Few-body physics: getting more effective

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Background

Over last $\sim$ 20 years: sea change in few-body physics

- ideas of universality, the renormalisation group and effective field theory have become central [König; Endo]
- demonstrated that EFT could be renormalised nonperturbatively (contrast to chiral perturbation theory for mesons, single baryons)
Identified nontrivial fixed point of the renormalisation group

- scale-free system $\rightarrow$ universal features
  (no “memory” of underlying physics)
- describes low-energy scattering of two non-relativistic particles
  with infinite scattering length, $a \rightarrow \infty$: unitary limit
- EFT for systems with scattering lengths much larger than range of interactions
- assumes good separation of scales: deviations from universality
  expanded in powers of ratios of low-energy scales to scales of
  underlying physics (“power counting”)
  $\rightarrow$ systematic calculations with estimates of errors
"Pionless" EFT (only short-range interactions)

Two-body systems

- theory of nonrelativistic particles with contact interactions, builds on effective-range expansion
  [Blatt and Jackson, Phys Rev 76 (1949) 18; Bethe, Phys Rev 76 (1949) 38]
- provides consistent effective operators for currents etc
- can be extended to include Coulomb potential (DW ERE)

Some lessons from EFT

- off-shell forms of potentials are not physics
- different choices of two-body potential compensated by use of consistent effective operators, many-body forces
- many-body forces, in particular three-body, are unavoidable
  [Sekiguchi; …]
Three-body systems

- three-body force needed at leading order (marginal perturbation)

$nd$ scattering length against $^3$H binding energy for model $NN$ potentials, compared with LO and NLO pionless EFT [from: Bedaque et al, Nucl Phys A714 (2003) 589]
Also describes Efimov effect [Efimov, Sov J Nucl Phys 12 (1971) 589]

- infinite tower of geometrically spaced bound states in unitary limit
- universal ratio: discrete relic of scale invariance (marginal, cycle)
- three-body force fixes energies [Kievsky]

Four- and more-body systems

- no further forces needed at leading order
- two four-body states below each Efimov three-body state
- explains Tjon line: correlation between $^3$H and $^4$He energies
  [Tjon, Phys Lett B56 (1975) 217 ]
- matches quite well two $0^+$ states of $^4$He [König]
- similar pairs of states found for $N \geq 4$ particles
- rely on impressive developments in computational methods
  [Lazauskas; Kievsky; Deltuva; . . . ]
Ultra-cold atoms in traps

- beautiful realisations of this physics [Chin; Endo]
- Feshbach resonances used to tune scattering length $\gg$ range
  ($\rightarrow$ really good separation of scales)
- first Efimov state observed in 2006
- periodic with Efimov factor 22.7
  [Huang et al, Phys Rev Lett 112 (2014) 190401]
- then four- and five-body states
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$^4$He atom clusters

- really shallow dimer: 140 neV [Dörner]
- spatial distribution imaged by laser ionisation
  [Zeller et al, PNAS 113 (2016) 14651]
- Efimov trimer observed [Kunitski et al, Science 348 (2015) 551]
Nuclear physics

Pionless EFT valid only for very low-energy states

- eg very weakly bound nuclei, halo $\gg$ core: “halo” or “cluster” EFT
  [Mathis; Carbonell; Filandri]

Higher energies require pion-exchange forces: chiral EFTs

- KSW added these perturbatively, but . . .
- expansion fails to converge in $S$ and $P$ waves
- common work-around: “Weinberg prescription”
- expand potential using perturbative counting and then treat it nonperturbatively [Epelbaum et al, Rev Mod Phys 81 (2009) 1773]
- works quite well in practice [Golak; Viviani], but . . .
- no guarantee of consistent counting for observables
- alternative power countings have been proposed
Recent developments

Pionless EFT: no leading four-body force, but exact order unknown
  • needed at same order as effective range (NLO)

Coulomb potential in pionless EFT
  • universal effects in few-body systems being studied
    [Schmickler et al, arXiv:1904.00913]

Chiral EFTs
  • counting of three-body interactions in presence of pion exchange
    (and other long-range two-body forces)

Weinberg approach
  • now implemented to 5th order and beyond
    [Reinert et al, Eur Phys J A54 (2018) 86; Golak; Reinert]
Hadronic molecules

- $X(3872)$ less than 200 keV from $D^0\bar{D}^{*0}$ threshold
- other states like $Z_c(3900)$, $Z_c(4020)$...close to $D\bar{D}$ or $B\bar{B}$ thresholds
- also pentaquarks $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ near $\bar{D}\Sigma_c$
- could be molecules or have large molecular components
  
  (universal features if close enough to threshold)

Cluster scattering and reactions

- resonating group method for scattering in pionless EFT
  [Naidon et al, J Phys B 49 (2016) 034002]
- $(d,p)$ transfer reaction in halo EFT
Lessons from EFT

Potentials and wave functions are not observables
• depend on theorists’ choices (off-shell form, field representation)
Instead, focus on quantities that are “theory independent”:
• asymptotic normalisation coefficients, not spectroscopic factors
• Tan’s “contact” terms, not short-range correlations
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Many-body forces expected in any effective theory

- Phillips line in $^6\text{Li}$: hint of three-body force?
  [Lei et al, Phys Rev C98 (2018) 051001; Elster]
- three-body forces in transfer reactions [Dinmore et al, Phys Rev C99 (2019) 064612; Dinmore]
Nuclear reactions

Would like an effective theory for nuclear reactions

- but issues with separation of scales
- inverse core radii are small, $1/R_c \lesssim 60$ MeV $\sim$ low-energy scale?
- Coulomb scale can be large, $\alpha Z_1 Z_2 M_{\text{red}} \sim 140$ MeV ($p + ^{40}\text{Ca}$) $\rightarrow$ contact interactions ineffective
- cannot apply halo EFT to most reactions, even at low energies $\rightarrow$ perhaps add $1/R_c$ to list of low-energy scales?
- $\delta$-shell potentials, finite-$r$ boundary conditions often used as regulators, but perhaps we should take them more seriously
- for example: use a version of $R$-matrix theory? [Tennyson]
Many areas of few-body physics with no good separation of scales → ideas of universality and effective field theory do not apply

But where they do...
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We should strive to be as effective as possible!