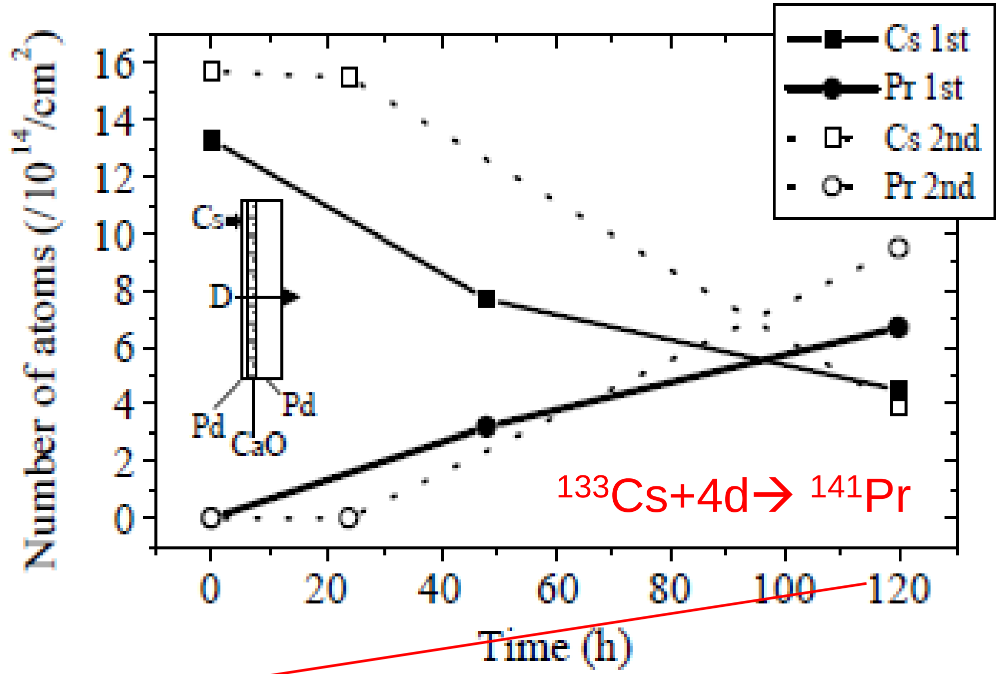




$^{135}Cs+2d \rightarrow {}^{139}La$

$^{133}Cs+4d \rightarrow {}^{141}Pr$

$$n = 1.4 \times 10^{15} \ /cm^2$$

$$T = 343K \approx 70^{\circ}C$$

・No long range pot. No electron-ion pot.

$$\sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx 7.1 \times 10^{7} / cm^2$$

・With long range No electro-ion pot.

$$\sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.5 \times 10^{16} / cm^2$$

