Partial wave mixing and $\rho\ \pi$ scattering

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Scattering involving hadrons with non-zero spin

Most hadrons appear as resonances.

All but the simplest couple to final states (or intermediate states) containing hadrons with non-zero spin.

E.g. $a_1(1260) \rightarrow \rho\,\pi \rightarrow \pi\,\pi\,\pi$ $\quad$ $b_1(1235) \rightarrow \omega\,\pi \rightarrow \pi\,\pi\,\pi\,\pi$

$X(3872) \rightarrow D\bar{D}^* \rightarrow D\bar{D}\pi$ $\quad$ $Z_c(3900) \rightarrow J/\psi\,\pi$

Nucleon resonances, *N N*, ...

$J = \ell \otimes S$ (total ang. mom. = orbital ang. mom. $\otimes$ spin)

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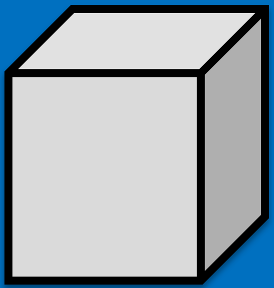
This talk: isospin-2 ρ π scattering (with a stable ρ)

Antoni's talk: b_1 in isospin-1 ω π

Scattering from lattice QCD – overview

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$



Large bases of interpolating operators and variational method (GEVP)

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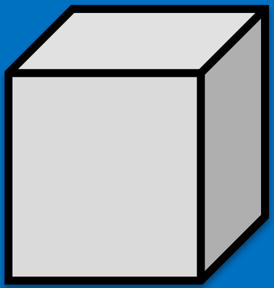


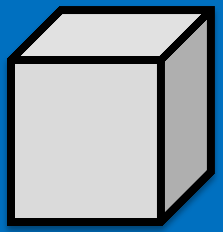
Large bases of interpolating operators and variational method (GEVP)

Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**.

See earlier talks (Max Hansen, Jo Dudek)

Reduced symmetry

Finite cubic lattice with periodic b.c.s – **reduced sym**.



$\vec{p} = \vec{0}$: 3D rotation group

→ octahedral group O_h^D

Finite number of *irreps* Λ:

A_1, A_2, T_1, T_2, E

(+ others for

half-integer spin)

Λ^P	J^P
$A_1^{\pm}$	$0^{\pm},\ 4^{\pm},\ \ldots$
$T_1^{\pm}$	$1^{\pm},\ 3^{\pm},\ 4^{\pm},\ \ldots$
$T_2^{\pm}$	$2^{\pm},\ 3^{\pm},\ 4^{\pm},\ \ldots$
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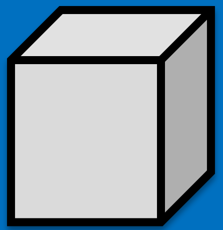

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$\vec{p} \neq \vec{0}$

Little group: $\mathrm{LG}(\vec{p}) \subset \mathrm{O}_h^{\mathrm{D}} \quad \left\{ R \in \mathrm{O}_h^{\mathrm{D}} \,|\, R\vec{p} = \vec{p} \right\}$

Finite-volume spectra

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

For each $\vec{P}$, Λ: large basis of interpolating operators with appropriate structures; variational method (GEVP)

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

$$\lambda^{(n)}(t) \sim e^{-E_n(t-t_0)} \qquad v_i^{(n)} \to Z_i^{(n)} \equiv \langle 0 | \mathcal{O}_i | n \rangle$$

($t >> t_0$)

Single-meson operators

[PRL 103, 262001; PR D82, 034508; D84, 074508; similarly for baryons PR D85, 014507]

Fermion-bilinear 'single-meson' operators:

$$\mathcal{O}^{J^P,M}(\vec{p},t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\, \bar{\psi}(x) \left[\Gamma \times \overleftrightarrow{D} \times \overleftrightarrow{D} \ldots\right] \psi(x)$$

Circular basis for *D* and Γ, couple using SU(2) Clebsch-Gordans → J^P, M

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'Subduce' operators into lattice irrep (Λ), row (μ):

$$\left[\mathcal{O}^{[J]}_{\Lambda^P,\mu}(\vec{0})\right]^\dagger = \sum_M \mathcal{S}^{J,M}_{\Lambda,\mu}\left[\mathcal{O}^{J^P,M}(\vec{0})\right]^\dagger$$

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For $\vec{p} \neq \vec{0}$ 'helicity ops':

$$\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_M \mathcal{D}^{(J)}_{M\lambda}(R) \left[\mathcal{O}^{J^P,M}(\vec{p})\right]^\dagger \qquad R:\ (0,0,|\vec{p}|) \to \vec{p}$$

Subduce → little group irreps

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Subduce → little group irreps

Many ops in each channel (up to 3 derivs at rest, 2 at non-zero mom.)

Variationally optimised meson ops

For each $\vec{p}$, Λ: variational analysis with large basis of fermion-bilinear ops.
n'th eigenvector → optimal operator for *n*'th state

$$\Omega^{(n)\dagger} \sim \sum_i v_i^{(n)} O_i^\dagger$$

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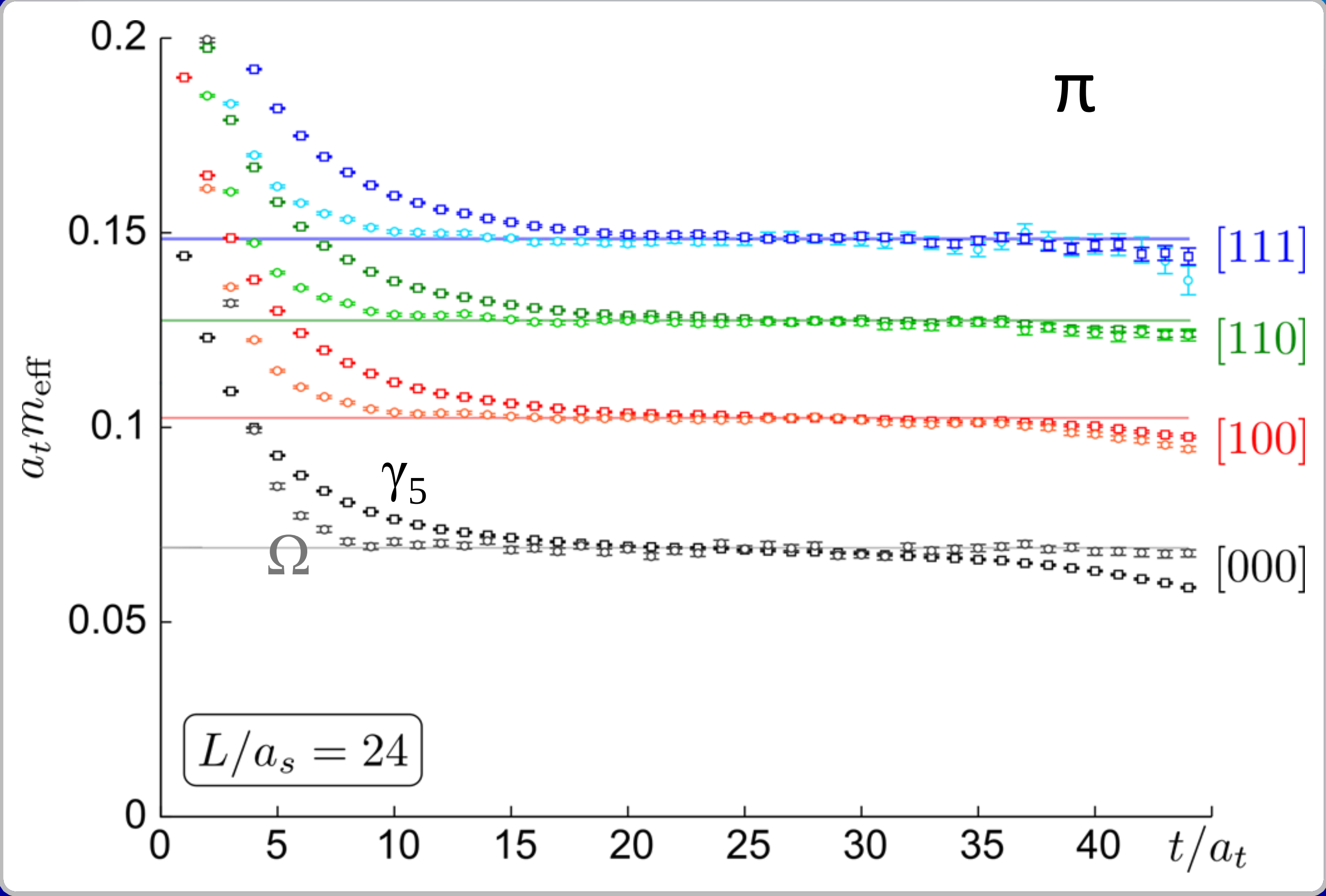

$m_\pi \approx 400$ MeV

[PR D86, 34031 (2012)]

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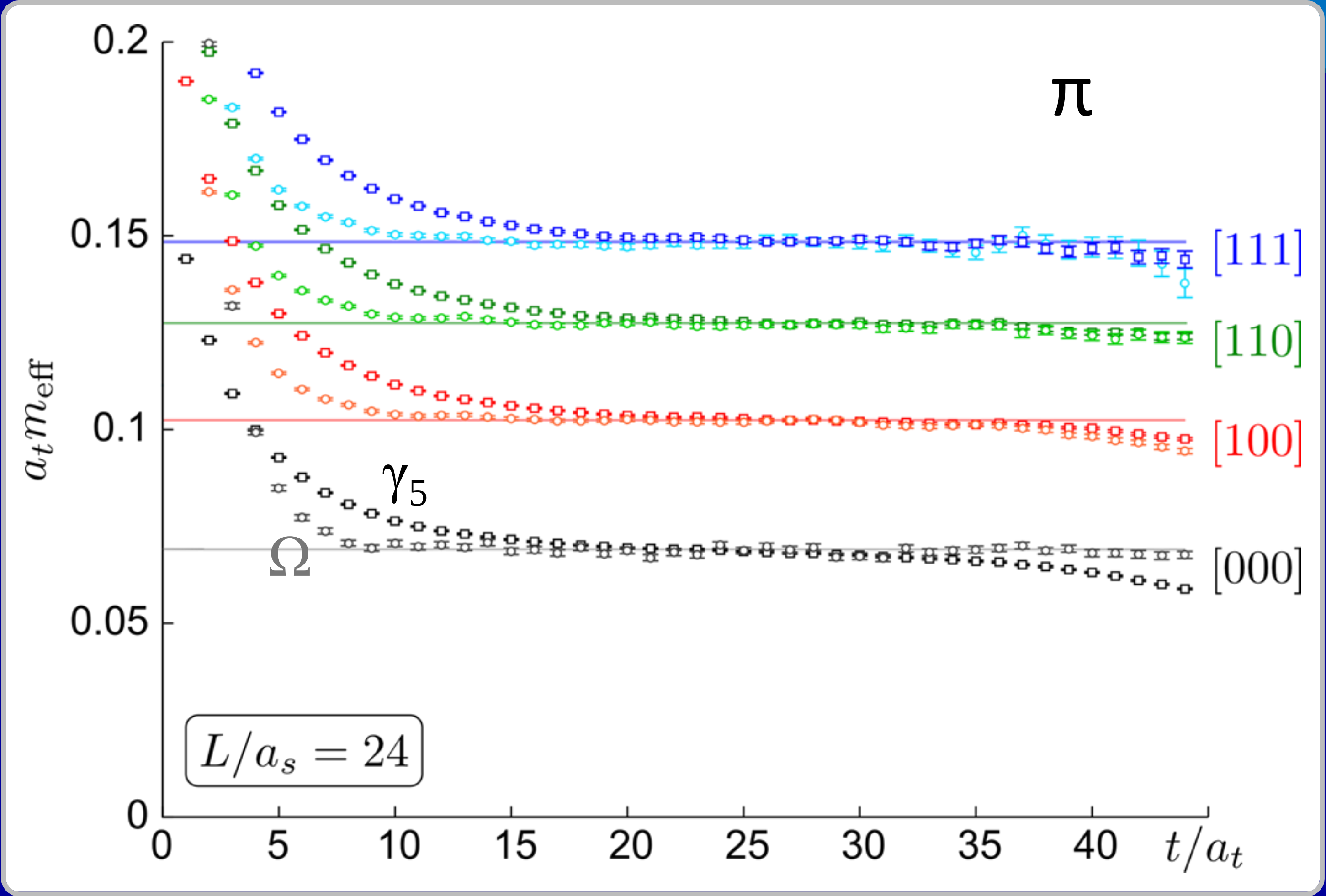

Could have multi-hadron ops

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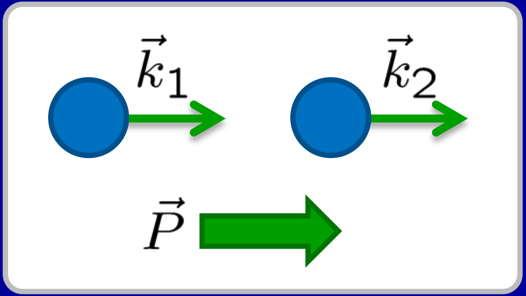
[PR D86, 34031 (2012)]

Hadron-hadron (or multi-hadron) operators

$$\mathbb{O}^{\dagger}_{\Lambda,\mu}(\vec{P}) = \sum_{\mu_1,\mu_2} \sum_{\vec{k}_1,\vec{k}_2} C(\vec{P},\Lambda,\mu;\vec{k}_1,\Lambda_1,\mu_1;\vec{k}_2,\Lambda_2,\mu_2)\, \mathbb{O}^{\dagger}_{\Lambda_1\mu_1}(\vec{k}_1)\, \mathbb{O}^{\dagger}_{\Lambda_2\mu_2}(\vec{k}_2)$$



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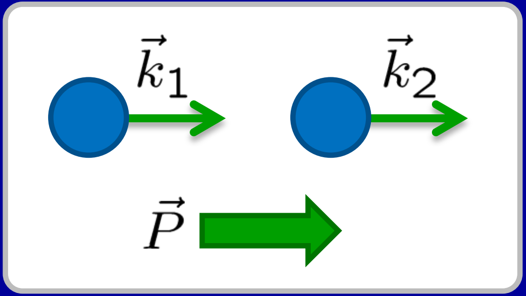
Could be: simple fermion-bilinears, optimised ops, multi-hadron ops, ...



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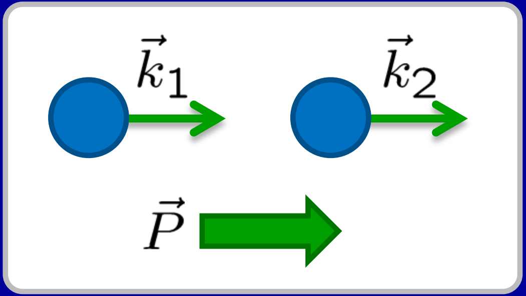
'Generalised Clebsch-Gordans' $\Lambda_1 \otimes \Lambda_2 \to \Lambda$

$\Lambda_1 \in \mathrm{LG}(\vec{k}_1),\ \Lambda_2 \in \mathrm{LG}(\vec{k}_2),\ \Lambda \in \mathrm{LG}(\vec{P})$

(calculate using induced representation)



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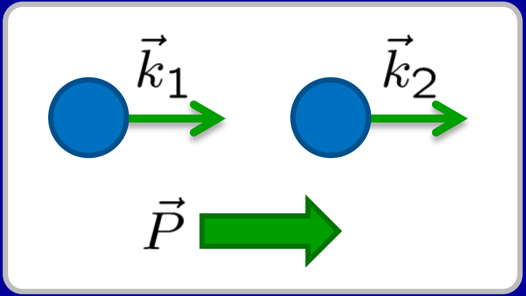

Sum over all $\vec{k}_1,\ \vec{k}_2$ related by allowed lattice rot. such that $\vec{P} = \vec{k}_1 + \vec{k}_2$
($R\,\vec{k}_{1,2}\ \forall\, R \in \mathsf{LG}(\vec{P})$)

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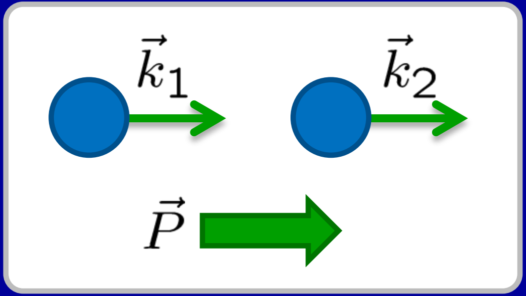

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Why this approach?

- Don't mix different Λ_i
- Can use optimised single-hadron ops
- Can iteratively construct >2 hadron ops

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Compare continuum formulation:

$$\mathcal{O}^{\dagger[S,\ell]}_{J,M} \sim \sum_{\lambda_1\lambda_2} \int d\hat{p}\, C(J,\ell,S,M;\vec{p},S_1,\lambda_1;-\vec{p},S_2,\lambda_2)\, \mathcal{O}^{\dagger S_1\lambda_1}(\vec{p})\, \mathcal{O}^{\dagger S_2\lambda_2}(-\vec{p})$$

$$C = \langle S_1,\lambda_1;S_2,-\lambda_2|S,\lambda\rangle\langle \ell,0;S,\lambda|J,\lambda\rangle\, \mathcal{D}^{(J)*}_{M\lambda}(\hat{p})$$

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Scattering from lattice QCD

Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**.

$$\det\left[1 + i\,\rho(E_{\rm cm})\,t(E_{\rm cm})\left(1 + i\,\mathcal{M}^{\vec{P}}(E_{\rm cm}, L)\right)\right] = 0$$

Infinite-volume scattering *t*-matrix

effect of finite volume (including reduced sym.)