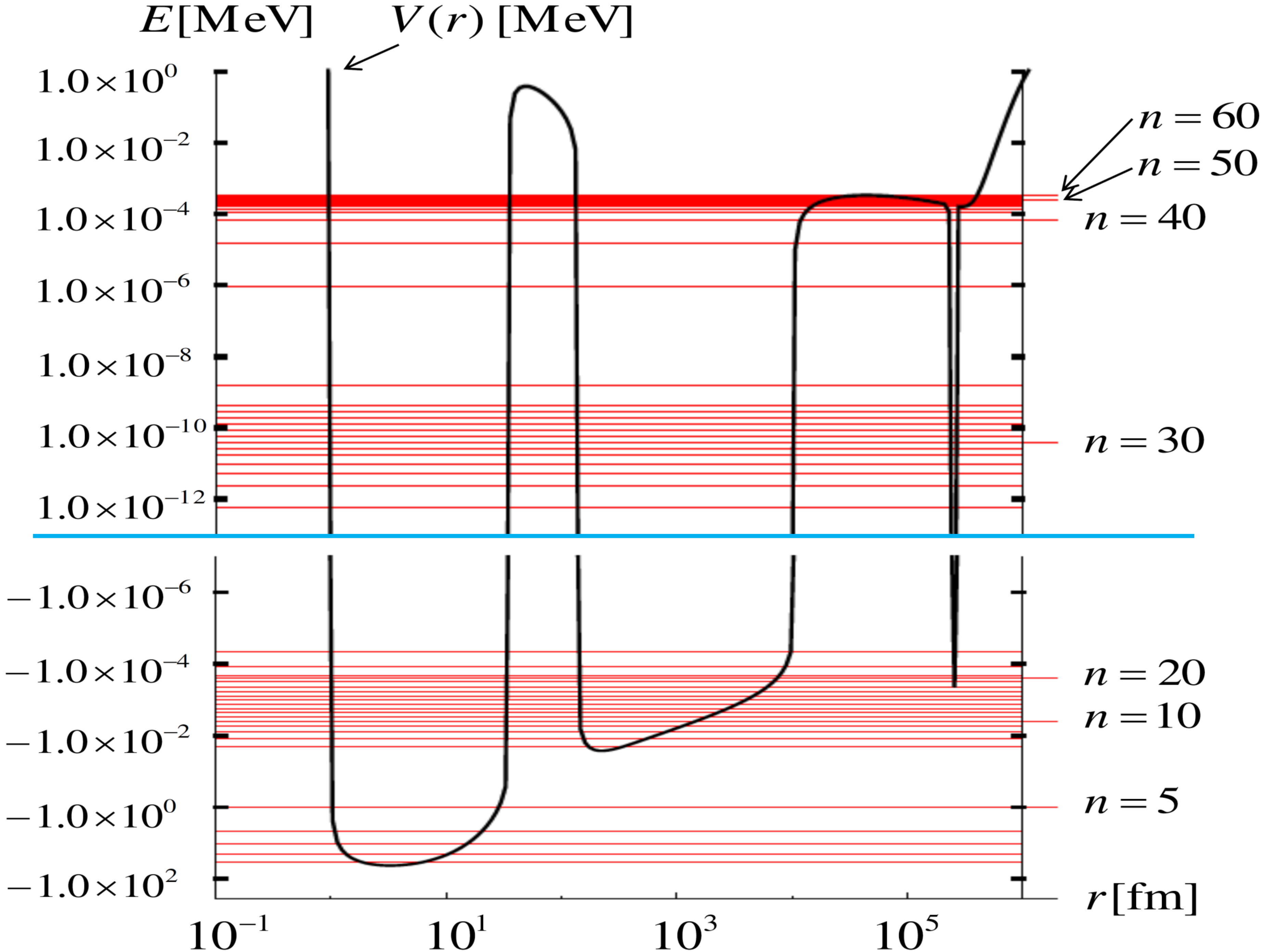
・No long range pot. With electron-ion pot.

$$\sum_{i=6}^{n_{max}} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx \sum_{i=6}^{60} \sum_{f=1}^{5} \frac{n}{100} \frac{dN_{i \to f}^{E2'}}{dt} \times 120 \times 3600 \approx 1.1 \times 10^{8} / cm^2 \sim \frac{7 \times 10^{14}}{73 \times 10^{8}} \approx 10^{5}\ y$$

GPT-potential is given by parametes a and γ *and* a potential depth $V_0 (< 0)$.