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Subduce: block-diagonalise *M* into irreps

ρπ isospin-2 scattering

[**Woss**, CT, Dudek, Edwards, Wilson, JHEP 1807, 043 (2018)]

Different partial waves with the same J^P can mix dynamically. E.g. $J^P = 1^+$ ($^{2S+1}\ell_J = {}^3S_1, {}^3D_1$), $J^P = 2^-$ ($^3P_2, {}^3F_2$)

$J^P = 1^+$

$$\mathbf{t} = \begin{bmatrix} t(^3S_1|^3S_1) & t(^3S_1|^3D_1) \\ t(^3S_1|^3D_1) & t(^3D_1|^3D_1) \end{bmatrix}$$

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Finite-volume lattice QCD calculations: reduced symmetry →additional 'mixing'

[0,0,0]	T_1^+
$J^+({}^3\ell_J)$	$1^+ \begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$ $3^+ \begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$ $4^+ \begin{pmatrix} {}^3G_4 \end{pmatrix}$

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[0,0,0]	T_1^+
$J^+(^3\ell_J)$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$ $3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$ $4^+ \ (^3G_4)$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

[00n] Λ	A_1	A_2	E
$J^P({}^3\ell_J)$		0^- $({}^3P_0)$	
		1^+ $\begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$	1^+ $\begin{pmatrix} {}^3S_1 \\ {}^3D_1 \end{pmatrix}$
	1^- $({}^3P_1)$		1^- $({}^3P_1)$
	2^+ $({}^3D_2)$		2^+ $({}^3D_2)$
		2^- $\begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$	2^- $\begin{pmatrix} {}^3P_2 \\ {}^3F_2 \end{pmatrix}$
		3^+ $\begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}$	3^+ $\begin{pmatrix} {}^3D_3 \\ {}^3G_3 \end{pmatrix}_{[2]}$
	3^- $({}^3F_3)$		3^- $({}^3F_3)_{[2]}$
	4^- $\begin{pmatrix} {}^3F_4 \\ {}^3H_4 \end{pmatrix}$	4^- $\begin{pmatrix} {}^3F_4 \\ {}^3H_4 \end{pmatrix}_{[2]}$	4^- $\begin{pmatrix} {}^3F_4 \\ {}^3H_4 \end{pmatrix}_{[2]}$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

Calc. at $SU(3)_F$ symmetric point (m_u=m_d=m_s)

$m_\pi \approx$ 700 MeV

$m_\rho \approx$ 1020 MeV (stable)

Large bases of ρπ $SU(3)_F$ operators:

constructed from 'optimised' ρ and π ops $\sim \bar{\psi}\Gamma D \ldots \psi$

Use 'distillation' [PR D80 054506 (2009)]

One lattice spacing, 2 volumes ($L \approx$ 2, 3 fm; $m_\pi L \approx$ 10, 12)

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

$1\ T_1^+$

$\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

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$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

1 T_1^+

$\rho[001]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow E_2$, $\pi[001] \rightarrow A_2$

$[001]\ A_1 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, A_1^-, E^-$

$[001]\ E_2 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, T_2^+, T_1^-, T_2^-$

2 T_1^+

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

ρ[000] → T_1^-, π[000] → A_1^-

[000] T_1^- x [000] A_1^- → [000] $\mathbf{T_1^+}$

1 T_1^+

ρ[001] λ=0 → A_1, λ=±1 → E_2, π[001] → A_2

[001] A_1 x [001] A_2 → [000] $\mathbf{T_1^+}$, A_1^-, E^-

[001] E_2 x [001] A_2 → [000] $\mathbf{T_1^+}$, T_2^+, T_1^-, T_2^-

2 T_1^+

$$E_{\text{n.i.}} = \sqrt{m_\rho^2 + |\vec{k}_\rho|^2} + \sqrt{m_\pi^2 + |\vec{k}_\pi|^2}$$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

$1\ T_1^+$

$\rho[001]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow E_2$, $\pi[001] \rightarrow A_2$

$[001]\ A_1 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, A_1^-, E^-$

$[001]\ E_2 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, T_2^+, T_1^-, T_2^-$

$2\ T_1^+$

$\rho[011]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow B_1, B_2$, $\pi[011] \rightarrow A_2$

$[011]\ A_1 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

$[011]\ B_1 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

$[011]\ B_2 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

$3\ T_1^+$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\, T_1^- \times [000]\, A_1^- \rightarrow [000]\, \mathbf{T_1^+}$

1 T_1^+

$\rho[001]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow E_2$, $\pi[001] \rightarrow A_2$

$[001]\, A_1 \times [001]\, A_2 \rightarrow [000]\, \mathbf{T_1^+}, A_1^-, E^-$

$[001]\, E_2 \times [001]\, A_2 \rightarrow [000]\, \mathbf{T_1^+}, T_2^+, T_1^-, T_2^-$

2 T_1^+

$\rho[011]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow B_1, B_2$, $\pi[011] \rightarrow A_2$

$[011]\, A_1 \times [011]\, A_2 \rightarrow [000]\, \mathbf{T_1^+}, \ldots$

$[011]\, B_1 \times [011]\, A_2 \rightarrow [000]\, \mathbf{T_1^+}, \ldots$

$[011]\, B_2 \times [011]\, A_2 \rightarrow [000]\, \mathbf{T_1^+}, \ldots$

3 T_1^+

Overall non-zero momentum

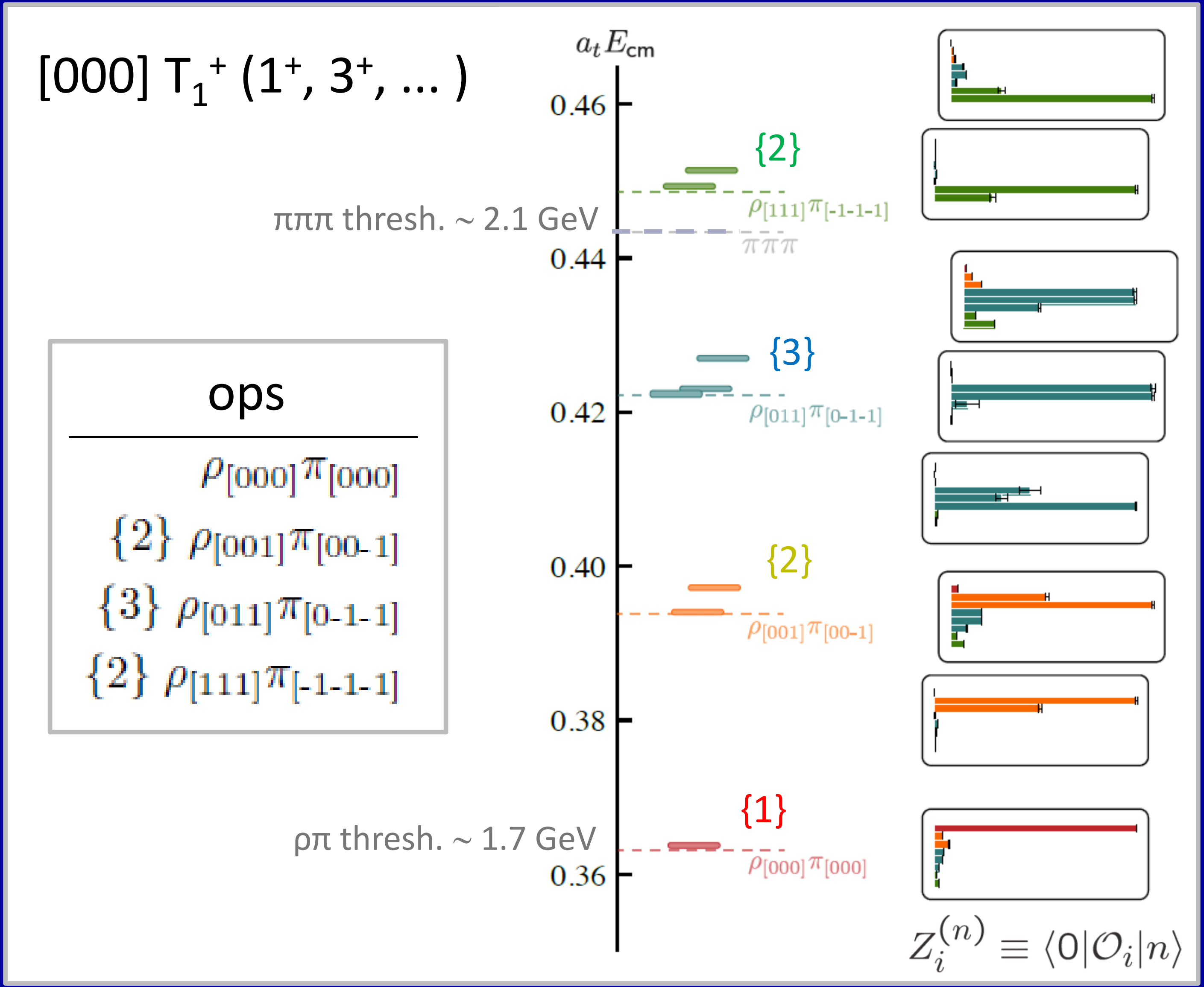
$[001]\, A_1 \times [011]\, A_2 \rightarrow [001]\, \mathbf{E_2}, \ldots$

$[001]\, E_2 \times [011]\, A_2 \rightarrow [001]\, \mathbf{E_2}, \mathbf{E_2}, \ldots$

3 E_2

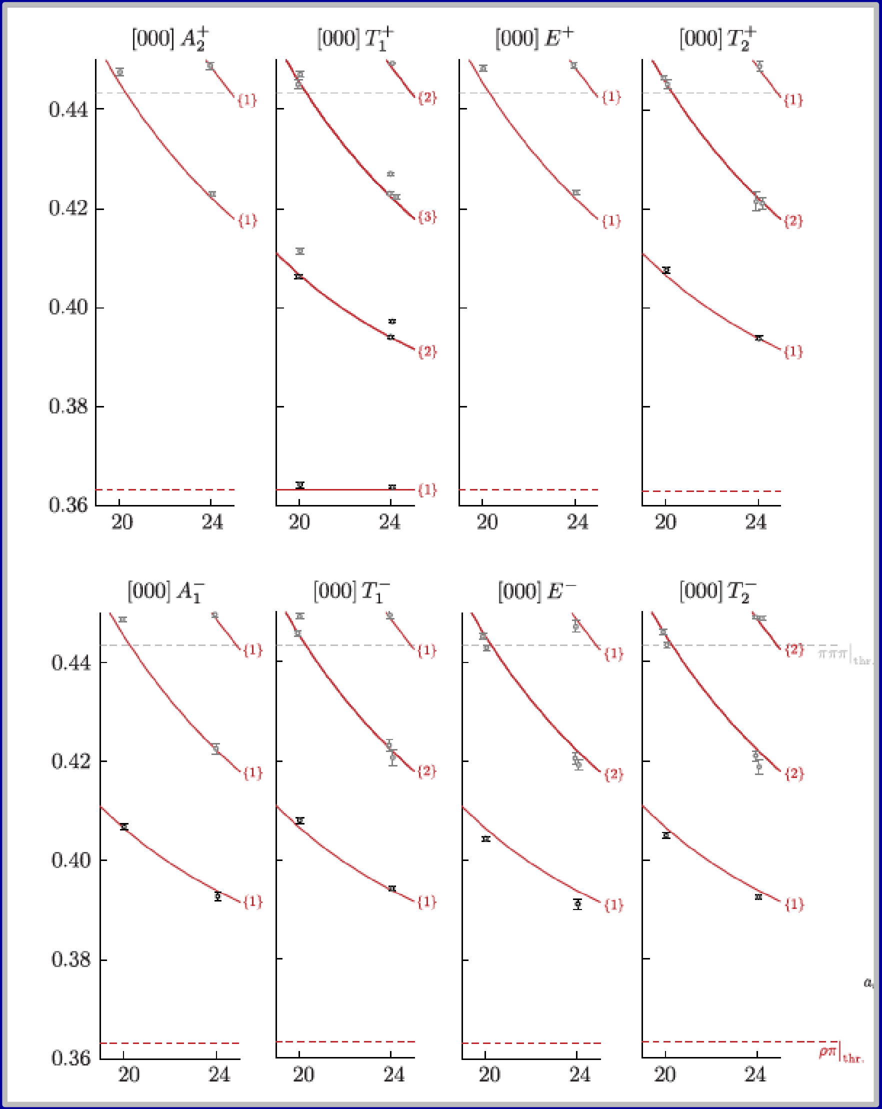
ρπ isospin-2 scattering – spectra

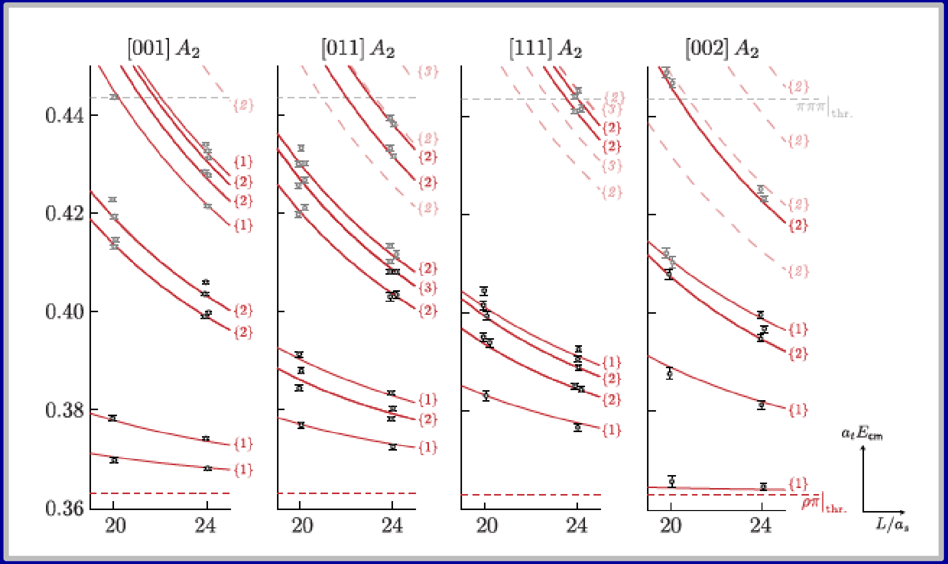
[JHEP 1807, 043 (2018)]



ρπ isospin-2 scattering – spectra

[JHEP 1807, 043 (2018)]





+ others

Used 141 energy levels for ℓ = 0, 1, 2

ρπ isospin-2 scattering – amplitudes

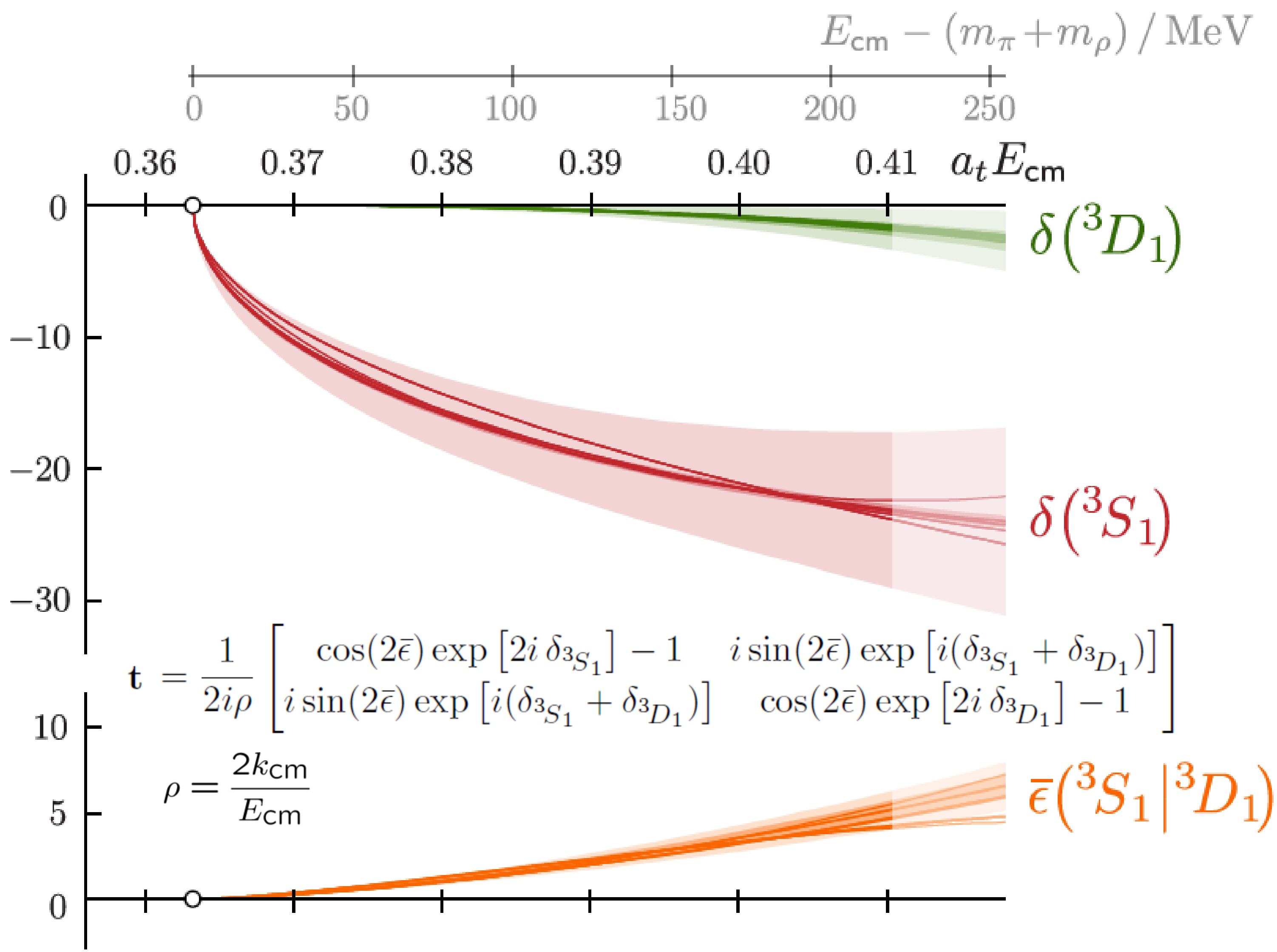
[JHEP 1807, 043 (2018)]

$$\left[t^{-1}(s)\right]_{\ell J,\ell' J} = \frac{1}{(2k_{\mathsf{cm}})^{\ell}} \left[K^{-1}(s)\right]_{\ell J,\ell' J} \frac{1}{(2k_{\mathsf{cm}})^{\ell'}} + \delta_{\ell\ell'}\, I(s)$$

$$K_{\ell J,\ell' J}(s) = \sum_{n\geq 0}^{N(^3\ell_J|^3\ell'_J)} c_n(^3\ell_J|^3\ell'_J)\, s^n$$

ρπ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]