$$V^{GPT}(r) = V_0 \frac{a^{2\gamma+2}}{r(r/2+a)^{2\gamma+2}}$$

γ	$r \ll a$	GPT-potential	$a \ll r$
-1	V_0/r	V_0/r	V_0/r
$-1/2$	$V_0 e^{-r/2a}/r$	$V_0(2a)/\left[r(r+2a)\right]$	$V_0(2a)/r^2$
0	$V_0 e^{-2r/2a}/r$	$V_0(2a)^2/\left[r(r+2a)^2\right]$	$V_0(2a)^2/r^3$
$1/2$	$V_0 e^{-3r/2a}/r$	$V_0(2a)^3/\left[r(r+2a)^3\right]$	$V_0(2a)^3/r^4$
1	$V_0 e^{-4r/2a}/r$	$V_0(2a)^4/\left[r(r+2a)^4\right]$	$V_0(2a)^4/r^5$
$3/2$	$V_0 e^{-5r/2a}/r$	$V_0(2a)^5/\left[r(r+2a)^5\right]$	$V_0(2a)^5/r^6$
2	$V_0 e^{-6r/2a}/r$	$V_0(2a)^6/\left[r(r+2a)^6\right]$	$V_0(2a)^6/r^7$
...			



E[MeV]
$V(r)$ [MeV]
1.0×10^{0}
1.0×10^{-2}
1.0×10^{-4}
1.0×10^{-6}
1.0×10^{-8}
1.0×10^{-10}
1.0×10^{-12}
-1.0×10^{-6}
-1.0×10^{-4}
-1.0×10^{-2}
-1.0×10^{0}
-1.0×10^{2}
$n=60$
$n=50$
$n=40$
$n=30$
$n=20$
$n=10$
$n=5$
r[fm]
10^{-1}
10^{1}
10^{3}
10^{5}

第１章

Efimov effect (エフィモフ効果)と 長距離ハドロンポテンシャルの予言

Review of Efimov-effect

Efimov V. Energy levels arising from resonant two-body forces in a three-body system, Phys. Lett**.** **B33**, 563 (1970)

Efimov V. Energy levels of three resonantly interacting particles, Nucl. Phys. **A210** 157 (1973)

1) the scattering length of the sub-system should be $a \to \infty$ (**the first criterion**)

2) three-body binding energies condense on the three-body break-up threshold (3BT) where the energy level structure is given by

$E_n / E_{n+1} = \text{constant} > 1$ (n: quantum number)
(the second criterion)

3) energy level can be obtained by r^{-2} potential (the third criterion)

Nicholson A.F., Bound states and scattering in an r^{-2} potential Australian J. Phys **15**, 174-179 (1962)

Review of Efimov-effect

Efimov V. Energy levels arising from resonant two-body forces in a three-body system, Phys. Lett**.** **B33**, 563 (1970)

Efimov V. Energy levels of three resonantly interacting particles, Nucl. Phys. **A210** 157 (1973)

Kraemer T. et al., Evidence for Efimov quantum states in an ultracold gas of caesium atoms. **Nature** vol. **440**, pages **315**–318 (**2006**)

In the hadron systems:

1) The first criterion : $a_{NN} \neq \infty$, $a_{N\pi} \neq \infty$

ほぼ考えられない！

2) The second criterion is that there are some instances that energy levels come near the threshold region. However, it is very hard to confirm whether they are Efimov levels or not.

エネルギー0近傍では実験的検証が困難！

3) The third criterion is that the nuclear potential is usually a short-range: one pion exchange Yukawa potential etc.

湯川ポテンシャルは短距離力である！

$1/r^2$ のようなポテンシャルは考えられない。

I. General particle transfer (GPT)-potential

We reevaluate the Efimov physics

by the thresholds.

1) Pay attention to the three-body break-up thresholds (3BT) : appear in reactions:

$$d + p \rightarrow n + p + p,$$

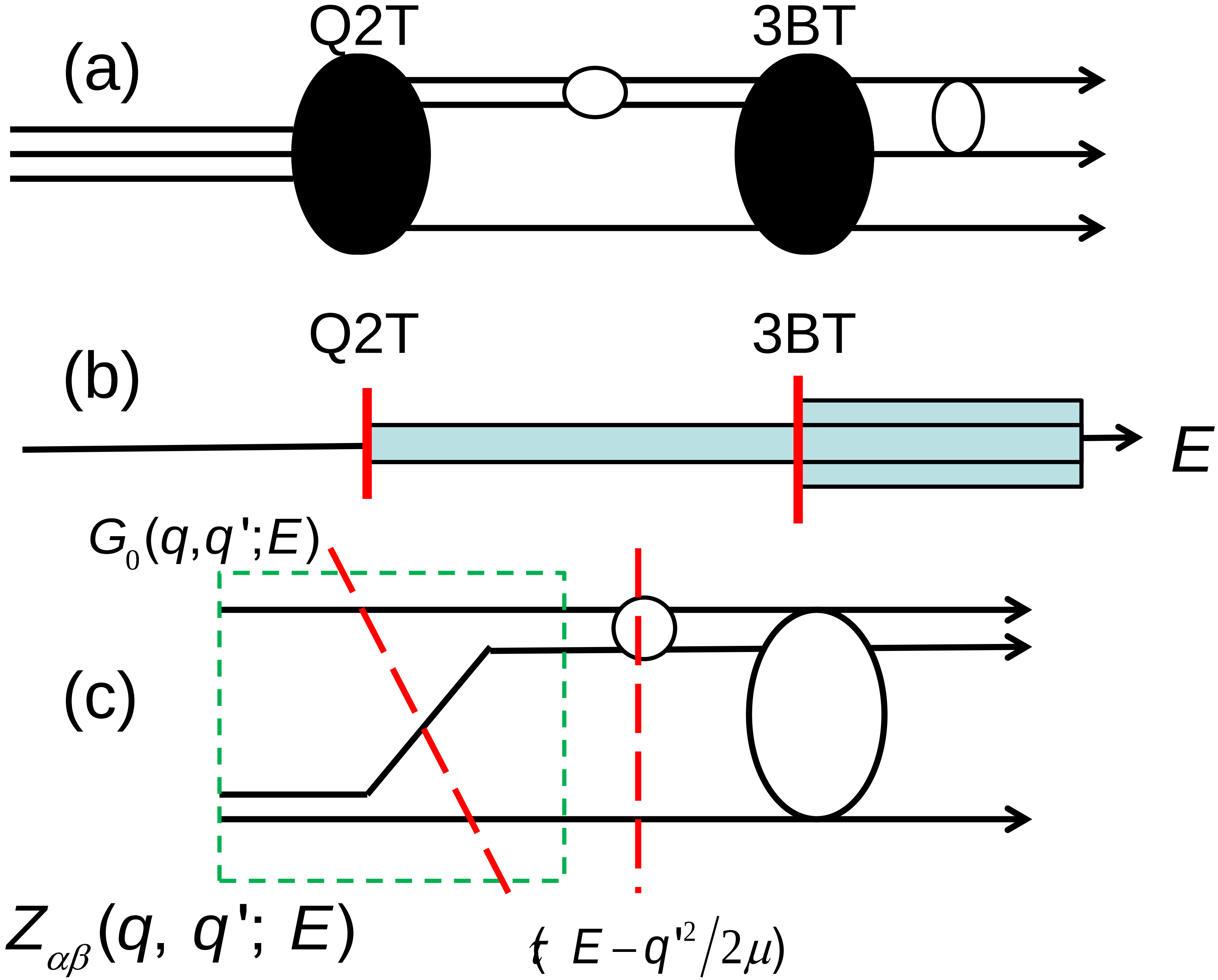
$$N + N' \rightarrow N + N + \pi, \ etc.$$

2) From 3-body bound state to the quasi two-body system:

quasi two-body threshold (Q2T) :