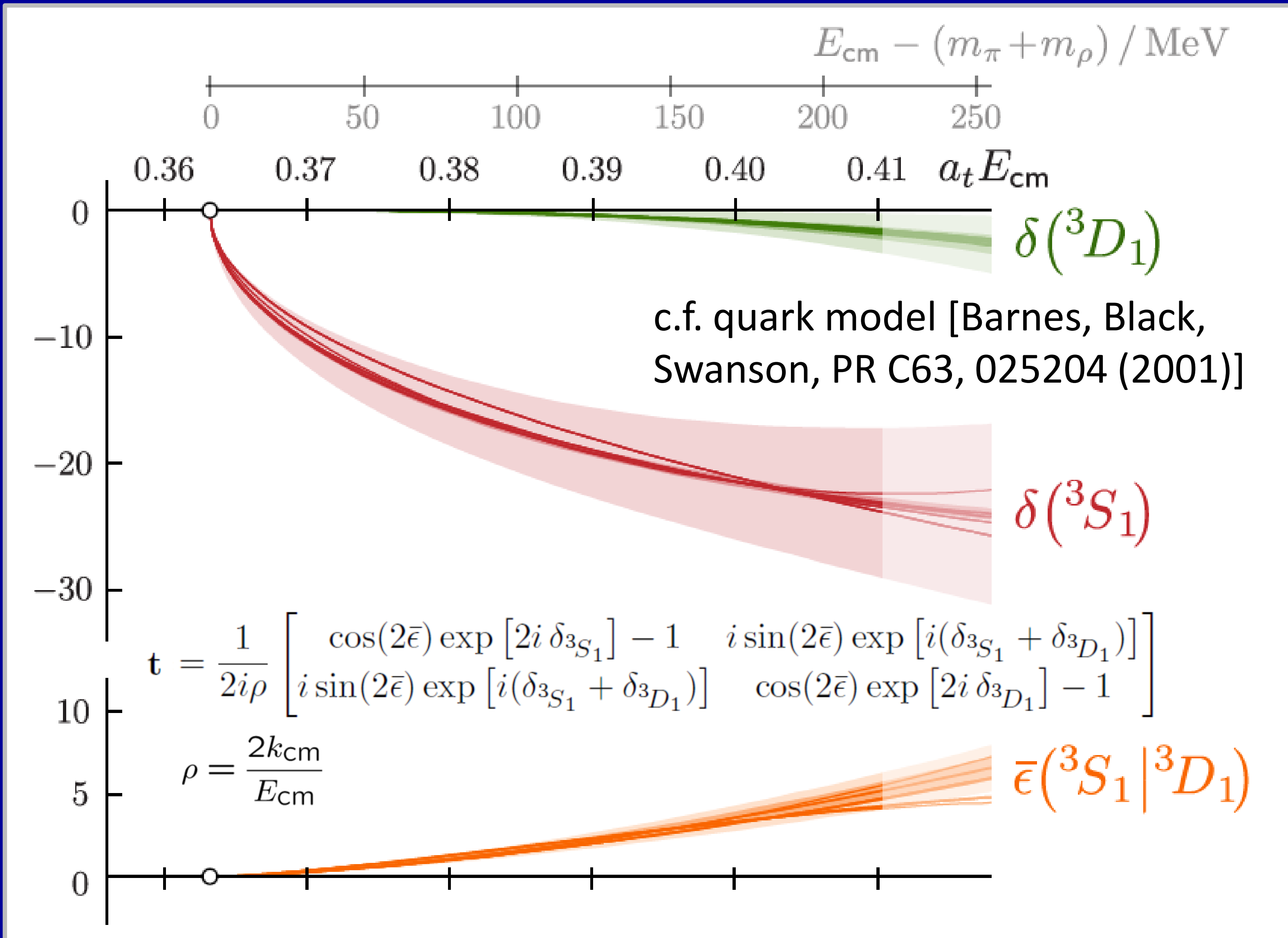


$$\mathbf{t} = \frac{1}{2i\rho}\begin{bmatrix} \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3S_1}\right]-1 & i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1}+\delta_{^3D_1})\right] \\ i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1}+\delta_{^3D_1})\right] & \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3D_1}\right]-1 \end{bmatrix}$$

$$\rho = \frac{2k_{cm}}{E_{cm}}$$

ρπ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]



c.f. quark model [Barnes, Black, Swanson, PR C63, 025204 (2001)]

$$\mathbf{t} = \frac{1}{2i\rho}\begin{bmatrix} \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3S_1}\right] - 1 & i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1} + \delta_{^3D_1})\right] \\ i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1} + \delta_{^3D_1})\right] & \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3D_1}\right] - 1 \end{bmatrix}$$

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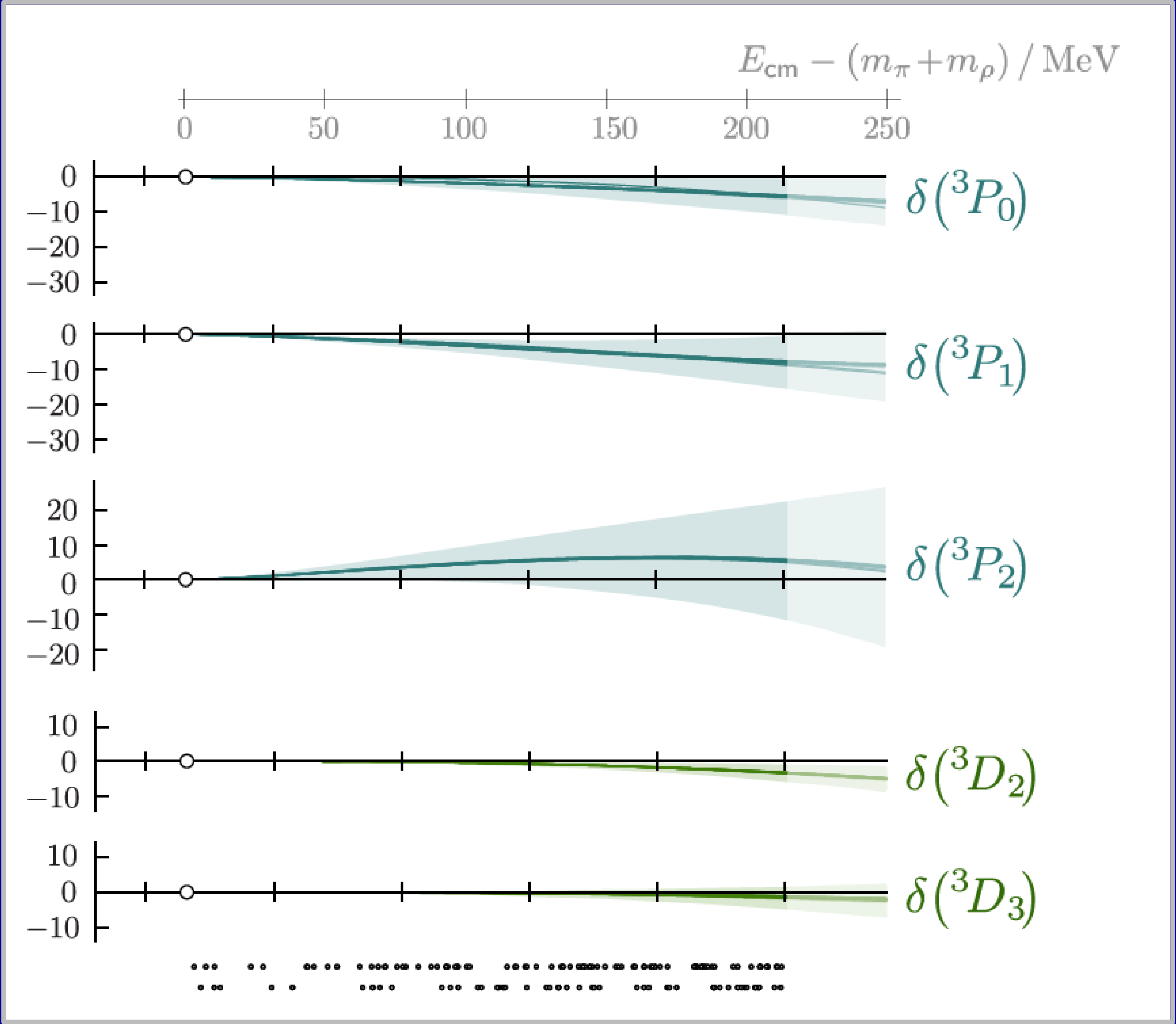
ρπ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]

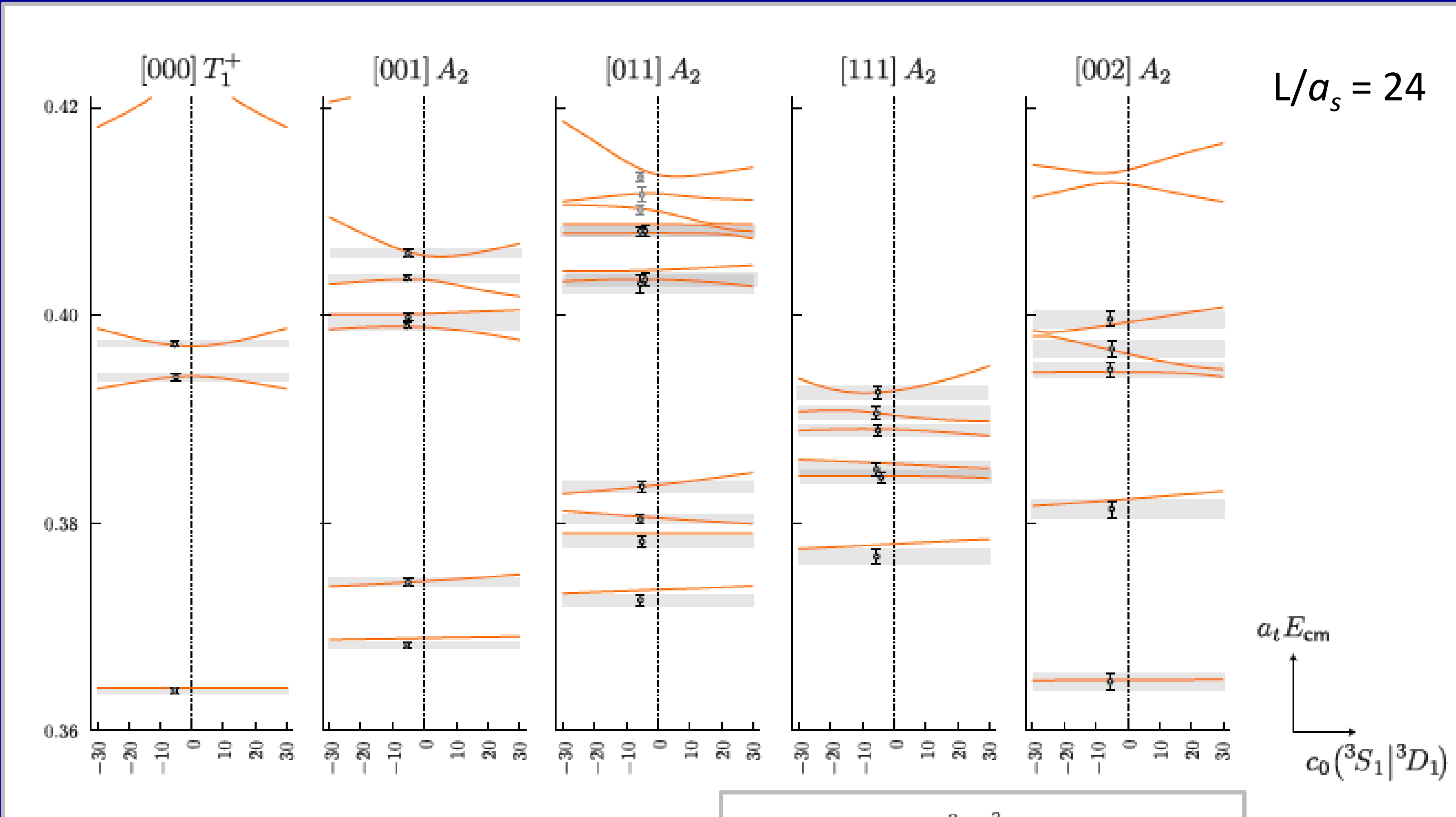


ρπ isospin-2 scattering – $^3S_1$,$^3D_1$ mixing angle

$$K_{\ell J,\ell' J}(s) = \sum_{n\geq 0}^{N(^3\ell_J|^3\ell'_J)} c_n(^3\ell_J|^3\ell'_J)\, s^n$$

[JHEP 1807, 043 (2018)]

ρπ isospin-2 scattering – $^3S_1$,$^3D_1$ mixing angle



$$K_{\ell J,\ell' J}(s) = \sum_{n\geq 0}^{N(^3\ell_J|^3\ell'_J)} c_n(^3\ell_J|^3\ell'_J)\, s^n$$

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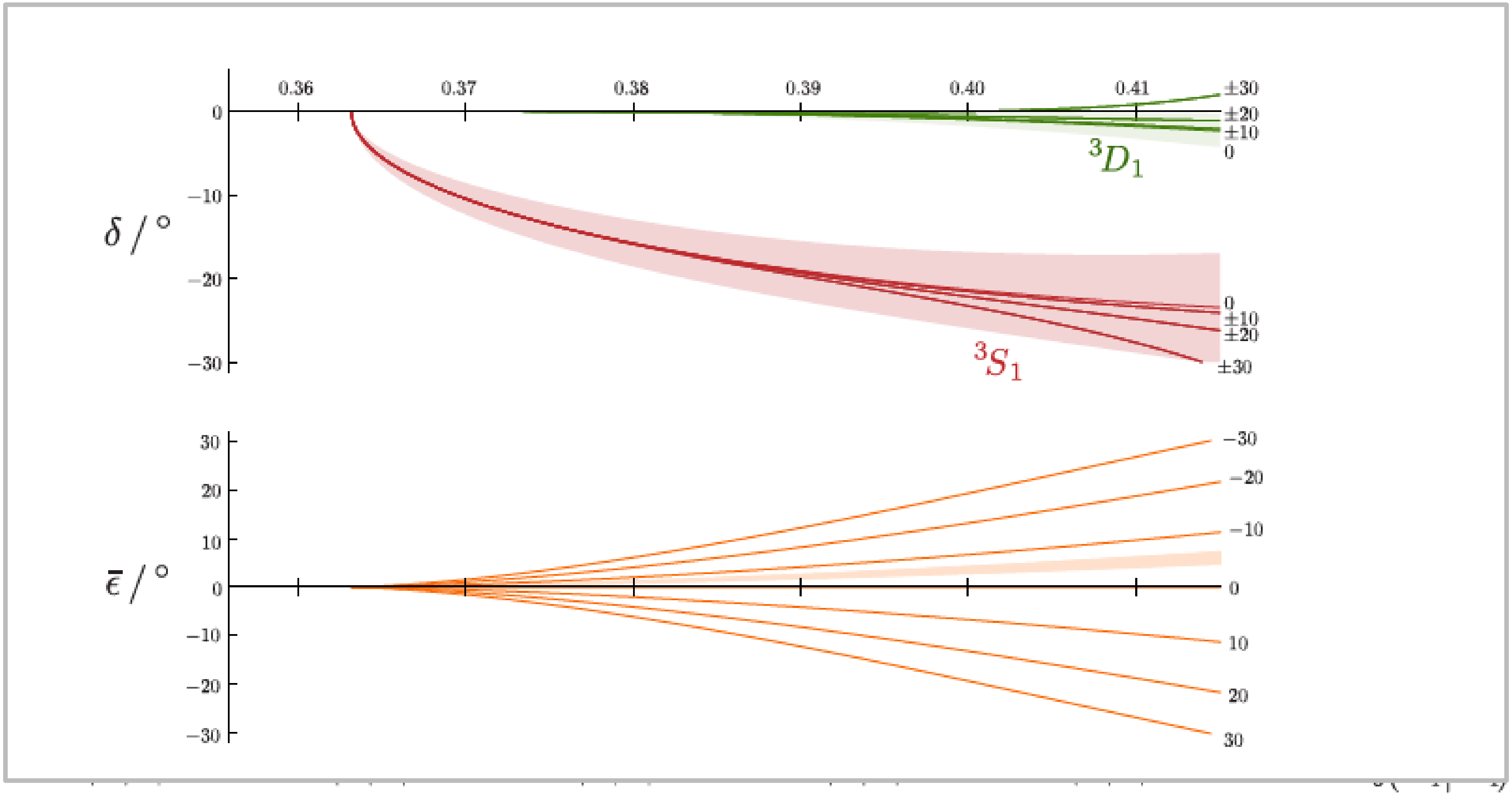

$$K_{\ell J, \ell' J}(s) = \sum_{n \geq 0}^{N(^3\ell_J|^3\ell'_J)} c_n(^3\ell_J|^3\ell'_J)\, s^n$$

[JHEP 1807, 043 (2018)]

Summary

- Progress in determining scattering amplitudes from lattice QCD – hadrons with non-zero spin
- $\rho\,\pi$ isospin-2 scattering with dynamically-coupled $^3S_1$, $^3D_1$ partial waves
- Antoni (next talk): $\omega\,\pi$ isospin-1 and the b_1
- Other channels...

hadspec

UNIVERSITY OF CAMBRIDGE

Science & Technology Facilities Council

Charm tetraquarks

Cheung, CT, Dudek, Edwards (HadSpec) [JHEP 1711, 033 (2017)]

Compute spectra in some **exotic-flavour** charm channels

Use a range of **'meson-meson'** operators**,**
and **'tetraquark'** (diquark-antidiquark) operators,

$$\sim \sum_{a,d} C_{ad} \left[c_{abc}\, q_b^T\, \Gamma_1\, q_c \right] \left[c_{def}\, \bar{q}_e\, \Gamma_2\, \bar{q}_f^T \right]$$

1 volume (≈ 2 fm), $m_\pi \approx 391$ MeV

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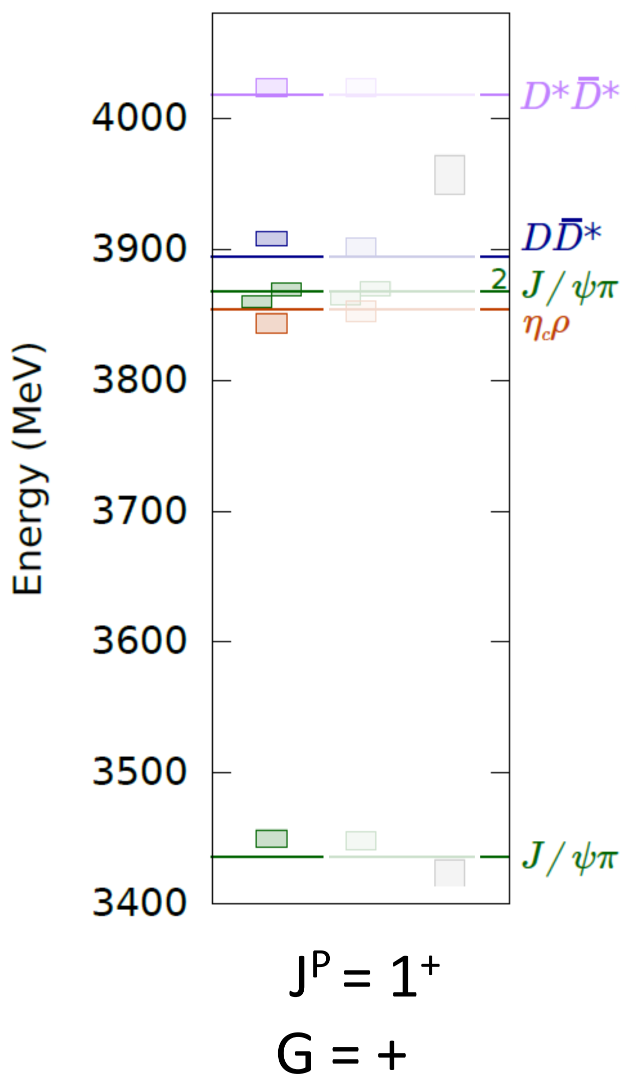
1 volume (≈ 2 fm), $m_\pi \approx 391$ MeV