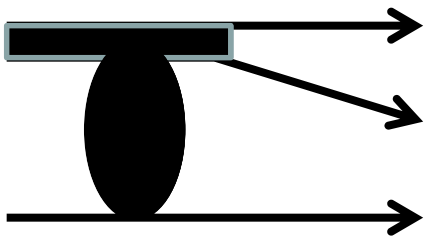
$$^{3}He \rightarrow d + p,$$

$$D \rightarrow N + N' \equiv N + (N\pi), \quad etc.$$

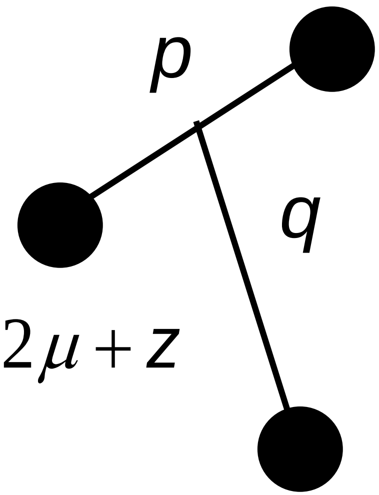


(a)
Q2T
3BT
(b)
Q2T
3BT
E
(c)
$G_0(q,q';E)$
$Z_{\alpha\beta}(q, q'; E)$
$t(E - q'^2/2\mu)$

1) At the 3BT,



the Born term Z of the Faddeev or the Alt-Grassberger-Sandhas (AGS) equation, and the propagator have singularities;



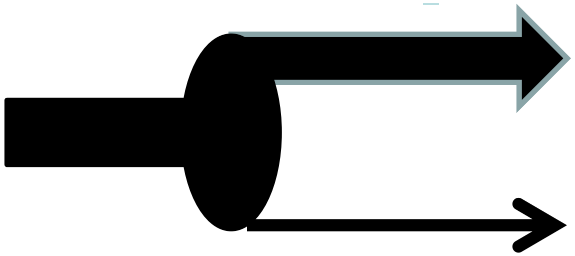
At 3BT ($E=0,\, q=p=0$) : by using $E = q^2/2\mu + z$

$$Z_{\alpha\beta}(q,\, q';\, E) = \frac{g_\alpha(p)g_\beta(p')(1-\delta_{\alpha\beta})}{E - q^2/2\mu - p^2/2\nu} \rightarrow \infty$$

$$t(z) = t(E - q^2/2\mu) = \frac{f(z)}{\varepsilon_B + z} \Rightarrow \frac{f(z)}{z} = \frac{f(z)}{E - q^2/2\mu} \rightarrow \infty.$$

$$\text{or}\quad t(z) \propto \frac{1}{-1/a - ik} \rightarrow \lim_{a\to\pm\infty} i\frac{\sqrt{2\nu}}{\sqrt{E - q^2/2\mu}} \rightarrow i\infty.$$

2) At the Q2T :



a) Propagator : at Q2T, with $E = q^2/2\mu + z$

$$\tau_B(z) = \frac{f(z)}{\varepsilon_B + z} = \frac{f(z)}{(\varepsilon_B + E) - q^2/2\mu} \rightarrow \infty$$

for $E_{cm} = E + \varepsilon_B = 0, \quad q = 0$

Apart from AGS, an Energy dependent Two - body Quasi(E2Q) potential with two - body bound state (or $a \neq \infty$) becomes by using on - shell condition for Q2T.

Numerical calculation for GPT- Efimov-like potential

n	E_n	E_n/E_{n+1}	$<r_n^2>^{1/2}$	$<r_n^2>^{1/2}/<r_{n+1}^2>^{1/2}$
1	-2.222		2.516	
2	-1.271×10^{-2}	174.8	3.652×10^{1}	14.52
3	-7.433×10^{-5}	171.0	4.812×10^{2}	13.18
4	-4.347×10^{-7}	171.0	6.296×10^{3}	13.08
5	-2.543×10^{-9}	171.0	8.233×10^{4}	13.08
6	-1.487×10^{-11}	171.0	1.077×10^{6}	13.08
7	-8.697×10^{-14}	171.0	1.408×10^{7}	13.08
8	-5.087×10^{-16}	171.0	1.841×10^{8}	13.08
9	-2.975×10^{-18}	171.0	2.407×10^{9}	13.08
10	-1.740×10^{-20}	171.0	3.147×10^{10}	13.08

Our analytic prediction fits to the numerical solution.