Previous LQCD work on charm tetraquarks or relevant channels:

- Ikeda *et al* [PL B729, 85 (2014); PRL 117, 242001 (2016)]
- Prelovsek *et al* [PR D91, 014504 (2015)]
- Guerrieri *et al* [PoS (LATTICE2014) 106]
- Padmanath *et al* [PR D92, 034501 (2015)]
- Chen *et al* [PR D89, 094506 (2014); PR D92, 054507 (2015)]

Hidden-charm I=1 ($c\bar{c}l\bar{l}$)

[JHEP 1711, 033 (2017)]



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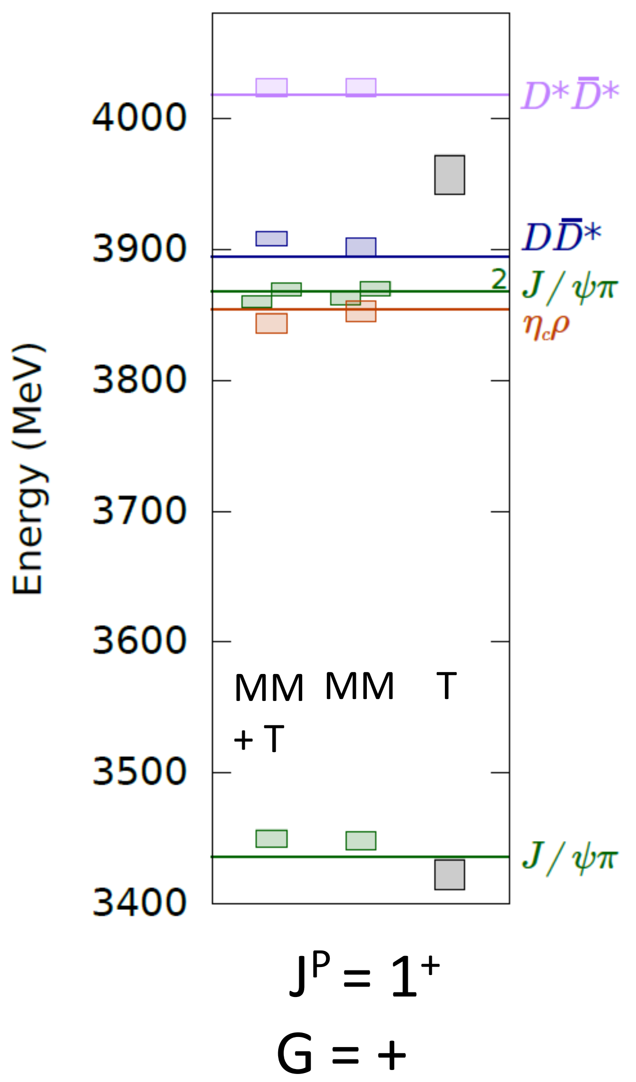

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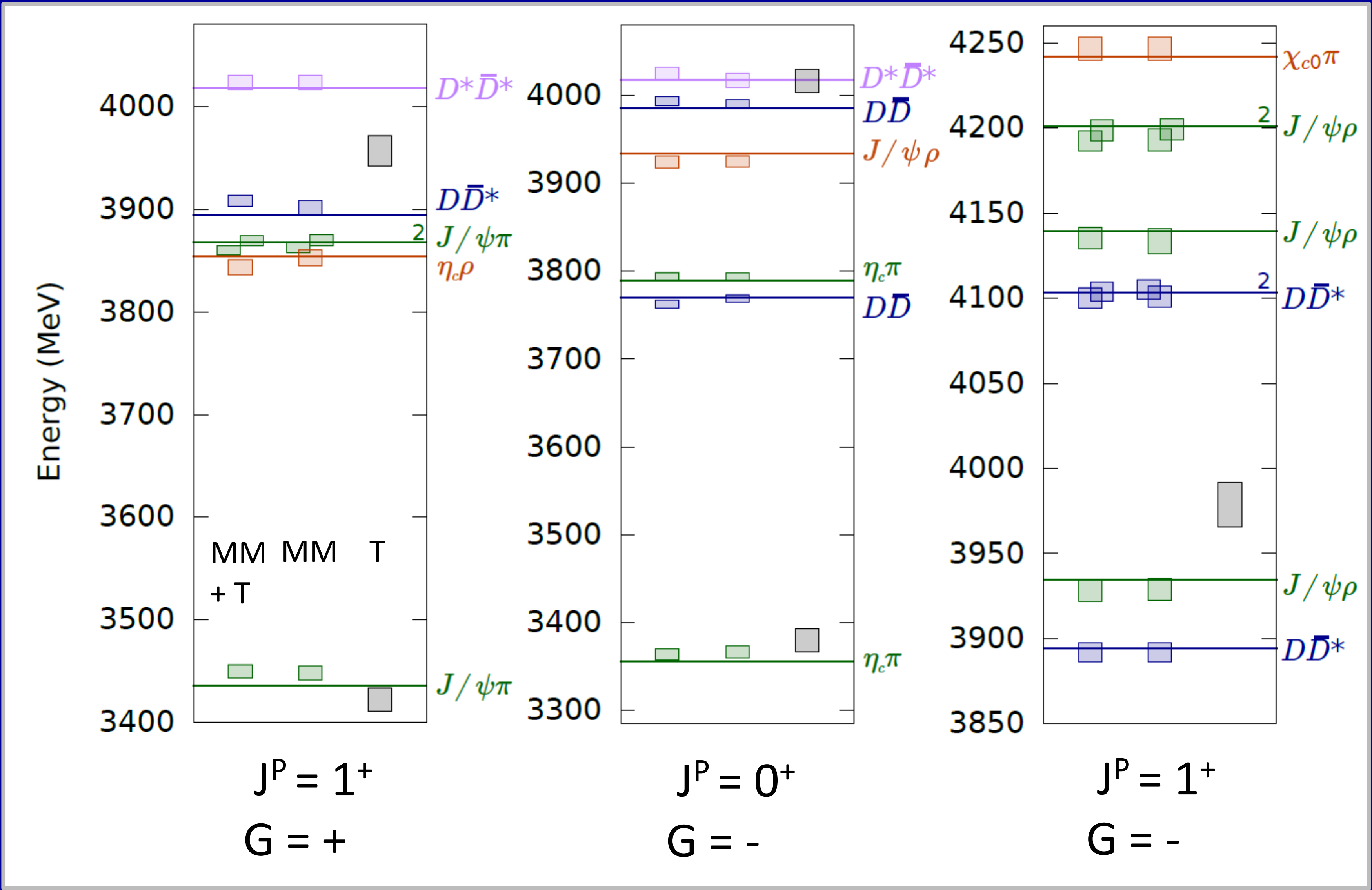
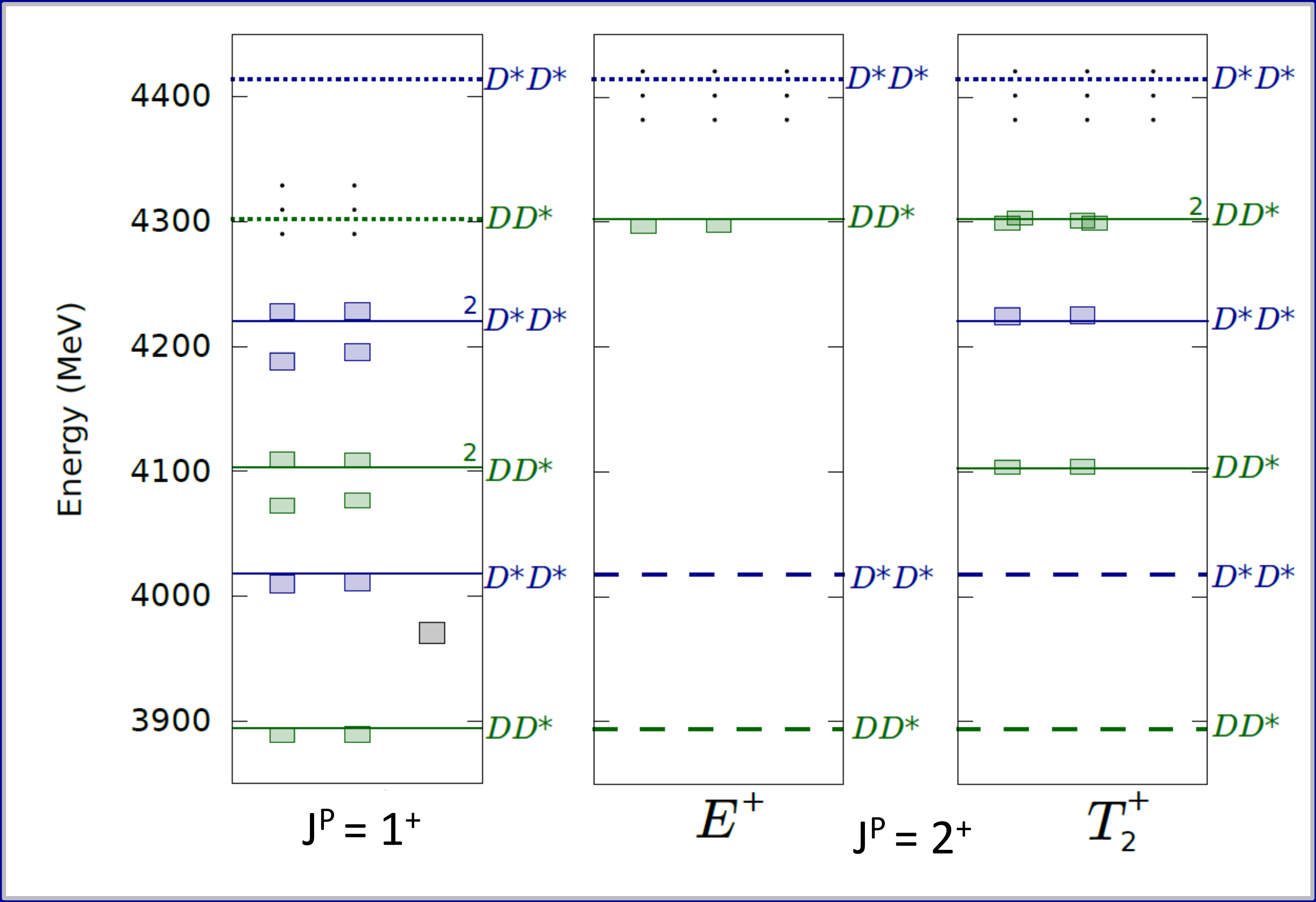


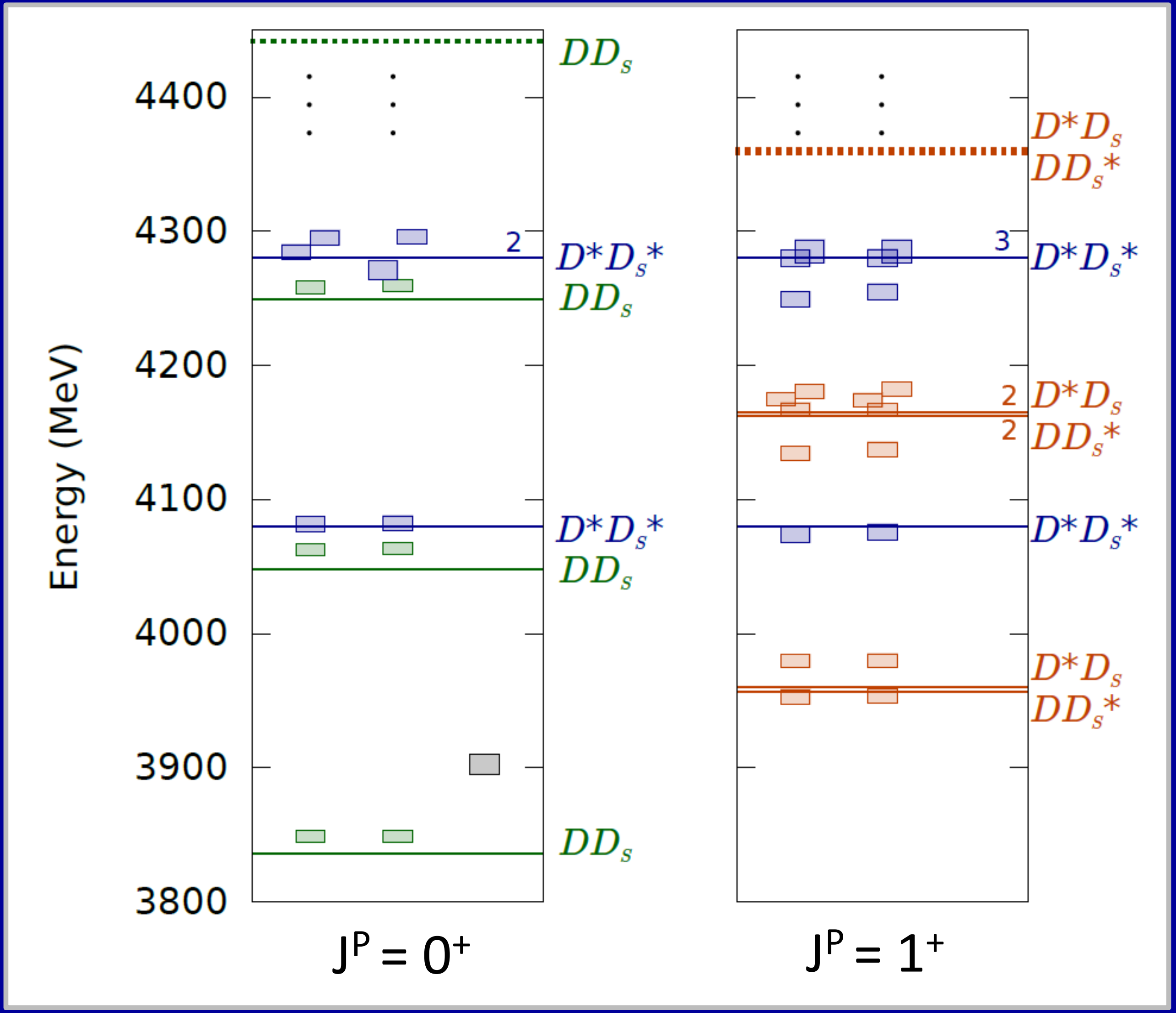
Doubly-charmed I=0 ($cc\bar{l}\bar{l}$)

[JHEP 1711, 033 (2017)]

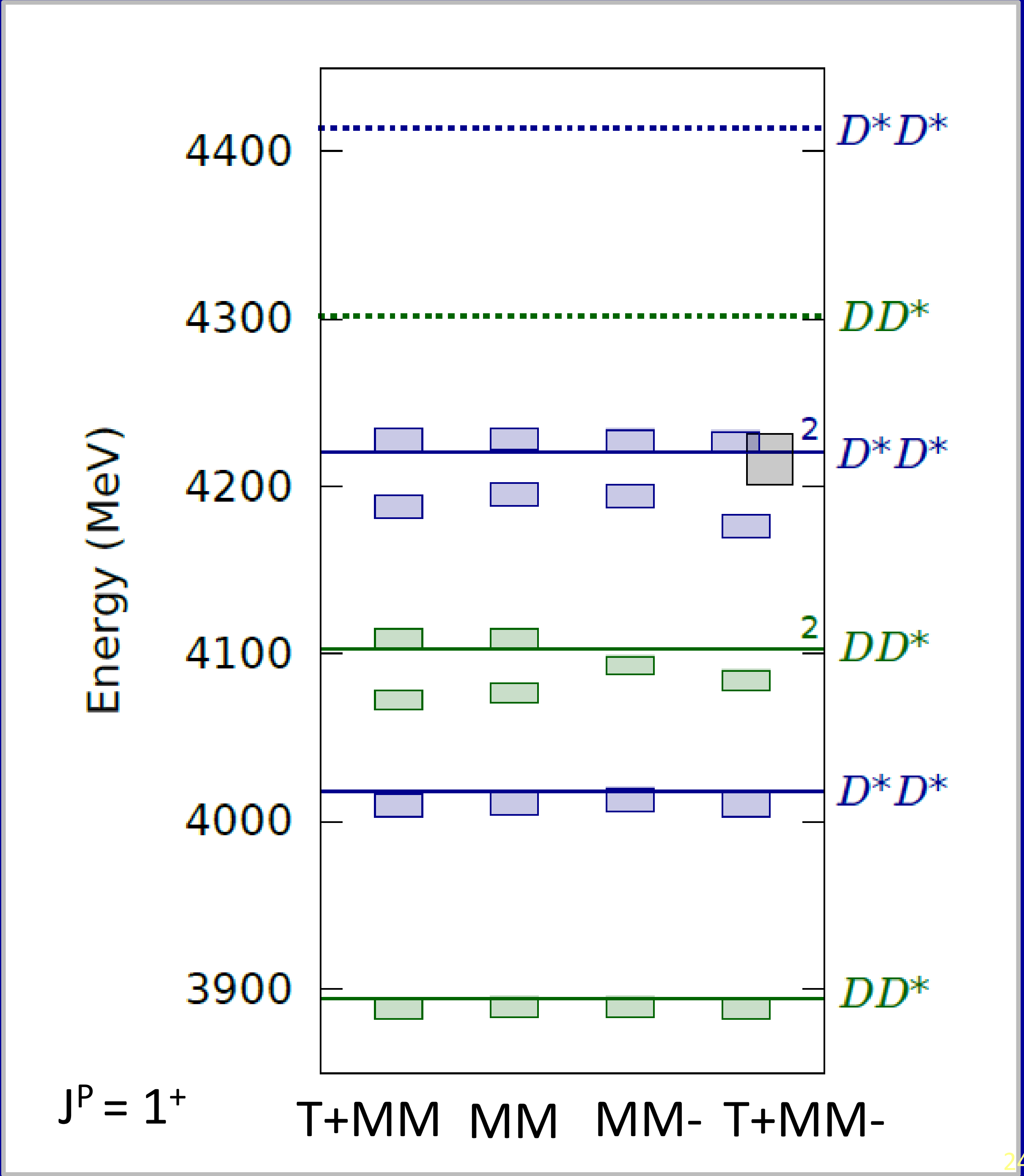


Doubly-charmed I=½ ($cc\bar{l}\bar{s}$)

[JHEP 1711, 033 (2017)]



Doubly-charmed I=0 ($cc\bar{l}\bar{l}$)



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